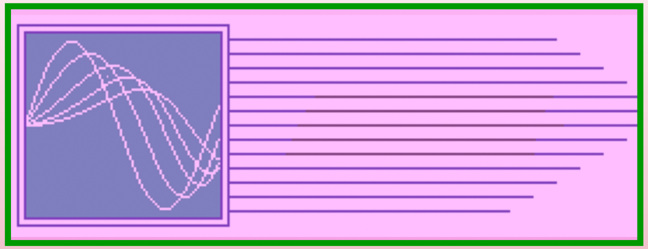
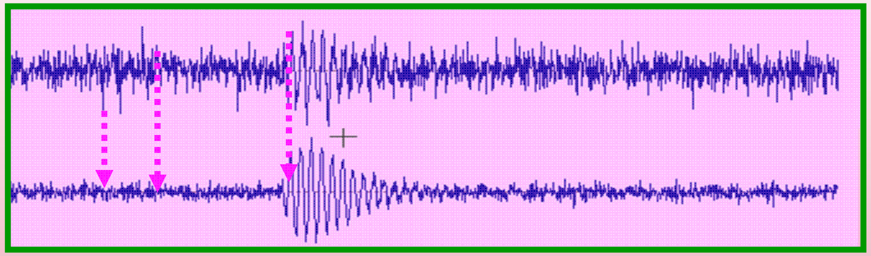
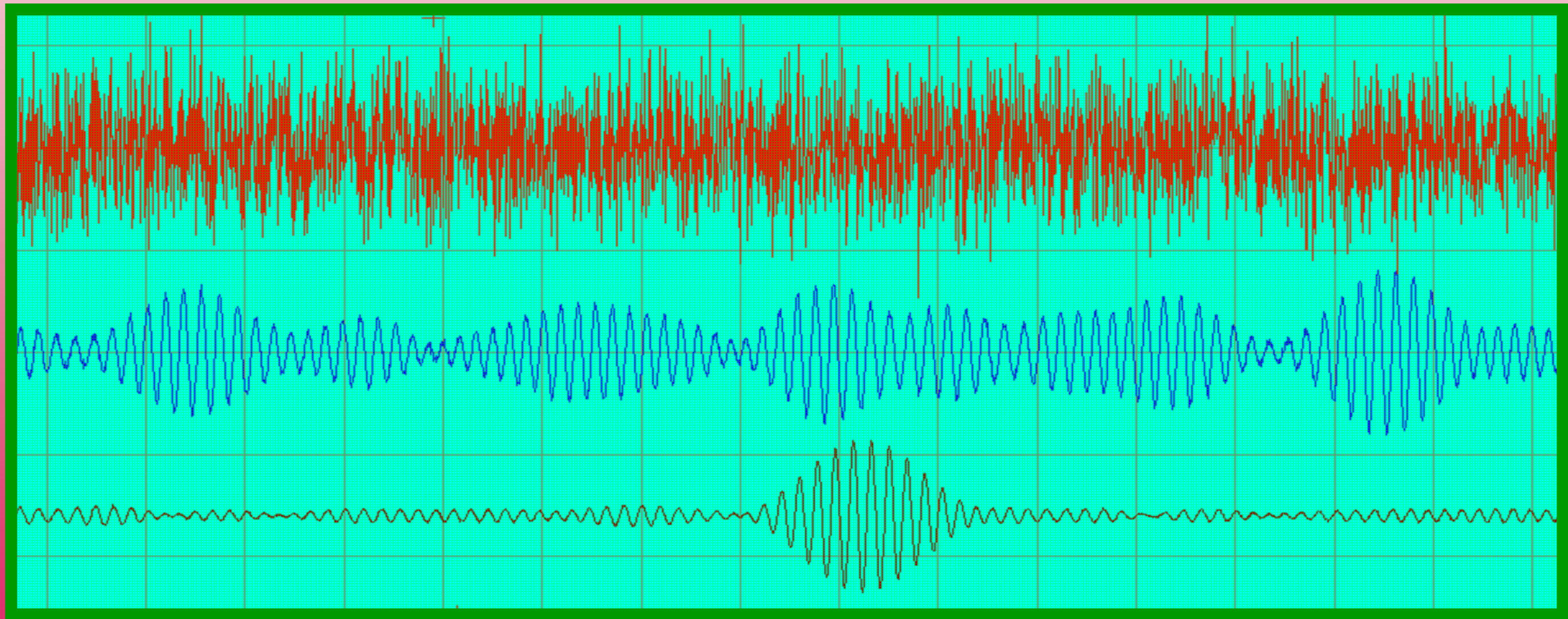
ANALOG AND DIGITAL SIGNAL PROCESSING
ADSP – Chapter 9







Chapter 9 Signal-to-Noise Ration Improvement:
Basic concepts

Introduction

Preamplifier noise

Band-pass filtering

Averaging

Correlation – Windowing

Spectral Analysis - Windowing

Problems

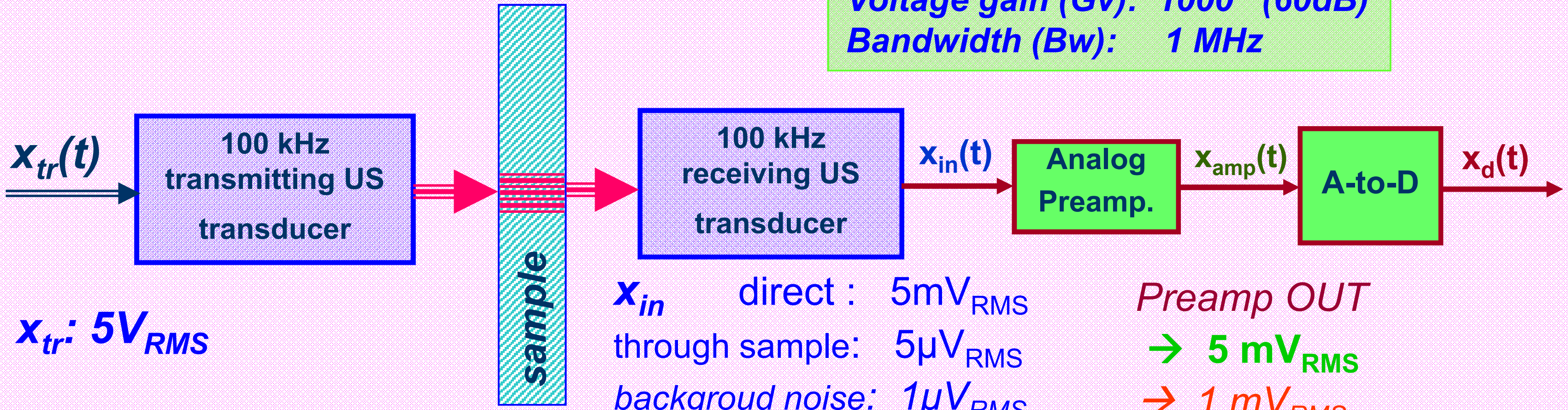
What is the biggest challenge?

INTRODUCTION

Example from acoustics:

Analog Preamp. Specs:

Voltage gain (Gv): 1000 (60dB)
Bandwidth (Bw): 1 MHz



x_{tr}: $5V_{RMS}$

x_{in} direct : $5mV_{RMS}$
through sample: $5\mu V_{RMS}$
backgroud noise: $1\mu V_{RMS}$

Preamp OUT
→ $5\ mV_{RMS}$
→ $1\ mV_{RMS}$
→ $16\ mV_{RMS}$ (LM6142 noise)

Sample attenuation: 60 dB

A-to-D voltage range: ± 1 V (14bits)
➔ AD_{noise}: $35\ \mu V_{RMS}$ (not significant in this case)

Analog Preamp: LM6142

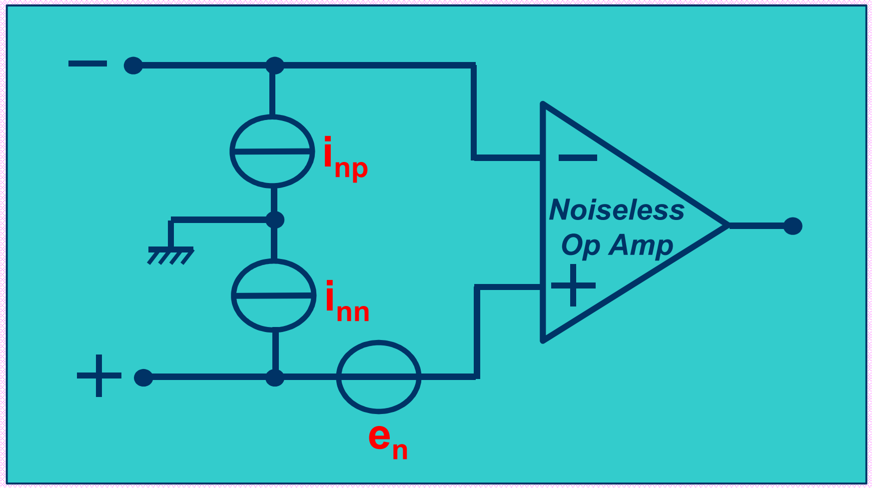
Equivalent input noise: $16\ nV_{RMS}/\sqrt{Hz}$
➔ $16\ \mu V_{RMS}$ (Bw = 1 MHz)

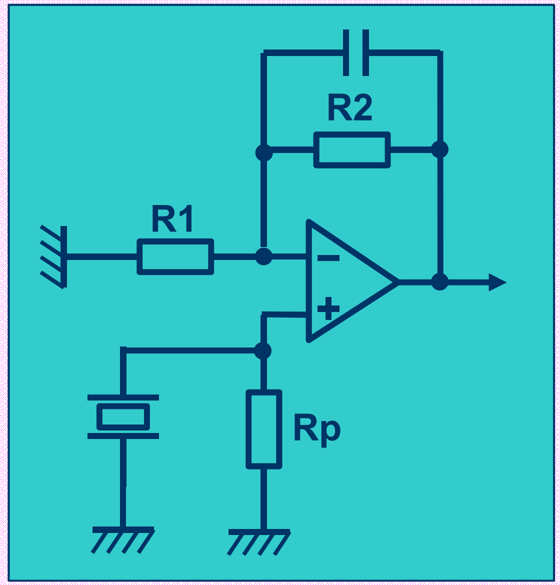
➔ Usefull signal is apparently lost in the noise

PREAMPLIFIER NOISE

Op Amp Noise Voltage or Current Spectral Density

An operational amplifier (OP Amp) is characterized by the spectral densities of its noise voltage (e_n) and noise current (i_{nn}-i_{np}) per root hertz, i.e. V/√Hz or A/√Hz. Spectral densities are commonly used to specify noise parameters.





In a non-inverting voltage preamplifier, the equivalent input noise voltage spectral density is determined as follows (R1 << Req):

$$e_{equi} = (e_n^2 + e_{Req}^2 + i_{np}^2 \cdot Req^2)^{0.5}$$

e_n: Op Amp Input Noise-Voltage Density - ***LM6142: 16 nV/√Hz, MAX412: 2.4 nV√Hz***

i_{np}: Op Amp Input Noise-Current Density - ***LM6142: 0.2 pA/√Hz, MAX412: 1.2 pA√Hz***

Analog and Digital Signal Processing

PREAMPLIFIER NOISE (cont')

e_{Req}: Equivalent voltage spectral density of Req

Req : the real part of the source impedance ***Zs*** at the frequency of operation in parallel with Rp

Voltage spectral density of typical resistor values (at 300⁰C): ***1 kΩ → 4 nV/√kΩ/√Hz***

===> 50Ω → 0.9 nV/√Hz, 10 kΩ → 12.6 nV/√Hz

Equivalent preamplifier ouput noise of bandwidth Bw: $uout_{RMS} = Gv \cdot \sqrt{Bw} \cdot e_{equi}$

Example: Gv = 100, Bw = 1 MHz, ***LM6142 and*** R_{Req} ***= 1 kΩ***

Max412 and R_{Req} ***= 1 kΩ***

LM6142 $$\sqrt{\left(16\cdot 10^{-9}\right)^2+\left(4\cdot 10^{-9}\right)^2+\left(0.2\cdot 10^{-12}\cdot 10^{3}\right)^2}\cdot 100\cdot\sqrt{1\cdot 10^{6}} = 1.65\times 10^{-3}\ V_{RMS}$$

Max412 $$\sqrt{\left(2.4\cdot 10^{-9}\right)^2+\left(4\cdot 10^{-9}\right)^2+\left(1.2\cdot 10^{-12}\cdot 10^{3}\right)^2}\cdot 100\cdot\sqrt{1\cdot 10^{6}} = 481.66\times 10^{-6}\ V_{RMS}$$

Appropriate OPAMP choice and resistor values optimization
→ SNR improvement

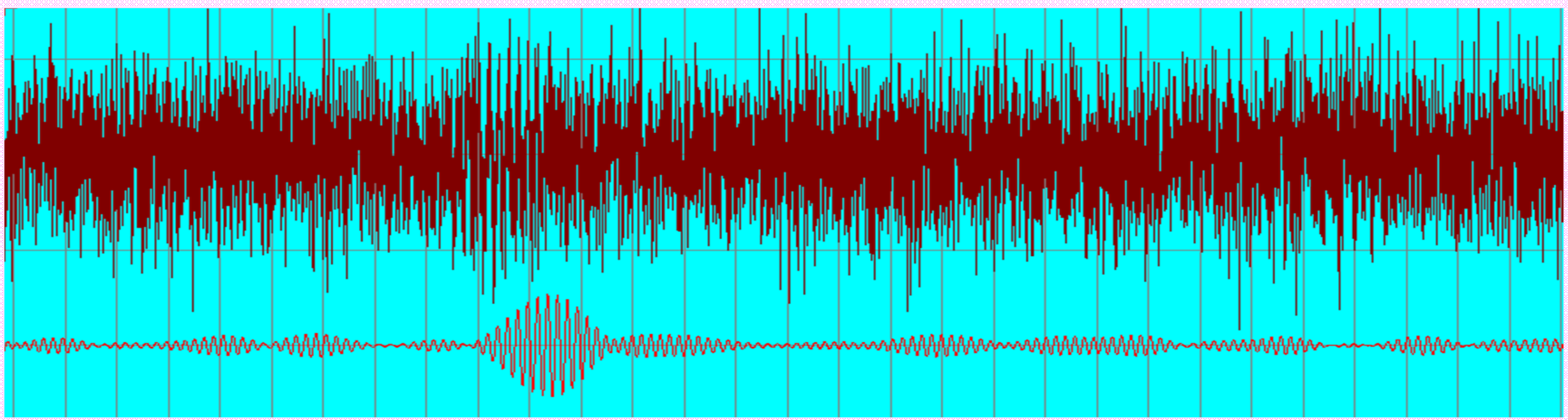
BAND-PASS FILTERING

Noise bandwidth: Bw_{noise}, Band-pass filter bandwidth: Bw_{BPF}

$$SNR(dB)_{out} = SNR(dB)_{in} + 10 \log [Bw_{noise} / Bw_{BPF}]$$

➔ In a uniformely distributed noise, its power is proportional to the bandwidth

Example: Bw_{noise} = 1 MHz, Bw_{BPF} = 10 kHz → SNR improvement of 20 dB



➔ ***Drawback: Rise-time and fall-time inversely proportional to Bw_{BPF}***

➔ Bw_{BPF} is bounded by ***Desired Signal Time Position Estimation Accuracy***

AVERAGING: Multiple periodic excitation response averaging

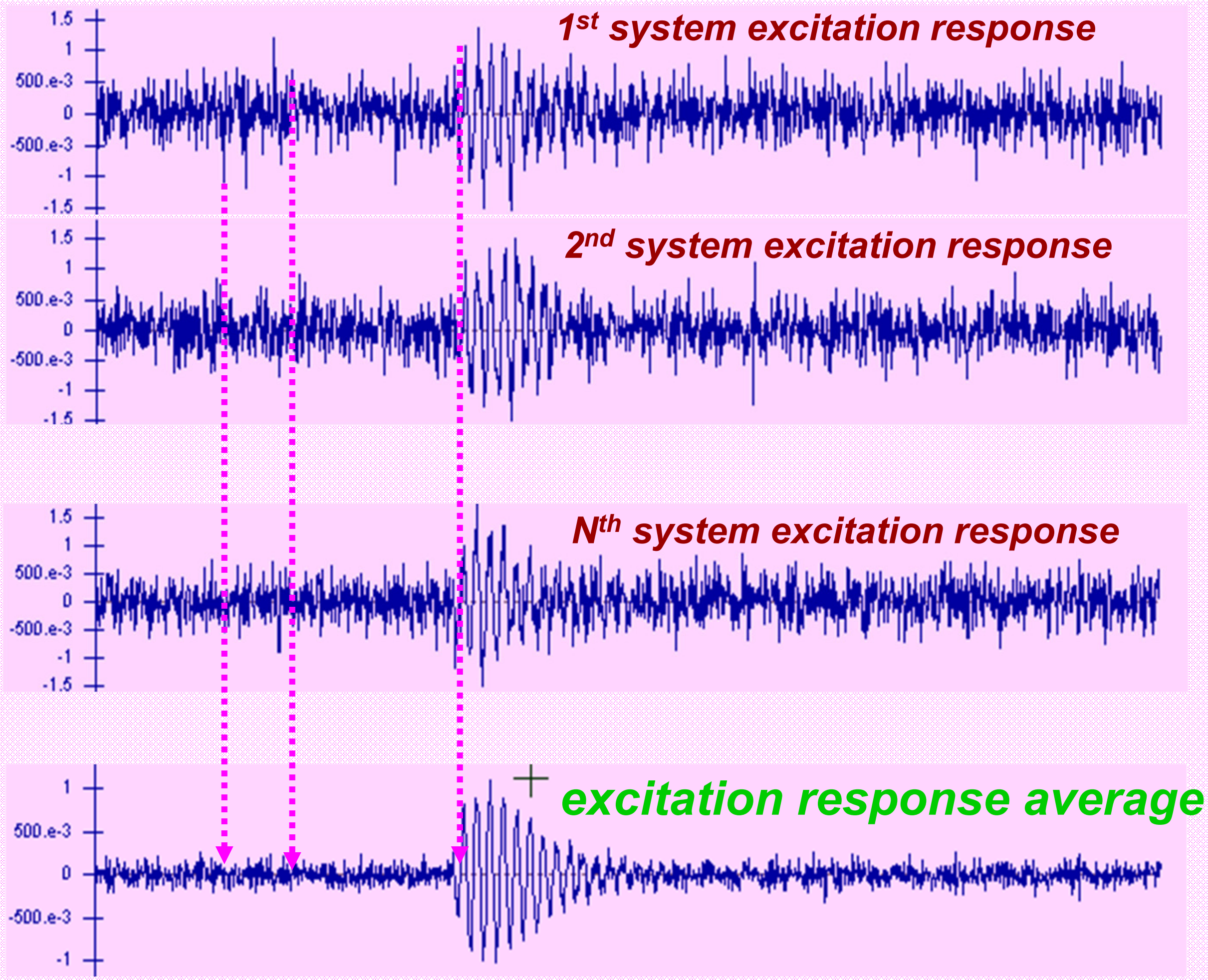
Concept:

$x_1(n) = S(n) + N_1(n)$

$x_2(n) = S(n) + N_2(n)$

$x_N(n) = S(n) + N_N(i)$

$$x_{Aver}(n) = \frac{1}{K} \cdot \sum_{i=1}^{K} x_i(n)$$

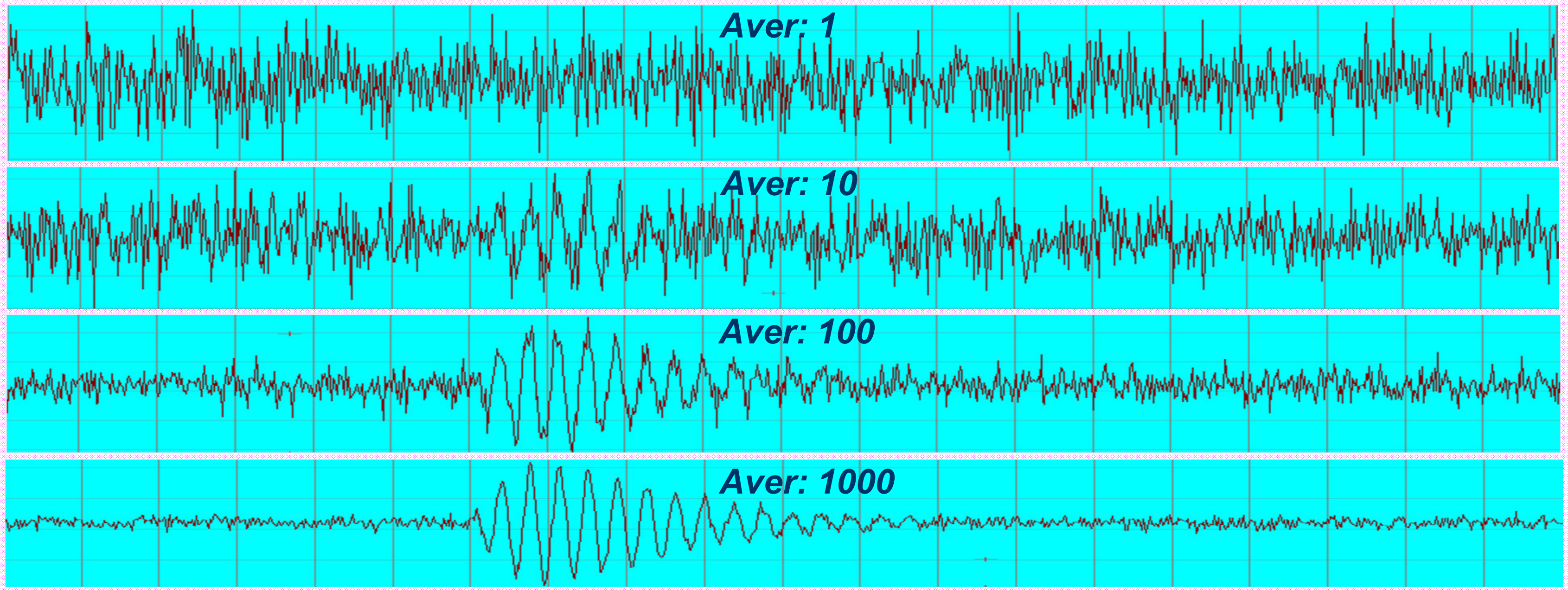


AVERAGING (cont')

If the noise vectors $N_i(n)$ are independant than it can be shown that:

$$SNR_N(dB) = SNR_1(dB) + 10 \log_{10} N$$

In words: the SNR improvement is proportional to the number of repetition N of the system excitation



If the noise is NOT random ➔ Averaging is USELESS!

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CORRELATION - WINDOWING

Definition: The correlation between waveforms is a measure of the **similarity or resemblance** between the waveforms. The correlation between x(t) and y(t), or more precisely the average ***cross-correlation*** is defined as :

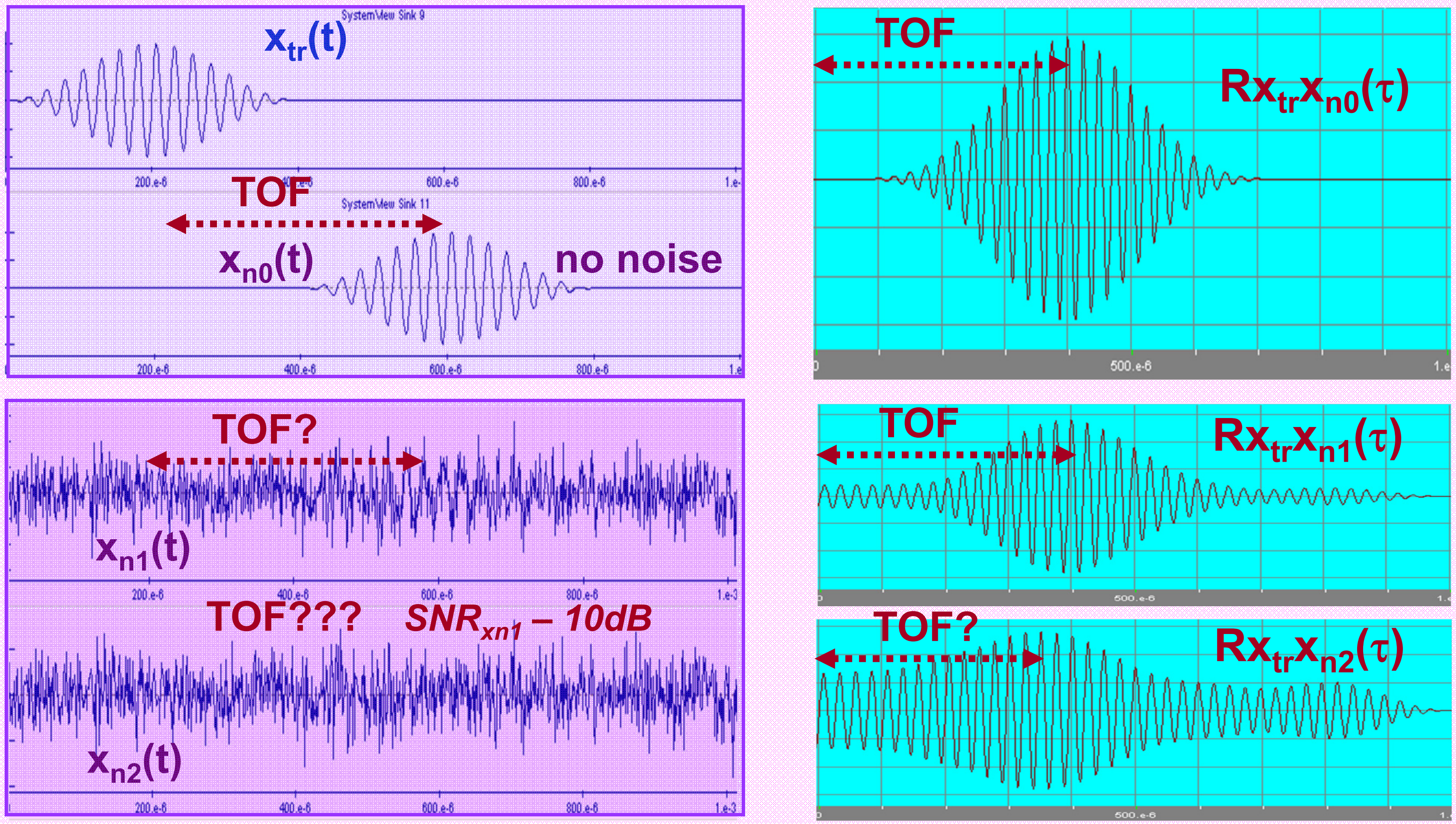
$$Rxy(\tau) = E\big(x(t)\cdot y(t+\tau)\big) = \lim_{t_2 - t_1 \to \infty} \frac{1}{t_2 - t_1} \cdot \int_{t_1}^{t_2} x(t)\cdot y(t+\tau)\,dt$$

With x(t) → x(n) and y(t) → y(n), their **cross-correlation** is usually defined as follows:

$$R_{XY}(n) = \frac{1}{L-K+1} \cdot \sum_{m=K}^{L} x(n)\cdot y(n+m)$$

where K and L define a realistic interval over which $R_{XY}(n)$ is computed.

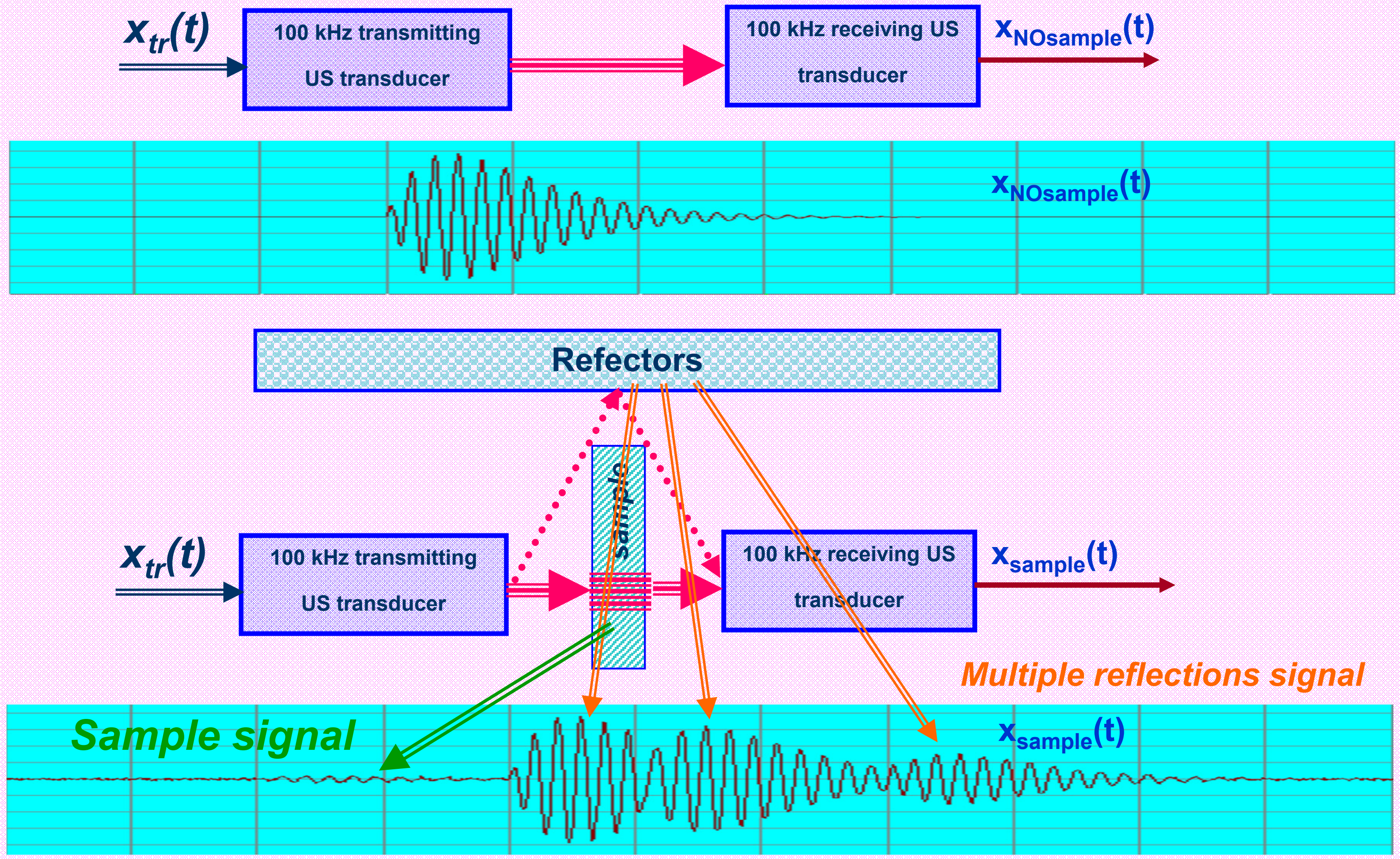
CROSS-CORRELATION APPLICATION: Time-of-flight (TOF) estimation



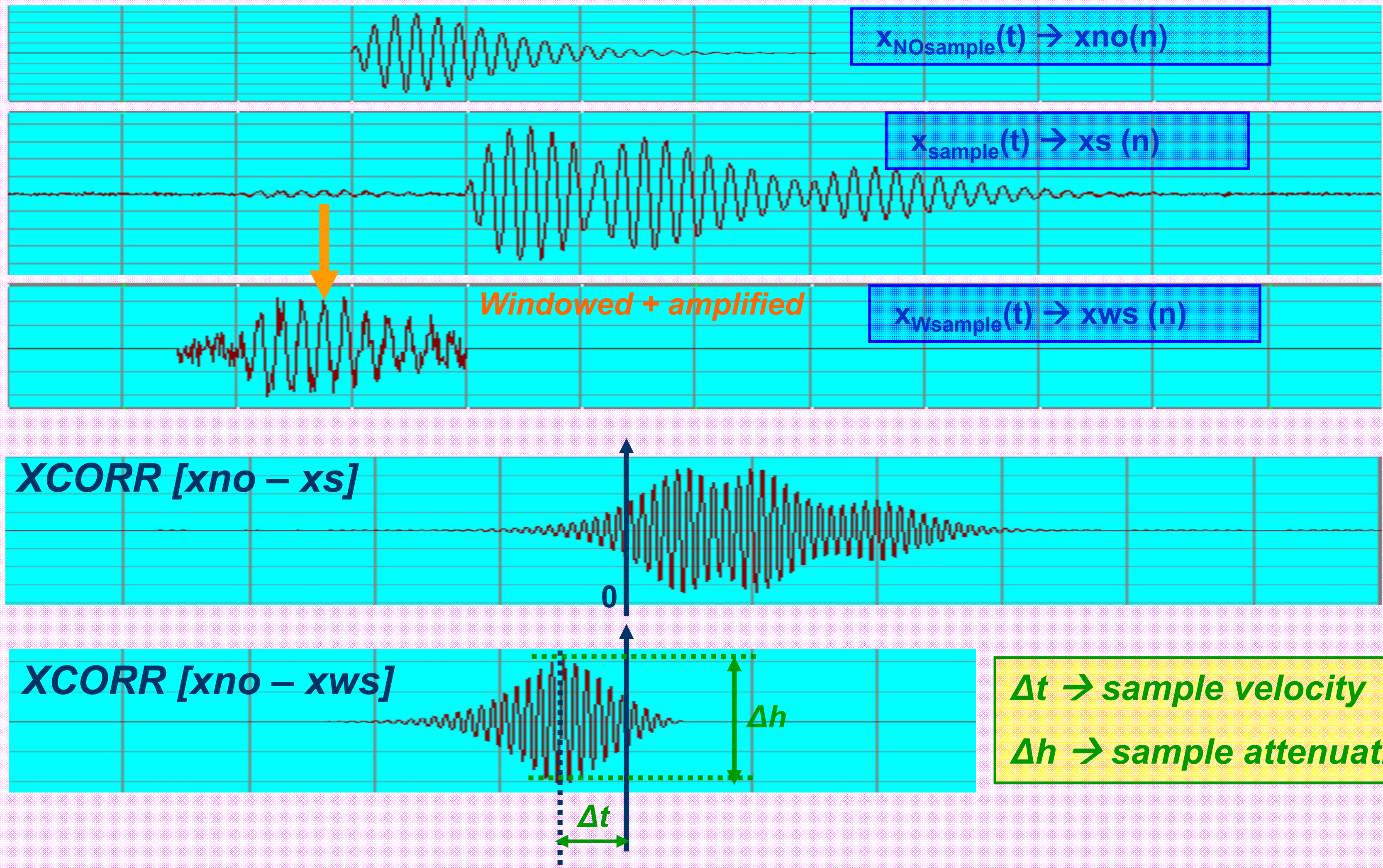
Problem: The maximum is NOT reliably determined when the noise gets very strong!

Analog and Digital Signal Processing

CORRELATION - WINDOWING



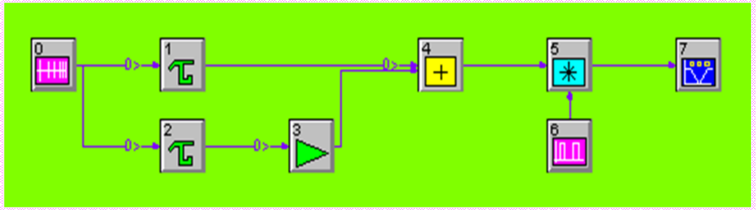
CORRELATION – WINDOWING (cont')



Δt → sample velocity

Δh → sample attenuation

SPECTRAL ANALYSIS - WINDOWING: *Example*



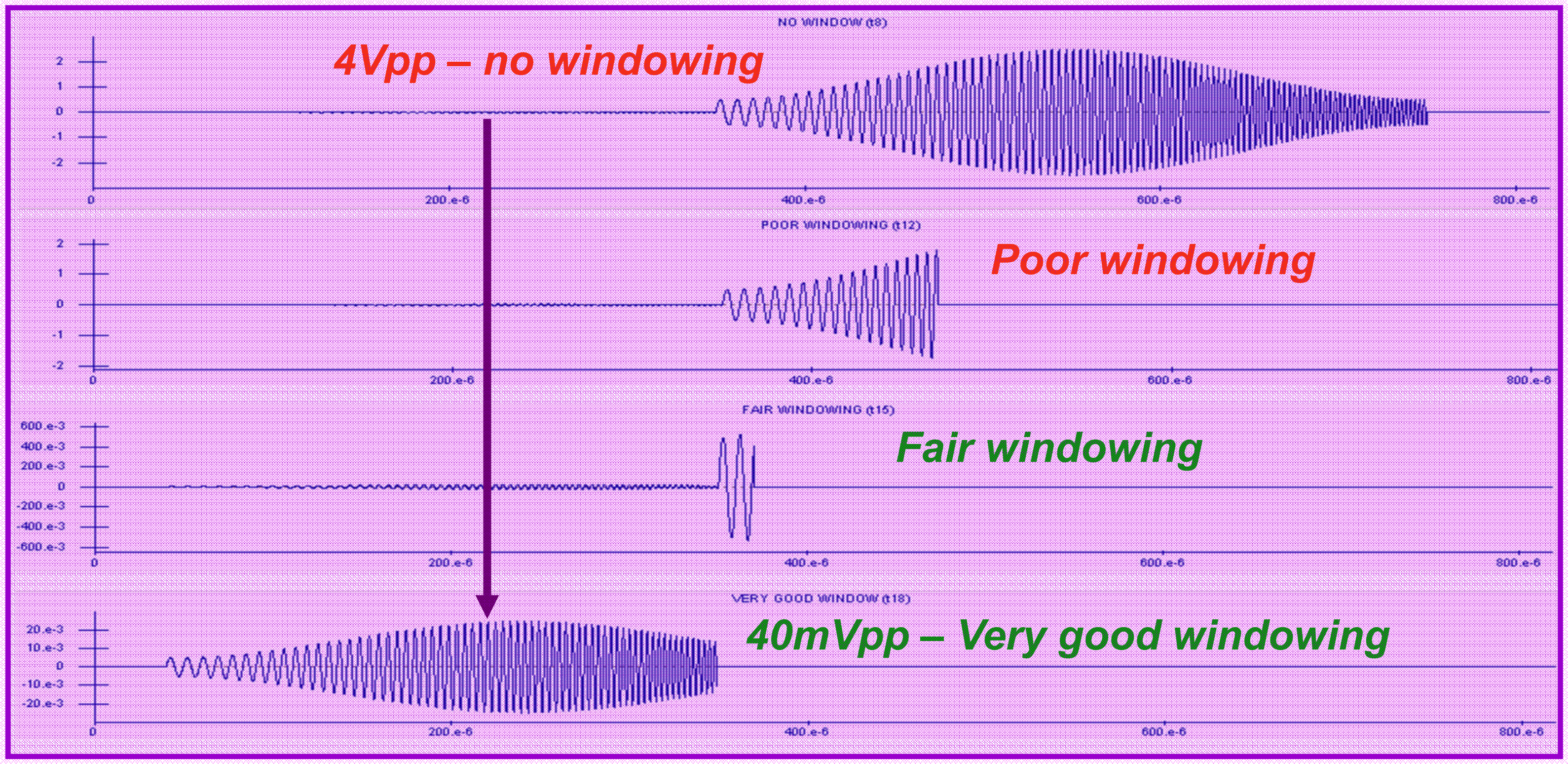
0 - chirp – 100 kHz → 400 kHz in 400µs

1 - Propagation delay unwanted signal: 350 µs

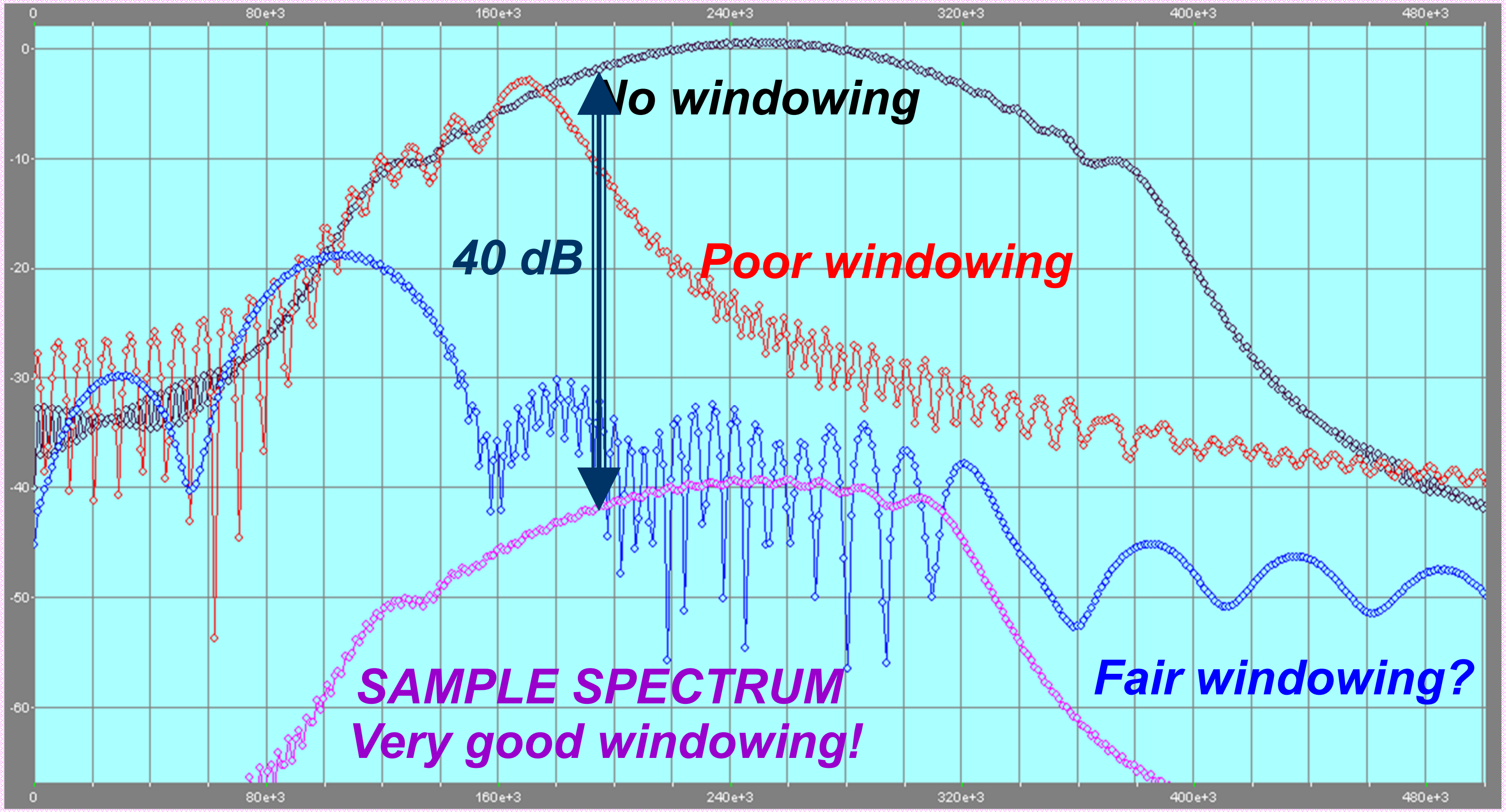
2 - Propagation delay desired signal: 40 µs

3 - Sample attenuation: 40 dB

5/6- Windowing



SPECTRAL ANALYSIS - WINDOWING: *Example (cont')*



Adequate WINDOWING is crucial in spectral analysis

PROBLEMS

Problem 9.1 (p. 9.4)

In a non-inverting voltage preamplifier, the equivalent input noise voltage spectral density is determined as follows (R1 << Req and Rp >> Rs):

$$e_{equi} = (e_n^2 + e_{Rs}^2 + i_{np}^2 \cdot Rs^2)^{0.5} \quad \text{where } Rs = real[Zs]$$

You have the choice between two Op Amps: Max412 and LM6142. Which one do you use if:

a) $Rs = 400\Omega$ *b)* $Rs = 100k\Omega$ with $e_{Rs} = 4\ nV \cdot (Rs/1000)^{0.5} /(Hz)^{0.5}$ **(300⁰K)**

Problem 9.2 (p 9.3)

In this example the through sample signal is equal to ***5 mV***$_{RMS}$.

➔ Preamp OUT SNR(dB) ≈ 20 log(5/16) ≈ - 10dB

The 100kHz ultrasound signal ***rise-time*** and ***fall time*** is approximately equal to ***70µs***.

a) How many repetition do you need in order to obtain a SNR_{Proc}(dB) = 20dB

$$\rightarrow Usample_{RMS}/Unoise_{RMS} = 10$$

b) What improvement do we get if we replace the ***LM6142*** by a ***Max412***?

Consider that the effect of $\boldsymbol{u_n}$ *largely dominates the effect of* $\boldsymbol{i_{np}}$.

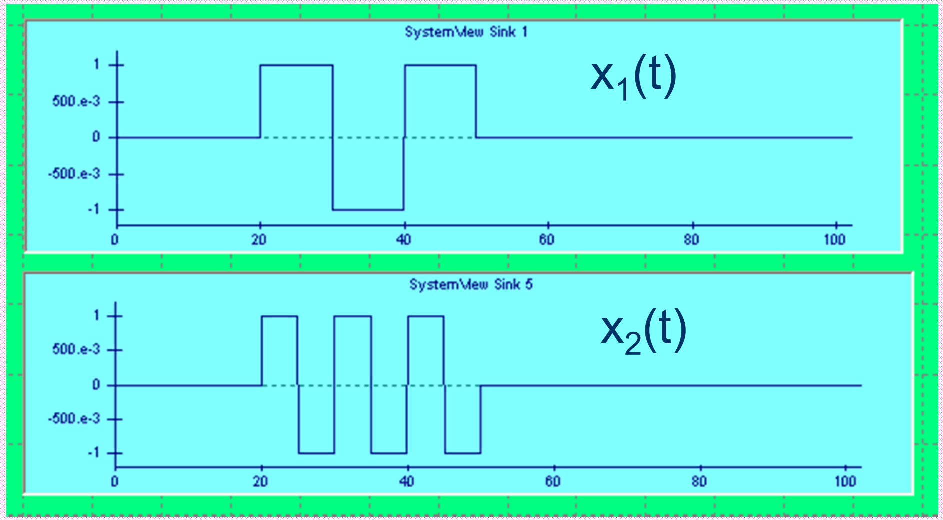
Problem 9.3 (Correlation)

Plot and determine the maximum of:

a) $Rx_1x_1(\tau)$

b) $Rx_2x_2(\tau)$

c) $Rx_1x_2(\tau)$



Problem 9.4 (SystemView)

Redo the example of page 9-10 with:

$x_{tr}(t) = 0.01 \cdot \sin(2\pi 100000\ t) \cdot (1-\cos(2\pi 10000\ t), 0 \leq t < 100\mu s$

TOF = 250µs

Determine the maximum value of ***Std.Dev*** (Gaussian Noise) such that TOF (Time-of-Flight) is estimated with an acceptable accuracy.