3D Finite Element Analysis of the Langevin Transducer

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Abstract – In this paper, a 3–D finite element analysis of the vibrational behaviour of the Langevin transducer and of the dependence of its performances on the length to diameter ratio is presented. Several transducers with total length greater than, comparable to, and smaller than the diameter have been simulated, and the frequency spectrum is drawn in order to identify the regions where coupling between thickness and radial modes exists, and to highlight the limit of the classical 1–D theory. Performances of transducers with any length to diameter ratio, but all working at the same frequency (153 kHz), are evaluated by comparing the product between the output mean displacement and the area of the radiating surface. Results have shown that Langevin transducers with comparable longitudinal and lateral dimensions have performances comparable to or even better than transducers with total length much greater than diameter.

I. Introduction

The Langevin transducer is generally designed to vibrate along its length in pure thickness–extensional mode. This vibration mode can be excited by realizing the lateral dimensions less than a quarter of the longitudinal wavelength and can be adequately analyzed with the Mason one-dimensional theory [1]. In order to extend the analysis to transducers with any length to diameter ratios, analysis tools able to describe also the radial modes and the coupling between radial and thickness modes are required [2],[3]. In this work a 3–D finite element analysis of the Langevin transducer has been carried out in order to evaluate potential performances and suitability to applications of transducers with diameters comparable to or greater than the total length. A commercial FE package has been implemented for calculations. The analyzed structure is composed of two piezoceramic disks sandwiched between two identical titanium cylinder-shaped masses. Several transducers with total length greater than, comparable to and less than the diameter have been modelled and simulated. The frequency spectrum, i.e., the map of the resonance frequencies, is computed by varying the aspect ratio G between mass thickness and diameter. The analysis of the spectrum permits to identify the regions where coupling between thickness–extensional and radial modes exists. In order to investigate the dependence of transducer's performances on its aspect ratio, several transducers all designed to work at the same frequency (about 153 kHz) are simulated. In order to compare transducers' performances, the product γ between the mean displacement and the area of the end surface seems more appropriate that the mean displacement only. As an example, in several applications, the Langevin transducer is used as a driver for a displacement amplifier. In these applications the end displacement of the amplifier depends not only on the output displacement but also on the area of the output surface of the Langevin transducer.

II. Finite Element Model

Fig. 1 shows the analyzed Langevin transducer configuration; it is composed of a couple of piezoceramic disks (PZT-4 by Morgan-Matroc [4]) with radius a and thickness t_c, which are poled along z direction but with opposite polarity and electroded on their flat surfaces, and of two identical titanium cylinder–shaped masses (mass density $\rho = 4680$ kg/m^3, Young modulus $E = 10.93 \cdot 10^{10}$ N/m^2, Poisson ratio $\sigma = 0.3$), which have radius identical to that of the disk.

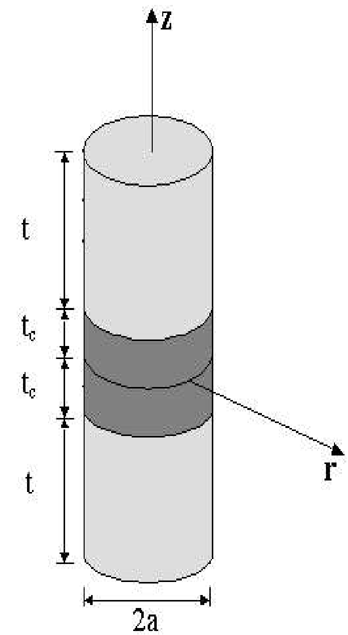

Figure 1: Schematical view of the Langevin transducer.

A FE model of that structure was constructed by using the commercial package ANSYS.

As the analyzed transducer configuration was axially symmetric, a 3–D analysis was performed by using 2–D quadratic elements. The 2–D axial symmetric model was validated by comparing its results with those obtained with a 3-D model which uses hexahedral quadratic elements. Due to the excellent agreement, the 2–D model was employed because computations are at least 2.5 times faster.

The structure was modelled by imposing the continuity of the displacements both in radial and in axial directions at the interfaces between piezoceramic disks and loading masses, as well as at the interface between the two piezoceramic disks. In order to simulate metallizations, two methods were investigated. The first method consists in creating two new thin layers of aluminium for each piezoceramic disk and putting them in contact with the flat surfaces, by imposing the continuity of displacements and voltage. The second method simply consists in setting to the same electrical potential all nodes belonging to each flat surface of the piezoceramic. Because the thickness of the aluminum layer was negligible with respect to that of the piezoceramic disks actually used in simulations, the two methods provided almost identical results and the second one has been adopted due to its less complexity and its time saving.

In order to characterize the vibrational behaviour of the Langevin transducer for any aspect ratio, several transducers with total length greater than, comparable to, and smaller than the diameter have been simulated.

For each transducer a modal analysis has been first performed in order to find the natural resonance frequency and the corresponding modal shapes of the structure. Successively an harmonic analysis has been performed by applying to the piezoceramic disks, electrically connected in parallel, an ideal harmonic current generator with variable frequency. In this way the plot of the electrical input impedance versus frequency was computed. The harmonic analysis was also exploited to quantitatively compare displacements of transducers with different aspect ratios. To this end internal structural losses of the transducer have been taken into account by applying to the structure an appropriate damping (1%).

III. The vibrational behaviour

Figure 2 shows the frequency spectrum, i.e. the map of the resonance frequencies of the structure, computed by FE simulations. It was obtained by performing harmonic analysis on several transducers with masses thickness varying from 40 mm down to 0.5 mm. For each transducer the radius a and the thickness t_c of the piezoceramic disks were set to 10 mm and 1 mm, respectively. In the plot, on the x axis the ratio G between a single mass thickness and the diameter is reported, while on the y axis the parallel resonance frequency f_p is multiplied by the diameter. For each resonance mode the frequencies of minimum and maximum electrical impedance were computed. Due to the low losses these frequencies can be assumed to correspond to f_s and f_p, respectively. The diameter of the circles is proportional to the effective electrome-

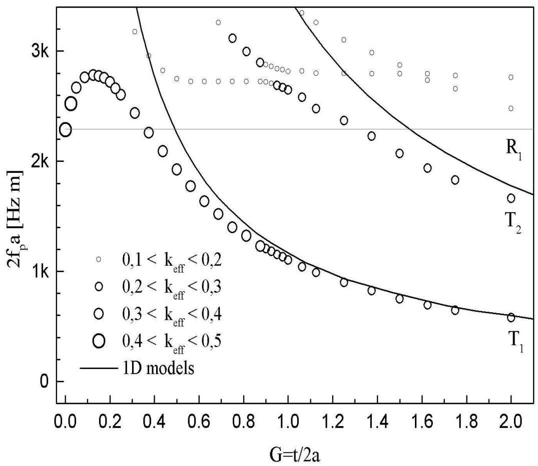

Figure 2: The frequency spectrum.

chanical coupling factor (k_{eff}), which is defined as:

$$k_{eff} = \sqrt{\frac{f_p^2 - f_s^2}{f_p^2}} \quad , \qquad (1)$$

For comparison, Figure 2 also shows the frequency spectrum of the Langevin transducer computed with the classical 1-D thickness extensional model[1] (solid curves T_1, T_2), and a straight line representing the resonance frequency of the pure radial modes of the piezoceramic R_1, computed under the hypothesis of thin disks [5]. By increasing G, the resonance frequency of the pure thickness mode T_1 decreases, as well as the harmonic T_2, while the resonance frequency corresponding to R_1 is constant because it only depend on the diameter of the structure.

As can be seen from the figure, the one dimensional model is able to predict resonance frequencies of the fundamental thickness mode T_1 only when the length of the transducer is much greater that its diameter.

As far as FE results are concerned, for G decreasing from 2 down to about 1, the first two resonance frequencies increase when the mass thickness decrease, while the third mode resonance frequencies are nearly constant. Therefore, the first and second modes can be identified with the fundamental thickness mode and its first harmonic, respectively, while the third mode can be recognized as a radial mode. It should be noted that the resonance frequencies for this last mode are slightly higher than that of the pure radial mode of the unloaded piezoceramic disk, represented in the figure by the straight line R_1.

For G decreasing from about 1 down to about 0.9, the resonance frequencies of the second and third modes become very close and coupling between these two modes occurs.

For G decreasing from about 0.9 down to about 0.6, the second mode may be recognized as a radial mode, while the resonance frequencies of the third mode rapidly increase beyond the range of the highest computed frequency.

The first mode can be clearly identified with the fundamental thickness extensional mode whenever G is greater than 0.1. For further G reductions, the first mode couples with the second one and for G less than 0.3, it may be considered a radial mode; its resonance frequency reaches a maximum value (at about the resonance frequency of the radial mode of the transducer) and successively decreases and tends to the radial resonance frequency of the sole piezoceramic.

IV. Performance analysis

In order to investigate the possibility to exploit Langevin transducers with any aspect ratio in applications, the performances of several transducers with G ratios varying from 0.1 to 2, all designed to work at the same frequency (about 153 kHz), have been evaluated. Each transducer was assumed to operate in vacuum and was driven by a harmonic current generator with magnitude of 1 A. A 1% damping, defined as the ratio between loss energy and kinetic energy, was imposed for all transducers.

Figure 3 shows the modal shapes computed for three transducers with aspect ratios G equal to 0.1 (a=9.625 mm, t=1.925 mm), 0.6 (a=5.575 mm, t=6.69 mm) and 2 (a=1.745 mm, t=6.981 mm), respectively. As can be seen the displacement of the external surface is almost flat for the transducer with G=2, showing that this transducer works in a almost pure thickness extensional mode. On the

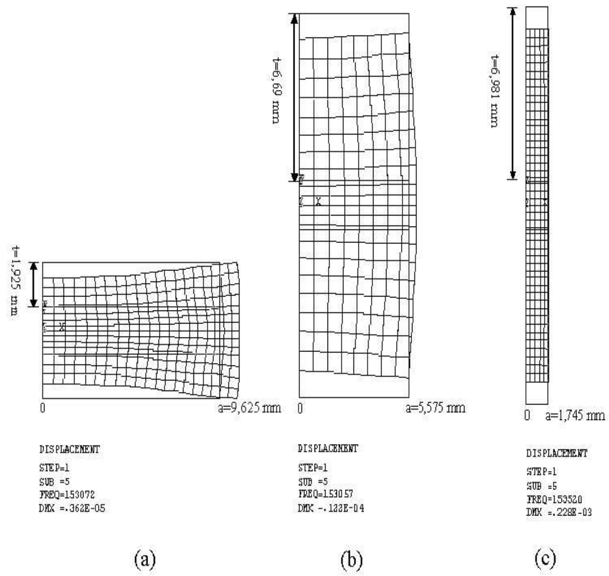

Figure 3: Modal shapes of Langevin transducers with G=0.1 (a), G=0.6 (b), G=2 (c).

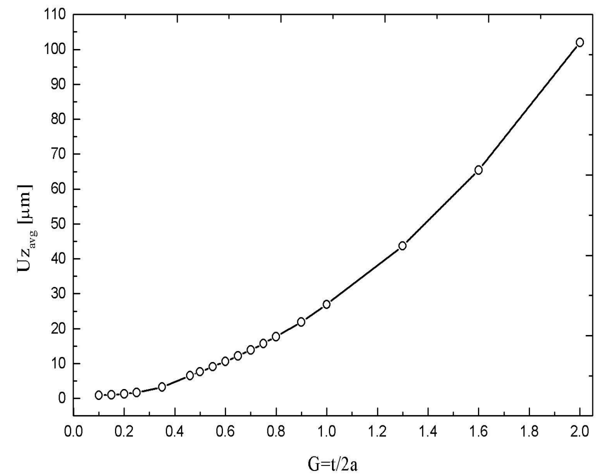

Figure 4: Mean displacement versus G.

other end for the transducer with G=0.1, which works in an almost pure radial mode (see Figure 2), the axial displacement has a maximum in r=0 and decreases by increasing r, becoming almost null in r=a. The transducer with G=0.6 presents a modal shape that is intermediate among the two others. As can be seen from Figure 2, for this geometry, coupling between thickness and radial modes occurs.

In order to investigate the dependence of transducer's performances on its aspect ratio, the mean displacement Uz_{avg} of the external surface has been computed and is shown in Figure 4 as a function of G. As can be seen the mean displacement increases by increasing the G ratio. This results is in agreement with the classical 1-D theory of the Langevin transducer that suggests that transducers with high G have to be used in applications. However, by observing Figure 3, one should note that a transducer with low G has a higher radiation surface, and hence an higher power capacity, than a low G one working at the same frequency. As a consequence, a parameter that seems more adequate to compare performances of transducer with different G is the product γ between the mean displacement Uz_{avg} and the area of the radiating surface.

Figure 5 shows the plot of γ as a function of G. As can be seen, when the transducer vibrates in pure thickness mode (high G values) γ is practically constant. By decreasing G, γ slightly increases and reaches its maximum value at $G \cong 0.6$. This value is about 6% greater than that obtained for high G. For this aspect ratio the transducer presents a modal shape that is a mixture between a thickness mode and radial mode (see Figure 3b) and its resonance frequency is fairly less than that predicted by the 1-D thickness mode (see Figure 2). By further decreasing G, the influence of the radial mode is more and more accentuated and a rapid decrease of γ is observed in the plot.

The results shown in Figure 5 highlight that Langevin transducers with G values very lower than those suggested by the classical 1-D theory (but greater than 0.45) can be exploited in applications, allowing a greater design flexibility. Furthermore, the maximum γ value is achieved just in the G region where coupling between thickness and radial mode occurs.

V. Conclusions

In this work a 3–D finite element analysis of the Langevin transducer has been carried out. The

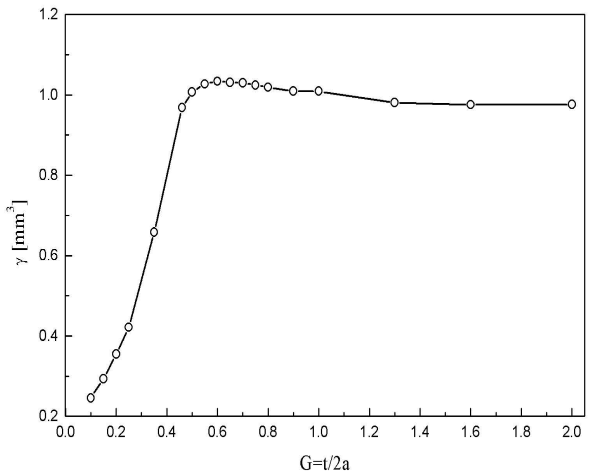

Figure 5: γ versus G.

3–D approach has been able to study the vibrational behaviour and to evaluate the performances of transducers with total length greater than,

comparable to, and smaller than the diameter, overcoming the limit of the classical 1–D theory that is able to accurately describe only transducers with total length much greater than the diameter. Results have shown that also Langevin transducers with comparable longitudinal and lateral dimensions can be exploited in applications, and, furthermore, transducers with such aspect ratios have slightly better performances than transducers with total length much greater than the diameter.

Next steps of the study will be the analysis of the influence of the static pre–stress on the performances of the transducer and the 3–D analysis of displacement amplifiers that are often joined to the Langevin transducer in several applications.

VI. References

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