2.3.3. Macro-Cosmological Matter-Waves and Gravitation

Present General Relativity theory when addressing Gravitation is still not far from the old Newton and classical mechanics foundations and predictions. For what we can measure, verify, and use within our planet and inside our solar system, Newton and Kepler framework of gravitation seems still sufficiently or particularly good. Here is convenient to mention the existence of much earlier foundations of Kepler laws established and published in a period of flourishing of Arab science and culture, much before Kepler translated and revitalized such old concepts. For a larger scale of cosmic laboratories (like an open space with many galaxies), astronomic observations are indicating that we need to have certain updated and redesigned theory of gravitation. A. Einstein with his General Relativity theory did not go too far from Newton theory predictions. He introduced kind of mathematically complicated and almost inoperative spatial-temporal or modified geometry-related interpretation of gravitation (saying that a space is curved around mass formations and that we need to use more convenient, curved-space geometry in order to describe uniform motions within such spatial-temporal deformations caused by presence of masses). In other words, masses are anyway formations of concentrated, agglomerated or stored and stabilized energy, or atoms, and we already know from Classical Mechanics that forces are defined by gradients of energy concentrations. This way Einstein replaced such universal force definition with an equivalent or isomorphic concept that spatial-temporal geometry around masses should be appropriately deformed. This way, geometrically guided planetary and satellites motions, where our mechanical and cosmic engineering is presently working, are still mutually isomorphic and comparable, or almost identical, to Newton theory results (except applied mathematics is much more complex in Einstein's case). When Einstein's General Theory of Relativity is applied within our solar system, everything works perfectly well, as in Newton case, but predictions of General Relativity related to vast cosmic spaces with number of spiral galaxies are in some cases incomplete, doubtful, incorrect and requesting to take into account some unknown, virtual, missing and undetectable dark masses and dark energy… This is valid for predictions based both on Newton's and theory of Relativity.

The mainstream of contemporary scientific authorities, dealing with Gravitation and modern Physics, are showing a tendency to artificially extend, forge and fit present Relativity and Quantum theories (by introducing superficially missing, virtual and artificial items, like dark energy and dark masses) into something where we still do not have certain final, stable, commonly accepted theoretical and verifiable framework in operation. This looks like prematurely taking contemporary Relativity and Quantum theories too seriously, almost as perfectly-well established, final and solid step-stones or facts (or like hesitating and forbidding to modify something what is falsely considered as already being well constructed). Consequently, everything else in Physics, like surrounding scientific theories and concepts, should be (almost ideologically and dogmatically) subordinated to mentioned and postulated, by consensus created, artificial grounds (of Relativity and Quantum theory), and if something is missing or presenting problems (to contemporary Relativity and Quantum theory), we will simply and conveniently invent whatever missing, and postulate it as something that should exist (like dark matter and dark energy, tunnel-effects, different stochastic miracles, and what could only be some arbitrary, imaginative names and labels on black-boxes without real, verifiable or detectable meaning).

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER…** ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Obviously, that associated mathematical modeling and processing of mentioned "Quantum-Relativistic" and Newton-Gravitation mechanical grounds, works sufficiently well, but only in laboratories within our planet, or within any stable solar system (in our part of Cosmic space). When applying mentioned theories to much larger astronomic distances of our universe, or to extremely small subatomic environment, we will realize that such theories are most-probably not complete, not sufficiently and mutually compatible, not well applicable, and that we really need certain new and better theory of Gravitation (instead postulating, by consensus accepting, and/or inventing virtual, missing dark matter and energy, as presently practiced). Consequently, we need to have an appropriately modified and updated Relativity and Quantum theory (including to have a better, more convenient and larger mathematical environment for dealing with such problematic; -on some way this being analogically comparable to the evolution of Mathematical Analysis from the domain of Real numbers towards Complex numbers, and later towards Hypercomplex numbers and Analytic Signal functions).

Classical mechanics, Newton theory of Gravitation, and Relativity theory are presently defined and operating well within a smooth, continuous, deterministic environment and associated mathematics. Contemporary Quantum theory is conceptualized on a way that relevant matter domains' motions and energy-states are statistically and probabilistically modeled (as averaged expectations), being combined with some generalized and not mathematically very clear, multi-parameter discretization and quantization of everything what there presents energy-moments states, motions, interactions..., including implementation of some virtually related, but nonexistent ontological, supposed to be theory foundations (everything of that being too much fuzzy). Such artificial, theoretical, mathematical, and conceptual practices and platforms (as known in present Relativity and Quantum theory) cannot be easily and naturally united without certain conceptual, theoretical, and innovative redesign work. It will also be necessary to apply more general, unifying mathematical concepts, like "Analytic Signal", joint time-frequency analysis based on using Hilbert Transform (established by Denis Gabor; -see [57] Michael Feldman), and "Kotelnikov-Shannon, Whittaker-Nyquist Sampling and Signals Recovery Theory" (see more in chapters 4.0 and 10), etc.

One of the interesting trends in a new understanding of Gravitation is related to the very successful conceptualization, modeling and quantizing of planetary systems, orbits, and motions, analogically to N. Bohr atom modeling (what will be specifically addressed in this chapter, and it is well elaborated in Chapter 8.). Familiar (to N. Bohr atom structure) modeling is also producing correct and verifiable results based on relevant observations and measurements in different planetary or solar systems (see more, later, in the same chapter around (2.11.13) - (2.11.13-5) - (2.11.14) including T.2.8.). Even Micro-World, Wave-Particle Duality and Matter-Waves concepts (that are already well implemented and integrated into atoms' and micro-world events modeling) can be analogically extended to macro systems like planetary systems, ***but this time without big or any need to use stochastic and probabilistic modeling*** (as practiced in the contemporary Quantum theory). *In this book, the concept of particle-wave duality and (de Broglie) matter waves is being extended and enriched with ideas about complementarity of linear and rotational (or spinning) motions, being analogically and universally applicable to micro and macro world of physics, or to subatomic and astronomic spaces (somewhat analog to complementarity of electric and magnetic fields; -see much more about wave-particle duality in Chapters 4.0, 4.1.*

and 10., and about electromagnetic and mechanical analogies, symmetry, couplings, and interactions in Chapter 3.).

New understanding of Gravitation

Briefly saying, gravitation (as promoted in this book; -see "2.2.1. WHAT THE GRAVITATION REALLY IS", and Chapter 8. about extended planetary-atom modeling) is the phenomenology ***linked to spatially complex, stationary, and standing, cosmic matter waves***, manifesting within structurally and multi-dimensionally resonating universe, where orbital, linear and spinning inertial motions, are complementarily and structurally united. Formations of masses in such stabilized, structurally-oscillating (and rotating) universe are occupying nodal zones of highest energy-mass densities and highest accelerations, while and where oscillating amplitudes are minimal (very similar to half-wave resonators in High Power Ultrasonics technology, and in cases of acoustic levitation, where we can easily notice the presence of ***attractive forces***, acting towards nodal zones). Of course, such spatially resonating, stationary and standing-waves of orbiting structures are taking forms of atomic, solar, stellar and galactic systems, respecting laws of energy-momentum conservations, while involved linear and rotational (or spinning) motions are specifically united and mutually complementary, like in cases of electromagnetic fields, mass-spring and/or inductance-capacitance oscillatory circuits. Matter waves and Particle-Wave Duality manifestations, including associated and strongly coupled electromagnetic complexity (as elaborated in this book in Chapter 3., and on a similar way as presented in [71], from Dr. Jovan Djuric, "Magnetism as Manifestation of Gravitation"), are enabling energy-momentum communications within such dynamic and self-stabilizing, standing-waves and resonant spatial formations. What is the principal "external source" of mentioned vibrations, and how such universal and overwhelming, structural (micro and the macro world), standing-waves oscillations, spinning and rotations are being created and maintained within our Universe, are still unanswered questions (see familiar elaborations in *[99] from Konstantin Meyl).*

Set of updated and generalized Wave Equations (evolving from Classical Wave Equation and redesigned Schrödinger equation), formulated using the Analytic Signal, Complex and Hypercomplex functions, based on Hilbert transformation, are presently the best mathematical framework to address structural oscillations within our micro and macro Universe (as exercised in this book; -see Chapter 4.3). If we imaginatively extend the same framework and concepts, we will realize that Nikola Tesla's ideas about Dynamic Gravity theory are familiar to here summarized thoughts, and compatible or complementing with Rudjer Boskovic's universal Force descriptions (see more in [6], [97], [98], [99] and [117]).

We could significantly simplify the understanding of Gravitation as follows: Since we know that content of gravitational masses are atoms, and atoms have number of internal constituents performing orbital and spinning motions, this way creating magnetic moments, the most probable source of gravitational attraction should be such (mutually-interacting) internal magnetic moments, presenting some spatially distributed, and specifically polarized elementary magnets. There is always a certain number of not self-compensated internal magnets inside macro masses, and thanks to global (and accelerated) macro-motions within our Universe, mentioned internal

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

magnetic elements are mutually polarizing (or orienting) on a way that forces between masses are respecting attractive Coulomb or Newton laws (also being applicable for forces between magnets). Also, spinning, and rotating electrons and protons have different masses, and associated electric dipoles polarization will be facilitated, supporting attractive Coulomb forces. Such accelerated, motion-induced polarizations and standing waves formations, and associated effects of attractive forces, contemporary physics, "unintentionally and unknowingly" recognized as Gravitation, and as being based only on masses attraction. Masses' motions (including oscillations) are additionally producing matter-waves that will create surrounding stationary and standing matter-waves and fields that necessarily (and dominantly) should have an electromagnetic nature, but we are still incorrectly conceptualizing such mixed and dynamic effects as being only an independent and self-standing force of Gravitation, based only on mutual and inexplicable, static masses attraction. In the first chapter about analogies in Physics it is demonstrated that only static and electromagnetically neutral mases cannot be sources of Gravitation on an analogical way as electric charges and magnetic moments of fluxes are, based only on respecting Coulomb force law, but oscillating mases are able to produce gravitational attraction. Consequently, agglomerated atoms or masses in motion, and with associated and mutually coupled electromagnetic, electromechanical, and mechanical moments and charges, should be the real sources of gravitation (see more in Chapters 3. and 8.). Consequently, most of theories based on old (and still practiced) concepts about gravitation and other natural forces should significantly evolve or be completely replaced with better and new concepts.

In the case of micro-universe of atoms and elementary particles, de Broglie matter waves are manageable using the following relations (see more in chapter 4.1 and chapter 10., concerning PWDC):

Wave or motional energy (=) $\tilde{E} = hf = E_k$,
Matter-waves wavelength (=) $\lambda = h/p$,
Phase velocity (=) $u = \lambda f = \omega/k = E_k/p$,
Group or particle velocity (=) $v = d\omega/dk = dE_k/dp$.

Let us now try to construct or exercise what could be the *macro-universe equivalent* to de Broglie matter-waves concept. The idea here is to show that solar systems, planets, satellites and similar macro-objects (in orbital and spinning motions) are also analogically respecting certain periodicity and "standing macro matter-waves packing rules", like de Broglie matter waves in a micro-universe (especially like in N. Bohr atom model; -see more in Chapter 8.), but instead of Planck constant **h**, new constant $\mathbf{H} \gg> \mathbf{h}$ is becoming relevant in a similar way as **h** is in a micro world of Physics. See an introduction to such concept given by equations (2.11.5) - (2.11.9), (2.11.9-1) - (2.11.9-4) and (2.9.5-1) - (2.9.5-5).

The best for exercising such brainstorming is to start from the Kepler's third law (of planetary orbital motions), which is also applicable to all satellite and lunar, inertial movements around a specific planet or big mass. Let us temporarily focus our attention only on idealized circular motions, where the radius of rotation is R, to be able to use simpler mathematical expressions (approximating elliptic, orbital planetary motions, where **R** is a planet semi-major orbital radius). Kepler's third law is showing that the period T of a planet (or satellite) with mass **m**, orbiting around a big mass $M >>> m$ (or its sun), is given by (2.11.10),

$$T^2 = \left(\frac{4\pi^2}{G(M+m)} \right) R^3 \cong \left(\frac{4\pi^2}{GM} \right) R^3 \quad . \tag{2.11.10}$$

We will later also need to take from Newton-Kepler theory the expression for a maximal orbital or escape speed v_e and escape kinetic energy E_e (when planet, rocket or satellite would escape its stationary, circular orbit), which can be found as (2.11.11),

$$\left(E_e = \frac{1}{2} m v_e^2 = \frac{GmM}{R} \right) \Leftrightarrow v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e^2 R = 2GM = \text{constant.} \cdot \tag{2.11.11}$$

Escape velocity is a flexibility-parameter of boundary orbital stability limit of all motions within and around specific planet or sun (where planet or sun could be approximated as a local center of mass for such mutually related motions). Later, we will see that similar relation "$v^2 R = v_n^2 R_n = \text{constant}$" is also valid for all planets of certain solar system. For instance, for all planets of our solar system, we always have the same constant, $v^2 R = v_n^2 R_n = \mathbf{1.3256E+20}$ (see later in the same chapter T.2.3.3). For other planetary and satellite systems, such constants will be different. We will then see that mentioned relations are consequences of periodicity and standing, macro-matter-waves structures within stable planetary and satellite systems.

Kepler laws are also showing the intrinsic tendency of (mutually approaching) motional masses, planets and satellites, to eventually stabilize in some form of elliptic, rotational, orbital and inertial motions around certain big mass (local star or sun), which is in the same time very close to a local center of mass (applicable for such planetary or solar system). For having an additional background, see introductory elaborations in this chapter, around equations (2.4-11) – (2.4-17)), where we can find that for the stability of specific orbital motion (planetary system), the main request is that its total orbital momentum is conserved (meaning constant). If this (about orbital motions) were not the global tendency of mutually approaching masses, our universe would collapse in the process of permanent masses agglomeration. We also know that the conservation of orbital and spin moments is equally valid and important on a micro-world scale (analogically as we find in N. Bohr atom model). The dominant tendency, also valid for micro and elementary particles is to create stable (standing matter-waves), periodic and orbital motions, based on balancing between involved attractive forces with repulsive centrifugal forces (see new trends in modeling atoms and elementary particles in literature references from [16] to [22], Bergman, Lucas, Kanarev and others). Natural, non-forced, orbital, planetary motions are in the same time inertial (uniform, continuous, self-closed and self-standing), periodical motions, which are coincidently conserving their linear and orbital moments, and potentially hosting standing matter waves formations, as shown in (2.9.1). Also, consequences of stable self-closed standing matter wave's orbital formations (that are directly related to periodical motions) are various energy, spin and orbital momentum relations and quantizing situations, being very much analogical to N. Bohr atom model (see more in Chapters 8., 10., and in literature references under **[63]**).

Citation from [63], under ***25) - Spin - orbit coupling in gravitational systems:***
We employ in this work the analogy existing between electromagnetism and gravitation [1]. We extend this analogy to include all phenomena occurring at atomic level and assume that they also do occur at the gravitation level and are governed by analogous rules (equations). The spin-orbit interaction that exists in hydrogen atom, due to the magnetic field if we introduce the concept of gravitomagnetic field that is analogous to the ordinary magnetic field. We have seen that the spin-orbit interaction is the same interaction that Einstein attributed to the curvature of the space [2]. And since all planets do have spin, the spin-orbit interaction is intrinsically prevailing in all star-planet systems. Bear in mind that some atoms can have zero total spin angular momentum. Note that the spin of planets remained a kinematical quantity in Newton and Kepler formulation of planetary motion. But we will show here that the spin is a dynamical quantity without which the planets would not remain stable in their orbits. Moreover, without spin there is no orbital motion. How much a planet should spin will depend on how much it is needed to conform with the orbital one. Equating the gravitational energy of a star-planet system to the spin-orbit interaction yields a formula that relates the primary star-planet system parameters to each other. Moreover, we found that such a system exits only if the spin and orbital angular momenta are proportional to the planet mass to the star mass ratio. This condition represents a dynamical balance between the two angular momenta. We call the resulting equation the Kepler's fourth law which represents the missing equation (law) to determine a star-planet system completely.
………..
As in hydrogen atom, which is analogous to the solar system, there is an interaction between the internal magnetic field arising from the electron orbital momentum, and its spin angular momentum. This is normally known as the spin-orbit interaction. The gravitomagnetic field is analogous to the magnetic field arises from the motion of the electron around the nucleus.

Let us now attempt to show that (like in case of de Broglie matter waves applied on Bohr's hydrogen, or planetary atom model) a circular planetary (or satellite) orbit, or its perimeter, is susceptible to host some gravitational (meaning electromagnetic or inertial), orbital standing matter-wave. *Such macro matter-wave should have an orbital frequency* $f_o = f_{on}$, *wavelength* $\lambda_o = \frac{2\pi R}{n} = \lambda_{on}$, $n = \text{Integer}$, *group or orbital speed v (equal to planet semi-major orbital radius velocity), and associated phase speed* $u = \lambda_o f_o = u_n$. *Integer n should serve as a principal quantum number, being mainly related to the number of days in one year of certain planet orbiting its sun. The same quantum number could be also related to presence of satellites, moons, and to involved angular and spinning moments, since relevant (and associated) standing-waves structure will be affected by all of mentioned items (what introduces additional quantum numbers for arranging structural and spatial, standing waves packing and synchronization). Familiar matter-waves related conceptualization and results are addressed in the chapters 4.1, 10, and in this chapter among equations (2.9.1), (2.9.2), (2.11.5), (2.11.14)-h, and tables T.2.3.3-a, T.2.3.3-1, and such conceptualization is shown widely applicable, both in a micro and macro world of our Universe. One of good examples showing coupling of linear and spinning motions are cases of vortex-shedding flow-meter, presented around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,I, from the chapter 4.1. Many ideas showing or constructing very rich and well-operating (astronomic observations verifiable) analogy between planetary systems and N. Bohr atom model, can be found in [63], Arbab I. Arbab, [64], Marçal de Oliveira Neto, and in [67] Johan Hansson.*

In Chapter 8. of this book, "8.3. Structure of the Field of Subatomic Forces" (see equations from (8.64) until (8.74)), we can find proposals how to conceptualize spatial standing-waves-structured forces, emanating from internal atom field structure, where both orbital and radial quantizing rules are applicable and mutually synchronized. It will be challenging to apply similar modeling to stable planetary systems. One of the consequences of such modeling could be that gravitational forces between agglomerated atoms and masses are the result of Lorentz-forces and Coulomb attractions between half-wave resonating (and rotating) mass-dipoles (or better to say attractions between electromagnetically polarized, charged and mutually oscillating dipoles with masses).

Effectively, here we attempt to present motional energy of an orbiting planet as an equivalent macro matter-wave packet or wave group (which is the concept often and successfully applied in the micro-world physics). Specific planetary rotation around certain sun (see below (2.11.12)) has a period $T = T_m$ (or its one-year duration) and frequency of such (mechanical) rotation is $f_m = 1/T = 1/T_m$. f_m is not necessarily the frequency f_o of the associated, orbital, standing and macro matter-wave. For understanding the difference between mechanical (mass or particle) revolving frequency f_m and orbital macro matter-wave frequency f_o, we will first assume (and prove later) that $f_m \neq f_o$. Since the framework of this exercise implicitly accepts that relevant planetary or satellite (orbital) speeds are much lower compared to the light speed ($v << c$), we could safely say that certain planetary or group velocity (or its orbital velocity) should be two times higher than its phase velocity, $v = 2u = 2\lambda_o f_o$. See better explanation why and when $v = 2u$ in chapter 4.0, with equations (4.0.78) – (4.0.81). In other words, if the analogy with de Broglie matter-waves hypothesis also applies to planetary orbital motions, then the kinetic energy of specific planet should be equal to its equivalent matter-wave energy (or its matter-wave packet), $E_k = \tilde{E} = \frac{1}{2}mv^2 = Hf_0$, where H is a kind of gravitational, Planck's-analog, constant (all of that being very much analogical to matter waves and PWDC, as presented in Chapter 10; -see "10.00 DEEPER MEANING OF PWDC").

Now we can find mentioned orbital frequency, wavelength, group, and phase speed of such (hypothetical), planetary standing matter-wave as (2.11.12),

$$\left\{\begin{array}{l} 2\pi r = n\lambda_o,\ T = \dfrac{1}{f_m} = \dfrac{2\pi}{\omega_m} = T_m,\ v = \dfrac{2\pi R}{T} = 2\pi R f_m = \omega_m R \cong 2u,\ n = \text{Integer} \\ u = \lambda_o f_o = u_n \cong \dfrac{1}{2}v = \dfrac{\pi R}{T} = \pi R f_m,\ T^2 = \left(\dfrac{4\pi^2}{GM}\right)R^3 = \dfrac{1}{f_m^2}, v_e = \sqrt{\dfrac{2GM}{R}}, \\ v = u - \lambda\dfrac{du}{d\lambda} = -\lambda^2\dfrac{df}{d\lambda} = v_n, u = \dfrac{v}{1+\sqrt{1-v^2/c^2}} = u_n \end{array}\right\} \Rightarrow$$

$$\lambda_o = \frac{2\pi R}{n} = \lambda_{on},\ f_o = \frac{u}{\lambda_o} = f_{on} = n\frac{u}{2\pi R} = n\frac{f_m}{2} = \frac{n}{2T} = \frac{n\sqrt{GM}}{4\pi R^{3/2}}, f_m = \frac{2f_o}{n} = \frac{1}{T} = \frac{\sqrt{GM}}{2\pi R^{3/2}}, \quad (2.11.12)$$

$$u = \frac{v}{2} = \frac{1}{2}\sqrt{\frac{GM}{R}} = \frac{1}{2\sqrt{2}}v_e,\ v = 2u = \sqrt{\frac{GM}{R}} = \frac{1}{\sqrt{2}}v_e << c,\ m << M, \forall n = \text{Integer}.$$

Based on the group or planet's orbital speed, $v = \frac{2\pi R}{T} = 2u = 2\lambda_0 f_0 = \left(2\frac{H}{p}f_0\right)$ from (2.11.12), the wave energy or kinetic energy and gravitational Planck constant H, of an orbiting planet, which has mass $m << M$, and naturally keeps its angular momentum **L** = constant, can be mutually supporting and connected as:

$$\begin{gathered}
\tilde{E} = E_k = \frac{1}{2}mv^2 = mvu = pu = 2mu^2 = \frac{1}{4}mv_e^2 = \frac{GmM}{2R} = \frac{E_e}{2} = \frac{1}{2}\cdot\left(\frac{GmM}{R^2}\right)\cdot R = \frac{1}{2}\cdot F_{m-M}\cdot R = \\
= \frac{m}{2}\left(\frac{2\pi R}{T}\right)^2 = \frac{8m\pi^2R^2}{n^2}f_o^2 = 2m(\pi Rf_m)^2 = (2\pi mR^2 f_m)\cdot(\pi f_m) = \mathbf{L}\pi f_m = (\frac{2\pi}{n}\mathbf{L})\cdot f_o = Hf_o, \\
\left\{\begin{array}{l} \mathbf{L} = n\frac{H}{2\pi} = 2\pi mR^2 f_m,\ p = mv = \frac{\tilde{E}}{u} = \frac{Hf_o}{u} = \frac{H}{\lambda_o} = n\frac{H}{2\pi R} = m\sqrt{\frac{GM}{R}},\ G = 6.67\cdot 10^{-11}Nm^2kg^{-2}, \\ F_{m-M} = F_g = \frac{GmM}{R^2} = \frac{mv^2}{R},\ \lambda_o = \frac{H}{p} = \frac{2\pi r}{n},\ f_o = \frac{nf_m}{2} = \frac{n}{2T} = \frac{n\sqrt{GM}}{4\pi R^{3/2}}, H = \frac{2\pi}{n}\mathbf{L} = const.. \end{array}\right\}. \\
\Rightarrow \left\{\begin{array}{l} E_k = \tilde{E} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(2\frac{H}{p}f_0\right)^2 = 2\frac{H^2}{mv^2}f_0^2 = Hf_0 \\ 2\pi R = n\lambda_0 = n\frac{H}{mv},\ H = Const.\ n = 1,2,3,... \end{array}\right\} \Rightarrow H = \frac{2\pi mvR}{n} = \frac{2\pi m_i v_i R_i}{n_i} \Rightarrow \\
\Rightarrow \frac{m_i v_i R_i}{n_i} = \frac{m_j v_j R_j}{n_j} = \frac{H}{2\pi}.
\end{gathered} \tag{2.11.13}$$

If we formulate an Analytic Signal, power-related function, that represents matter wave of a specific orbiting planet (since we know its kinetic or wave energy), the same results, (2.11.12) and (2.11.13), should be associable to such complex matter-wave (see more about Analytic Signal in the chapters 4.0, 4.1 and 10).

[♣ COMMENTS & FREE-THINKING CORNER:

In order to get more tangible feeling what should be measurable effects of mentioned "standing gravitational waves" (or gravitational field phenomena that has phase velocity $u = \lambda_o f_o \cong \frac{1}{2}v = \frac{\pi R}{T} = \pi R f_m$ *), we could analyze tidal waves on our planet Earth in relation to the Earth spinning and Moon rotation around the Earth, as well as Earth-Moon rotation around our local Sun. This way we should be able to establish predictable and measurable correlations (which should comply with (2.11.12) and (2.11.13)). Of course, it will also be necessary to consider proper values for* $u, \lambda_o, f_o, v, R, T, f_m$*, valid for Moon's rotation around Earth.*

We could also consider the orbital, matter wave frequency f_o *as the **time-frequency-train reference** for measuring our real-time flow. In reality, we are effectively using such time reference on different ways, since it has very high stability, like quartz crystal or atomic clock oscillators, and it is the most significant for measuring our time flow (what will become more evident later; -see (2.11.14)-g)).*

---

*Let us address **gravitational field intensity and potential**. The **gravitational intensity or gravitational field*** $\mathbf{I} = \mathbf{E_g}$ *is traditionally defined as the gravitational force experienced by a unit mass* $\mathbf{m}$ *when placed in the gravitational field of another mass. From Newton's gravitational force we will get,*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$F_{m-M} = F_g = G\frac{mM}{R^2}$, *we can get,* $I = \frac{F_g}{m} = G\frac{M}{R^2}$.

*In this book (see the first chapter about Analogies), we know, based on analyzed electromechanical analogies, that **mass itself should not be the real and only source of gravitation. Only mass, which has linear and orbital moments, is (analogically and still hypothetically) better fitted to present gravitational charge** (see later (2.11.13-6) - (2.11.13-8), and more complete explanations in chapter 10, equations (10.1.4) - (10.1.7)). Effectively similar or familiar conclusions (what real sources of gravity are) can be drawn from the number of works of Dr. Jovan Djuric, [71]. Consequently, Newton gravitational force should have specific hidden or intrinsic, linear, and angular velocity parameters, embedded inside gravitational constant **G** (here, this unknown linear velocity parameter is* $v_0 = \text{const.}$*).*

We could now formulate another equivalent expression for Newton gravitational force, as being dependent of the product of relevant moments of attracting masses,

$$F_{m-M} = F_g = \frac{GmM}{R^2} = \frac{G}{v_0^2}\frac{(mv_0)\cdot(Mv_0)}{R^2} = \frac{G}{v_0^2}\frac{(p_m)\cdot(p_M)}{R^2} = K_G\frac{p_m \cdot p_M}{R^2},\ K_G = \frac{G}{v_0^2} = \text{const.}.$$

Based on such modified formulation of Newton force, and newly introduced gravitational charges p_m *and* p_M*, we can redefine an adjusted **gravitational intensity (or gravitational field** I* = E*$_g$**)** as,*

$$I^* = \frac{F_g}{p_m} = \frac{K_G p_M}{R^2} = \frac{G}{v_0^2}\frac{p_M}{R^2} = \frac{G}{v_0^2}\frac{Mv_0}{R^2} = \frac{G}{v_0}\frac{M}{R^2} = \frac{I}{v_0}.$$

We see, after we compare the ordinary (traditional) definition of gravitational intensity, and modified gravitational intensity, that qualitatively nothing significant changed (since $v_0 = \text{const.}$*).*

Consequently, if we respect analogical predictions (from the first chapter about analogies in Physics), we need to admit (still hypothetically) that real sources of gravitation are relevant and mutually coupled, linear and angular momenta. In such cases, Gravitational field intensity will be given by expressions (2.11.13-6) - (2.11.13-8).

*The **gravitational potential** **V** at a certain point in the gravitational field is traditionally defined as the work done in taking a unit mass **m** from that point to infinity against the force of relevant gravitational attraction. From such definition, we have,*

$$V(R) = \frac{\text{Energy}}{\text{unit mass}} = -\frac{1}{m}\int_r^\infty F dR = -\frac{1}{m}\int_r^\infty \frac{GMm}{R^2}dR = -\int_r^\infty \frac{GM}{R^2}dR = -\int_r^\infty I\,dR = -\frac{GM}{R}(=)\left[\frac{m^2}{s^2}\right].$$

*If we now consider that real charges (or sources) of gravitation are masses with linear and/or orbital moments, as in (2.11.13-6) - (2.11.13-8), we can redefine **modified gravitational potential** as,*

$$V^*(R) = \frac{\text{Energy}}{\text{unit moment}} = -\frac{1}{mv_0}\int_r^\infty F dR = -\frac{1}{mv_0}\int_r^\infty \frac{GMm}{R^2}dR = -\frac{1}{v_0}\int_r^\infty \frac{GM}{R^2}dR = -\int_r^\infty I^* dR = -\frac{G}{v_0}\frac{M}{R}(=)\left[\frac{m}{s}\right],$$

$$V^*(R) = \frac{E_{km}}{mv} = -G\frac{M}{2Rv} = -\frac{v}{2} = -\frac{R}{2}\omega_m = -\frac{G}{v_0}\frac{M}{R}, v_0 = 2\frac{G}{v}\frac{M}{R} = 2G\frac{M}{R^2\omega_m} = 2v$$

$$\left\{\begin{array}{l} E_{km} = \frac{R}{2}\left|F_g\right| = G\frac{mM}{2R} = G\frac{m_r m_c}{2R} = \frac{mv^2}{2} = \frac{J_m\omega_m^2}{2} \\ \left|F_g\right| = G\frac{mM}{R^2} = G\frac{m_r m_c}{R^2} = \frac{mv^2}{R} = \frac{J_m\omega_m^2}{R} = \frac{2E_{km}}{R} \end{array}\right\},$$

alternatively, if we consider that only angular moments are dominantly relevant, we will get,

$$V^*(R) = \frac{E_{km}}{J_m\omega_m} = \frac{1}{2}\omega_m , \left(E_{km} = \frac{J_m\omega_m^2}{2} \right)..$$

We can again see, when we compare ordinary (traditional) definition of gravitational potential, and modified gravitational potential, that qualitatively nothing significantly changes, and that a relevant angular velocity (or angular moment) should be the most significant for hypothetically and analogically innovated expressions of gravitational field intensity and potential (see later the same resume in (2.11.13-6) - (2.11.13-8)).

*Now we can redefine **gravitational potential energy**. The work obtained in bringing a body from infinity to a point in the gravitational field is called the gravitational potential energy of the body at that point. Potential energy* $U = E_p$ *is usually presented mathematically as,*

$$U = E_p = -F_g \cdot R = G\frac{mM}{R} = K_G\frac{p_m \cdot p_M}{R} .$$

As we can see, the traditional definition of gravitational potential energy is identical to the modified definition of gravitational potential energy (and it is evident that introducing the new meaning of gravitational charges should not be a problem). The gravitational potential energy at infinity is assumed (to be) zero.

Here we also see that we similarly define gravitational intensity and potential as we do with electric charges and fields. This could be an intuitive argument in relation to relevant electromechanical analogies, in a direction that gravitation could be a specific hidden manifestation of electromagnetic forces, since rotating and spinning motions are producing electric charges separation (or electric dipoles), and spinning matter states are related to elementary magnets that are getting properly-arranged during mentioned rotation.

As the support to (2.11.13), it is convenient to mention that total mechanical energy (without rest-mass energy), $E_m = E_k + E_p$*, for an object with mass* m*, in a closed circular orbit with radius* R*, in a central gravitational field around a body with mass* M*, such as a planet orbiting about local sun, is equal to the sum of its kinetic energy* $E_k = \frac{1}{2}mv^2$*, and its potential, or "positional" energy* $E_p = -G\frac{mM}{R}$*. The gravitational potential energy is defined as a negative value, equal to the kinetic energy that the object would gain by falling from an infinite distance to its current position. At considerable distances from the Sun, the object would have zero potential energy (since it would not have picked up any speed, by falling). Objects close to the Sun have considerable (although negative) potential energies, corresponding to the speed they would gain by dropping a long way.*

$$E_m = E_k + E_p = \frac{1}{2}mv^2 - \left(G\frac{mM}{R^2}\right)\cdot R,\ F_c = F_g = m\frac{v^2}{R} = G\frac{mM}{R^2} \Rightarrow 2E_k = mv^2 = G\frac{mM}{R} \Rightarrow$$

$$\Rightarrow E_m = E_p + E_k = \frac{1}{2}G\frac{mM}{R} - G\frac{mM}{R} = -G\frac{mM}{2R} = \frac{1}{2}E_p,\ E_k = G\frac{mM}{2R} = -E_m = -\frac{1}{2}E_p\ .$$

Very similar, or better to say identical result for E_m *also holds for an elliptical orbit, after we generalize it by replacing the radius of orbit R by the relevant orbital semi-major axis, as usually applied for Newton's derivation of Kepler's third law. Now, such total mechanical energy is constant and has similar forms for circular and elliptical orbits (*$E_m = -G\frac{mM}{2R} = \frac{1}{2}E_p$*, where R is semi-major axis). In ideal circular orbits, since there, speed v is constant, kinetic, and potential energies are constant. In elliptical orbits, the kinetic and potential energy is not constant, but somewhat variable on the way that one is large when the other is small and vice versa. For elliptic orbits, where* $0 < e < 1$ *is the eccentricity of the elliptical orbit, the following equations can be derived:*

T.2.7.

Energy type	**R = R$_{min.}$**	**R = R$_{max.}$**
Potential, E_p	$2E_m/(1-e)$	$2E_m/(1+e)$
Kinetic, E_k	$-E_m(1+e)/(1-e)$	$-E_m(1-e)/(1+e)$

We see that when orbit eccentricity $e = 0$*, all latest results for elliptical orbits again correspond to a circular orbit result. The larger the eccentricity,* ***e****, the larger is the variation of the potential and kinetic energies during each period of the orbital motion.*

What is the meaning of the gravitation force and associated matter waves energy between two masses ***m*** *and* ***M*** *can be briefly explained based on simplified two-body problem analysis? Here, we will use the same symbolic and meanings for associated parameters, as in (2.11.10) – (2.11.13). If we have two isolated, static (or standstill) masses,* ***m*** *and* ***M****, in the same inertial, reference frame, where a distance between them is equal to* R*, we can say that the total energy,* $E_{tot.}$*, of such system and Newton force of gravitation* F_g*, between them, are,*

$$E_{tot.} = mc^2 + Mc^2 = (m+M)c^2 = m_c c^2,\ F_g = -G\frac{mM}{R^2}, m_c = m + M\ . \qquad (2.11.13\text{-}1)$$

If such masses are in the same reference frame and have specific mutually relative motion (where ***m*** *has velocity* $\vec{v}_1$ *and* ***M*** *has velocity* $\vec{v}_2$*), the total energy of such two-body system and force of gravitation between them are,*

$$\begin{aligned} &E_{tot.} = mc^2 + \frac{1}{2}mv_1^2 + Mc^2 + \frac{1}{2}Mv_2^2 = \\ &= (m+M)c^2 + \frac{1}{2}(m+M)v_c^2 + \frac{1}{2}m_r v_r^2 = m_c c^2 + \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_r v_r^2, \\ &F_g = -G\frac{mM}{R^2} = -G\frac{m_r m_c}{R^2},\ m_c = m + M,\ m_r = \frac{mM}{m+M},\ \vec{v}_r = \vec{v}_1 - \vec{v}_2, \vec{v}_c = \frac{m\vec{v}_1 + M\vec{v}_2}{m+M}. \end{aligned} \qquad (2.11.13\text{-}2)$$

If, also, each of masses is self-spinning (has its spin moment), the same situation with the total energy (similar to elaborations around equations (2.5.1-4) - (2.5.1-7) from the same chapter) and force of gravitation will be,

$$\left\{\begin{array}{l} E_{tot.} = mc^2 + \frac{1}{2}mv_1^2 + \frac{1}{2}J_{s1}\omega_{s1}^2 + Mc^2 + \frac{1}{2}Mv_2^2 + \frac{1}{2}J_{s2}\omega_{s2}^2 = \\ = (m+\Delta m_s)c^2 + \frac{1}{2}(m+\Delta m_s)v_1^2 + (M+\Delta M_s)c^2 + \frac{1}{2}(M+\Delta M_s)v_2^2, \\ \Delta m_s c^2 = \frac{1}{2}J_{s1}\omega_{s1}^2,\ \Delta M_s c^2 = \frac{1}{2}J_{s2}\omega_{s2}^2 \end{array}\right\} \Rightarrow \quad (2.11.13\text{-}3)$$

$$\Rightarrow \left\{\begin{array}{l} E_{tot.} = m^*c^2 + \frac{1}{2}m^*v_1^2 + M^*c^2 + \frac{1}{2}M^*v_2^2 = m_c^*c^2 + \frac{1}{2}m_c^*v_c^2 + \frac{1}{2}m_r^*v_r^2 \\ m^* = m + \Delta m_s,\ M^* = M + \Delta M_s, \\ F_g = -G\frac{m^*M^*}{R^2} = -G\frac{m_r^*m_c^*}{R^2}, m_c^* = m^* + M^*,\ m_r^* = \frac{m^*M^*}{m^*+M^*},\ \vec{v}_r = \vec{v}_1 - \vec{v}_2, \vec{v}_c = \frac{m^*\vec{v}_1 + M^*\vec{v}_2}{m^*+M^*} \end{array}\right\}.$$

Within one or the other option (for masses without, or with self-spinning), we are coming to the possibility to express reduced-mass kinetic energy $\frac{1}{2}m_r^*v_r^2 = E_r$ as specific orbital, rotation energy $\frac{1}{2}J_r^*\omega_r^2$ of relative mass m_r^* about its center of mass m_c^*, and this energy corresponds to the associated matter wave energy $\tilde{E}$ of relative mas in its (would or could be) orbital-like motion (as in (2.11.13)),

$$E_{tot.} = m_c^*c^2 + \frac{1}{2}m_c^*v_c^2 + \frac{1}{2}m_r^*v_r^2 = m_c^*c^2 + \frac{1}{2}m_c^*v_c^2 + \frac{1}{2}J_r^*\omega_r^2,\ E_r = \frac{1}{2}m_r^*v_r^2 = \frac{1}{2}J_r^*\omega_r^2 = \tilde{E}. \quad (2.11.13\text{-}4)$$

Now, we will introduce the decisive, essential, questionable, and innovative assumption regarding gravitation. ***If we assume that rotating (orbital-like) motional energy $\frac{1}{2}J_r^*\omega_r^2$, of the relative mass $m_r^* = \frac{m^*M^*}{m^*+M^*} = \mu$, about $m_c^* = m^* + M^*$, is equal to one half of the work (or energy) of Newton gravitational force $F_g \cdot R$ (between masses m^* and M^*, along the distance R, necessary to unite masses m^* and M^*, as already seen in (2.11.13)), we will be able to develop, or reformulate the same (already known) form of the Kepler Third Law*** *(2.11.10), as follows,*

$$\left[\begin{array}{l} 2\tilde{E} = 2E_r = 2\cdot\frac{1}{2}J_r^*\omega_r^2 = 2\cdot\frac{1}{2}m_r^*v_r^2 = F_g\cdot R = G\frac{m^*M^*}{R^2}\cdot R = G\frac{m_r^*m_c^*}{R} = -E_p = -2E_m, \\ E_m = E_p + E_k = -G\frac{m^*M^*}{2R} = \frac{1}{2}E_p = -E_r,\ E_k = G\frac{m^*M^*}{2R} = -E_m = -\frac{1}{2}E_p = E_r \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} 2\cdot\frac{1}{2}v_r^2 = G\frac{m_c^*}{R} = 2\cdot\frac{1}{2}(\omega_r R)^2 = 2\cdot\frac{1}{2}(2\pi f_r R)^2 = 4\pi^2 f_r^2 R^2 = \frac{4\pi^2R^2}{T_r^2}, \\ v_r = \omega_r R = 2\pi f_r R = \frac{2\pi R}{T_r} = \frac{2\pi R}{T},\ f_r = f_m = \frac{1}{T} = \frac{1}{T_r} \end{array}\right] \Rightarrow \quad (2.11.13\text{-}5)$$

$$\Rightarrow T_r^2 = \frac{4\pi^2}{Gm_c^*}\cdot R^3 \Leftrightarrow T^2 = \frac{4\pi^2}{G(M+m)}\cdot R^3 \cong \frac{4\pi^2}{GM}R^3.$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\tau = 2\pi\sqrt{\frac{m}{k}}\,a^{3/2}.$$

For the family of planets orbiting the sun, $k/m = GM$ where M is the mass of the sun. Thus, the period of a planet is proportional to the $3/2$ power of the semi-major axis of the planet's orbit; it does not depend on the mass of the planet or the eccentricity of the orbit. This is **Kepler's third law** of planetary motion.[4] See Fig. 1.11 and note that the slope of the log-log plot is $3/2$.

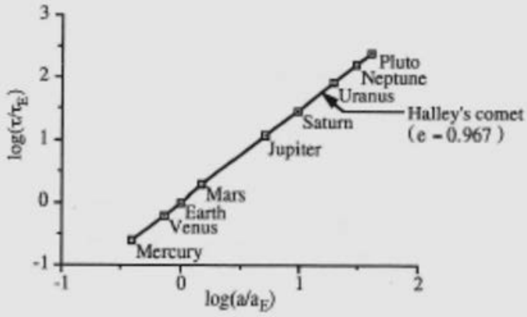


Fig. 1.11. Kepler's third law

Kepler's second law, plus some geometry, can also give the way the particle moves around the orbit as a function of time.[5] However, for modern readers more familiar with calculus than with geometry, it is probably easier to obtain this by integrating the equation for radial motion

The picture was taken from Lagrangian and Hamiltonian Mechanics, M. G. Calkin
ISBN: 978-981-02-2672-5

With (2.11.13-5), (2.4-13), (2.4-5.1) and with many familiar elaborations in Chapter 4.1, we are supporting and defending the concept of the real existence of planetary, macro matter waves, as introduced in (2.11.12) and (2.11.13). ***We also see that natural inertial, meaning orbital motions and self-closed standing matter-waves structures are appearing coincidently.*** *Different quantizing (or integer dependent) formulas are also the consequence of standing waves formations. Later, we will collect more arguments in a direction that gravitation-related matter waves (and gravitation) are most probably the consequences and effects of fundamentally electromagnetic, electromechanical, electro and magnetostrictive background.*

From widely elaborated electromechanical analogies, (see chapter 1), and from common definitions for the electric (and magnetic) field, we can analogically speculate about a new meaning for the field of gravitation, where real sources of gravity will be involved angular and linear moments (and associated electromagnetic charges and fluxes), instead of masses. *For instance, the gravitational force between masses m and M, where m is orbiting about M, can be formulated as,*

$$\begin{aligned}
&\left|F_g\right| = G\frac{mM}{R^2} = G\frac{m_r m_c}{R^2} = \frac{mv^2}{R} = \frac{J_m\omega_m^2}{R} = \frac{2E_{km}}{R}, \left(v = v_m = \omega_m R\right),\\
&v_m = \sqrt{G\frac{M}{R}} = \omega_m R,\ \omega_m = \sqrt{G\frac{M}{R^3}},\\
&v_r = \sqrt{G\frac{m_c}{R}} = \omega_{mr} R,\ \omega_{mr} = \sqrt{G\frac{m_c}{R^3}}, \frac{v_m}{v_r} = \frac{\omega_m}{\omega_{mr}}.
\end{aligned} \quad (2.11.13\text{-}6)$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Using the analogical field-intensity definition, where a field is developed from its force divided by its source-charge, and if the relevant gravitational charge is either linear or angular momentum, (not a mass, like in traditional Newtonian gravitation), we will have,

$$E_g = \begin{Bmatrix} \frac{F_g}{mv} \\ \text{or} \\ \frac{F_g}{J_m \omega_m} \end{Bmatrix} = \begin{Bmatrix} \frac{mv^2}{mvR} \\ \text{or} \\ \frac{J_m \omega_m^2}{J_m \omega_m R} \end{Bmatrix} = \begin{Bmatrix} \frac{v}{R} = \omega_m \\ \text{or} \\ \frac{1}{R} \omega_m \end{Bmatrix}. \qquad (2.11.13\text{-}7)$$

*In both cases of (*2.11.13-7)*, here modified field of gravitation E_g is directly proportional to the proper angular (or orbital) velocity ω_m (as we see in (2.11.13-7)).*

*The standard and traditional definition of the gravitational field (where only a mass is the source of gravitation) is much different from (*2.11.13-7), for example,

$$E_g = \frac{F_g}{m} = G\frac{M}{R^2} = \frac{v^2}{R} = \omega_m^2 R. \qquad (2.11.13\text{-}8)$$

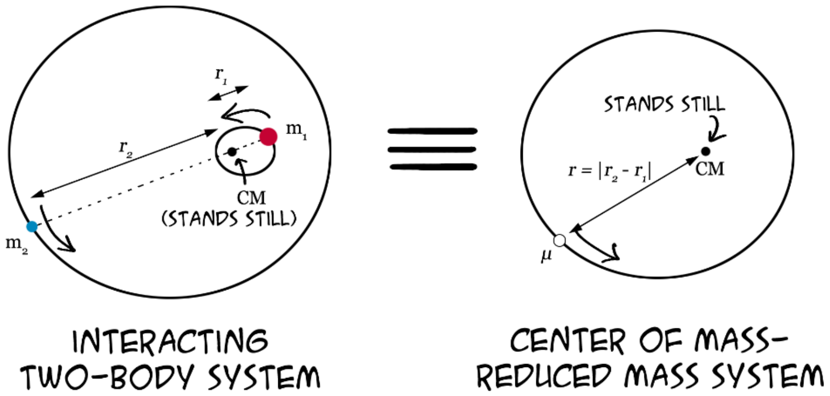
Consequently, here we are on some intuitive and brainstorming (or hypothetical) way generating conclusions that the most relevant gravitation-related sources should be angular, orbital and spinning moments (see more in chapter 4.1). This could be additionally supported if we try to specify what is common, for both micro-world of atoms and subatomic entities, and the macro-world of planetary systems and galaxies. The typical common items are micro and macro systems, and events with rotations, spinning states, and orbital motions, all of them characterized by angular moments and dimensionally or directly proportional to certain relevant angular velocity, very much similar as we see in the new definition of the gravitational field (2.11.13-7). In chapter 1. of this book (about analogies), we also find that relevant sources of gravitation (based on analogical conclusions) are not masses, but linear and orbital moments, or electric charges and magnetic moments and fluxes (related to rotation and spinning). This is similar or at least sufficiently common theoretical platform about the dominant place of rotation and spinning in our universe, as we can find in publications under [36], Anthony D. Osborne & N. Vivian Pope.

Practically and briefly summarizing, we could say that:
1° All-natural motions in our universe are curved, and
2° All stable, stationary, periodical, and inertial motions should be orbital motions.
3° Also, our Universe is, globally, intrinsically, and holistically rotating (oscillating and resonating) on all micro and macro structural levels.

Consequently, only angular, or orbital and spinning moments should be the most relevant regarding phenomenology we understand and describe under gravitation. Since spinning, rotation and orbiting are very often coupled with associated magnetic fields and moments, here is the place for understanding electromagnetic background nature of gravity.

Until here, we mostly analyzed somewhat static and simple-motion situations between two (electromagnetically neutral, and internally balanced) masses, but similar conclusions can be drawn from the general two-body problem. For instance, the same energy balance and gravitational attraction between two bodies, as given with (2.11.13-2) and (2.11.13-6), is also applicable in cases of motional masses, when we transform such two-body system to an equivalent orbiting motion of the reduced mass around its central mass. ***The more***

appropriate conceptualization here will be to treat the two-body problem as an impact situation and to search for evolving angular and rotating elements in such mutually interacting motions. *In other words, every two-body motional system is generating elements of angular, or somewhat circular (spiral and orbital) accelerated movements, what we can see when we play with different, mutually related reference frames, as presented with (2.11.13-9), and on the picture, below.*



Taken from: *https://quantumredpill.files.wordpress.com/2013/01/two-body-cm-systems.png*

Mentioned two or multibody systems with evolving elements of orbiting and circular motions are naturally creating matter-waves. If involved masses are also electromagnetically charged, and have spinning moments, mentioned revolving, circular, and resulting spinning motions will be intensified. In addition, if electromagnetically neutral or non-charged masses are getting closer, because of specified self-generated and evolving elements of angular and rotational movements, we can naturally expect to get internal electromagnetic dipoles polarizations (inside of masses, because masses are composed of molecules, atoms, electrons, protons and neutrons, all of them are on some way oscillating, rotating and spinning). This way, certain kind of electric and magnetic dipoles-related internal currents and magnetic fields will be created (since masses m and M are also performing linear and circular motions, depending on the point of view, what is influencing and stimulating spontaneous, but organized electromagnetic dipoles polarization). Mentioned internal electromagnetic dipoles-related currents and fluxes are effectively creating a spatial situation like parallel wires with electric currents passing in the same direction, this way magnetically attracting each other, what we detect as gravitation. In the same time, because of periodical, spatial and temporal motions of involved masses and because of associated elements of angular, rotational, spinning and helical movements, matter waves will be naturally created.

Two-body energy balance and involved gravitational force with elements of linear and circular motions can be summarized as,

$$E_{tot} \cong mc^2 + \frac{1}{2}mv_m^2 + Mc^2 + \frac{1}{2}Mv_M^2 = m_c c^2 + \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_r v_r^2 =$$

$$= \boxed{m_c c^2 + \frac{1}{2}J_m \omega_m^2 + \frac{1}{2}J_M \omega_M^2 = m_c c^2 + \frac{1}{2}J_c \omega_c^2 + \frac{1}{2}J_r \omega_r^2},$$

$$\left|F_g\right| = G\frac{mM}{R^2} = G\frac{m_r m_c}{R^2} = \frac{mv^2}{R}\left(=\frac{m_r v_r^2}{R}\right) = \frac{2E_{km}}{R} = \boxed{\frac{J_m \omega_m^2}{R}},$$

$$\left(\begin{array}{l} E_{km} = \frac{1}{2}mv_m^2 = \frac{1}{2}J_m\omega_m^2 = \frac{R\left|F_g\right|}{2} = G\frac{mM}{2R} = G\frac{m_r m_c}{2R},\ v = v_m = \omega_m \cdot R \cong 2u, \\ \Psi_m^2(t)dt = d\tilde{E}_m = dE_{km} = v_m dp_m = d\left(\frac{1}{2}mv_m^2\right) = mv_m dv_m = p_m dv_m = \\ = \omega_m d\left(J_m\omega_m\right) = \left(J_m\omega_m\right)d\omega_m,\ \Delta\Psi_m(x,t) - \frac{1}{u_m^2}\frac{\partial^2\Psi_m(x,t)}{\partial t^2} = 0,\ u_m = \lambda_m f_m = v_m / 2 \end{array}\right), \quad (2.11.13\text{-}9)$$

$$\omega_m = 2\pi f_m,\ n\lambda_m = 2\pi R,\ \vec{v}_c = \frac{m\vec{v}_m + M\vec{v}_M}{m+M} = K\frac{J_m\vec{\omega}_m + J_M\vec{\omega}_M}{J_m + J_M} = K\vec{\omega}_c,\ K = \text{Const.}$$

See much more in chapter 4.1, where similar problems of helical matter waves are additionally elaborated. In addition, if mutually-approaching and interacting particles or masses already have spinning and electromagnetic moments and charges, and if specific spontaneous electromagnetic dipoles polarization is produced, beside gravitational forces and effects, we will need to account presence of Coulomb force interactions, indicating that all of mentioned forces and fields essentially have an electromagnetic origin or background.

Now is a right place to mention an analogy between here-introduced concepts (of couplings and equivalency between linear and angular motions) as already presented in this chapter and later in chapter 4.1 (on illustrations on Fig.4.1, Fig.4.1.2, Fig.4.1.3, Fig.4.1.4, Fig.4.1.5, and within equations under (4.3) and later).

Citation took from the Internet: *http://www.school-for-champions.com/science/gravitation_orbit_center_of_mass_derivation.htm#.V6SSvKL4i6E*

Derivation of Circular Orbits Around Center of Mass

by Ron Kurtus (revised 14 May 2011)

Circular orbits of two objects around the center of mass (CM) between them require tangential velocities that equalize the gravitational attraction between the objects.

Tangential velocities tend to keep the objects traveling in a straight line, according to the Law of Inertia. If gravitation cases an inward deviation from straight-line travel, the result is an outward centrifugal force. By setting the gravitational force equal to the centrifugal forces, you can derive the required tangential velocities for circular orbits.

The orbit equations can be in simplified forms when the masses of the two objects are the same and when the mass of one object is much greater than that of the other.

Questions you may have included:

- *What are the factors involved in the derivation?*
- *What are the equations for the velocities of the objects?*
- *What happens when one object is much larger than the other is?*

This lesson will answer those questions. Useful tool: Units Conversion

Factors in determining orbital velocities

The linear tangential velocities required for two objects to be in circular orbits around the center of mass (CM) between them is found by comparing their gravitational force of attraction with the outward centrifugal force for each object.

***Note**: A linear tangential velocity is a straight-line velocity is perpendicular to the axis between the two objects. It is tangent to the curved path and is different from rotational speed.*

(See Center of Mass and Tangential Gravitational Motion for more information.)

Assume no initial radial velocities

When two objects in space are traveling toward the general vicinity of each other, they both have radial and tangential velocities concerning the center of mass (CM) between them. However, to simplify the derivation for circular orbits, we will only look at the case where there are no inward or outward radial velocities and be concerned about the tangential velocities.

This is similar to the case of Newton's cannonball going into orbit or sending a satellite into orbit around the Earth.

(See Gravity and Newton's Cannon for more information.)

Since there is no radial motion, a separation between the objects remains constant, which is a requirement for circular orbits.

The gravitational force of attraction

The gravitational force of attraction between two objects is:

$$F = GMm/R^2$$

where

- ***F** is the force of attraction between two objects in newtons (N)*
- ***G** is the Universal Gravitational Constant = $6.674 \cdot 10^{-17}$ N-km^2/kg^2*
- ***M** and **m** are the masses of the two objects in kilograms (kg)*
- ***R** is the separation in kilometers (km) between the objects, as measured from their centers of mass*

***Note**: Since force is usually stated in newtons, but motion between astronomical bodies is usually stated in km/s, an adjusted value for **G** is used, with N-km^2/kg^2 as the unit instead of N-m^2/kg^2. **G** is also sometimes stated as $6.674 \cdot 10^{-20}$ km^3/kg-s^2.*

(See Universal Gravitation Equation for more information.)

Separation of objects

*As the objects orbit the CM, their total separation, **R**, remains constant. The individual separations between the objects and CM are also constant and determined by **R** and their masses:*

$R = R_M + R_m$

where

- R_M *is the separation between the center of object **M** and the CM in km*
- R_m *is the separation between the center of object **m** and the CM in km*

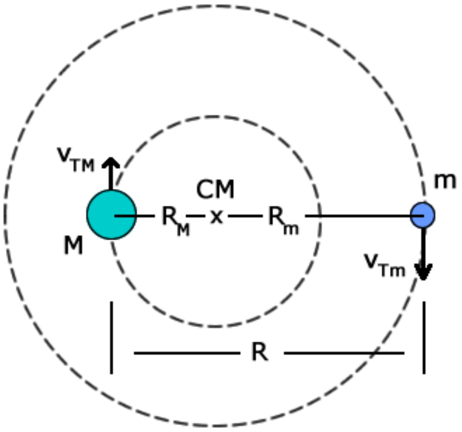
The values of R_M *and* R_m *are according to the equations:*

$R_M = mR/(M + m)$

$R_m = MR/(M + m)$

(See Center of Mass Definitions for more information.)

The factors involved can be seen in the illustration below:



Factors in objects orbiting CM

***Note**: Although the Earth orbits the Sun in a counterclockwise direction, we usually indicate motion in a clockwise direction.*

(See Direction Convention for Gravitational Motion for more information.)

Centrifugal force

The centrifugal inertial force on each object relates to its circle of travel:

$F_M = Mv_{TM}^2/R_M$

$F_m = mv_{Tm}^2/R_m$

where

- ***F_M*** *is the centrifugal inertial force on mass* ***M***
- ***v_{TM}*** *is the tangential velocity of mass* ***M***
- ***F_m*** *is the centrifugal inertial force on mass* ***m***
- ***v_{Tm}*** *is the tangential velocity of mass* ***m***

Note*: Centrifugal force is caused by inertia and is not considered a "true" force. It is sometimes called a pseudo- or virtual force.*

Substituting $R_M = mR/(M + m)$ *and* $R_m = MR/(M + m)$ *in the above equations gives you:*

$F_M = Mv_{TM}^2(M + m)/mR$

$F_m = mv_{Tm}^2(M + m)/MR$

Solve for individual velocities

Since the centrifugal force equals the gravitational force for a circular orbit, you can solve for the velocity.

The object with mass m

In the case of the object with mass ***m****:*

$F_m = F$

Substitute equations:

$mv_{Tm}^2(M + m)/MR = GMm/R^2$

Multiply both sides by ***MR*** *and divide by* ***m****:*

$v_{Tm}^2(M + m) = GM^2/R$

Divide both sides by ***(M + m)****:*

$v_{Tm}^2 = GM^2/R(M + m)$

Take the square root:

$v_{Tm} = \pm\sqrt{[GM^2/R(M + m)]}$

This means the velocity can be in either direction for a circular orbit. Since direction is not relevant here:

$v_{Tm} = \sqrt{[GM^2/R(M + m)]}$ *km/s*

The object with mass M

Likewise, for the object of mass ***M****:*

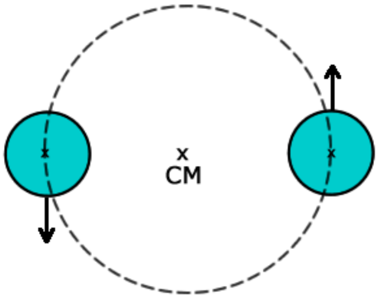
$v_{TM} = \sqrt{[Gm^2/R(M + m)]}$ *km/s*

Sizes of objects

The equations for the tangential orbital velocities can be simplified when both objects are the same size, as well as when one object has a much greater mass than the other does.

Objects have the same mass

There are situations in space where two stars have close to the same mass and orbit the CM between them. Astronomers call them double stars.



Double stars follow the same orbit around CM

If the objects are the same mass, then ***M = m*** *and the velocity equation for each becomes:*

$v_{TM} = \sqrt{[Gm^2/R(m + m)]}$ km/s

The equation reduces to,

$v_{TM} = \sqrt{[Gm/2R]}$ km/s

Since both objects or stars have the same orbital velocity and the same separation from the CM, they follow the same orbit around the CM.

One object much more massive than other

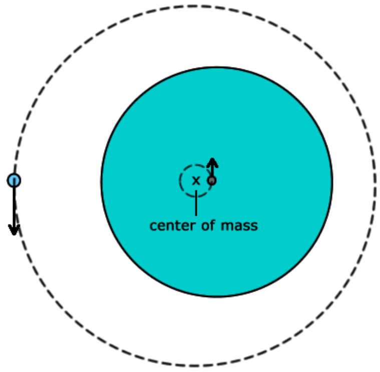
Another situation often seen in space is, when one object is much larger than the other is. In this case, the CM between them is almost at the more massive object's geometric center. This results in simplifying the equation for orbital velocity. The small object then seems to orbit the more massive object.

For example, the CM between a satellite orbiting the Earth is near the geometric center of the Earth. Likewise, the CM between the Earth and the Sun is near the center of the Sun.

Suppose ***M >> m*** *(**M** is much greater than **m**). Then:*

$M + m \approx M$

where ≈ means "approximately equal to".



Orbits, when one object is much larger than other, is

Substitute $\boldsymbol{M + m \approx M}$ *into the equation for the velocity of the smaller object:*

$$v_{Tm} = \sqrt{[GM^2/R(M + m)]}$$

$$v_{Tm} = \sqrt{(GM^2/RM)}$$

Reducing the equation results in:

$$v_{Tm} = \sqrt{(GM/R)} \text{ km/s}$$

This is the same as the standard equation for the orbital velocity of one object around another.

(See Orbital Motion Relative to Other Object for more information.)

Summary

When two objects are moving at the correct tangential velocities, they will go in circular orbits around their CM. The velocity equations are determined by setting the gravitational force equal to the outward centrifugal forces caused by their tangential velocities.

The velocity equations are:

$$v_{Tm} = \sqrt{[GM^2/R(M + m)]} \text{ km/s}$$

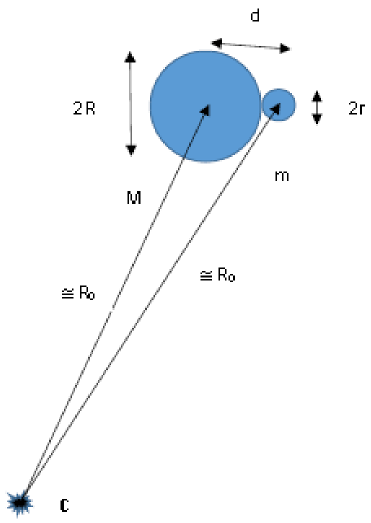
$$v_{TM} = \sqrt{[Gm^2/R(M + m)]} \text{ km/s}$$

When the mass of each object is the same, the velocity equation is simplified. The same is true when the mass of one object is much greater than that of the other.

To illustrate (or approximate on a highly speculative way) the meaning of global, universal, cosmic, or holistic rotation (which could be hidden or invisible for us, but we suppose that it effectively exist and produces gravitational force), let us imagine that small mass m is sitting on a big mass M, being attracted by mutual gravitational force. For instance, M could be certain planet and m will be a small spherical object, where the following approximations are applicable: $M >>> m$, $R >>> r$, $d <<< R_0$. *Here, d is the distance between centers of M and m, and* R_0 *is the distance from certain distant and common, dominant, or significant center C of*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

global or holistic (universal) rotation (see the picture below). We do not know where exactly the center of universal cosmic rotation is to place our reference system there, but mathematically we could operate with such imaginative (and still hypothetical) center of rotation.



Both masses m and M, are effectively rotating synchronously (or coincidently) around their global and universal cosmic center of rotation, here marked with C. Both masses also have the same angular or revolving frequency around center C. Such (on some way) hidden, and mathematically effective rotation could also be specific complicated angular motion locally presentable as rotation (since center C should not always be center of mass of the relevant local, planetary, or solar system). Between masses, M and m, we can safely say that it exists an attractive gravitational force, as follows,

$$\left|F_g\right| = G\frac{mM}{d^2} = \frac{mv_0^2}{R_0} = \frac{2E_{km}}{R_0} = \frac{J_m\omega_m^2}{R_0} = \frac{J_m\omega_0^2}{R_0} = m\,\omega_0^2\,R_0,\ \omega_M = \omega_m = \omega_0 = \frac{v_0}{R_0}.$$

If we now assume that gravitational force between m and M is the consequence of global, universal rotation (about center C), where relevant orbital moments are dominant factors (instead of involved masses), we will have,

$$G\frac{mM}{d^2} = G_1\frac{J_m\omega_m \cdot J_M\omega_M}{d^2} = G_1\frac{J_mJ_M\omega_0^2}{d^2} = G_1\frac{mR_0^2 \cdot MR_0^2}{d^2} = G_1\frac{mMR_0^4}{d^2}$$
$$= \frac{mv_0^2}{R_0} = \frac{J_m\omega_m^2}{R_0} = \frac{J_m\omega_0^2}{R_0} = m\,\omega_0^2\,R_0,\ \omega_M = \omega_m = \omega_0 = \frac{v_0}{R_0} \Rightarrow$$
$$\Rightarrow R_0 = \frac{GM}{d^2\omega_0^2}, G_1 = \frac{G}{R_0^4}...$$

We could also speculate that universal cosmic rotation (probably combined with linear, spinning and helical motion) is the primary cause of internal electrostatic and magnetic polarizations of involved masses, this way giving grounds to explain gravitational attraction as an electromagnetic dipoles' attraction. This example is just a brainstorming draft of a future, more elaborated modeling. ♣]

The magnitude of the angular momentum $\mathbb{L}$ from (2.11.13), of a periodically orbiting planet or satellite, its relevant, orbital mean-radius $\mathbf{R}$ (or semi-major axis), and associated characteristic speeds are quantized. Such orbital quantization is based on a planet-associated resonant and standing matter-waves (having a particular group and phase velocity, like in any periodical wave motion), respecting (in average) simple geometrical fittings (such as $2\pi R = n\lambda_o$), and can be summarized as results shown in (2.11.14), below.

Similar concepts and results are presented in [43], M. Pitkänen; [38], [39], F. Florentin Smarandache and Vic Christianto; [40], D. Da Rocha and Laurent Nottale; [64], Marçal de Oliveira Neto; and in **[125],** Markus J. Aschwanden; (see in (2.11.14)).

Here is a place to underline that nobody of mentioned authors is interpreting such results in a direct and robust relation to group and phase velocity of associated matter-waves groups, or wave-packets obtained by superposition of significant number of mutually agglomerated harmonic, elementary waves, like in the micro-world physics (what is an innovative contribution in this book).

$$\left[\mathbb{L} = pR = mvR = mR^2\omega_m = mR^2\frac{2\pi}{T} = 2\pi mR^2 f_m = m\sqrt{GMR} = n\frac{H}{2\pi} = n\hbar_{gr.} = const,\ m \cong \mu = \frac{mM}{m+M}\right] \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \left[\begin{array}{l} R = R_n = n^2\dfrac{H^2}{4\pi^2 GMm^2} = n^2\dfrac{GM}{v_0^2} = \dfrac{GM}{v_n^2} = n\dfrac{\lambda_o}{2\pi} = n\dfrac{\hbar_{gr.}}{mv},\ \lambda_o = \dfrac{H}{mv} = \dfrac{H}{p} = \dfrac{2\pi R}{n} = \lambda_{on}, \\ T = \dfrac{2\pi R}{v} = \dfrac{1}{f_m} = \dfrac{nT_0}{2} = \dfrac{n}{2f_o} = \dfrac{(nH)^3}{4\pi^2 G^2 M^2 m^3} = T_m,\ v = u - \lambda_o\dfrac{du}{d\lambda_o} = -\lambda_0^2\dfrac{df_0}{d\lambda_0} \cong 2u = 2u_n, \\ f_o = n\dfrac{f_m}{2} = n\dfrac{1}{2T} = n\dfrac{\sqrt{GM}}{4\pi R^{3/2}} = f_{on},\ T_o = \dfrac{1}{f_o} = \dfrac{2T}{n} = \dfrac{4\pi R}{nv},\ E_k = \tilde{E} = \tilde{E}_n = Hf_o = n\dfrac{Hf_m}{2}, \\ v = v_n = \dfrac{v_0}{n} = \dfrac{2\pi}{nH}GMm = 2u = \dfrac{v_e}{\sqrt{2}} = \sqrt{\dfrac{GM}{R_n}},\quad u = u_n = \lambda_o f_o = \dfrac{1}{2}\sqrt{\dfrac{GM}{R_n}} = \dfrac{v_0}{2n} \cong \dfrac{v}{2}, \\ v_0 = nv_n = \dfrac{2\pi}{H}GMm = \dfrac{GMm}{\hbar_{gr.}} = 2un = \dfrac{nv_e}{\sqrt{2}} = n\sqrt{\dfrac{GM}{R_n}},\ n, n_i = \text{Integers}],\ v << c. \end{array}\right] \Rightarrow \\ \Rightarrow \left[\begin{array}{l} \left\{\begin{array}{l} \text{For two planets on orbits 1 and 2: } H = const. \Rightarrow \\ \dfrac{R_1}{R_2} = \dfrac{n_1^2}{n_2^2}\dfrac{m_2^2}{m_1^2} = \left(\dfrac{n_1 m_2}{n_2 m_1}\right)^2, \dfrac{T_1}{T_2} = \left(\dfrac{n_1 m_2}{n_2 m_1}\right)^3 = \left(\dfrac{R_1}{R_2}\right)^{3/2} = \dfrac{v_2}{v_1}\cdot\dfrac{R_1}{R_2} = \dfrac{T_{1m}}{T_{2m}} = \dfrac{n_1}{n_2}\cdot\dfrac{f_{o2}}{f_{o1}} = \dfrac{n_1}{n_2}\cdot\dfrac{T_{o1}}{T_{o2}} \\ \dfrac{v_1}{v_2} = \dfrac{u_1}{u_2} = \dfrac{\lambda_{o1}f_{o1}}{\lambda_{o2}f_{o2}} = \dfrac{n_2}{n_1}\cdot\dfrac{m_1}{m_2} = \sqrt{\dfrac{R_2}{R_1}} \Leftrightarrow \underline{R_i v_i^2 = R_j v_j^2},\ \dfrac{\lambda_{o1}}{\lambda_{o2}} = \dfrac{n_2}{n_1}\cdot\dfrac{R_1}{R_2} \end{array}\right\} \\ ------------------- \\ \left\{\begin{array}{l} \text{For the same planet passing between two orbits: } H = const. \Rightarrow \\ (m_1 \cong m_2), \dfrac{R_1}{R_2} = \left(\dfrac{n_1}{n_2}\right)^2, \dfrac{T_1}{T_2} = \left(\dfrac{n_1}{n_2}\right)^3 \end{array}\right\} \end{array}\right] \end{array}\right\} \quad (2.11.14)$$

Effectively, sitting on results and assumptions from (2.11.14), Titus – Bode's law (related to quantization of planetary orbits) is significantly rectified and optimized by Markus J. Aschwanden; **[125]** -"Self-organizing systems in planetary physics: Harmonic resonances of planet and moon orbits". Mentioned reference, **[125]**, is strongly supporting and reinforcing here elaborated model of planetary, standing matter-waves, and additionally giving more of legitimacy to the corrected and upgraded Titus-Bode law.

[♣ COMMENTS & FREE-THINKING CORNER:

The same or equivalent, quantizing-like results, and conclusions (as in (2.11.14)) can be formulated almost directly, analogically and much faster if we consider that in certain solar system, the Sun analogically presents a proton, and planets are like electrons orbiting around. Quantizing is applicable if the system can be approximately treated as a *2-body problem*. If we exploit the mathematical identity between the electrostatic Coulomb force in the hydrogen atom, and Newton's "static" gravitational force, and systematically substitute $\frac{Ze^2}{4\pi\varepsilon_0} \to GmM$ (or $Ze^2 \leftrightarrow mM$, $\frac{1}{4\pi\varepsilon_0} \leftrightarrow G$) in all relevant results known from hydrogen atom analyzes, where M is mass of the sun, m is mass of certain planet, H is macrocosmic planetary constant analog to Planck constant h, ($H >>> h$) and G is gravitational constant. See much more of such background in [63] Arbab I. Arbab, and in [67], including other familiar publications from Johan Hansson [77], Newtonian Quantum Gravity. "*Gravito-static versus electrostatic analogy*" should not be only a mathematical curiosity, coincidence and academic discussion option, after we consider as realistic the possibility that solar system elements are mutually electrically (and magnetically) polarized like mutually-attracting electric (and magnetic) dipoles and multi-poles (since there are electromagnetic fields and forces around them). Such electromagnetic polarization option is already presented in this chapter (see ***2.2. Generalized Coulomb-Newton Force Laws***; -equations from (2.3) until (2.4-10)). In addition, the Chapter 8. of this book (Bohr Model) develops and presents most of analogical, quantized results (see results from (8.23) to (8.33)), as found in (2.11.14), where mutual correspondence and full analogy of such results can be established by applying "*Gravito-static versus electrostatic analogy*". Of course, here analogy means more than mathematical similarity or identity, since in the case of gravitation within planetary systems relevant results are also correct, verified by astronomic measurements, and other theoretical and experimental observations. Consequently, here we deal with accurate empirical, natural and scientific facts and the main consequence should be that gravitation and associated electromagnetic complexity are coincidently present and mutually coupled, at least in cases of solar or planetary systems (see also elaborations around equations (2.11.20) - (2.11.22) and Fig.2.6.).

Let us directly apply analogical substitutions of quantized expressions relevant for orbiting electrons, and their associated standing matter waves (as shown in [77], Johan Hansson, Newtonian Quantum Gravity, in results from the Chapter 8., Bohr model…, and in (2.4-8)), to results (8.4), and (8.26) - (8.30). This way, we will create analogical and quantized, standing-waves expressions as summarized in T.2.8., comparable to results from (2.11.14), valid and correct for orbiting planets (including planetary macro matter waves),

T.2.8. N. Bohr hydrogen atom and planetary system analogies

We will apply the following analogies and formal replacements:

$$\left\{\frac{1}{4\pi\varepsilon_0}\frac{q\cdot Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(ze)\cdot(Ze)}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{zZ\cdot e^2}{r^2} \Leftrightarrow G\frac{m\cdot M}{r^2}\right\} \Rightarrow$$

$$\left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}, \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2}\right] \Leftrightarrow \left[G\frac{m^2}{r^2}, G\frac{M^2}{r^2}\right], q = ze \Leftrightarrow m\sqrt{4\pi\varepsilon_0 G},\ Q \Leftrightarrow Ze = M\sqrt{4\pi\varepsilon_0 G},$$

$$zZe^2 \Leftrightarrow mM4\pi\varepsilon_0 G,\ e^2 \Leftrightarrow \frac{mM}{zZ}4\pi\varepsilon_0 G,\ e\sqrt{zZ} \Leftrightarrow \sqrt{mM}\sqrt{4\pi\varepsilon_0 G}, \frac{1}{4\pi\varepsilon_0} \leftrightarrow G,\ h \leftrightarrow H, \quad \Rightarrow$$

$$\left[\frac{e}{\sqrt{4\pi\varepsilon_0 G}}\sqrt{\frac{zZ}{m+M}} \Leftrightarrow \sqrt{\frac{mM}{m+M}} = \sqrt{\mu}\right] \Rightarrow \frac{e}{\sqrt{4\pi\varepsilon_0}}\sqrt{\frac{zZ}{m+M}} \Leftrightarrow \sqrt{\frac{mM}{m+M}}\cdot\sqrt{G} = \sqrt{\mu}\cdot\sqrt{G}$$

$$\frac{ze}{\sqrt{4\pi\varepsilon_0}} \Leftrightarrow m\cdot\sqrt{G},\ \frac{Ze}{\sqrt{4\pi\varepsilon_0}} \Leftrightarrow M\cdot\sqrt{G},\ \mu = \frac{mM}{m+M}$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\Rightarrow \left[\begin{array}{l} \left[\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} \Leftrightarrow G\frac{m\cdot M}{r^2}\right] \Rightarrow \left[\frac{Ze^2}{4\pi\varepsilon_0} = \frac{e}{\sqrt{4\pi\varepsilon_0}}\cdot\frac{Ze}{\sqrt{4\pi\varepsilon_0}} \Leftrightarrow m\sqrt{G}\cdot M\sqrt{G}\right] \Rightarrow \\ \left[\frac{e}{\sqrt{4\pi\varepsilon_0}} \Leftrightarrow m\sqrt{G}, \frac{Ze}{\sqrt{4\pi\varepsilon_0}} \Leftrightarrow M\sqrt{G}\right] \Rightarrow Z \Leftrightarrow \frac{M}{m}; \frac{1}{4\pi\varepsilon_0} \leftrightarrow G, h \leftrightarrow H, \frac{Ze^2}{4\pi\varepsilon_0} \Leftrightarrow Gm\cdot M \end{array} \right]$$

Effectively, here we assume that every mass or agglomerated atoms have the corresponding amount of polarizable electric dipoles or electric charges in a way that Coulomb and Newton's laws are mutually equivalent or replaceable (see (2.4-4.1) - (2.4-4.3) developed earlier in this second Chapter). This way, masses attraction can be treated as adequately oriented electric dipoles attraction (or by analogically extending the same conceptualization to associated magnetic dipoles attraction, since in such dynamically stable and self-closed systems electric and magnetic performances are mutually balanced like in capacitance-inductance or mass-spring resonant circuits). Here we also assume the existence of some omnipresent, holistic angular cosmic motion (including oscillatory and resonant states), being the ultimate cause or source of mentioned intrinsic and volumetric masses, electromagnetic polarization. In case if our Universe has more dimensions than 4 [more than (x, y, z, t)], we could imagine that mentioned holistic motion would be recognizable from, for us still not detectible, higher dimensional spaces. In some distant parts of our universe (for instance concerning spiral galactic formations...), such universal angular motion, and associated electromagnetic polarization could produce stronger Coulomb or Newtonian attractions (than in our part of Cosmos), and we maybe wrongly associate improvable "dark mass and dark energy" mystifications to such very much clear, natural and explicable phenomenology (see [121], Raymond HV Gallucci).

Analogically crated relations between N. Bohr atom model and planetary systems are, as follows:

the phase velocity of an electron wave	the phase velocity of a planetary wave
$u_s \approx \frac{1}{2}v = \frac{Ze^2}{4nh\varepsilon_0} = \lambda_s f_s$, $n = 1, 2, 3...$ (8.26)	$u_n \approx \frac{1}{2}v = \frac{\pi GmM}{nH} = \lambda_n f_n, n = 1, 2, 3, ...$ $(u_n = \lambda_{on} f_{on} = \frac{1}{2}\sqrt{\frac{GM}{R_n}} = \frac{v_0}{2n} \cong \frac{v}{2}$, (2.11.14))

the group velocity of an electron wave	the group velocity of a planetary wave
$v_s \approx 2u_s = \frac{Ze^2}{2nh\varepsilon_0}$, (8.27)	$v_n \approx 2u_n = \frac{2\pi GmM}{nH}$, $(v_n = \frac{v_0}{n} = 2u = \frac{v_e}{\sqrt{2}} = \sqrt{\frac{GM}{R_n}} = \frac{2\pi}{nH}GmM$, (2.11.14))

a frequency of an orbital electron wave	a frequency of an orbital planetary wave
$f_s \approx \frac{mZ^2e^4}{8n^2h^3\varepsilon_0{}^2}$ (8.28)	$f_n \approx \frac{2\pi^2G^2m^3M^2}{n^2H^3}$, $(f_n = f_{on} = n\frac{f_m}{2} = n\frac{1}{2T_n} = n\frac{\sqrt{GM}}{4\pi R^{3/2}} = \frac{\tilde{E}_n}{H}$, (2.11.14))

a wavelength of an orbital electron wave	a wavelength of an orbital planetary wave
$\lambda_s = \frac{h}{p} \approx \frac{2nh^2\varepsilon_0}{mZe^2}$ (8.29)	$\lambda_n = \frac{H}{p} \approx \frac{nH^2}{Gm^2M}$,

	$(\lambda_n = \lambda_{on} = \frac{H}{mv} = \frac{H}{p} = \frac{2\pi R}{n}$, (2.11.14))

the energy of a stationary electron wave	the energy of a stationary planetary wave
From N. Bohr atom model: $\varepsilon_s = \varepsilon_n = hf_s = \frac{1}{2}\mu v^2 = \frac{\mu Z^2 e^4}{8n^2h^2\varepsilon_0^2}$ (8.30) --------------------------------------- From the solutions of the Schrödinger equation in spherical coordinates: $\varepsilon_s = \varepsilon_n = \frac{(\frac{h}{2\pi})^2}{2\mu a_0^2 n^2}$, $\left\{ a_0 = \frac{(\frac{h}{2\pi})^2}{\mu e^2} = \text{Bohr radius}, \quad \frac{2\mu e^2}{(\frac{h}{2\pi})^2\delta} = \frac{2j+k+1}{2} = j+\ell+1 = n, \quad \ell(\ell+1) = \frac{k^2-1}{4}, \left(\frac{\delta}{2}\right)^2 = -\frac{2\mu\varepsilon_s}{(\frac{h}{2\pi})^2}, k = 2\ell+1, \quad \delta = \frac{2\mu e^2}{(\frac{h}{2\pi})^2 n} = \frac{2}{a_0 n}, \quad (k, j, \ell, n) = \text{integers}. \right\}$	$\varepsilon_n = \tilde{E}_n = Hf_n \approx \frac{1}{2}\mu v^2 = \frac{2\pi^2G^2m^3M^2}{n^2H^2}$, $(E_k = \tilde{E}_n = Hf_{on} = n\frac{Hf_m}{2}$, (2.11.14)) --- Analogically formulated, hypothetical: $\varepsilon_n = \frac{(\frac{H}{2\pi})^2}{2\mu a_{0g}^2 n^2}$, (hypothetical; -analogically created) $\left\{ a_{0g} = \frac{(\frac{H}{2\pi})^2}{G\mu^2 M} = \text{Gravitational, Bohr radius}, \quad \frac{2\mu^2 M}{(\frac{H}{2\pi})^2\delta_g} = \frac{2j+k+1}{2} = j+\ell+1 = n, \quad \ell(\ell+1) = \frac{k^2-1}{4}, \left(\frac{\delta_g}{2}\right)^2 = -\frac{2\mu\varepsilon_s}{(\frac{H}{2\pi})^2}, k = 2\ell+1, \quad \delta_g = \frac{2\mu^2 M}{(\frac{H}{2\pi})^2 n} = \frac{2}{a_{0g} n}, (k, j, \ell, n) = \text{integers}. \right\}$

a radius of an electron orbit	a radius of a planetary orbit
$r_n = \frac{n^2h^2\varepsilon_0}{\pi\mu e^2 Z}$ (8.4) -------------------------------- $\langle r_{n,\ell} \rangle = \frac{a_0}{2}\left[3n^2 - \ell(\ell+1)\right]$, $\langle r^2_{n,\ell} \rangle = \frac{a_0^2 n^2}{2}\left[5n^2 + 1 - 3\ell(\ell+1)\right]$, $\left\langle \frac{1}{r_{n,\ell}} \right\rangle = \frac{1}{a_0 n^2}, \left\langle \frac{1}{r^2_{n,\ell}} \right\rangle = \frac{1}{a_0^2 n^3(\ell+1/2)}$, $n = k+\ell = 1,2,3...$ $\ell = 0,1,2,...,n-1$ $m = -\ell, -\ell+1, ..., \ell-1, \ell$	$R_n = \frac{n^2H^2}{4\pi^2Gm^2M}$, $(R_n = n^2\frac{GM}{v_0^2} = \frac{GM}{v_n^2} = n\frac{\lambda_{on}}{2\pi} = \frac{n^2H^2}{4\pi^2Gm^2M}$, (2.11.14)) ---------------------------- (hypothetical; -analogically created) $\langle R_{n,\ell} \rangle = \frac{a_{0g}}{2}\left[3n^2 - \ell(\ell+1)\right]$, $\langle R^2_{n,\ell} \rangle = \frac{a_{0g}^2 n^2}{2}\left[5n^2 + 1 - 3\ell(\ell+1)\right]$, $\left\langle \frac{1}{R_{n,\ell}} \right\rangle = \frac{1}{a_{0g} n^2}, \left\langle \frac{1}{R^2_{n,\ell}} \right\rangle = \frac{1}{a_{0g}^2 n^3(\ell+1/2)}$, $n = k+\ell = 1,2,3...$ $\ell = 0,1,2,...,n-1$ $m = -\ell, -\ell+1, ..., \ell-1, \ell$

Also, for the specific planet, we can analogically determine gravitational fine-structure constant,

Fine-structure constant	Gravitational fine-structure constant
$\alpha = \frac{e^2}{4\pi\varepsilon_0 (\frac{h}{2\pi})c} \cong \frac{1}{137}$	$\alpha_g = \frac{Gm^{*2}}{(\frac{H}{2\pi})c}$

Of course, in T.2.8., reduced masses $\mu = \frac{m_1 m_2}{m_1 + m_2}$ are different in case of Bohr atom model and for planets of a specific solar system (because different masses m_1 and m_2 are involved).

We can see that almost all (easily verifiable) analogically formulated results in T.2.8. are correct, since we already have such results from different astronomic observations, publications, and analyses. Here, we have striking analogical and quantitative, direct proportionality between involved masses and (analogically) involved electric charges. How could this be possible, when we know that gravitational attraction also exists between electromagnetically neutral or internally, spatially, and statistically, electromagnetically compensated masses (when macroscopically measurable magnetic and electric fields do not exist)? One of the logical and straightforward explanations is that involved masses, meaning involved agglomerations of atoms are (for some reason) only slightly, electrically and magnetically, internally polarized (almost beyond our experimental recognition), and creating (in average) uniformly organized very weak electric and magnetic dipoles. Such dipoles, when spatially correctly organized, are exercising Coulomb types of attractions (since the same form of Coulomb law is equally applicable to electric charges and magnets, and in the same time this is analogically equivalent to Newton gravity force law, where only involved masses are taken into account). We assume here that quantitatively certain mass is directly proportional to the number of its internal constituents, atoms, or the number of involved electrostatic dipoles and/or elementary magnets belonging to atom constituents. Since we also know (from the first chapter of this book) that direct analogies between electric and magnetic charges (or fluxes) are not masses, but linear and spinning or orbital moments, to complete the analogical picture and understanding (concerning Coulomb and Newton laws), here we only miss certain (relatively constant, intrinsic or background) linear and/or angular speed of our Universe. This, about global background velocity, could be specific global, omnipresent, holistic, cosmic motion (where linear and angular movements, rotation and spinning are combined). Mentioned rotation, and associated centrifugal force are influencing weak (spatially properly polarized, organized or aligned) electromagnetic dipoles (thanks to an enormous mass difference between electrons and protons), this way producing Coulomb type of electromagnetic forces, and we analogically (but essentially mistakenly) describe such effects as Newtonian interaction between involved masses. As we know, electrons and protons, and almost all subatomic entities have spinning moments and magnetic properties. In cases of totally macro-neutral or macro-compensated masses, involved internal electromagnetic states will cancel mutually and spatially. However, if specific global, background rotation (of our Universe) anyway exist, small electrostatic dipoles polarization will appear on surfaces of involved masses and involved internal masses domains will (in average) experience some spinning or elementary-magnets alignments. Both, electric dipoles, and internal magnetic moments (or magnetic dipoles) are respecting Coulomb law forces, and in some cases, we mistakenly consider such effects as a manifestation of gravitation. In examples of significant rotating galactic masses, mentioned electromagnetic dipoles are much stronger, favorably and dominantly polarized, producing stronger Coulomb or (looks-like) gravitational attraction, and we wrongly consider this as an argument that some associated, additional dark, invisible mass (matter or energy) should exist in order to defend observed and inexplicable masses attraction, and to prolong the theoretical and conceptual validity of Newton and A. Einstein gravity-related theories. In other words, all of that serves to stop or prevent searching for new theory, which is better explaining what the Gravitation really is. Effectively, only oscillating masses, atoms and externally extended atomic force fields are real sources of Gravitation (instead of static masses), and all of such structures are mutually coupled and synchronized, since in cosmic systems we find applicability of such atoms modeling (see much more in Chapter 8.).

Planetary systems, as mechanical and periodical-motions systems are on some way creating or respecting stationary and standing waves structure, like mechanical resonators (as already demonstrated in this chapter with equations from (2.11.10) until (2.11.13). Also, planetary systems behave as being very much analogically predictable (and presentable) with N. Bohr's atom model, or *like electromagnetic resonators, where similar stationary waves also exist, and where electrostatic forces are dominant. Consequently, we should expect the existence of substantial and direct electromechanical couplings (between associated electric and magnetic dipoles, or multipoles) within such dualistic resonant systems (on some way like the piezoelectric effect). Such mechanical and electromagnetic resonant planetary systems are anyway united. Every stable planetary system should behave as a single, internally and externally, synchronized resonant system, having specific and mutually harmonized, structural, radial and orbital (standing waves) resonant frequencies, when observed only as a mechanical, or just as an electromagnetic resonator (also being analogical to manifestations of acoustic levitation). Every mechanical or electromagnetic perturbation of such coupled resonators will synchronously produce similar electromagnetic or mechanical disturbance (like in piezoelectric devices).*

This electromechanical coupling should explain the essential (or ontological) electromagnetic nature of Gravitation since within stationary or standing matter waves we will have nodal zones with effects of attractive forces, which are creating masses-agglomerations (like in Newtonian attractions). To satisfy unity of radial, axial, orbital and transversal, resonant behaviors of structurally resonant planetary systems, electromagnetic waves should have both transversal and longitudinal components, meaning that Maxwell equations should be conveniently upgraded to support longitudinal waves. One attempt or proposal for such upgrading is initiated in the third chapter of this book. Nikola Tesla measured mentioned structural and stationary planetary, resonant waves, both as mechanical and electromagnetic waves, and this way, most probably, formulated his ideas about new Dynamic Theory of Gravitation (but unfortunately never published or finalized it). See literature references under [97], [98], [99] and [117].

Ling Jun Wang; -Citation (see [122]), … "presented a theory of unification of gravitational force and the electromagnetic force based on the generalization of Newton's law of gravitation to include a dynamic term inferred from the Lorentz force of electromagnetic interaction. The inclusion of this dynamic term alone in the gravitational force is enough to develop the entire dynamic theory of gravitation parallel to that of electrodynamics".

Familiar ideas about the extension of the Lorentz force are elaborated in the third chapter of this book. If we connect electric and magnetic (dipoles and multipoles) polarizations, and global (holistic) motions and rotations within our Universe, with Lorentz force effects, the picture of unity between gravitation and electromagnetism will be much clearer and more indicative.

The bottom-line simplified explanation about masses coupling is related to the fact that masses are composed of atoms. Atoms internally have number of discrete, stationary, and standing waves energy states, meaning resonant states, or we could also say physical resonators. Resonators with mutually overlapping spectral characteristics are being naturally synchronized (in zones where they have the same resonant frequencies). This way, compact and united macro mass starts behaving like a big, united atom with number of internal, discrete (atomic and molecular) energy states. Since all macro masses are such kind of complex resonant states, it is natural to expect that mentioned (mutually overlapping) resonant states from any of two separate masses will again mutually synchronize and on some way energetically communicate (by creating standing electromagnetic waves between them), producing the effects of Gravitation (see more in Chapter 8.). Familiar innovative concept about Gravitation, where mases of planets are in states of permanent mutual electromagnetic energy exchanges and coupling, being in the same time transmitters and receivers of electromagnetic energy, and where relevant solar system structure is creating standing waves fields between the sun and planets, including many of additional imaginative and challenging excursions towards other domains of modern Physics, can be found in [144], Poole, G. (2018) Cosmic Wireless Power Transfer System and the Equation for Everything.

Citation from [144] "**Abstract**: By representing the Earth as a rotating spherical antenna several historic and scientific breakthroughs are achieved. Visualizing the Sun as a transmitter and the planets as receivers the solar system can be represented as a long wave radio system operating at Tremendously Low Frequency (TLF). Results again confirm that the "near-field" is Tesla's "dynamic gravity", better known to engineers as dynamic braking or to physicists as centripetal acceleration, or simply (g). …
A new law of cosmic efficiency is also proposed that equates vibratory force and pressure with volume acceleration of the solar system. Lorentz force is broken down into centripetal and gravitational waves. …

Spherical antenna patterns for planets are presented and flux transfer frequency is calculated using distance to planets as wavelengths. The galactic grid operates at a Schumann Resonance of 7.83 Hz, …
The Sun and the planets are tuned to transmit and receive electrical power like resonating Tesla coils”.

As we know, original N. Bohr’s Planetary Atom Model is upgraded, and successfully exploited, by applying Schrödinger’s equation, and ideas of particle-wave duality. Consequently, we should be able, because of the validity of mentioned "Gravito-static versus electrostatic analogies” (based on $\frac{Ze^2}{4\pi\varepsilon_0} \to GmM$), to analogically apply relevant wave functions and Schrödinger equation to familiar planetary, and other astronomic situations (with periodic and circular, orbital and inertial motions). Elaborations and analyzes of planetary systems, starting from (2.11.12) until (2.11.14), are anyway clearly indicating that planetary systems, presented as macro-cosmological matter-waves, behave very much analogically as known in microphysics. Consequently, here we have enough grounds to apply Schrödinger equation (see such attempts, later, around equations (2.11.20) - (2.11.22), and Fig.2.6.). The Analogy in question is not at all establishing different, non-doubtful, definitive grounds that *probabilistic methodology* of quantum theory is relevant here (opposite to what certain authors are implicitly forging as the fact). Schrödinger equation is applicable here mostly because stable, self-closed orbits, hosting periodical and standing waves, create necessary conditions to formulate and apply such equation (and there is nothing to connect it with probabilities and statistics). Much more striking, challenging and significant here is the fact that gravitation and electromagnetic field are on some way (more than only analogically) connected, and that sources of gravity are most probably of electromagnetic nature (see much more of similar ideas in [72], Dr. László Körtvélyessy. The Electric Universe).

Citation taken from **[63]**, under **24)**: Arbab Ibrahim Arbab. The Generalized Newton’s Law of Gravitation versus the General Theory of Relativity. **Journal of Modern Physics.**

“We have shown that the gravitomagnetism and the general theory of relativity are two theories of the same phenomenon. This entitles us to accept the analogy existing between electromagnetism and gravity fully. Hence, electromagnetism and gravity are unified phenomena. The precession of the perihelion of planets and binary pulsars may be interpreted as a spin-orbit interaction of gravitating objects. The spin of a planet is directly proportional to its orbital angular momentum and mass, weighted by the Sun's mass. Alternatively, the spin is directly proportional to the square of the orbiting planet's mass and inversely proportional to its velocity".

Since in T.2.8., we have the analogical expression for gravitational, fine structure constant $\alpha_g = \frac{Gm^{*2}}{(\frac{H}{2\pi})c}$, and since ordinary (atomic and electromagnetic) fine structure constant is known as extraordinarily stable, α = 7.2973525698(24)x10^{-3}, α^{-1}=137.035999074(44), we could search for (analogic) conditions when gravitational fine structure constant will be equal to atomic, or electromagnetic fine structure constant, meaning,

$$\alpha_g = \frac{Gm^{*2}}{(\frac{H}{2\pi})c} \cong \alpha = 7.2973525698(24)\cdot 10^{-3} \cdot \qquad (2.11.14\text{-}1)$$

From (2.11.14-1) we can determine the hypothetical (analogically founded) value for macro-cosmological or gravitational Planck constant H, as,

$$H = \frac{2\pi G}{\alpha c} m^{*2} \cong 137.035999074 \frac{2\pi G}{c} m^{*2} . \qquad (2.11.14\text{-}2)$$

On a similar way, as we are converging gravitational fine-structure constant to the atomic or electromagnetic fine structure constant $\alpha_g \to \alpha$, $m^* \to m^*_{minimal}$, it is clear that gravitational or macro cosmological Planck constant H, from (2.11.14-2), would be in the same time (analogically) converted into microworld Planck constant h, meaning $H \to h$. The minimal value of the mass $m^* = m^*_{minimal}$, which has specific gravitational mass, when is still meaningful or possible to detect and measure effects of gravitation in the Newtonian framework, could be one among masses of the proton, neutron, or electron, but here we will find that this is not the case.

$$\left(H \rightarrow h = \frac{2\pi G}{\alpha c} (m^{*}_{minimal})^2 \cong 137.035999074 \frac{2\pi G}{c} (m^{*}_{minimal})^2 \right) \Rightarrow$$
$$m^{*}_{minimal} \cong \sqrt{\frac{hc}{137.035999074 \cdot 2\pi G}} = 1.44308 \cdot 10^{-19} \ [kg] \qquad (2.11.14\text{-}3)$$

We can calculate from (2.11.14-3) that the minimal mass, which has specific gravitational meaning (under here introduced analogical framework) $m^* \cong 1.44308 \cdot 10^{-19}$ [kg], is $8.62766 \cdot 10^{7}$ times bigger than the mass of the proton, or $8.61592 \cdot 10^{7}$ times more significant than the mass of the neutron, and $1.58417 \cdot 10^{11}$ times bigger than the mass of an electron.

Anyway, it is experimentally known that the validity of Newton gravitational force law (between two masses) is testable and provable until the lower distance limits of 300 micrometers (approximately). One of the conclusions here could be that gravitation has a meaning only for a relatively large group of electromagnetically polarizable atoms (for instance, for a minimum of $8.62766 \cdot 10^{7}$ hydrogen atoms), above certain threshold mass amount ($m^* \cong 1.44308 \cdot 10^{-19}$ [kg]), and for distances between two masses higher than 300 micrometers. All of that is indicating that gravitation could be a manifestation of electromagnetic forces between masses with specific electric dipoles polarization (as speculated at the beginning of this chapter, around equations from (2.4-7) to (2.4-10)). If electromagnetic forces and charges are essential sources of gravitation, consequently, what we expect to detect as gravitational waves should be some very low-frequency electromagnetic waves. In cases of stable solar or planetary systems, we should be able to find such standing and stationary, macro electromagnetic field structures between planets and a local sun. It is still too early to draw definite conclusions, but at least, here we got specific indicative numbers (regarding validity of gravitation), under certain sufficiently well-defined and challenging conditions.

In T.2.8. we explored formal analogies based on a comparison between the Bohr planetary atom model and a real planetary system such as,

$$\left\{ \frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow \frac{GmM}{r^2}, \ Ze \Leftrightarrow M, e \Leftrightarrow m \cong \frac{mM}{m+M} = \mu, Z \Leftrightarrow \frac{M}{m}, \frac{1}{4\pi\varepsilon_0} \Leftrightarrow G, h \Leftrightarrow H \right\} \Rightarrow \left\{ \frac{Ze^2}{4\pi\varepsilon_0} \rightarrow GmM, Ze^2 \rightarrow mM \cong \mu M \right\}. \qquad (2.11.14\text{-}4)$$

From the first chapter of this book (see T.1.2 until T.1.8) we know that when respecting Mobility system of electromechanical analogies, electric charges are analog to linear and orbital moments, meaning that in reality, Newton law of Gravitation should present specific force-field between linear and orbital moments, like already exercised and summarized in T.2.2, T.2.2-2 and (2.4-5.1). If we consider that in the Newton law of gravitational force, instead of mutually attracting masses, we should have attraction of corresponding linear and orbital moments (which are on some way implicitly present, but still hidden, and somewhat hypothetical), we can reformulate mentioned initial analogies (2.11.14-4) on the following way,

$$(G = gv_c^2 = Const) \Rightarrow \qquad (2.11.14\text{-}5)$$
$$\left\{ \frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow g\frac{(mv_c)(Mv_c)}{r^2} = g\frac{p_c \cdot P_c}{r^2}, Ze \Leftrightarrow Mv_c = P_c, e \Leftrightarrow mv_c = p_c \cong \frac{mM}{m+M} v_c = \mu v_c, \ \frac{1}{4\pi\varepsilon_0} \Leftrightarrow g, h \Leftrightarrow H \right\} \Rightarrow$$
$$\Rightarrow \left\{ \frac{Ze^2}{4\pi\varepsilon_0} \rightarrow g \cdot (p_c \cdot P_c) \Rightarrow Ze^2 \Leftrightarrow p_c \cdot P_c = mMv_c^2 \cong \mu M v_c^2 \right\}$$

Practically, in (2.11.14-5) we see a way to extract (or expose) missing or hidden velocities of interacting masses from Newton gravitational constant, $G = gv_c^2 = Const.$, where speeds of both masses **m** and **M** are the same and equal to $v_c \cong const.$. This way, instead of static masses product, mM we created

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

the outcome of associated linear moments $p_c \cdot P_c = (mv_c)(Mv_c)$, satisfying analogy that electric charge corresponds to linear momentum (like in T.1.8), and consequently transforming Newton law of gravitation to be the force between involved mechanical (linear and/or orbital) moments. In other words, both masses, **m** and **M,** are globally moving concerning specific reference system with a certain velocity, and making certain, linear and/or oscillatory motion (like oscillating dipoles). It is also evident that such a speculative and intuitive situation (regarding hidden, or background velocity parameters) should be better elaborated and explained. ***Of course, later we also need to find a way to involve angular and spin moments of interacting masses in a Newton law, but what is important here is to show that Newton force of gravitation could evolve towards richer conceptualization***. In chapter 10 of this book, we can find the complete explanation of the same situation regarding unknown or background velocity parameters and Newtonian attraction between important linear and angular moments (see (10.1.4) - (10.1.7)).

Analogies between the Bohr atom model and planetary systems are very much striking and indicative (see T.2.8. and Chapter 8.). Since, an extended atom modeling is also related to magnetic, orbital and spin moments, we can (analogically and hypothetically) expect that planetary systems should operate within a similar environment of involved orbital moments, spins and surrounding electric and magnetic fields, and standing matter waves (as a consequence of stable periodical motions). Similar explorations and modeling is exercised all over this book, and in number of publications from [36], Anthony D. Osborne, & N. Vivian Pope, [63], Arbab I. Arbab, and in [71], from Jovan Djuric; "Magnetism as Manifestation of Gravitation". If we have necessary technical and observational means, we should be able to visualize relevant electromagnetic matter-waves, and other masses-distribution-related standing waves structure of planetary and galactic systems. Another option that is hypothetically radiating from T.2.8. is that this is not only a system of analogies between electromechanical, gravitational, and electromagnetic entities, but much more something like describing the same (anyway united) phenomenology using combined mechanical and electromagnetic concepts. In other words, here we may have mutually analog and equivalent, mechanically, electromagnetically and electromechanically coupled entities of the same force that is in physics (by chance and in different historical periods) independently conceptualized either as Newtonian mechanics and gravitation, and as electromagnetism related Coulomb force. Here, we are on the way to propose possible unification of electromechanical phenomenology, as already exercised around equations and expressions (2.4-7) - (2.4-10), earlier in this chapter. Different motions of masses are on some way creating internal, spatially (or volumetrically) distributed electromagnetic entities, dipoles, moments and charges, and this way we get a chance to describe planetary or mechanical motions on different ways; either dominantly mechanically or using electromagnetic conceptualization. ♣]

It can also be roughly (numerically) verified that quantum number n, which appears in $2\pi R = n\lambda_o$, (2.11.12)-(2.11.14), is taking the same order of magnitude as number of days in a year of relevant planet, indicating where we should search for the meaning of planetary, standing matter waves quantizing (see (2.11.14)-g, h).

Analog to vortex shedding phenomenology, known in fluid motions as "Karman Vortex Street", we could say that certain kind of "Planetary Karman Street" is on some way following planets and astronomic size objects, like a helix oscillatory tail. As shown in the chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,i such vortex shedding is in agreement with matter waves quantization $\lambda = H/p, \tilde{E} = Hf$, or on a similar way equivalent to (2.11.12), (2.11.13) and (2.11.14). ***For instance, the frequency of vortex shedding is directly proportional to relevant fluid or particle velocity, meaning that in (2.11.14) we may have*** $f_o = f_{on} = const \cdot v = C(n) \cdot v$***, what is explaining that*** v_0 ***for the specific planetary system could be constant, as follows,***

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$f_o = const \cdot v = C(n) \cdot v = C(n) \cdot v_n \Rightarrow$$

$$\left\{\begin{array}{l} R_n = n^2 \frac{GM}{v_0^2} = \frac{GM}{v_n^2} = n\frac{\lambda_{on}}{2\pi} = \frac{n^2H^2}{4\pi^2 Gm^2 M},\ v_n = \frac{v_0}{n} = 2u = \frac{v_e}{\sqrt{2}} = \sqrt{\frac{GM}{R_n}} = \frac{2\pi}{nH}GmM \\ R_i v_i^2 = R_j v_j^2 \Rightarrow R_n v_n^2 = R_n \frac{v_0^2}{n^2} = GM \Leftrightarrow R_n v_n^2 = Rv^2 = constant, \end{array}\right\} \Rightarrow \quad (2.11.14)\text{-a}$$

$$v_0 = \frac{nv_e}{\sqrt{2}} = nv_n = \frac{2\pi}{H}GMm = \frac{GMm}{\hbar_{gr.}} = 2un = n\sqrt{\frac{GM}{R_n}},\ n = 1, 2, 3...$$

$$f_o = n\frac{f_m}{2} = n\frac{1}{2T} = n\frac{\sqrt{GM}}{4\pi R^{3/2}} = \frac{n}{4\pi R_n}\sqrt{\frac{GM}{R_n}} = \left(\frac{n}{4\pi R_n}\right)v_n = \left(\frac{v_0^2}{4\pi nGM}\right)v_n = \left(\frac{1}{4\pi R_n}\right)v_0 = C(n) \cdot v_n.$$

[♣ COMMENTS & FREE-THINKING CORNER:

There is an increasing evidence from astronomical measurements (spectral, Doppler redshifts of the electromagnetic radiation passing about galactic centers; - [37] Tifft, [40] Nottale, [41] Rubćić, A., & J. Rubćić, [43] M. Pitkänen) that v_0 (appearing in (2.11.14) and (2.11.14)-a) is a characteristic velocity parameter applicable for many planetary systems (like other universal or fundamental constants known in Physics) having the value $v_0 = n\sqrt{\frac{GM}{R_n}} = 144.7 \pm 0.7$ km/s. ***Nottale is showing in [40] that such fundamental velocity constant is observed from the planetary scales to the extragalactic scales (see the diagram below). His theoretical predictions, based on "Scale Relativity Theory" agree very well with the observed values of the actual planetary orbital parameters, including those of the asteroid belts.*** *Mentioned observations are supporting the legitimacy of all other quantized parameters (from (2.11.14)) like orbital radius, phase and group velocity R_n, u_n, v_n, etc..*

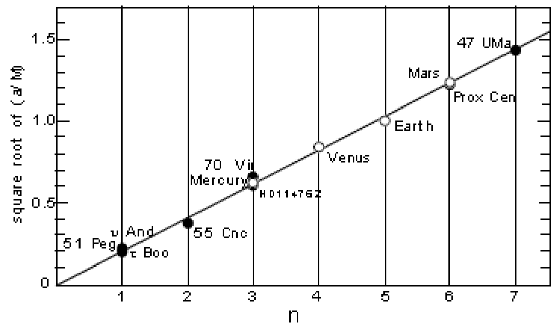


Figure 2. Square root of the ratio (a/M) (a in AU and M in $M_\odot$), where a is the semi-major axis of a planet and M the mass of its star, versus n integer, for inner solar system planets and extra-solar companions. The line corresponds to the law $\sqrt{a/GM} = n/w_0$, with $w_0 = 144$ km/s.

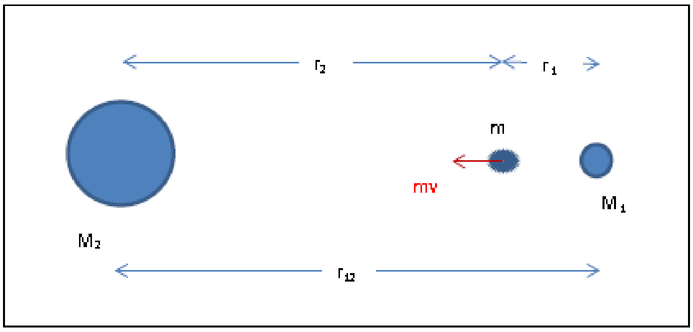
This picture is taken from [40]; -the "Letter to the Editor; Scale-relativity and quantization of extra-solar planetary systems. L. Nottale", DAEC, CNRS et Université Paris VII, Observatoire de Paris-Meudon, F-92195 Meudon Cedex, France, Received 9 May 1996 / Accepted 5 September 1996.

Some gravitational, or inertial standing waves field structure, which also has specific electromagnetic nature (whatever that means), really exist around and behind planets in stationary orbital motions. See more of supporting remarks later, related to measured red shifts, around equations (2.11.15) to (2.11.19), and [63], Arbab I. Arbab, [67], Johan Hansson and [68], Charles W. Lucas, Jr. … All of that is giving chances that some "Planetary Karman Street" should exist behind every planet in orbital motion. Since certain electromagnetic nature is intrinsically incorporated (see [63]) into such planetary and gravitational formations (and quantization of planetary systems is the fact), the imprints and traces of "Planetary Karman Streets" should exist and be measurable as electromagnetic and Doppler-shifts spectral signatures (as Tifft measured), and, very probably, as some kind of charged particles currents and plasma-related manifestations.

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Let us observe two astronomic objects with masses M_1 and M_2. One of them $M_1 << M_2$ could be small planet or satellite, and the other M_2 could be a bigger planet or local sun, or we could have two independent and "self-standing" cosmic masses M_1 and M_2. The distance between M_1 and M_2 is $r_{12} = r_1 + r_2$, as presented in the following picture.

Let us now imagine that specific small mass $m <<< M_1$ is projected (like a gun bullet) from M_1 towards M_2. We could assume that initial mass, speed, and linear moment of the bullet-mass m are constant and known ($(r_1 = \text{minimum}) \Rightarrow m = m_0 = \text{const. } v = v_0 = \text{Const.}, \ p = p_0 = m_0 v_0$). We could also consider that M_1 and M_2 are relatively stable and static masses.



There is also an attractive field of gravitation between all involved masses m, M_1 and M_2, making that bullet mass m will have an increasing speed and linear moment. This time, we will neglect the possible presence and influence of electromagnetic fields and forces. To be more general, we could imagine that all of the involved masses should have certain linear and angular moments (what will be correct in cases of planetary systems), but for analyzing this example, we will assume that masses M_1 and M_2 are sufficiently static (or approximately standstill) and stable. The objective here will be to find evolving effective mass, velocity, energy and momentum of a small gun bullet m.

The first step in such analyzes is to apply energy and momentum conservation laws (this time neglecting possible involvement of angular moments and electromagnetic fields and forces),

$$\begin{aligned}
&\vec{P}_{total} = \vec{p}_m + \vec{p}_1 + \vec{p}_2 = \text{const.} \\
&\vec{p}_m = m\vec{v}, \vec{p}_1 = M_1\vec{v}_1, \vec{p}_2 = M_2\vec{v}_2 \Leftrightarrow (\vec{p}_m = \gamma m_0 \vec{v}, \vec{p}_1 = \gamma_1 M_{10}\vec{v}_1, \vec{p}_2 = \gamma_2 M_{20}\vec{v}_2) \\
&(\vec{p}_{m0} = m_0\vec{v}_0) \Rightarrow \vec{p}_{10} = M_{10}\vec{v}_{10}, \vec{p}_{20} = M_{20}\vec{v}_{20}, \\
&E_{total} = E_m + E_1 + E_2 = (m_0c^2 + E_{km}) + (M_{01}c^2 + E_{kM1}) + (M_{02}c^2 + E_{kM2}) = \text{const.} \\
&(\vec{P}_{total}, \frac{E_{total}}{c}) = \text{invariant.} \Rightarrow \vec{P}^2_{total} - \frac{E^2_{total}}{c^2} = -\frac{E_0^2}{c^2}, \ E_0 = m_0c^2 + M_{01}c^2 + M_{02}c^2, \\
&E_{km} = (\gamma - 1)m_0c^2 \cong \frac{1}{2}mv^2, E_{kM1} = (\gamma_1 - 1)M_{01}c^2 \cong \frac{1}{2}M_1v_1^2, E_{kM2} = (\gamma_2 - 1)M_{02}c^2 \cong \frac{1}{2}M_2v_2^2
\end{aligned} \qquad (2.11.14)\text{-a-1}$$

It is evident that bullet mass m, its velocity, and momentum will be dependent on speeds and moments states of M_1 and M_2. The small mass m is in the field of attractive gravitational forces of masses M_1 and M_2 (acting in mutually opposite directions and being distance-dependent, based on Newton law), meaning that all the involved masses will have evolving and mutually dependent moments.

It is also possible to present motional mass m as a matter-wave packet or photon, where we could start exploiting associated group and phase speed, wave energy, matter-wave wavelength, and matter-wave frequency. For instance, in cases of microparticles like electrons, protons, positrons, etc. it will be, $\tilde{E}_m = hf = E_{km} = (\gamma - 1)m_0c^2 \cong \frac{1}{2}mv^2, \ p = \gamma m_0 v$ *and* $v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\tilde{E}}{dp}, \ u = \lambda f = \frac{\tilde{E}}{p}$. *We will find that useful (and analogical) matter-wave characteristics of the bullet mass m, or an equivalent photon* *(like wavelength and frequency) will also evolve, being distance, velocity and all initial masses dependent (basically getting certain observer-dependent Doppler, red and blue frequency shifts). Here is a place to underline that Planck's constant h is applicable only for cases involving microparticles and photons. Typical examples where we can verify the existence of such analogical particle-wave parallelism situations are innovative analyzes of Compton, and photoelectric effects, including the continuous spectrum of x-rays (or photons), caused by impacts of electrons accelerated in an electrical field between two electrodes (see such analyzes in chapter 4.2). For other macroparticles, certain new, and analogical H constant will be more appropriate instead of Planck constant h (especially in cases when self-closed, matter-waves structures are being involved or created).*

The next step can be to imagine that the kinetic energy of a small bullet-mass m (as in analogical cases of elementary microparticles and photons) will be replaced or cinematically represented by an equivalent matter-wave or photon energy $\tilde{E}_m$ (see equations under (4.2) and T.4.0 from the chapter 4.1),

$$\begin{bmatrix} (\mathbf{0} \le \mathbf{2}u \le \sqrt{uv} \le v \le c, m = \gamma m_{\mathbf{0}}) \Rightarrow E_{km} = \tilde{E}_m = hf = (m - m_{\mathbf{0}})c^2 = \frac{pv}{\mathbf{1} + \mathbf{1}/\gamma}, \gamma = \mathbf{1}/\sqrt{\mathbf{1} - \frac{v^2}{c^2}}, \\ (\mathbf{0} \le v \cong \mathbf{2}u << c) \Rightarrow E_{km} = \tilde{E}_m = hf \cong \frac{\mathbf{1}}{\mathbf{2}} mv^2 = \frac{\mathbf{1}}{\mathbf{2}} pv \end{bmatrix}, \quad (2.11.14)\text{-a-2}$$

$$\begin{bmatrix} (v \cong u \cong c) \Rightarrow \left(\frac{\tilde{E}_m}{p} \cong \frac{d\tilde{E}_m}{dp} \right) \Leftrightarrow \frac{dp}{p} \cong \frac{d\tilde{E}_m}{\tilde{E}_m} \Rightarrow \tilde{E}_m = hf \cong \frac{E_{\mathbf{0}}}{p_{\mathbf{0}}} p \cong cp, \frac{E_{\mathbf{0}}}{p_{\mathbf{0}}} = \frac{E_a}{p_a} = \frac{E_b}{p_b} = ... = \frac{E_m}{p} \\ (E_{\mathbf{0}} = m_{\mathbf{0}}c^2, p_{\mathbf{0}} = m_{\mathbf{0}} c) = \text{constants} \Rightarrow v = \frac{d\tilde{E}_m}{dp} \cong \frac{E_{\mathbf{0}}}{p_{\mathbf{0}}} \cong c, \ u = \lambda f = \frac{\tilde{E}_m}{p} \cong c \end{bmatrix}.$$

From results in (2.11.14)-a-2 we see that relevant matter wave model (in cases of microparticles and photons) could have its initial particle-like part (like a non-zero rest mass, $(E_{\mathbf{0}}, p_{\mathbf{0}}) = \text{constants}$), and waving-tail part concerning its phase velocity. For macro masses or macroparticles, we should be able to construct similar mathematical modeling with new H constant (like already exercised for planetary or solar systems, in this chapter). Of course, this mathematically challenging situation could be much better elaborated, and we should not forget that until here we neglected angular moments and associated electromagnetic complexity.

Interactions between Photons and Gravitation

If we detect and analyze photons emitted from very distant sources (as stars or galaxies), we will be in a position to make any judgments about evolving photons' parameters related to gravitational influences of masses, which are on the way between photons source and photons receiver. Light waves coming to our astronomic observatories are carrying imprints, modulations, or signatures of planetary and galactic systems that are between a distant source of light and our observatory. Tifft, [37], performed number of spectral analyzes of light from remote sources, and found that in such cases we often have certain quantized or discrete frequency shifts (or "redshifts"), which are most probably gravitational (or electromagnetic) imprints of astronomic objects that are on the way of involved light waves. Of course, we should not exclude the possibility to detect "blue shifts" within the same framework. Here, we are directly faced with the very probable existence of quantized gravitational and electromagnetic structures, and macro-cosmological matter waves (as exercised in this book), interfering, and interacting with photons propagating around. If Gravitational intensity, related velocities and orbital diameters of planetary systems (along with light ways propagation) are naturally quantized, this will produce that received light waves from such distant sources will also be on certain similar way quantized (towards red or blue Doppler shifts).

Effectively, here we assume that it should exist certain gravitational force-interaction between photons and big gravitational masses around. ***Initial conditions****, relations, assumptions, and necessary mathematical relations (see chapter 4.1) applicable to a photon propagating from a very distant source (towards its observer or receiver) are:*

$\left[\tilde{E} = hf = \tilde{m}c^2, V = V(r) = V_0 = \mathbf{0}\right] \Leftrightarrow \left[u = v = u_0 = v_0 = c\right] \Rightarrow \tilde{E}_0 = hf_0 = \tilde{m}_0 c^2$ (=) initial photon,

$V = V(r) = \frac{GM}{r} =$ gravitational potential (=) $\left[\left(\frac{m}{s}\right)^2\right.$, velocity squared $\left.\right]$, G = gravitational const.

$\tilde{E}_0$ = photon energy on its distant source where $V = V_0 = \mathrm{Lim}(V)_{r\to\infty} = \mathbf{0}$, $V = V(r) = \frac{GM}{r}$,

Index "0" means that certain value relates to its distant radiation source, where $V = V_0 = \mathbf{0}$,

Photon source or emitter (in this case) has negligible mass and zero gravitational intensity.

$\tilde{E} = hf = \tilde{m}c^2$ = wave energy of a photon,

$\tilde{m}$ = mass-equivalent of a photon = $\tilde{E}/c^2$,

$d\tilde{E} = hdf = c^2 d\tilde{m} = vd\tilde{p} = Fdr = -V\tilde{m}\frac{dr}{r}$,

F = Force acting on a photon $= \frac{d\tilde{p}}{dt} = -G\frac{M\tilde{m}}{r^2} = -V\frac{\tilde{m}}{r}$,

$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\tilde{E}}{d\tilde{p}}$ = group speed,

$u = \lambda f = \frac{\tilde{E}}{\tilde{p}}$ = phase speed,

$\mathbf{0} \le \mathbf{2}u \le \sqrt{uv} \le v \le c$,

$\lambda = \frac{h}{\tilde{p}}$ = photon wavelength

$F = \frac{d\tilde{E}}{dr} = h\frac{df}{dr} = c^2\frac{d\tilde{m}}{dr} = v\frac{d\tilde{p}}{dr} = -G\frac{M\tilde{m}}{r^2}$

*Possible accurate approximations (regarding propagating photon) that can be quickly developed from just stated **initial conditions** are:*

$$\tilde{m} = \tilde{m}_0 \cdot e^{\frac{V}{c^2}} \cong \tilde{m}_0 \cdot (\mathbf{1} + \frac{V}{c^2}) \cong \frac{\tilde{m}_0}{\sqrt{\mathbf{1} - \frac{\mathbf{2}V}{c^2}}} \ge \tilde{m}_0,\ V << c^2$$

$$f = f_0 \cdot e^{-\frac{V}{c^2}} \cong f_0 \cdot (\mathbf{1} - \frac{V}{c^2}) \cong \frac{f_0}{\sqrt{\mathbf{1} + \frac{\mathbf{2}V}{c^2}}} \le f_0,$$

$$\lambda = \frac{h}{\tilde{p}} = \frac{\lambda_0}{u_0}\frac{\tilde{E}}{\tilde{p}} \cdot e^{\frac{V}{c^2}} = \lambda_0\frac{\tilde{E}}{c\tilde{p}} \cdot e^{\frac{V}{c^2}} \cong \lambda_0\frac{\tilde{E}}{c\tilde{p}} \cdot (\mathbf{1} + \frac{V}{c^2}) \cong \frac{\lambda_0\frac{\tilde{E}}{c\tilde{p}}}{\sqrt{\mathbf{1} - \frac{\mathbf{2}V}{c^2}}} \ge \lambda_0\frac{\tilde{E}}{c\tilde{p}},$$

$$\lambda_0 = \frac{h}{\tilde{p}_0} = \frac{c}{f} \cdot e^{-\frac{V}{c^2}} \cong \frac{c}{f} \cdot (\mathbf{1} - \frac{V}{c^2}) \cong \frac{\frac{c}{f}}{\sqrt{\mathbf{1} + \frac{\mathbf{2}V}{c^2}}} \le \frac{c}{f}$$

$$\tilde{E} = hf_0 \cdot e^{-\frac{V}{c^2}} = \tilde{E}_0 \cdot e^{-\frac{V}{c^2}} = hf = \tilde{m}c^2 \cong \tilde{E}_0 \cdot (\mathbf{1} - \frac{V}{c^2}) \cong \frac{\tilde{E}_0}{\sqrt{\mathbf{1} + \frac{\mathbf{2}V}{c^2}}} \ge \tilde{E}_0 = hf_0,$$

$$u = \frac{\tilde{E}}{\tilde{p}} = \lambda f = \lambda f_0 \cdot e^{-\frac{V}{c^2}} = \frac{\lambda}{\lambda_0} u_0 \cdot e^{-\frac{V}{c^2}} = c\frac{\lambda}{\lambda_0} \cdot e^{-\frac{V}{c^2}} \cong c\frac{\lambda}{\lambda_0} \cdot (1 - \frac{V}{c^2}) \cong \frac{c\frac{\lambda}{\lambda_0}}{\sqrt{1 + \frac{2V}{c^2}}} \geq c\frac{\lambda}{\lambda_0},$$

$$u_0 = \lambda_0 f_0 = \frac{\tilde{E}_0}{\tilde{p}_0} = c = v_0,$$

$$\left\{\begin{array}{l} \tilde{p} = \tilde{p}_0 + \frac{\tilde{m}_0 V}{v_0} = \tilde{p}_0 + \frac{\tilde{m}_0 V}{c} = \frac{hf}{c} \cdot e^{\frac{V}{c^2}} + \frac{\tilde{m}_0 V}{c} = \frac{hf}{c} \cdot e^{\frac{V}{c^2}} + \frac{hf_0 V}{c^3} = \tilde{m}c \cdot e^{\frac{V}{c^2}} + \tilde{m}_0 c \frac{V}{c^2} \\ \tilde{p} = \tilde{m}_0 c \cdot e^{2\frac{V}{c^2}} + \tilde{m}_0 c \frac{V}{c^2} = \tilde{m}_0 c \cdot \left(e^{2\frac{V}{c^2}} + \frac{V}{c^2} \right) \cong \tilde{p}_0 \cdot \left(1 + 3\frac{V}{c^2} \right) \cong \frac{\tilde{p}_0}{1 + \frac{V}{c^2}} \cong \tilde{p}_0 \sqrt{1 - 2\frac{V}{c^2}}, \\ \tilde{p}_0 = \tilde{m}_0 c = \frac{hf_0}{c} = \frac{hf}{c} \cdot e^{\frac{V}{c^2}} \cong \frac{hf}{c}\left(1 + 3\frac{V}{c^2} \right) = \tilde{m}c \cdot \left(1 + 3\frac{V}{c^2} \right) \cong \frac{\tilde{m}c}{1 + \frac{V}{c^2}} \cong \tilde{m}c\sqrt{1 - 2\frac{V}{c^2}}, \\ \tilde{m} = \tilde{m}_0 \cdot e^{\frac{V}{c^2}} \cong \tilde{m}_0 \cdot (1 + \frac{V}{c^2}) \cong \frac{\tilde{m}_0}{\sqrt{1 - 2\frac{V}{c^2}}} \cong \frac{\tilde{m}_0}{1 + 3\frac{V}{c^2}}. \end{array}\right\} \Rightarrow$$

$$\Rightarrow \tilde{p} = \frac{h}{\lambda} \cong \tilde{m}c \cdot \left(1 + 3\frac{V}{c^2} \right)^2 = \tilde{p}_0 \cdot \left(1 + 3\frac{V}{c^2} \right) \cong \tilde{m}c \cdot \left(1 - 2\frac{V}{c^2} \right) \cong \tilde{p}_0 \cdot (1 + \frac{V}{c^2}) \cdot \left(1 + 3\frac{V}{c^2} \right)^2 \cong$$

$$\cong \frac{\tilde{p}_0}{\sqrt{1 - 2\frac{V}{c^2}}} \cdot \left(1 + 3\frac{V}{c^2} \right)^2 \cong \tilde{m}c\sqrt{1 - 2\frac{V}{c^2}} \cong \frac{\tilde{m}c}{1 + \frac{V}{c^2}} \cong \frac{\tilde{p}_0}{\sqrt{1 - 2\frac{V}{c^2}}} \cong \tilde{m}c.$$

$$v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = \frac{d\tilde{E}}{d\tilde{p}} = \frac{\tilde{E}_0}{c^2} \frac{dV}{d\tilde{p}} \cdot e^{-\frac{V}{c^2}} = \tilde{m}_0 \frac{dV}{d\tilde{p}} \cdot e^{-\frac{V}{c^2}} \cong \tilde{m}_0 \frac{dV}{d\tilde{p}} \cdot (1 - \frac{V}{c^2}),$$

$$v_0 = \tilde{m}_0 \left(\frac{dV}{d\tilde{p}} \right)_{V \to 0} \Rightarrow v_0 d\tilde{p} = \tilde{m}_0 dV \Rightarrow v_0(\tilde{p} - \tilde{p}_0) = \tilde{m}_0 V \Rightarrow v_0 = \frac{\tilde{m}_0 V}{\tilde{p} - \tilde{p}_0} = c,$$

$$v = \tilde{m}_0 \frac{dV}{d\tilde{p}} \cdot e^{-\frac{V}{c^2}} = c \cdot e^{-\frac{V}{c^2}} \cong \tilde{m}_0 \frac{dV}{d\tilde{p}} \cdot (1 - \frac{V}{c^2}) = c(1 - \frac{V}{c^2}) \cong \frac{c}{\sqrt{1 + 2\frac{V}{c^2}}},$$

*Several of here obtained results are similar or identical to published results about **Gravitational Redshift and Gravitational Effects on Light Propagation** concerning General relativity Theory. What is significant here is that we confirm that big cosmic, gravitational systems are an integral part of universal **P**article-**W**ave **D**uality **C**oncept (also confirmable on different ways).*
..............♣]

As the direct support to quantizing concepts, assumptions and results found in (2.11.14), and (2.11.14)-a, ($v_n^2 R_n = GM = \underline{const.}$), we can verify (based on very long time known, and many times published measurements) that product between Semi-major Axis of planet revolution **R**, and square of a mean Semi-major orbital (or group) velocity **v**, for each of members of certain stable planetary, or satellite system, is a constant number, $v^2 R = v_n^2 R_n = \text{constant}$ (see the table T.2.3.3, below).

T.2.3.3

Planets	m, Planet mass [kg]	R, Semi-major Axis of revolution around the Sun (mean radius of rotation) [m]	Mean Semi-major Orbital (or group) velocity, v (=) [m/s]	v^2R [m^3/s^2]	m/v [kg s/m]	nH = C_1(m/v)
Mercury	3.3022.E+23	**5.80E+10**	**4.7828E+04**	**1.3256E+20**	6.9043238270469.E+18	5.7506047172213.E+39
Venus	4.8690.E+24	**1.08E+11**	**3.5017E+04**	**1.3256E+20**	1.3904674872205.E+20	1.1581190412636.E+41
Earth	5.9742.E+24	**1.50E+11**	**2.9771E+04**	**1.3257E+20**	2.0067179469954.E+20	1.6715195460789.E+41
Mars	6.4191.E+23	**2.28E+11**	**2.4121E+04**	**1.3256E+20**	2.6612080759504.E+19	2.2165176631950.E+40
Jupiter	1.8988.E+27	**7.78E+11**	**1.3052E+04**	**1.3256E+20**	1.4547961998161.E+23	1.2116983645068.E+44
Saturn	5.6850.E+26	**1.43E+12**	**9.6383E+03**	**1.3256E+20**	5.8983430687984.E+22	4.9127243050721.E+43
Uranus	8.6625.E+25	**2.87E+12**	**6.7951E+03**	**1.3253E+20**	1.2748156760018.E+22	1.0615524611320.E+43
Neptune	1.0278.E+26	**4.50E+12**	**5.4276E+03**	**1.3256E+20**	1.8936546539907.E+22	1.5772231515805.E+43
Pluto	1.5000.E+22	**5.91E+12**	**4.7365E+03**	**1.3257E+20**	3.1668953868891.E+18	2.6379031231061.E+39
AVERAGE	2.9650.E+26	**1.78087E+12**	**1.9599E+04**	**1.3256E+20**	2.6280461756991.E+22	2.1888788009444.E+43

The relation $v^2R = v_n^2R_n = \text{constant}$ is originally discovered only mathematically, by finding strong numerical relationships between involved factors (based on measured data), but here is theoretically and conceptually founded (as the consequence of standing matter waves formations), getting much higher significance and generalized weight. It can be additionally confirmed on many similar examples and looks as generally applicable to all stable solar and satellite systems, and it is substantially related to the satellite escape velocity (2.11.11). There is a big chance that such relation could already be considered as the law of contemporary Physics if it was properly understood and respected before the establishment of Kepler and Newton laws (at least, it is not inferior compared to Kepler and Newton laws). It is possible to show that Newton gravitational force between two masses (one of them, $\mathbf{m} = \mathbf{m_1}$, rotating on a stable circular orbit around bigger mass $\mathbf{M} = \mathbf{m_2}$) can be postulated, invented, or analogically formulated from mentioned relation $v^2R = v_n^2R_n = \text{constant}$. Based on analogies from the first chapter, summarized in T.1.8., and Coulomb-Newton force laws, as given in T.2.2, T.2.2-2, (2.1), (2.2), (2.4-5.1), (2.11.14-5), we can see that what should be analog to Coulomb electrostatic force between two electric charges q_1 and q_2 is similar relationship between two linear moments p_1 and p_2,

$$\left\{\begin{array}{l} F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{R^2} \left(\begin{array}{c}\leftrightarrow \\ \text{analogical}\end{array}\right) F_g = \frac{1}{4\pi g_p}\cdot\frac{p_1p_2}{R^2} = \frac{1}{4\pi g_p}\cdot\frac{m_1v_1\cdot m_2v_2}{R^2} \\ q_{1,2} \leftrightarrow p_{1,2} = m_{1,2}v_{1,2} \end{array}\right\} \Rightarrow$$

$$F_g = \frac{v_1v_2}{4\pi g_p}\cdot\frac{m_1m_2}{R^2} = G\frac{m_1m_2}{R^2} = G\frac{mM}{R^2}, \left(G = \frac{v_1v_2}{4\pi g_p} = \text{const.} \leftrightarrow \frac{1}{4\pi\varepsilon_0}\right) \Leftrightarrow$$

$$\Leftrightarrow \left\{ \begin{array}{l} v^2 R = \text{constant} \Leftrightarrow v^2 = \frac{\text{constant}}{R} \Leftrightarrow mv^2 = m \cdot \frac{\text{constant}}{R} \Rightarrow \\ F_c = \frac{mv^2}{R} = m \cdot \frac{\text{constant}}{R^2} \begin{pmatrix} \leftrightarrow \\ \text{analogical} \end{pmatrix} F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{R^2} \Rightarrow \\ F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{R^2} \begin{pmatrix} \leftrightarrow \\ \text{analogical} \end{pmatrix} F_g = \frac{1}{4\pi g_p} \cdot \frac{p_1 p_2}{R^2} = G \frac{mM}{R^2} = F_c(R) \\ G = \frac{v_1 v_2}{4\pi g_p} = \frac{v_c^2}{4\pi g_p} = \frac{c^2}{4\pi g} = \text{const.} \Rightarrow v_1 v_2 = v_c^2 = \text{ constant} \end{array} \right\} . \quad (2.11.14)\text{-b}$$

The way expression for Newton gravitational force developed here (in (2.11.14)-b) is implicating that anyway, in any case of the gravitational attraction of masses, we should have elements of stationary rotations (on stable and inertial-motion orbits with standing matter wave structures) to be able to apply such force law. If in some instances we do not see such elements of stable orbiting (between attracting masses), this is most probably because we are the part of specific complex or more substantial scale rotation (concerning a larger or more general reference frame). In other words, the gravitational force is not, and should not only be a central force $F_c(R)$ between static masses. Dynamic parameters like linear and/or angular moments should also be on some essential way involved here, in the broader reference frame (as it is very well supported in [36]).

We still do not have solid arguments to be undoubtedly and generally sure in $(v^2 R = \text{constant}) = GM$, but this looks very convincing, based on astronomic observations (see T.2.3.3), and as such is intuitively (and analogically) invented or postulated by Newton (see [61], Mark McCutcheon).

From published literature is known that Gravitational force and Coulomb force are two familiar examples with $F_c(R)$ being proportional to $1/R^2$. Both, neutral and electrically charged masses in such force field with negative $F_c(R)$ (presenting an attractive force) obey Kepler's laws of planetary motion.

-Also, the force-field of a spatial harmonic oscillator is central, with $F_c(R)$ proportional to **R,** and negative.

-Bertrand's theorem formulates more significant support to Kepler-Newton Laws, when saying, $F_c(R) = -k/R^2$, and $F(R) = -kR$, are the only possible central force fields with stable and closed orbits.

We could explore other consequences of "$v^2 R = v_n^2 R_n = \text{constant}$"concerning orbital quantization, to estimate numerical value of gravitational Planck constant H as follows (see T.2.3.3, (2.11.13) and (2.11.14)),

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$v^2R = v_n^2R_n = \text{constant} \Leftrightarrow 2\pi R v^2 = 2\pi \cdot \text{constant} = C_1 = 2\pi GM \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} 2\pi R = n\lambda_0 \\ \lambda_0 = H/p \\ n\lambda_0 v^2 = C_1 \end{Bmatrix} \Rightarrow \begin{Bmatrix} H = \frac{2\pi}{n}\cdot(v^2R)\cdot(\frac{m}{v}) = \frac{C_1}{n}(\frac{m}{v}) = \frac{2\pi m\sqrt{GMR}}{n} \\ nH = C_1(\frac{m}{v}) = 2\pi GM(\frac{m}{v}) = 2\pi m\sqrt{GMR}, n = \text{Integer} \end{Bmatrix},$$

$$\Rightarrow \begin{Bmatrix} H = \frac{C_1}{n}(\frac{m}{v}) = \text{Const.} \\ \tilde{E} = E_k = \frac{mv^2}{2} = Hf_0 \end{Bmatrix} \Rightarrow \frac{m}{nv} = \text{const.} \Leftrightarrow \frac{m_i}{m_j} = \frac{n_i}{n_j}\cdot\frac{v_i}{v_j}, \frac{m_i v_i^2}{m_j v_j^2} = \frac{n_i v_i^3}{n_j v_j^3} = \frac{f_{0i}}{f_{0j}}. \qquad \text{(2.11.14)-c}$$

From data in T.2.3.3 it is possible to find that an "average gravitational Planck constant" **H** applicable in case of our planetary system could be somewhere inside the following estimations:

$$\begin{Bmatrix} H = \frac{2\pi m\sqrt{GMR}}{n}, n = \text{Integer}, \\ \langle H \rangle \in \left(\frac{9.33\cdot E+42}{n}, \frac{2.1888788009444\cdot E+43}{n} \right) \end{Bmatrix} \Rightarrow \left(2\pi m\sqrt{GMR} \le 2.1888788009444\cdot E+43 \Rightarrow R \le \frac{0.121362271}{m^2GM}\cdot E+86 \right).$$

(2.11.14)-d

It is almost obvious from (2.11.14)-a,b,c,d that in $v^2R = v_n^2R_n = \text{constant} = GM$ something could be wrong with **G**, since estimated **H** is too far from being the universal constant, meaning that Newton law of gravitation should have certain weak sides. What remains is that we could creatively exploit relations:

$$\left(\frac{m}{nv} = \text{const.} \right) \Leftrightarrow \left(\frac{m_i}{m_j} = \frac{n_i}{n_j}\cdot\frac{v_i}{v_j} \right) \Leftrightarrow (H = \text{Const.}) \Rightarrow \left(\frac{m_i v_i^2}{m_j v_j^2} = \frac{n_i v_i^3}{n_j v_j^3} = \frac{f_{0i}}{f_{0j}} \right), \qquad \text{(2.11.14)-d1}$$

and draw new conclusions and consequences regarding relations between gravitation, **H**-constant, and planetary masses.

We can also exploit $H = \frac{2\pi mvR}{n} = \frac{2\pi m_i v_i R_i}{n_i}$ from (2.11.13), and calculate the set of possible (or approximate) values for the gravitational Planck-like constant H, as,

T.2.3.3-a

Planets	m, Planet Mass,	R, Semi-major Axis of revolution around the Sun (mean radius of rotation)	v, Mean Semi-major Orbital (or group) velocity	n, number of days in one planetary year	$\frac{m_i v_i R_i}{n_i}$	H (=) Gravitational Planck constant
	[kg]	[m]	v (=) [m/s]	[1]	[H/2p] (=) [kg m²/s]	[H] (=) [kg m²/s]
Mercury	3.3022E+23	5.8000E+10	4.7828E+04	1.5000E+00	6.1069E+38	3.8351E+39
Venus	4.8690E+24	1.0800E+11	3.5017E+04	9.2500E-01	1.9907E+40	1.2501E+41
Earth	5.9742E+24	1.5000E+11	2.9771E+04	3.6600E+02	7.2893E+37	4.5777E+38
Mars	6.4191E+23	2.2800E+11	2.4121E+04	6.7000E+02	5.2690E+36	3.3089E+37
Jupiter	1.8988E+27	7.7800E+11	1.3052E+04	1.0500E+04	1.8363E+39	1.1532E+40
Saturn	5.6850E+26	1.4300E+12	9.6383E+03	2.4200E+04	3.2378E+38	2.0333E+39
Uranus	8.6625E+25	2.8700E+12	6.7951E+03	4.2700E+04	3.9563E+37	2.4846E+38
Neptune	1.0278E+26	4.5000E+12	5.4276E+03	8.9700E+04	2.7986E+37	1.7575E+38
Pluto	1.5000E+22	5.9100E+12	4.7365E+03			
Average	*2.9650E+26*	*1.7813E+12*	*1.9599E+04*	*2.1017E+04*	*2.8529E+39*	*1.7916E+40*

Obviously that n, as the principal quantum number (from T.2.3.3-a, temporarily specified as the number of days during one planetary year), because of existence of moons and satellites, should be combined or composed from different quantum numbers in relation to planets' orbital and spinning moments, what analogically also exist in the N. Bohr atom model (see T.2.8. N. Bohr hydrogen atom and planetary system analogies).

Another aspect of $v^2R = v_n^2R_n = \text{constant}$ is that this is also the way to determine the planetary (or satellite) escape speed, based on (2.11.11). For instance, for planets of our Solar system, we have $v^2R \cong 1.3256E{+}20$, (see T.2.3.3), and similar escape speed for every particular planet can be found as $v_{en} \cong 1.41 \cdot v_n$ (meaning that specific planet can be removed from its stable orbit if its orbital velocity will be suddenly increased 1.41 times):

$$\left(E_e = \frac{1}{2}mv_e^2 = \frac{GmM}{R}\right) \Leftrightarrow v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e^2R = 1.3256E{+}20 = 2GM \Rightarrow$$
$$\Rightarrow v_{en} = \sqrt{\frac{2GM}{R_n}} = \sqrt{\frac{1.3256E{+}20}{R_n}} = v_n\sqrt{2} = \frac{v_0}{n}\sqrt{2} = u_n 2\sqrt{2},$$
$$\frac{v_{en}}{v_n} = \frac{1}{v_n}\sqrt{\frac{2GM}{R_n}} = \frac{1}{v_n}\sqrt{\frac{1.3256E{+}20}{R_n}} = \frac{v_0}{nv_n}\sqrt{2} = \frac{u_n 2\sqrt{2}}{v_n} = \sqrt{2},\ v_{en} \cong 1.41 \cdot v_n \ . \qquad \text{(2.11.14)-d2}$$

[♣ COMMENTS & FREE-THINKING CORNER:

If matter-wave Earth's wavelength is related only to the period of one Earth day, *such Earth wavelength will be:*

$$\lambda_{1-day} = vT_{(1-day)} = \frac{H}{p} = \frac{H}{mv} = 2.56617E+9\ [m] \Rightarrow H = mv^2T_{(1-day)} \cong 4.56E+38,$$

$T_{(1\text{-}day)}$ = 8.62E+04 [s],

v = 29771 [m/s] (=) Earth Mean Semi-major Orbital (or group) velocity,

m = 5.9742 E+24 [kg] (=) Earth mass,

R = 1.4959826 x 10^{11} m (=) Earth Mean, or Semi-major Orbital radius.

Since one Earth year has 365.26 Earth days, it should also have 365.26 single wavelengths, and we can easily verify that, $365.26 \cdot \lambda_{1-day} \cong 2\pi R \cong 9.399E+11\ [m]$ *, meaning that n = 365.26, and associated, relevant macrocosmic Planck constant (in this case) could be* $H \cong 4.56E+38$*. If we check on a similar way, the same situation regarding other planets in our solar system, we will get indicative and encouraging, not extremely large divergences, or dispersion of results related to orbital perimeters, relevant matter wavelengths, and this way calculated macrocosmic Planck constant. Naturally, after certain process-refinement, we could generalize such concepts and results (as elaborated in the Appendix, under Chapter 10). See later T.2.3.3-1, where the same idea is applied to all planets of our solar system. ♣]*

It is almost evident that the more complete picture about quantization in stable planetary systems should also take care about additional angular and spinning quantum numbers (of involved planets, moons, asteroids, meteorites, and satellites). In [64], Marçal de Oliveira Neto, we can find (effectively based on (2.11.14)) very precisely and convincingly presented, fitted, and calculated, quantizing results, applied to our planetary system.

The integer "**n**", or some kind of quantum number (appearing in all expressions from (2.11.12) until (2.11.14)-a,b,c,d) could be an arbitrarily high number, and this is presenting a difficulty regarding understanding and using precise and meaningful quantization of planetary systems. It will be much easier if we could say, for instance considering our Solar system, that Mercury is the first and closest planet orbiting our Sun, and it should be characterized as the orbit number 1 (one). The same way, Venus is on the second planetary orbit around the Sun, and it should be characterized as the orbit number 2. Earth and Mars will have orbits 3 and 4, etc. Such orbital numbers can be considered as principal, significant quantum numbers. Here, we will use symbol "i" for mentioned numbers (**i = 1, 2, 3, …**). Obviously, such orbital numbers are not at all equal to integer "**n**" appearing in (2.11.12) - (2.11.14)-d. We can try to present the

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

integer **n** in relation to orbital quantum number i, as $n = iN$, where **N** is a certain constant number (also integer), being the same (and valid) for all planets of certain planetary system. ***We will consider that every planetary system has its own characteristic number N, and its own, unique constant H, while*** $v_n^2 R_n = \text{constant}$. Now, relations developed under (2.11.14) will evolve to,

$$\Rightarrow \left[\begin{array}{c} \left\{\begin{array}{l} \text{For two planets on orbits 1 and 2:} \\ H = \text{const.} \Rightarrow \frac{R_1}{R_2} = \frac{n_1^2}{n_2^2}\frac{m_2^2}{m_1^2} = \left(\frac{n_1 m_2}{n_2 m_1}\right)^2, \frac{T_1}{T_2} = \left(\frac{n_1 m_2}{n_2 m_1}\right)^3 \end{array}\right\} \\ ------------------- \\ \left\{\begin{array}{l} \text{For the same planet passing between two orbits:} \\ (m_1 \cong m_2), H = \text{const.} \Rightarrow \frac{R_1}{R_2} = \left(\frac{n_1}{n_2}\right)^2, \frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3 \end{array}\right\} \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\left\{\begin{array}{l} \text{For two planets on orbits i and j: } H = \text{const.} \Rightarrow \\ \frac{R_i}{R_j} = \left(\frac{n_i m_j}{n_j m_i}\right)^2 = \left(\frac{i \cdot N \cdot m_j}{j \cdot N \cdot m_i}\right)^2 = \left(\frac{i \cdot m_j}{j \cdot m_i}\right)^2, \\ \frac{T_i}{T_j} = \left(\frac{n_i m_j}{n_j m_i}\right)^3 = \left(\frac{i \cdot N \cdot m_j}{j \cdot N \cdot m_i}\right)^3 = \left(\frac{i \cdot m_j}{j \cdot m_i}\right)^3 \\ i, j \in [1, 2, 3, ...],\ n_i = i \cdot N, n_j = j \cdot N, n_i = \frac{i}{i-1} n_{i-1} \end{array}\right\}, \left\{\begin{array}{l} \text{For the same planet passing between two orbits:} \\ (m_i \cong m_j), H = \text{const.} \Rightarrow \\ \frac{R_i}{R_j} = \left(\frac{n_i}{n_j}\right)^2 = \left(\frac{i}{j}\right)^2, \frac{T_i}{T_j} = \left(\frac{n_1}{n_2}\right)^3 = \left(\frac{i}{j}\right)^3 \\ i, j \in [1, 2, 3, ...],\ n_i = i \cdot N, n_j = j \cdot N, n_i = \frac{i}{i-1} n_{i-1} \end{array}\right\}\right]. \tag{2.11.14-e}$$

Relations (2.11.14)-e are also identical, analogical, or equivalent to links found in [64], Marçal de Oliveira Neto. Here is the chance to exploit and extend results presented in [64]. As we can see in [64], mentioned relations are creatively and "ad hock" fitted (by Marçal de Oliveira Neto), to satisfy relevant astronomic observations, to the following forms,

$$\begin{array}{l} \left(\frac{R_i}{R_j} = \left(\frac{i}{j}\right)^2, \frac{T_i}{T_j} = \left(\frac{i}{j}\right)^3\right) \Leftrightarrow \left(R_i = R_j \cdot \left(\frac{i}{j}\right)^2, T_i = T_j \cdot \left(\frac{i}{j}\right)^3\right) \Rightarrow \\ \Rightarrow (\text{as in [64], Marçal de Oliveira Neto}) \Rightarrow \\ \Rightarrow R_{i+1} = R_i \cdot \left(\frac{i+1}{i}\right)^2, T_{i+1} = T_i \cdot \left(\frac{i+1}{i}\right)^3 \Rightarrow \\ \Rightarrow R_{i,j} = R_{j,j} \cdot \left(\frac{i^2 + j^2}{j^2 + j^2}\right), T_{i,j} = T_{j,j} \cdot \left(\frac{i^2 + j^2}{j^2 + j^2}\right)^{3/2} \\ i, j \in [1, 2, 3, ...],\ n_i = i \cdot N, n_j = j \cdot N, n_1 = 1 \cdot N, n_2 = 2 \cdot N, n_3 = 3 \cdot N, ... \end{array} \tag{2.11.14-f}$$

Citation from [64]:

*"The application of Eq. (2.11.14)-f can be illustrated by considering the mean planetary radii and orbital periods, in astronomical units, within the solar system. The semi-major axis, which is the same as the mean planetary distance to the Sun, is expressed in terms of the mean distance from the Earth to the Sun (designated as one astronomic unit or AU). In astronomical units, the orbital period of the Earth (one year) defines the unit of time. Mercury is the closest planet to the Sun (hence its orbit corresponds to n = 1), and its observed mean radius (R_1) and orbital period (T_1) are 0.387 AU and 0.241 years, respectively. Based on these values, the other mean planetary radii and orbital periods can be calculated from Eq. (2.11.14)-f by setting **content i** in the range between 1 and 13 (Table 1).*

Regarding planetary orbits where i = 2 and i = 3, the sum of the squares of i , and a second ad-hoc integer j, taking values from 0 to i, must be considered using expressions analogous to those of Eq. (2.11.14)-f. This procedure may be illustrated by reference to the series of orbits with i = 2. Starting from the state i = 2, j = 2, which is associated with the orbit of Mars, the state i = 2, j = 1 corresponds to Earth's orbit and i = 2, j = 0 *corresponds to that of Venus. The respective mean planetary radii and orbital periods are given by the following calculations:*

$$R_{2,1} = R_{2,2}\left(\frac{2^2+1^2}{2^2+2^2}\right) = 0.9677,\ R_{2,0} = R_{2,2}\left(\frac{2^2+0^2}{2^2+2^2}\right) = 0.7742,$$

$$T_{2,1} = T_{2,2}\left(\frac{5}{8}\right)^{3/2} = 0.953,\ T_{2,0} = T_{2,2}\left(\frac{4}{8}\right)^{3/2} = 0.682.$$

in which the values R2,2 = 1.548 and T2,2 = 1.928 correspond to the parameters for Mars. It is worth noting that the splitting of states associated with i equal to 2 or 3 is analogous to the spectral series derived from modern atomic theory. Moreover, this splitting of states occurs in the region corresponding to the distance of Jupiter from the Sun (i = 4) and may be linked with the unusual characteristic of this planet that, together with its many rings and satellites, almost constitutes a mini-solar system in its own right.

Regarding the terrestrial and the gas giant planets, as well as the dwarf planets Pluto, Makemake and Eris, the theoretical mean radii and orbital periods predicted by this model are in reasonable agreement with the observed values (Johnston's Archive; Space and Astronomy, 2010).
Furthermore, there is a significant agreement between the theoretical and observed results (Table 1) regarding the positions of some asteroids found in the solar system. The model predicts, for example, the orbits of the inner (i = 3; j = 0; HIL) and outer (i = 3; j = 1; HOL) limits of the Hungaria asteroids at mean observed radii between 1.780 and 2.000 AU. The asteroids Vesta (i = 3; j = 2) and Camilla (i = 3; j = 3) are correctly located in the inner (2.361 AU) and outer (3.477 AU) rings of the main asteroid belt, which lies between the orbits of Mars and Jupiter and contains approximately 2000 objects orbiting the Sun. The asteroid Chiron, a Centaur object, is positioned between the orbits of Saturn and Uranus at an observed mean radius of 13.698 AU. Moreover, the calculated mean radius of 24.768 AU is associated with the recently discovered asteroids, also Centaur bodies, named Nessus and Hylonome, whose mean distances are 24.617 and 25.031 AU from the Sun, respectively. Additionally, this model predicts the orbit of trans-Neptunian objects in the region of space where the Plutoids are found, including that of the asteroid 1999 DE9 (i = 12) at an observed mean radius of 55.455 AU, a value that accords very well with the theoretical result of 55.728 AU (Johnston's Archive; Space and Astronomy, 2010).”

Table 1 from [64]

Orbital position		Planet / Asteroid	R (=) Mean radius (AU),		T (=) Orbital period (years)	
i	j		Calculated	Observed	Calculated	Observed
1	1	Mercury	0.387	0.387	0.241	0.241
2	0	Venus	0.774	0.723	0.682	0.615
2	1	Earth	0.968	1.000	0.953	1.000
2	2	Mars	1.548	1.523	1.928	1.881
3	0	HIL	1.742	1.780	2.300	2.375
3	1	HOL	1.935	2.000	2.694	2.828
3	2	Vesta	2.515	2.361	3.994	3.630
3	3	Camilla	3.483	3.478	6.507	6.487
4		Jupiter	6.192	5.203	15.424	11.864
5		Saturn	9.675	9.537	30.125	29.433
6		Chiron	13.932	13.698	52.056	50.760
7		Uranus	18.963	19.191	82.663	83.530
8		Nessus	24.768	24.617	123.392	122.420
9		Neptune	31.347	30.069	175.689	163.786
10		Pluto	38.700	39.808	241.000	251.160
11		Makemake	46.827	45.346	320.771	309.880
12		1999 DE9	55.728	55.455	416.448	412.960
13		Eris	65.403	68.049	529.477	558.070

At least, with (2.11.14)-e and (2.11.14)-f, and results from [64], Table 1, we have specific, indicative and intuitive, justification of the relation that connects newly introduced, orbital quantum number $i = 1, 2, 3, ...$, and initial quantum number **n**, as $n_i = i \cdot N$, where **n** is figuring in T.2.3.3, and in equations (2.11.12) - (2.11.14)-a,b,c,d. Here, **N** is specific constant (and integer), valid for all planets of its solar system. This way, we also have particular insight regarding a deeper understanding of quantization in stable planetary and asteroid systems, with the overwhelming analogy with Bohr planetary atom model (see also [63], Arbab I. Arbab, and [67] Johan Hansson). Since now we have orbital quantum numbers (taken from [64]) associated to planets of our Solar system, $i \in [1, 10], n_i = i \cdot N$, it will be possible to make additional numerical speculations about gravitational, macrocosmic, Planck-like constant **H**, by introducing specific values of $i \in [1, 10]$ into last, right column of T.2.3.3.

Since constant N can be almost arbitrary big integer (and **n** is like number of days in a year for relevant planet; -see T.2.3.3-1), we can also conclude from (2.11.14), that for specific stable planetary system, exist the common, sufficiently high-frequency time-train (or frequency carrier), which is universally applicable for time-flow counting, for all planets belonging to the same solar system, as follows.

$$\left\{ \begin{array}{l} f_o = n\frac{f_m}{2} = n\frac{1}{2T} = n\frac{\sqrt{GM}}{4\pi R^{3/2}} = f_{on}, \ T_o = \frac{1}{f_o} = \frac{2T}{n} = \frac{4\pi R}{nv} \\ H = \frac{2\pi m\sqrt{GMR}}{n}, \quad n = i \cdot N, \ i = 1, 2, 3 ... \ N = \text{integer} \end{array} \right\} \Rightarrow$$

$$f_o = \frac{1}{T_o} = i \cdot N \cdot \frac{f_m}{2} = i \cdot N \cdot \frac{1}{2T} = i \cdot N \cdot \frac{\sqrt{GM}}{4\pi R^{3/2}} = f_{on} \ (=), \ f_m = \frac{2f_0}{n}, \tag{2.11.14-g}$$

$$T_o = \frac{1}{f_o} = \frac{2T}{i \cdot N} = \frac{4\pi R}{i \cdot N \cdot v}, \ T = T_m = T_y = \frac{1}{f_m} = \frac{2\pi R}{v} = \frac{nT_0}{2} = \frac{n}{2f_o} = \frac{(nH)^3}{4\pi^2 G^2 M^2 m^3} (=) \text{one year},$$

$$\tilde{E} = Hf_o = \tilde{E}_n = n\frac{Hf_m}{2} = E_k = \frac{1}{2}mv^2 = mvu = pu = 2mu^2 = G\frac{mM}{2R}.$$

This looks like establishing a precise mathematical way for understanding planetary systems synchronization, discretization, gearing, and digitalization, enriching our understanding of stability and integration of planets inside their solar systems (still without any need for using probability and statistics as in modern quantum theory).

As an example, let us creatively apply (2.11.14)-a,b,c, (2.11.14)-g, and standing matter waves concept from Chapter 10, to (all planets of) our Solar system, as found in T.2.3.3-1, which is created (as the spreadsheet, MS Excel table, with 22 columns), using known astronomic data and observations (mostly from very recent NASA publications). We will just start from the obvious fact that the number of days in a year N_{dy} (for every planet), multiplied with one-day time-duration $T_{(1-day)} = \frac{T_y}{N_{dy}} = \frac{T_y}{n} = \frac{T}{n} = \frac{T_0}{2} = \frac{1}{2f_0}$ is equal to the whole year time-duration $T_y = T = T_m$ (of a relevant planet). Consequently, number of days in a year $N_{dy} = \frac{T_y}{T_{(1-day)}} = n = i \cdot N$ (for every planet), multiplied with one planetary matter-wave wavelength $\lambda_{(1-day)}$ (found for every planet as, $\lambda = \lambda_{(one\ day)} = \frac{H}{p} = V \cdot T_{(one\ day)}$) is equal to the orbital circumference of a relevant planet $2\pi R = n\lambda = N_{dy}\lambda$, calculated using the following relations,

$$\left\{\begin{array}{l} 2\pi r = n\lambda_0 = N_{dy}\lambda_{(1-day)}, \lambda_0 = H/p,\ v_n^2R_n = v^2R\ = GM = \underline{const.},\ v \cong 2u = 2\lambda_{(1-day)}f_{(1-day)} \\ H = \frac{2\pi mvR}{n} = \frac{2\pi m\sqrt{GMR}}{n} = \frac{2\pi mGM}{nv} = \lambda_0 p = \lambda_{(1-day)}mv,\ f_0 = f_{(1-day)} = \frac{1}{2T_{(1-day)}} \\ \lambda = \lambda_0 = \lambda_{(1-day)} = vT_{(1-day)} = 2uT_{(1-day)} = \frac{H}{p} = \frac{H}{mv} \Rightarrow H = mv^2T_{(1-day)}, N_{dy} = \frac{T_y}{T_{(1-day)}} = n \\ T_{(1-day)} = \frac{\lambda_{(1-day)}}{v} = \frac{\lambda_{(1-day)}}{2u}, u = \lambda_{(1-day)}f_{(1-day)} = \frac{\lambda_{(1-day)}}{2T_{(1-day)}}, v_0 = nv_n = nv = N_{dy}v = n\sqrt{\frac{GM}{R_n}} \end{array}\right\} \Rightarrow$$

$$\Rightarrow H = \frac{2\pi mvR}{n} = \frac{2\pi m\sqrt{GMR}}{N_{dy}} = \frac{2\pi mGM}{nv} = mv^2T_{(1-day)} = \frac{mv^2T_y}{N_{dy}} = \qquad (2.11.14)\text{-h}$$

$$= 2E_kT_{(1-day)} = \frac{E_k}{f_{(1-day)}} = \lambda p = \lambda mv.$$

Relations (2.11.14)-h are almost in the full agreement with the helically spinning matter waves concept (associated with moving masses), as elaborated mainly in chapter 4.1. The table T.2.3.3-1 is created using relations from (2.11.14)-h, by applying relevant planetary data (taken from NASA publications). There, we can see that specific data initially known only from astronomic measurements (and from other observations) are getting completely verifiable, calculable and confirmable from the here-established conceptual framework of planetary standing matter-waves (see Appendix, Chapter 10, where standing matter-waves concept is additionally summarized). We can also see that only planet Saturn is still an exception (for an order of magnitude) related to predictions from (2.11.14)-h and results from the table T.2.3.3-1 (see columns 19 and 22). If we would like to make Saturn behaving as other planets in relation to T.2.3.3-1, its mean orbital radius should be about 10 times larger, compared to what we presently know regarding Saturn (but the final answer related to the planet Saturn will be more complicated than such simple solution).

T.2.3.3-1 Gravitational Planck Constant & Standing Matter Waves of our Solar System (see columns from 1 to 22)

1	2	3	4	5	6	7	8	9	10
Planets	**Mean Radius of rotation / Semi-major orbital radius around the Sun, R (=) [m]**	**Orbit Circumference = 2πR (=) [m]**	**Planet mass, m (=) [kg]**	**Sun mass, M (=) kg**	**G**	**π**	**Average, Orbital (group) velocity, v (=) [m/s]**	**Average, Orbital phase velocity, u = $\lambda_{(1-day)}f_{(1-day)}$ = v/2 (=) [m/s]**	**Linear, orbital moment, p = mv (=) [kg·m /s]**
Mercury	5.79E+10	3.60E+11	3.30E+23	1.99E+30	6.67E-11	3.14	4.74E+04	2.37E+04	1.56E+28
Venus	1.08E+11	6.80E+11	4.87E+24	1.99E+30	6.67E-11	3.14	3.50E+04	1.75E+04	1.70E+29
Earth	1.50E+11	9.40E+11	5.97E+24	1.99E+30	6.67E-11	3.14	2.98E+04	1.49E+04	1.78E+29
Mars	2.28E+11	1.43E+12	6.42E+23	1.99E+30	6.67E-11	3.14	2.41E+04	1.20E+04	1.54E+28
Jupiter	7.78E+11	4.89E+12	1.90E+27	1.99E+30	6.67E-11	3.14	1.31E+04	6.53E+03	2.48E+31
Saturn	1.43E+12	8.96E+12	5.68E+26	1.99E+30	6.67E-11	3.14	9.64E+04	4.82E+04	5.48E+31
Uranus	2.87E+12	1.80E+13	8.68E+25	1.99E+30	6.67E-11	3.14	6.80E+03	3.40E+03	5.90E+29
	4.50E+12	2.83E+13	1.02E+26	1.99E+30	6.67E-11	3.14	5.43E+03	2.72E+03	5.57E+29
AVERAGE	**1.24E+12**	**7.80E+12**	**3.33E+26**	**1.99E+30**	**6.67E-11**	**3.14**	**3.22E+04**	**1.61E+04**	**1.01E+31**

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER...** ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

11	12	13	14	15	16	17
Sidereal Orbit period/Period of full rotation around the Sun/Length of Year, T_y (=) [Earth days]	**Sidereal Orbit period/Period of full rotation around the Sun/Length of Year, T_y (=) [s]**	**Sidereal Rotation Period / One self-revolution period / Length of Day (=) Rotation period, $T_{(1\text{-day})}$ (=) [Earth days]**	**Sidereal Rotation Period / One self-revolution period / Length of Day (=) Rotation period, $T_{(1\text{-day})}$ (=) [s]**	**$\lambda_{(1\text{-day})} = vT_{(1\text{-day})} = \lambda = H/mv$ (=) [m]**	**$H = mv^2T_{(1\text{-day})} = 2E_k \cdot T_{(1\text{-day})} = E_k/f_{(1\text{-day})}$ (=) [kg·m²/s]**	**Sidereal Number of days in a year (=) [Columns 11,12,13, 14] (=) $T_y/T_{(1\text{-day})} = N_{dy}$**
87.969	7.58E+06	58.646	5.05E+06	2.39E+11	3.74E+39	1.50E+00
224.700	1.94E+07	243.018	2.09E+07	7.33E+11	1.25E+41	9.25E-01
365.260	3.15E+07	0.997	8.59E+04	2.56E+09	4.55E+38	3.66E+02
686.980	5.92E+07	1.026	8.84E+04	2.13E+09	3.29E+37	6.70E+02
4332.820	3.73E+08	0.414	3.56E+04	4.65E+08	1.15E+40	1.05E+04
10755.700	9.27E+08	0.444	3.83E+04	3.69E+09	2.02E+41	2.42E+04
30687.150	2.64E+09	0.718	6.19E+04	4.21E+08	2.48E+38	4.27E+04
60190.030	5.19E+09	0.671	5.78E+04	3.14E+08	1.75E+38	8.97E+04
1.34E+04	**1.16E+09**	**3.82E+01**	**3.29E+06**	**1.40E+11**	**2.02E+40**	**2.10E+04**

18	19	20	21	22
$C = N_{dy}\lambda_{(1\text{-day})} = N_{dy}\, vT_{(1\text{-day})}$ (=) (Column-17) * (Column-15) (=) Orbit Circumference (=) [m]	**Orbit Circumference/Orbit Circumference (=) (Column-3)/(Column-18) (=) $2\pi R/C$**	**Orbital, Kinetic energy, $E_k = mv^2/2 = H \cdot f_{(1\text{-day})}$ (=) [kg·m²/s²]**	**$H = \frac{2\pi m\sqrt{GMr}}{N_{dy}}$ (=) [kg·m²/s]**	**H-constant (Column-21)/ (H-constant Column-16) (=) H/H**
3.59E+11	1.002825	3.70E+32	3.83E+39	1.024627
6.78E+11	1.002785	2.98E+33	1.25E+41	1.002949
9.37E+11	1.002751	2.65E+33	4.57E+38	1.003012
1.43E+12	1.002693	1.86E+32	3.31E+37	1.007272
4.87E+12	1.002842	1.62E+35	1.16E+40	1.003635
8.93E+13	0.100281	2.64E+36	2.03E+39	0.010042
1.80E+13	1.002928	2.01E+33	2.49E+38	1.003490
2.82E+13	1.002624	1.51E+33	1.75E+38	1.002312
7.78E+12	**1.002779**	**2.45E+34**	**2.02E+40**	**1.006757**

In the table T.2.3.3-1, column 17, and earlier in T.2.3.3-a, we can find that calculated number of days in a year, $n = N_{dy}$ (in relation to H constant) is not an integer (as under ideal and mathematically preferable conditions should be), since here we are operating with mean or average values of related orbital parameters (and still neglecting involved spin characteristics). This is also linked to the reference platform from where our astronomic measurements are valid, and to the fact that solar or planetary systems are dynamically stable, space and time-evolving motions.

It is evident that in T.2.3.3-1, we are getting significant results (see Columns 19 and 22) by implicitly because all planets of our solar system (including the Sun) have orbital and spinning moments. In column 19, we find that values of planetary orbits-circumferences, calculated on two different ways (compared to known astronomic measurements, as in column 3, and to standing macro matter-waves concept, as in column 18) are producing almost identical values. In column 22, we can also find that macrocosmic **H** constant values, calculated on two different ways (one based on known astronomic data, and the other based on standing macro matter-waves concept), are mutually almost identical. This way, we are building the legitimacy of standing macro matter-waves concept in relation to planetary systems. Such a situation should be much better exploited to enrich our understanding of periodicity and standing matter waves quantization within stable solar systems. For instance, planet Earth's Moon is (helically) rotating around planet Earth, and its mean orbit circumference is 2.41E+09 m. In the column 15 of T.2.3.3-1, we can find that "1-day" Earth wavelength, $\lambda_{(1\text{-day})} = \mathbf{v}\mathbf{T}_{(1\text{-day})} = \lambda = \mathbf{H}/\mathbf{mv}$ is 2.56E+09 m, not very much different from 2.41E+09 m, meaning that Earth's Moon should be on some way captured or channeled by helical macro-matter-wave field associated to planet Earth's orbital motion. ***Since calculated H constants (columns 16 and 21) are still too much mutually different, this is indicating that additional, new quantizing, or new standing waves parameters should be considered, meaning that presented modeling is still oversimplified. The most promising strategy here would be to consider specific electromagnetic background involved in the structuring of planetary systems.***

[♣ COMMENTS & FREE-THINKING CORNER:

We can see that solar or planetary systems are respecting (or fully complying to) standing waves, spatial arrangements. This is on some more complicated way also valid for galaxies. Standing waves in question are radial and angular or circular formations of waves, meaning that for every solar system, all of its planets and local sun are properly participating, being part of well-integrated and mutually synchronized structure of spatial, macrocosmic standing waves. Temporarily, we could say that mentioned standing waves are waves of gravitational field, but, the most probable nature of such waves is within an electromagnetic phenomenology, naturally coupled with acoustical or mechanical oscillations.

The nature of all standing waves is that in relevant nodal spots or zones, there are only attractive, agglomerating forces, acting towards nodal spots. Such effects of attraction can easily be demonstrated (and measured) when experimenting with ultrasonic, half-wavelength resonators (that are producing standing waves), and in cases of acoustic or ultrasonic levitation effects realized within standing acoustic waves in air, or in other fluids. In such cases, we always see that masses (or particles) are agglomerating and achieving stable, standstill positions within nodal spots of standing waves. Such analogy should be also valid for planets in orbital motions. The hypothetical assumption here is that every planet or mass presents on some way agglomerating spatial-temporal and standing-waves nodal-formation (when it is defendable to use such conceptualization). Acoustic standing waves levitation effects should be extendable, or naturally coupled and synchronized to similar, intrinsically associated, standing electromagnetic waves (since masses are composed of electromagnetically polarizable atoms). See similar elaborations in [99] from Konstantin Meyl.

Existence of standing waves is requesting to have certain (external) source of vibrations. It should exist, in a surrounding cosmic background (of our universe), something what is producing resonant oscillations, and driving such complex 3-dimensional or multidimensional, resonating universe. For macro systems, resonant frequencies could be extremely low (even below 1 Hz). Of course, we know that standing cosmic matter-waves exist concerning solar systems, since we can verify this mathematically and by comparison with known astronomic measurements (as already elaborated in this chapter).

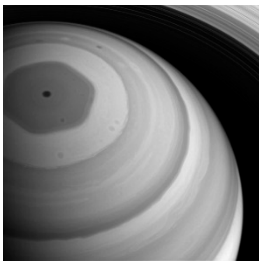
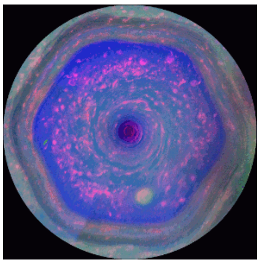
Every time when we have standing waves, with matter or masses fitting into such formations (like in cases of planetary systems), we also have associated electromagnetic dipoles (or multipoles) polarizations, organized within the same structure of standing waves, since masses are composed of atoms, and atoms internally have electric charges, with spinning and magnetic properties. Electric charges inside electrically (and magnetically) neutral atoms can be polarized creating spatially oriented electric dipoles (and multipoles) because of effects of accelerated motions, since electrons have almost 2000 times smaller mass compared to protons. Internal spinning within atoms (of electrons, protons, neutrons...) is creating number of small magnets, and such magnets (or magnetic moments) will be aligned or organized within the same structure of externally structured, macroscopic standing waves. In the same time, we will have macrocosmic or macroscopic 3D formations of ***gravitation and big masses related*** standing matter-waves, and similar (coincidently time-space synchronized) standing waves structure with electric and magnetic dipoles. It is obvious that for creating standing waves we also need to have certain material medium or fluid.

Let us now hypothetically assume that an ideal vacuum is anyway filled with certain fine fluidic medium (having some small particles), which is behaving as an ideal gas, and we will call such medium an ether. Since mentioned ether anyway has measurable electromagnetic constants (magnetic and electric permeability or susceptibility constants), that means that such ether should present certain material medium, and it can carry electromagnetic oscillations, fields and forces (since it has some of electromagnetic properties, and it can be electromagnetically polarized). Of course, ether is a weak carrier-medium for magnetic and electric fields and waves. Since photons and electromagnetic waves are propagating in fluids, open space, and in an ideal vacuum, this means that such ether has certain exotic material nature (being even something what we are still not able to conceptualize, or something what belongs to multidimensional universes). Gravitation should also be a field structure acting within the spatial-temporal matrix of mentioned ether fluid. This is the reason why gravitation is not extraordinarily strong force when compared with electromagnetic forces known in our electromagnetic and engineering practices.

Professor, Dr. Jovan Djuric, [71], proved that every small, non-magnetic mass of different metals, inorganic and organic matter like a wood, is able to self-orient (or self-align) in the direction of local, dominant geomagnetic lines, meaning to align with an existing magnetic field of its local planet. Such effects are very weak, slow evolving and difficult to be noticed, but Prof. Djuric found a way to make successful experimental presentations of such effects, and presented relevant mathematical modeling, which can be additionally developed and optimized. That means all organic and inorganic mases (not only masses with ferromagnetic properties) are being slightly influenced (or physically oriented) by external magnetic field, effectively creating what we still conceptualize as forces of gravitation. In a macrocosmic environment (as certain planetary or solar system is), we will always find standing waves formations, and inside such standing matter-waves we should be able to detect presence of (synchronized) magnetic and electric fields, also structured as the same spatial, macrocosmic standing waves formations (being analog to acoustic levitation effects). Here we are closing the loop of explanations, knowing that nodal spots of all standing waves are manifesting as centers of attractive forces, and by assuming that such attractive forces are creating gravitation (see more in [99] from Konstantin Meyl). This way conceptualizing, we are approaching Nikola Tesla [97], and Rudjer Boskovic [6] ideas about dynamic gravitation and universal natural forces. N. Tesla speculated about certain (standing-waves structured) streaming, or some fine matter flow of “radiant energy” between mutually attracting masses (associating on A. Einstein accelerated elevator), this way explaining effects of gravitation, what we could compare with an ether streaming around and between attracting masses. Rudjer Boskovic, also gave his contribution to gravitation, qualitatively describing the shape of certain universal natural force that should act between and inside all masses, or other corpuscular matter structures that externally manifests as Newton force of gravitation. Of course, this is very short, and an oversimplified conceptual explanation related to understanding gravitation, and it can be combined with ether-flow effects between masses, since all standing waves also have circulation of mutually transforming (and oscillating) kinetic and potential energy amounts... ♣]

Saturn rings should present a perfect (observational) case of specific *standing-waves-like* mass density distribution, where we could search for “signatures” and effects of associated orbital standing waves (or gravity related matter waves). Since Saturn also has a strong magnetic field, and its rings are rotating (becoming somewhat electrically charged and polarized), we could conceptualize specific electromagnetic explanation of the structure of Saturn rings, analogical to N. Bohr model. We could also speculate that involved gravitational nature and attracting force effects (about Saturn and its rings) are direct consequences of a primarily electromagnetic phenomenology, since isolated and static magnetic and electric field components cannot exist as mutually separated in dynamic (motional) situations. This way we will imaginatively enter the space of Nikola Tesla Dynamic Gravity speculations.

Citation from https://solarsystem.nasa.gov/news/531/saturns-famous-hexagon-may-tower-above-the-clouds/ "A new long-term study using data from NASA's Cassini spacecraft has revealed a surprising feature emerging at Saturn's northern pole as it nears summertime: warming, a high-altitude vortex with a hexagonal shape, akin to the famous hexagon seen deeper down in Saturn's clouds.
The finding, published Sept. 3 in Nature Communications, is intriguing because it suggests that the lower-altitude hexagon may influence what happens above and that it could be a towering structure hundreds of miles in height.


"The edges of this newly-found vortex appear to be hexagonal, precisely matching a famous and bizarre hexagonal cloud pattern we see deeper down in Saturn's atmosphere," said Leigh Fletcher of the University of Leicester, lead author of the new study.
Saturn's cloud levels host the majority of the planet's weather, including the pre-existing north polar hexagon. This feature was discovered by NASA's Voyager spacecraft in the 1980s and has been studied for decades; a long-lasting wave potentially tied to Saturn's rotation, it is a type of phenomenon also seen on Earth, as in the Polar Jet Stream.

For more on the new study, visit the European Space Agency's story here: http://sci.esa.int/cassini-huygens/60589-saturn-s-famous-hexagon-may-tower-above-the-clouds/"

[♣ Anyway, Newton-Kepler foundations of gravitation can be presently understood mostly as the best intuitive guess about planetary orbits fitting, based on observations, while several of structural and theoretical miss-concepts and autocorrecting steps are approximately and creatively implemented, producing still sufficiently useful mathematical model. This will have significant impact on our future and improved understanding of Gravitation, orbital motions, and micro-world modeling of motions within atoms. See the following citation from [127].

Non-Conservativeness of Natural Orbital Systems

Slobodan Nedić

University of Novi Sad, Faculty of Technical Sciences, DEET

Trg Dostieja Obradovića 6, 21000 Novi Sad, Serbia, nedics@uns.ac.rs; nedic.slbdn@gmail.com

The Newtonian mechanic and contemporary physics model the non-circular orbital systems on all scales as essentially conservative, closed path zero-work systems and circumvent the obvious contradictions (rotor-free 'field' of 'force', in spite of its inverse proportionality to squared time-varying distance) by exploiting both energy and momentum conservation, along specific initial conditions, to be arriving at technically more or less satisfactory solutions, but leaving many of unexplained puzzles. In sharp difference to it, in recently developed thermo-gravitational oscillator approach movement of a body in planetary orbital systems is modeled in such a way that it results as consequence of two counteracting mechanisms represented by respective central forces, that is gravitational and anti-gravitational accelerations, in that the actual orbital trajectory comes out through direct application of the Least Action Principle taken as minimization of work (to be) done or, equivalently, a closed-path integral of increments (or time-rate of change) of kinetic energy. Based on the insights gained, a critique of the conventional methodology and practices reveals shortcomings that can be the cause of the numerous difficulties the modern physics has been facing: anomalies (as gravitational and Pioneer 10/11), three or more bodies problem, postulations in modern cosmology of dark matter and dark energy, the quite problematic foundation of quantum mechanics, etc. Furthermore, for their overcoming, indispensability of the Aether as an energy-substrate for all physical phenomena is gaining a very strong support, and based on recent developments in Aetherodynamics the Descartes' Vortex Physics may become largely reaffirmed in the near future.

1. Introduction

Following the Newton's fitting of elliptical planetary orbits to the single central force inversely proportional to the square of its distance to the Sun, all natural systems
- from atomic to galactic scales - have been treated as non-conservative (work over closed loop in the field of potential force equaling to zero). The exclusive reliance on gravitation as the only central force does not allow for the formally exact prediction of the planet's trajec- tories in accordance with the Kepler's First law [1], and furthermore orbit fitting to an elliptical shape is contingent on the initial conditions [2]. The basic shortcoming of Newton's theory of orbital motion is the presumed absence of the tangential acceleration component, quite contrary to well established observational results, which is deduced either from the 'naive' interpretation of the Kepler's

Third law, which actually is related to the aver- age values of the orbital radius and elapsed time, or from the improper interpretation of Kepler's Second law as angular momentum, its presumed constancy implying only the circular motion.

For theoretical foundations and practical calculations, the factual time-dependence of the force (thus non-zero rotor field) is neglected and one proceeds from the constancy of the sum of kinetic and potential energies, on one side, and the constancy of the angular momentum, on the other, although in actuality neither of the two is the case.

Only recently, within explorations of biological molecular systems, as well as in certain domains of particle physics, the need starts arising for looking at such systems as non-conservative, the so-called "open systems", which within the classical formalisms turn out to become the "non-integrable" orbital systems (inability to be reduced to "circular coordinates" by even applying the time-varying transformations of the coordinate systems). This has led to modifications and specializations of the formalisms of the classical axiomatic mechanics having been developed by Euler, Lagrange, Hamilton, Noether and others for essentially conservative systems to be applicable to the non-conservative ones. However, a critical analysis of the matters suggests that all the natural orbital systems are open, that is non-conservative (including the planetary, atomic and galactic ones), and that neither the energy nor the (angular) impulse is constant over the time, so that the very basic foundations turn out to be erroneous.

………………..

Another resurfacing of the work not intended for publication is Feynman's scrutinizing and attempting to over- come the noticed week point in Newton's geometrical fit- ting of elliptical orbits to the central force inversely proportional to the squared distance is the above first cited [1], where Feynman had attempted to correct the inconsistency of Newton's geometrical fitting of the elliptic path to the squared distance inverse central force. It is deplorable indeed, that Feynman did not persevere and was not able to apply his favorite Least Action Principle to that problem, instead of stepping into the further support the otherwise unsoundingly set-up quantum mechanics by calculation of the (notably non-zero!?) works on all possible paths of an electron and assigning their reciprocal values to the probabilities, and further going into quite controversial development of the "Quantum Gravity".

…………………

2. Critique of the conventional approach in solving the Kepler's/Newton's problems

When it comes to determining the intrinsic feature of an orbital system, that is whether is it conservative or non-conservative, by all means of prime importance is the topic of a system energy balancing — evaluation of difference between the de-facto performed work and the (knowingly) available applied energy (re)sources.

……………………….

If the former exceeds the latter, or if the traditionally conceived and established law of sum of kinetic and potential energy conservation does not 'hold', we must be missing the awareness of the true nature mechanisms and the availability of the unaccounted for 'environmental' effective energy input(s).

As the historically firstly considered, the Sun's planetary orbital system should indeed be the right one for these considerations, in particular that the established theory and its further developments have detrimentally affected all other physics' and in general science do- mains — form the atom- to galactic-levels, and from chemistry to biology. In direct relation to the orbital energy balancing stands the concept of energy conservation with the related work over a closed path being equaled to zero, as intrinsic feature of the so-called potential fields (the 'central' force vector field having form of gradient of a scalar potential field). …………..♣]

As an example how to take into account mutually coupled (and mutually-interacting) orbital and spinning moments (of solar systems) as vectors, it is sufficiently illustrative to see familiar conceptualization in chapter 4.1, presented by the table "T.4.2.1, Analogies Between **n**-Body Coupled Inertial Motions in a Laboratory System". Such approach should result in the more precise numerical estimation of macrocosmic Planck constant **H**, since here, for every particular planet, we have different **H** constant because we are neglecting orbital and spinning moments as mutually coupled vectors (see (2.11.14)-h and T.2.3.3-1, columns 16 and 21). In reality, all orbital and spinning moments of specific solar system are so well mutually integrated and coupled, that effective, particular planetary moments should be established somewhat similar as in two-body problem, where we will create central and reduced moment of inertia, as well as center of inertia angular velocity, and relative angular and spinning speeds for every particular planet. This will be like reduced and center of mass terms in the two-body problem, but now using terms of rotational and spinning motions.

For instance, the individual solar or planetary system can be characterized by the following set of parameters:

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

m_i (=) mass of certain planet, $i \in [1, 2, 3, ...]$
M_s (=) mass of the local sun
$\vec{v}_i$ (=) planet orbital velocity (=) $\vec{\omega}_i R_i$
$\vec{v}_s$ (=) sun orbital velocity (relative to local galaxy center)
$\vec{\omega}_i$ (=) planet angular velocity
R_i (=) planet mean orbital radius
I_i (=) planet moment of inertia (≅) $m_i R_i^2$
I_{sun} (=) sun moment of inertia
$\vec{L}_i (=) I_i\vec{\omega}_i \cong m_i R_i^2 \vec{\omega}_i = m_i R_i \vec{v}_i$ (=) planet orbital moment
$\vec{L}_{si}$ (=) planet spin moment (=) $I_s \vec{\omega}_s$
$\vec{L}_{sun}$ (=) total angular moment of local sun (=) $I_{sun}\vec{\omega}_{sun}$

Since solar systems are (sufficiently and very long time) stable, we can consider that some of the orbital and spin moments of all planets, and the local sun is conserved (or constant), and this way we will be able to determine the value of local macrocosmic Planck constant **H**, as,

$$\vec{L}_{total} = \vec{L}_{sun} + \sum_{(i)} (\vec{L}_i + \vec{L}_{si}) = I_c \vec{\omega}_c = \overrightarrow{const.} \Rightarrow \left|\vec{L}_{total}\right| = \frac{H}{2\pi} \Rightarrow$$
$$\Rightarrow \left|\vec{\omega}_c\right| = \frac{\left|\vec{L}_{total}\right|}{I_c} = \frac{H}{2\pi I_c} = 2\pi f_c = \frac{2\pi}{T_c} \Leftrightarrow H = 4\pi^2 I_c f_c = \frac{4\pi^2 I_c}{T_c}. \quad (2.11.14)\text{-i}$$

Apparently, in a larger picture, if we attempt to determine unique value of macrocosmic **H** constant, we should not neglect the contribution of all (involved) orbital and spin moments, as well as participation of associated, mutually coupled electromagnetic and other fields and forces within solar systems (as roughly conceptualized in (2.11.14)-i). If such **H** is a stable and constant value, we could speculate around "entanglement ideas" that all mutually coupled orbital and spin moments within the specific stable solar system are communicating with enormously high speed (or instantaneously).

With (2.11.14)-i we are effectively presenting (or replacing) a whole solar or planetary system with a single, central, spinning solar mass M_c, which also has its center of mass velocity $\vec{v}_c$ (relative to local galaxy center), as follows,

$$M_c = M_s + \sum_{(i)} m_i \ , \ \vec{v}_c = \frac{M_s\vec{v}_s + \sum_{(i)} m_i \vec{v}_i}{M_c}. \quad (2.11.14)\text{-j}$$

If we consider our local galaxy center as a new reference frame, our solar system M_c will make orbital motion around galaxy center (having orbital velocity $\vec{v}_c$), and planets will create progressive helical movements around mass M_c and direction of $\vec{v}_c$ (as in (2.11.14)-j). Orbital motions of planets about the local sun are elliptical or close to circular (in the reference system linked to the local sun), but this is because we neglect that complete solar system is also orbiting about its local galaxy center (observed from the reference system linked to a galaxy center in question). This way, we are again coming to clear helical motions concepts that are in any case associated with linear movements, like elaborated in chapter 4.1 (valid both for micro and macro physics world). The much simpler example for visualizing such helical motion is rotating (or orbiting) movement of the specific moon about its local planet (from the reference system linked to the local sun or local planetary system center of mass). Here is also the more in-depth background or nature of matter waves and particle-wave duality (and not at all in a probability or possibility of "*chances that something could happen*"). Modern micro particles accelerators and colliders are also generating streams of energy-momentum products that are respecting similar unity of linear and spinning motions, particle-waves duality, and helical motions framework.

On specific planetary system, we can attempt to create and apply the most common, universally valid, space-time measuring referential frame linked to the local galaxy center, on the following way. Mass M_c *from (2.11.14)-j will be placed in the center of the (newly created), orthogonal axes* (X-Y) *plane, and* Z*-axis will be perpendicular to such* (X-Y) *plane and collinear (or coaxial) with the center of mass velocity* $\vec{v}_c$. *Total spinning moment of mass* M_c *will have its value* $\vec{L}_{total}$ *relative to such* (X, Y, Z) *coordinate system (using similar mathematics as in (2.11.14)-i). In the same time, we can place another* $(\mathbf{x, y, z})$ *reference system (linked to the center of a local solar system) where* M_c *is again in the center of orthogonal axes* $(\mathbf{x}$-$\mathbf{y})$ *plane, and perpendicular* $\mathbf{z}$*-axis has the direction of* $\vec{L}_{total}$. *Of course,* $(\mathbf{X}$-$\mathbf{Y})$ *and* $(\mathbf{x}$-$\mathbf{y})$ *planes in general case are not overlapping, except having common (0,0) and (0, 0, 0) center, and* $\vec{L}_{total}$ *will be different in* (X-Y-Z) *and* (x-y-z) *coordinates. We will always be able to use coordinates rotation and make connections between orthogonal* $(\mathbf{x, y, z})$ *and* $(\mathbf{X, Y, Z})$ *reference frames. The proposed concept is effectively describing an equivalent (and big, remarkably high mechanical quality factor) gyroscope replacing the complete solar system and could be more complicated than here simplified example. Such equivalent gyroscope is spinning, making precession around different axes, and in the same time, creating a large-scale rotational motion or orbiting in the* $(\mathbf{X, Y, Z})$ *coordinate system (linked to galaxy center). Such solar-system gyroscope has its center of mass velocity* $\vec{v}_c$*, its mass* M_c*, and its linear, orbital, and spinning moments, including electromagnetic moments and associated electromagnetic properties and charges.*

Now, we can introduce the hybrid, four-dimensional, space-time coordinate system $(\mathbf{x, y, z, t})$ *where the time flow or time direction (or time axis) will be linked to the center of mass velocity* $\vec{v}_c$ *(from (2.11.14)-j), similar as in Minkowski space of Relativity theory, using the planetary coordinates basis* $(x, y, z, Iv_c t)$. *Here, "**I**" presents Hypercomplex imaginary unit (*$I^2 = -1$*), composed from three more elementary imaginary units, as introduced in chapter 4.0. This way, we will make the most natural space-time frame for describing and visualizing helical planetary motions in relation to local galaxy center (see the picture below, taken from [57], showing different aspects of an Analytic Signal of specific attenuated oscillatory process), or for visualizing helical motions of moons around particular planet.*

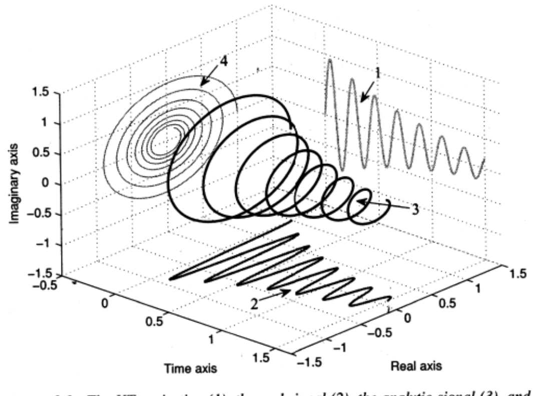
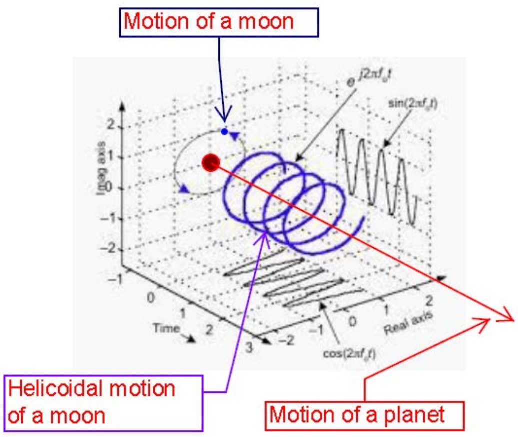


Figure 2.3 The HT projection (1), the real signal (2), the analytic signal (3), and the phasor in complex plain (4) (Feldman, ©2011 by Elsevier)



The ultimate evolution of such conceptualization is to arrive to unite and extended Minkowski, 4-vectors Hypercomplex space (x, y, z, Ict) *and Hypercomplex Analytic Signal functions (based on Hilbert transform, HT), to characterize different motions and associated energy-momentum relations (as elaborated in the chapters 4.0., 4.3. and 10.). If we intellectually, creatively, and philosophically extrapolate this situation, we will understand that similar conceptualization applies to the whole micro and macro universe.*

What is relatively new and original here (around equations (2.11.12) - (2.11.14)-a,b,c,…) is the introduction of a mutually related group and phase, orbital, planetary velocities (compatible to universally valid wave-motions concepts, including compatibility with Lorentz transformations and Relativity theory 4-vectors of energy-momentum relations). Such foundations implicitly introduce and defend the idea of a wave-group, or wave-packet about orbital, planetary matter-waves (very much analogical to Quantum theory wave packets and wave functions). This directly opens a way to analogical implementation of wave functions and wave equations concepts in modeling planetary systems (as Schrödinger and Heisenberg did in Quantum theory). Later, we could enrich and extend the same modeling by applying ideas like Bohr-Sommerfeld's quantization conditions (see Appendix, Chapter 10; - "PARTICLES AND SELF-CLOSED STANDING MATTER WAVES"). Creations of N. Bohr and his followers about planetary atom model (in the early steps of old quantum mechanics) are much more natural and applicable to here elaborated matter-waves of planetary systems, than to the atom model. It is also clear that certain direct and substantial analogy and the connection between such micro and macro world matter-waves conceptualization really exist (see [63], [64], [67] and [68]).

Using equations and relations from (2.11.14)-a,b,c…, we can predict and verify surprisingly exact quantization of celestial orbits in the specific solar system (see such extremely well-documented analyses in [38], [39] and [64]). This additionally confirms, or at least supports, the validity of planetary standing waves field structure (as elaborated in this book). Since planets' orbiting periods are exceptionally long, involved frequencies are small and almost meaningless. Of course, this is only a matter of our perception, scaling and measurement reference systems. Since the force of Gravitation is too weak, compared to electric and magnetic forces, and since Newton and Coulomb force laws are mathematically identical, this implicates that Gravitation could be indirect (still hidden behind our intellectual horizons) electromagnetic forces manifestation. The same idea has been suggested earlier in this book, based on small electric and magnetic dipoles (or multi-poles) polarizations, equivalent to relevant field charges displacements, resulting in certain unbalanced electromagnetic distributions (because masses of electrons and protons are enormously different, and always locally and globally rotating, spinning, and creating electromagnetic fields). This is giving chances to electromagnetic forces to work and show effects that are still classified as Gravitation (see (2.4-6) - (2.4-10)). Effects of planetary system quantization, which are becoming verifiable by proper conceptualization, observations, measurements, and calculations, that are mainly imitating quantization in electromagnetic fields, like in atoms, are indicatively showing that planetary systems should have much more of electromagnetic nature than presently considered (see [67]). The apparent consequence of quantization in planetary systems is that magnetic fields, electric currents and charges should be much more involved in maintaining the dynamic and stationary structure and stability of planetary systems, and this should be valid for all matter in our universe on micro and macro scale.

What matters here is that the same mathematics concepts are applicable and working on micro and macro world scale (of course, not taking it literally, and without intellectual flexibility), both for planetary systems with masses and gravitational forces, and within Coulomb and other electromagnetic interactions between electrically charged microparticles (inside atoms). After establishing such grounds and analogical platforms, it will be imaginable to apply the framework of Schrödinger's, wave quantum mechanics, backward to orbiting planetary systems (see such mathematical modeling later; -equations (2.11.20 – (2.11.23)). There is the significant difference between

Schrödinger's wave mechanics, and quantum mechanics of micro-world (as presently established, within Copenhagen interpretation), and its analogical application on planetary systems, as promoted here. We will find out that intrinsically-probabilistic, and ontologically-stochastic wave function, and associated mathematical concepts and practices, are not at all necessary, natural and best choice to address all micro and macro-world matter waves (except when mathematical conditions for such modeling are met, and when we do not have a better opportunity).

Anyway, impressive, seducing and, in its own work-frames, operative mathematics of Quantum Theory will be still applicable and complementary tool to any other modeling of matter waves (at least for an "in-average" addressing of wave motions). It is incredible how the group of creative people (founders of Quantum Theory) invested such enormous efforts and wonderful imagination, creating an isomorphic, non-realistic, fantastic and artificial mathematical structure (as a shadow or projection of real world around us), that is producing beneficial, and practically good results. Even more amazing is how they convinced or almost ideologically influenced a countless number of followers to admire such creation as the final one and the best made (and fire of such foundations is still burning). We too often find in literature, publications, and interviews (about microphysics), endlessly repeating statements, sounding like Buddhist mantra, that there is no one theory in humans' history such successful and useful as contemporary Quantum Theory (of course, including unmistakable Quantum Electrodynamics). Even doubts about it are forbidden (at least being unprofessional and unacceptable), and should be eradicated or punishable, as effectively (of course, not very explicitly, like here) interpreted by some of the ultraorthodox warriors of present days Quantum Theory. Others (some of them brilliant minds) who privately, most probably have doubts (in modern Quantum Theory), and see the same situation somewhat differently, are anyway staying on the temporary stable grounds of not going explicitly against officially established mainstream.

How good predictions in (2.11.14)-a,b,c… are, and could be, is related to the facts that (in contemporary mechanics) we are still approximating, neglecting or omitting certain elements of a real situation (concerning stable planetary systems) in the following aspects:

a) **We consider that all relevant planetary masses are tiny and homogenous, isotropic solid balls, compared to their common central mass or sun, and that the sun is in the state of rest (without rotation in their common center of mass). The reality is that any planetary system, including its sun, is effectively rotating around its common center of mass (and the center of the sun is not overlapping with the common center of mass). Also, some of the planets are close to solid balls, some of them still have a liquid core, and almost all of them are non-homogenous and have different forms of anisotropy and specific moments of inertia. In reality, much more correct is to say that any Solar System moves through space, orbiting the center of its galaxy, and the planets trace out spirally (or helically) looking paths in space (what is noticeable, for instance, if we place the reference coordinate system in the center of galaxy).**

b) **On some way, we (implicitly) approximate that all planets and sun are rotating in the same (almost) flat plane, what is not the general case (not valid for our or any other solar system), meaning that all orbital and spinning moments of**

participants should be adequately taken into account (as vectors) when calculating macrocosmic H constant.

c) Also, we neglect interplanetary, orbital, and associated electromagnetic interactions, considering that allied forces and fields between every planet and its sun are dominant.

d) Moreover, we are still not enough precisely considering planets spinning concerning the total angular moment conservation.

e) We also do not consider too seriously the consequences of the orbital motion of solar systems concerning its local galactic center (what is effectively, in a larger scale, producing helical planetary motions). Since all solid bodies, like planets and satellites, are electromagnetically behaving like conductive metal masses, mentioned helical pats could be on a proper way considered as electric wires (or conductors), as Nikola Tesla proposed long time ago. Consequently, gravitational forces between such moving masses could be on some way presented as electromagnetic forces between wires with electric currents (since imaginative helical wires or planets-paths are anyway experiencing influences of surrounding electromagnetic fields and fluxes).

f) If quantization of planetary orbits (based on complex standing waves arrangements) has specific real meaning, as it looks to be, we should introduce additional (angular) quantum numbers (concerning orbital and spin moments).

Let us go back to planetary systems, where our reference frame is linked to the common center of inertia (or center of mass) of such system (not considering motions relative to galactic center…). It will become evident that expression for planetary macro-wave (motional or kinetic) energy, $E_k = Hf_o$ from (2.11.13), is directly analog to Planck's wave-quantum energy of a photon, $\tilde{E} = hf$, as well as macro equivalent for a wavelength $\lambda = \frac{H}{p}$ is also analog to a micro-world de Broglie matter wavelength $\lambda = \frac{h}{p}$, where new "macro-world Planck-like constant" H is,

$$\begin{aligned} & H = H(m, R, n) = \frac{2\pi}{n} L = \frac{2\pi}{n} \frac{GMm}{v} = \frac{2\pi m \sqrt{GMR_n}}{n} = \frac{2\pi GMm}{v_0} = \\ & = \frac{2\pi R_n vm}{n} = \frac{\tilde{E}}{f_o} = \tilde{E}T = \text{const.} >> h, \ (n \in [1, 2, 3...], v << c, \ m << M) \\ & \Rightarrow \left(\hbar = \frac{h}{2\pi} \right) \text{analog to} \left(\hbar_{gr.} = \frac{H}{2\pi} = \frac{L}{n} = \frac{m\sqrt{GMR_n}}{n} = \frac{GMm}{v_0} \right). \end{aligned} \quad (2.11.15)$$

The difference between Planck's constant h and analog constant of planetary macro waves H is that h is already known as universally valid constant (for world of atoms elementary particles and photons), and H could be different for every planetary and satellite system…. Of course, for specific planetary system, and for a sufficiently high integer $n = n_{max.}$ in (2.11.15), we should be able to find when H will be equal to h, as for instance,

$$H = h = \frac{2\pi m\sqrt{GMR}}{n_{max.}} = \frac{2\pi GMm}{v_0} = 6.626\ 0693(11)\cdot 10^{-34}\,J\cdot s,$$
$$n_{max.} = v_0\sqrt{\frac{R}{GM}} = 0.948252278\cdot 10^{34}\cdot m\sqrt{GMR} = 0.77443828\cdot 10^{29} m\sqrt{MR},$$
$$(v_0 = 0.77443828\cdot 10^{29}\sqrt{G}\cdot m\cdot M),$$

but such high quantum numbers are obviously unrealistic for characterizing macrocosmic objects and planetary systems.

In fact, for the specific planetary system (where each of planets concerning a common Sun could be approximated as a Binary System), we should be able to find H that will be the same constant for each planet. For instance (see (2.11.14) and (2.11.14-20)), if the ratio between any two of H constants (2.11.16), applied for planets (with circular orbits) from the same solar system, should be equal to one, then we can say that H is at least locally applicable constant, as for instance,

$$H = H_1 = H_2 \Rightarrow \frac{H_1}{H_2} = \frac{H(m_1, R_1, n_1)}{H(m_2, R_2, n_2)} = \frac{n_2}{n_1}\frac{m_1}{m_2}\sqrt{\frac{R_1}{R_2}} = \frac{n_2}{n_1}\frac{m_1}{m_2}\frac{v_2}{v_1} = 1,\ (n_1, n_2)\in[1, 2, 3...),$$
$$\Rightarrow \left\{n_2\cdot m_1\cdot\sqrt{R_1} = n_1\cdot m_2\cdot\sqrt{R_2} \Leftrightarrow n_2\cdot m_1\cdot v_2 = n_1\cdot m_2\cdot v_1\right\}\Rightarrow$$
$$\frac{L_2}{L_1} = \frac{m_2}{m_1}\frac{v_1}{v_2} = \frac{n_2}{n_1},\ \frac{L_2}{n_2}\frac{n_1}{L_1} = \frac{n_1}{n_2}\frac{m_2}{m_1}\frac{v_1}{v_2} = 1 \ \Rightarrow \qquad (2.11.16)$$
$$\frac{H}{2\pi} = \frac{L_1}{n_1} = \frac{L_2}{n_2} \Rightarrow \frac{\left|\vec{L}_1 + \vec{L}_2\right|}{n_1 + n_2} = \frac{\left|\vec{L}_i\right|}{n_i} = \frac{GMm}{v_0} \Rightarrow \frac{H}{2\pi} = \frac{\sum_{(i)}\left|\vec{L}_i\right|}{\sum_{(i)} n_i} = \frac{GMm}{v_0}.$$

The limitations of **H** constant expressions related to (2.11.15) and (2.11.16) are that we still approximate all orbits with circles (which are in the same plane) and we do not consider any planetary "self-spinning" momentum. Another limitation involved here is that orbital and spin moments conservation is entirely valid only if we bring in consideration (as a vector) a resulting total moment (including particular spin moments) of all planets, moons, and satellites of a solar system in question (see [36], Anthony D. Osborne, & N. Vivian Pope).
Consequently, after implementing more elaborated analyses, we should be able to find more general and more precise expressions for **H** (see (2.11.14)-i,j). Present comments regarding planetary-world **H** constant are still indicative brainstorming directions serving to establish the grounds for defending the utility of such constants. Another exciting situation here is how to explain that micro-world **h**-constant (or Planck constant) is unique and universally valid for all atomic and subatomic entities (or we just consider it as universally valid). Do we have (in our Universe) a succession of **H**-constants, starting from certain big **H**-numbers (for galactic formations) which are gradually descending towards smaller numbers with unique and constant **h**-value at the opposite subatomic end, could be a question to answer? *One common fact is almost apparent: **h** or **H** constants are products of stable, periodical, circular, or closed domain (standing waves) motions where orbital moments are conserved (meaning constant).*

[♣ COMMENTS & FREE-THINKING CORNER (still in preparation and brainstorming phase)**:**

It is clear that here we are combining dynamics of orbital motions with certain kind of stable space packing expressed by the necessity of standing waves formation, which is in very close relation to proper angular (and spin) moments conservation, also applicable to inclinations of planetary orbits. In addition, since certain wave-like spatial-periodicity and stable packing in periodical planetary motions exist, it could be presentable as integer multiple "n_α" of angular segments $\alpha = \frac{2\pi}{n_\alpha}$, capturing the angle of a full circle that is in average equal $n_\alpha \alpha = 2\pi = n\lambda_o / R,\ n_\alpha = 1, 2, 3...$ (of course, indicative for an idealized and oversimplified situation, just to give a direction of thinking about angular quantizing),

$$\begin{aligned} L &= n\frac{H}{2\pi} = \frac{n}{n_\alpha}\cdot\frac{H}{\alpha} = \frac{H}{\lambda_o}R = \frac{2\pi\sqrt{GM}}{n\lambda_o}\cdot mR^{3/2} = m\sqrt{GMR}, \\ \alpha &= \frac{2\pi}{n_\alpha} = \frac{n}{n_\alpha}\cdot\frac{H}{L} = \frac{n}{n_\alpha}\cdot\frac{\lambda_o}{r} = \frac{2\pi\sqrt{GM}}{n_\alpha L}\cdot mR^{1/2},\ (n, n_\alpha) \in [1, 2, 3, ...). \end{aligned} \tag{2.11.17}$$

For instance, spherical coordinate system (which should naturally be most applicable here) has one radial coordinate and two angular, and we should use minimum three different quantum numbers for describing such spatial standing waves packing. The more general approach regarding inclinations of planetary orbits, instead (2.11.17) should be an angular or spatial quantizing of relevant orbital matter waves, like in A. Sommerfeld quantization and semi-classical quantization of angular momentum (see [40], D. Da Roacha and L. Nottale). See later more of supporting background under "Wavelength analogies in different frameworks", T.4.2, as well as extended matter-waves conceptualization with equations 4.3-1, 4.3-2, 4.3-3 and Fig.4.1.2, Fig.4.1.3 and Fig.4.1.4, all from the chapter 4.1.

The ideas, modeling and documented astronomic observations about solar systems quantization of orbital radius and relevant velocities (like results in (2.11.14)) are already known from the publications of William Tifft, Rubćić, A., & J. Rubćić, V. Christianto, Nottale, L. and their followers (see literature under [37], [38], [39], [40], [41], and [42]). The possibility, suggested by the observation of velocity quantization (72 km/s, Tifft, [37]) in the redshifts of galaxies, that wave-particle duality with a much larger value of Planck's constant may apply at galactic distances is also examined. For instance, in (2.11.14), we have the phase velocity found as, $u = u_n = \frac{1}{2}\sqrt{\frac{GM}{R}} = \frac{v_0}{2n} \cong \frac{v}{2}$. The galactic (phase) velocity redshifts measured by Tifft are often found to be around 72 km/s (see [37]), and the best-known estimate for specific (galactic) velocity v_0 is, $v_0 = 144.7 \pm 0.7\left[\frac{km}{s}\right] = 2 \times 72.35 \pm 0.35\left[\frac{km}{s}\right]$ (see (2.11.14) and (2.11.14)-a). Of course, for higher values of quantum numbers $n = 2, 3$, we should be able to detect other galactic (phase-velocity related) redshifts such as, $u = \frac{v_0}{2n} \cong \frac{144}{2n} = \frac{72}{n}\left[\frac{km}{s}\right],\ n = 1, 2, 3, ... \Rightarrow$ $u \in (\frac{72}{2} = 36, \frac{72}{3} = 24, \frac{72}{4} = 18...)[\,km/s\,]$. It is also found that orbital velocities (see (2.11.14)) of planets and satellites belonging to our Solar System, $v_n = \sqrt{\frac{GM}{R_n}}$ multiplied by **n** (**n = 1, 2, 3…**) are equal to the multiple of a fundamental velocity, which is close to 24$[\,km/s\,]$. Also, increments of the intrinsic galactic redshifts are found to be $\cong 24[\,km/s\,]$ (see [40] Nottale; [41] Rubćić, A., & J. Rubćić; [43] M. Pitkänen), very much similar to the predictable situation regarding quantized orbital, planetary and satellite velocities from (2.11.14). Surprisingly, in here mentioned literature regarding redshifts, nobody related such case to phase velocity on the way as conceptualized here (related to orbital, macro-cosmological matter waves). Since measured spectral redshifts are really affected by orbital phase velocity $u = u_n$, it is almost evident that here hypothesized standing-waves field structure should exist. ♣]

The same problematic (related to redshifts and, why not to "blueshifts") we can conceptualize concerning matter-wave duality in the following way:

-An observer on our planet (or on a satellite orbiting our planet) is analyzing spectral content of a light coming from a specific distant galaxy.
-Distant galaxy is composed of many stars, solar systems, asteroids, meteorites, etc.
-Majority of such galaxy entities are solid bodies in mutually relative motions (and in a motion in relation to the distant observer), and many of them have specific magnetic field and emitting light or photons (including direct and secondary emissions of light).
-Since mentioned galaxy entities are motional bodies, we could associate matter-waves fields and wave properties to such motions (as we did all over this book).
-Let us consider that specific and dominant mass **M** of the galaxy in question has certain center-of-mass or group velocity **v**. At the same time, such group velocity is presentable as,

$$v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}, \ \lambda = \frac{H}{Mv}, u = \lambda f \ .$$

-If the mass **M** is emitting photons, and if our observer detects and analyze spectral signature of such photons, we should naturally have an interference and superposition between light waves or photons and matter waves of motional galactic mass **M**.
-Any light waves should also be presentable as having active group and phase velocity,

$$v_p = u_p - \lambda_p \frac{du_p}{d\lambda_p} = -\lambda_p^2 \frac{df_p}{d\lambda_p}, u_p = \lambda_p f_p \, .$$

-In such situation, group, and phase speeds of the resulting light emission, being detected by the distant observer, are naturally modulated by matter-waves of the motional galactic body **M**. Such modulation should produce red and blue (Doppler), spectral-shifts, and our distant observer should be able to detect such frequency alterations. Let us consider (for mathematical simplicity, to avoid using vectors) only extreme cases when we would have just additions or only subtractions of corresponding (group and phase) velocities. Resulting, effective (modulated) light received by the distant observer will have new, modified group and phase velocity ($v^* \text{and } u^*$),

$$v^* = v \pm v_p - \lambda \frac{du}{d\lambda} \mp \lambda_p \frac{du_p}{d\lambda_p} = -\lambda^2 \frac{df}{d\lambda} \mp \lambda_p^2 \frac{df_p}{d\lambda_p} = u^* - \lambda^* \frac{du^*}{d\lambda^*} = -\lambda^{*2} \frac{df^*}{d\lambda^*} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} v^* = v \pm v_p, u^* = u \pm u_p \, , \\ \lambda^* \dfrac{du^*}{d\lambda^*} = \lambda \dfrac{du}{d\lambda} \pm \lambda_p \dfrac{du_p}{d\lambda_p}, \\ \lambda^{*2} \dfrac{df^*}{d\lambda^*} = \lambda^2 \dfrac{df}{d\lambda} \pm \lambda_p^2 \dfrac{df_p}{d\lambda_p}, \\ u = \lambda f, u_p = \lambda_p f_p, u^* = \lambda^* f^* \end{array} \right\} .$$

-Also, we could exercise Relativity Theory concepts stating that whatever we do with photons or light waves, resulting group speed (of modulated photons) will always stay constant, equal to the speed of light, $v^* = c = \text{constant}$. This will directly support the facts that our observer should detect mass-velocity related spectral (or frequency) shifts, and this is what William Tifft measured. Of course, here-established concept should be additionally developed and much better elaborated (for instance, concerning additions of velocities), but qualitative conclusions regarding the origin of red and blue shifts are already evident. A similar chain of thinking and findings could be applied to the Michelson-Morley experiment, possibly indicating why Earth motion is not influencing the speed of light inside of such old experimental framework, and how to organize more appropriate experiment, where immediate and entanglement coupling and synchronizations between light beams will be significantly eliminated. *Another reason for confusion produced by old Michelson-Morley experiments is maybe related to the situation that light beams or photons are oscillating only transversally and streaming or flow of ether should be kind of laminar or linear motion.*

All of that (about measured redshifts) is also, implicitly suggesting that electromagnetic dipoles polarizations between rotating (and spinning) astronomic objects could also be involved here, creating associated electromagnetic field structure (as speculated earlier in this chapter; -see equations from (2.4-6) to (2.4-10)). W. Tifft type of redshift can analogically be related to (still hypothetical) "**Planetary Vortex Shedding**" phenomenology, known in fluid motions as "Karman Vortex Street". If certain kind of "Planetary Karman Street" is on some way following planets and astronomic size objects (like an oscillatory, or helix tail), the light coming from distant sources, and passing such spatial zones of electromagnetic "Planetary Karman Streets" will be velocity-modulated, causing measured **redshifts**, and probably in some cases "**blue-shifts**". See more about vortex shedding in the chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,i. Measured frequency shifts of electromagnetic waves coming from distant sources and passing planetary systems (probably applicable to other cosmic rays, x-rays, maybe neutrinos…), are showing certain quantized and predictable, velocity-dependent repetition nature. Such quantization can be numerically related to quantizing of a planetary group and phase velocities (like in (2.11.12) - (2.11.14)), what is pawing an open way towards confirmation of innovative concepts regarding gravitation, as promoted in this book. Of course, here we should not neglect other types of signal modulations, like amplitude and phase modulations of light waves passing complex (*electromagnetic and gravitational*) structure of "Karman Streets" or helicoidally shaped field-tails associated to planetary orbital motions. Why we do not easily see such (spinning), helix field tails, is most probably related to incredibly low frequencies of such phenomena (calculated based on our SI time unit). Such slow spinning effects have much more chances to be influential (or detectable) on planetary orbits when a planet has a relatively big mass, high orbital speed and relatively small perimeter, like in case of planet Mercury (precession of the perihelion). We also know that rotational and spinning motions are often coupled with associated magnetic fields, and we know that gyromagnetic ratios are maintaining constant values both in a micro and in the macrocosmic world, making the same situation more complex, but also more defendable and self-sustaining. Many planets, moons, and other astronomic systems have relatively stable magnetic fields, meaning that rotation and/or spinning (or helix motion) should be somewhere in the background (regardless how low or high is associated spinning frequency). We already have all elements and facts supporting here conceptualized, an extended theory of gravitation; -we only need to learn how to recognize, understand and use such facts. *In chapter 10 of this book, we can find the most complete explanation of the familiar situation regarding unknown or background velocity parameters and Newtonian attraction between important linear and angular moments (see (10.1.4) - (10.1.7))*.

An extraordinary publication from Charles W. Lucas, Jr. (see [54]), "The Symmetry and Beauty of the Universe" is explaining spiraling quantized orbits of planets and moons about the planets, linked to the universally present chiral symmetry based on new, universal electrodynamics force law (which is also addressing the updated force of gravitation). This symmetry can be observed on all size scales from the smallest elementary particle to the structure of the universe. Such spiraling phenomenology (in macro astronomical systems) should be in direct relation to planetary orbits quantizing, Tifft redshifts, and "Planetary Vortex Shedding" phenomenology (describing the same reality).

Anyway, our macro-universe is known to behave like a big and very precise astronomic clock, where periodical motions are its intrinsic property. It will be just a matter of finding or fitting proper integers $(n, n_\alpha) \in [1, 2, 3, ...)$ into above given (or similar) macro matter-waves relations, to support here presented concept. Of course, the situation analyzed here is presently addressing only purely circular planetary orbits (for having mathematical simplicity and faster introduction), and in later analyses, we would need to take into account elliptic and other self-closed planetary orbits (and, most probably, we will generate additional quantum numbers or integers like n, n_α). Quantizing of planetary orbital motions presented here is realized using extremely simple, geometrical concepts analog to N. Bohr atom model, such as $n\lambda = 2\pi r_n, \lambda = H/p = H/mv_n$. The natural development of such quantized model of planetary systems will be in some ways similar to the evolution of Bohr's planetary atom model towards Sommerfeld's atom model (related to the period before the wave and probabilistic quantum mechanics and Schrödinger equations started to be dominant theoretical approach). For micro-world, we are merely implementing or associate universal quantization by default, since this is well-known practice tested in many cases (Planck, L. de Broglie, Einstein, Sommerfeld…). We should not forget that analogical planetary, orbital quantizing is also valid because of global periodical motions, and macro-universe conservation of important orbital and spinning moments (see (2.9.1) and (2.9.1)).

Also, as a significant theoretical background and support to the innovative concept of Macro-Cosmological stability and gravitation (presented here) the following reference should be taken into account: [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces".

[♣ COMMENTS & FREE-THINKING CORNER:

There is another, slowly emerging support (as well as empirical confirmations coming from satellites technology), to unity and coupling of linear, circular, and spinning motions, related to Liapunov (or Lyapunov) stability concept applied on spinning satellites. The following resume is partially taken from the Internet (Wikipedia):

*"A **spin-stabilized spacecraft** is a satellite which has the motion of one axis held (relatively) fixed by spinning the spacecraft around that axis, using the gyroscopic effect.*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

The attitude of a satellite or any rigid body is its orientation in space. If such a body initially has a fixed orientation relative to inertial space, it will start to rotate, because it will always be subject to small torques. The most natural form of attitude stabilization is to give the rigid body an initial spin around an axis of minimum or maximum moment of inertia. The body will then have a stable rotation in inertial space. Rotation about the axis of minimum moment of inertia is at an energy maximum for a given angular momentum, whereas rotation about the axis of maximum moment of inertia is at a minimum energy level for a given angular momentum. In the presence of energy loss, as is the case in satellite dynamics, the spin axis will always drift towards the axis of maximum moment of inertia. For short-term stabilization, for example, during satellite insertion, it is also possible to spin-stabilize the satellite about the axis of minimum moment of inertia. However, for long-term stabilization of a spacecraft, spin stabilization about its axis of maximum moment of inertia must be used".

Surprisingly or by coincidence, we can find supporting background to an appearance of helical matter waves associated with bullets motion in modern guns. Rifling is the process of making helical grooves in the barrel of a gun or firearm, which imparts a spin to a projectile around its long axis. This spin serves to “gyroscopically stabilize” the projectile, improving its aerodynamic stability and accuracy. Bullet stability depends primarily on gyroscopic forces, the spin around the longitudinal axis of the bullet imparted by the twist of the rifling. Once the spinning ball is pointed in the direction the shooter wants, it tends to travel in a straight line until outside forces such as gravity, wind and impact with the target influence it. Without spin, the bullet would tumble in flight. Modern rifles are only capable of such fantastic accuracy because the ball is stable in flight (thanks to the gyroscopic effect). Even spherical projectiles must have a spin to achieve any sort of acceptable accuracy.

Let us elaborate bullet-spinning connection with matter-waves associated with the same bullet. As we know, all linear motions (inertial and non-inertial) are relative motions. Depending on observer's reference frame we can conclude that body is in relative movement to something else, and often different observers can differently describe which object is moving and which one is static (because such motions are mutually relative).

We could say something similar for rotational and spinning motions. Spinning and rotation in a specific mechanical system is also related to something, meaning being a relative angular motion from observer’s point of view, concerning specific axis, to a fixed or moving frame of reference, etc. To say what is rotating and what is in a stable state, concerning certain angular motion, also belongs to specific relative movements, meaning one motional state has described another state.

Now we can go back to a spinning bullet in linear motion. Mentioned spinning is relative to the longitudinal axis of the ball, meaning its axis is stable (static, rigid) and a bullet is spinning around. If observer's reference frame is fixed to the longitudinal axis and revolving around the same axis, such observer will notice that bullet is not rotating, (just to say what here means relative spinning motion). Since bullet’s accuracy and path-stability is enormously increased (thanks to gyroscopic effects) consequently, guns-related technology found a way to give spinning to bullets. We could analogically reverse the cause and effect in the same situation, and say that particle in inertial, stationary, and stable motion should have helical, spinning matter wave around its longitudinal axis of propagation, because such matter wave and moving particle are in mutually relative spinning motion (either one or the other is spinning).

As we know, most of the astronomical objects that can be considered as satellites, or as behaving in a similar way (like orbiting planets, moons, asteroids…) are also naturally spinning. All of that is either complementary or in a direct agreement with here elaborated ideas about “Macro Cosmological Matter Waves”, supporting the idea that ***there is a natural tendency of all (micro and macro) objects in linear motions to be on some way coupled to spinning***. Something similar is additionally conceptualized in Chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,I,j,k... In addition, (in this book) we are also considering rest masses as energy “condensed, frozen or stabilized spinning motions” (as already exercised in this chapter: “2.3.2. Rotation and stable rest-mass creation”).

Familiar situations (concerning couplings between linear motions and associated spinning) we could also find by analyzing laser beams (or beams of photons), and beams of electrons in different interactions with matter states. In addition, we could say that electrons are just specific energy-momentum formatting and packing states of high-energy photons (since a high-energy photon can produce electron-positron couple, and electron and positron are mutually annihilating and producing photons). As we know, both, photons and electrons have spin attributes, and both of them are presenting perfect examples of dualistic Wave-Particle objects (as widely elaborated in this book; -just to mention some of them as, Compton Effect, Photoelectric Effect, interference effects, scatterings and diffractions situations…).

Since gravitational force is anyway too weak in comparison to other forces (like electromagnetic and nuclear), if our laboratory for such measurements is only on our planet, this will not be enough and correct approach. We need to consider the macro Universe or Space around us (with many of galaxies and other objects), as a much more relevant laboratory for observing and learning about gravitation, and for testing new theories related to gravitation. We already know that photons or light waves (high-frequency electromagnetic radiation) are interacting with gravitation of big masses, also interacting with electric and magnetic fields, as well as with matter of any kind. It is logical to consider that light should also interact with other (still hypothetical) aspects of gravitation. If motional masses have some kind of associated field complement to gravitation (for instance something like spinning helix tail, or matter wave, or Karman Street, Vortex Shedding entity, like waves behind the boat crossing large and quiet water surface), light beams passing through such *Vortex Shedding Zone* should interact with it. The result of spectral measurements will be that received light is on some specific way modified, or modulated; -for instance frequency, phase and amplitude modulated (including Doppler Shifts), as measured by W. Tifft. Now we can consider our Cosmos as a big laboratory for testing new theories about gravitation. For instance, first, we can search for remote, stable, clear, strong and distant source of light with known (measurable) spectrum, and we qualify it in a situation when we know (see) that there is an open and empty space between the light source and our Observatory. Such spectral measurements we can consider as our reference measurements. In the second step, we need to wait until the spatial position of the Observatory is changed and make similar spectral (time and frequency domain) measurements when the same light will pass through certain galactic or planetary zones and compare new measurements with reference measurements. The differences should be related to specific signal (or spectral) modifications and modulations caused by certain motional, gravitational, electromagnetic and other matter states present in the concerned "*Vortex Shedding"* spatial zones (since light will interact with such matter and fields states). Of course, if we replace distant light source with something else (with cosmic rays of another kind) we will again be able to apply a similar concept regarding spectral signatures comparison. Another (more controllable) approach will be to have two satellites in stationary orbits around our planet (preferably on mutually opposite orbital ends) and to send laser beams between satellites on a way that reference beams will pass empty spatial areas between them. This way we will establish (measure) reference spectral signatures. In a second step, we will target "Planetary Karman Streets" of other planets (from the same solar system), passing there the same laser beams and registering new spectral signatures. By simple comparisons with reference situations, we should be able to find if "Planetary Karman Streets" exist, and if and how laser beams are modulated by such toroidal and spiraling spatial zones (of planetary and satellite motions; - see [54]). The nature of "Planetary Karman Streets" and Tifft redshifts should be causally linked to a complex structure of associated spiraling electromagnetic and gravitation-related fields (*most probably having very low frequencies for our SI* unit of time).

The signal analysis that will be applied in such cases should show specific correlations with motional and field states that are present in galaxies and planetary systems under investigation. If we are lucky, good mathematicians with fresh concepts and modeling about new gravitation-related phenomenology, and if we are well equipped with measurements and signal processing tools, we will be able to prove new theories about gravitation.

In [52], (Rainer W. Kühne: Gauge Theory of Gravity Requires Massive Torsion Field), we can find another supporting theory which is promoting necessity of linear, torsion and spinning motions coupling on the following way (just the abstract): "One of the greatest unsolved issues of the physics of this century is to find a quantum field theory of gravity. According to a vast amount of literature, unification of quantum field theory and gravitation requires a gauge theory of gravity, ***which includes torsion and an associated spin field****. Various models including either massive or massless torsion fields have been suggested".*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER…** ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

For stable and planar solar systems, we already know quantization rules (2.11.14) applicable for an orbit radius and relevant planetary (or tangential) velocity. Most stable solar systems are not necessarily planar, and we should consider the existence of similar quantization (or spatial orbits packing) regarding angular orbits positions (or orbit inclination towards specific reference orbital plane).

To generalize the same concept (already elaborated with equations from (2.11.13) to (2.11.17)) for any closed planetary orbit, we can apply Wilson-Bohr-Sommerfeld action integrals (used in supporting N. Bohr's Planetary Atom Model). Wilson-Bohr-Sommerfeld action integrals (see [9]), related to any periodical motion on a self-closed stationary orbit C_n, applied over one period of the movement, present the kind of general quantifying rule (for all self-closed standing waves, which are energy carrying structures, having constant angular momentum). Sommerfeld (see chapter 5; equations (5.4.1)) extended Bohr atom model to cover elliptic (and circular) electron (or planetary) orbits, where the semi-major axis is **"a"** and semi-minor axis is **"b"**. We can (just to initiate brainstorming in that direction) analogically (also still hypothetically, and highly speculatively) apply the same strategy on a planet which has mass **m** and which is rotating around its sun, which has mass **M** >> **m**, on the following way,

$$\left.\begin{array}{l}\left\{\oint_{C_n} \mathbb{L}d\alpha = n_\alpha H,\ \mathbb{L} = \text{Constant},\ 0\le\alpha\le 2\pi\right\} \Rightarrow \mathbb{L} = n_\alpha \frac{H}{2\pi},\ n_\alpha = 1,2,3...,n \\ \left\{\oint_{C_n} p_r dr = n_r H\right\} \Rightarrow \mathbb{L}(\frac{a}{b}-1) = n_r\frac{H}{2\pi},\ n_r = 0,1,2,3...\end{array}\right\} \Rightarrow \tag{2.11.18}$$
$$\Rightarrow a = \frac{m+M}{(mM)^2}\frac{(\frac{H}{2\pi})^2}{G}n^2 \cong \frac{(\frac{H}{2\pi})^2}{GMm^2}n^2 = a_0 n^2,\ b = a\frac{n_\alpha}{n} = a_0 n_\alpha n,\ n \equiv n_\alpha + n_r = 1,2,3,4...$$

In addition to (2.11.18), for a certain stable (planar) planetary system with a number of planets (or even for our universe) it should also be valid that its total angular momentum is constant (including spinning moments of planets, moons, and asteroids),

$$\left\{\begin{array}{l}\left[\begin{array}{l}\vec{\omega}_c = \frac{\sum_{(i)} J_i\vec{\omega}_i}{\sum_{(i)} J_i} = \frac{\sum_{(i)}\vec{\mathbb{L}}_i}{\sum_{(i)} J_i} \\ \oint_{C_n}\mathbb{L}d\alpha = n_\alpha H,\ \oint_{C_n} pdr = n_r H \\ n_\alpha, n_r \in [1,2,3,...]\end{array}\right] \Rightarrow \left[\begin{array}{l}\vec{\mathbb{L}} = \sum_{(i)}\vec{\mathbb{L}}_i = \sum_{(i)} J_i\vec{\omega}_i = \vec{\omega}_c\sum_{(i)} J_i = \text{const.}(= n\frac{H}{2\pi}) \\ \sum_{(i)}\oint_{C_n}\mathbb{L}_i d\alpha = \sum_{(i)} n_i H_i = nH = \text{const.}\end{array}\right] \\ \& \\ \left[(\vec{p},\frac{E}{c})^2 = \text{inv.} \Rightarrow p^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c})^2 \Leftrightarrow p^2c^2 - E^2 = -E_0^2 \Leftrightarrow p^2c^2 - (\gamma mc^2)^2 = -(mc^2)^2\right]\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l}\left[\begin{array}{l}\left(\frac{\vec{r}}{r}\times(\vec{p},\frac{E}{c})\right)^2 = \left(\frac{1}{r}(\vec{\mathbb{L}},\frac{E}{c}\vec{r})\right)^2 = \text{inv.}, \\ \gamma = \gamma_1,\ \gamma_2 = 1,\ \vec{\mathbb{L}} = \overrightarrow{\text{const.}}\end{array}\right] \Rightarrow \frac{1}{r_1^2}\left[\vec{\mathbb{L}}_1^2 - (\frac{E}{c})^2 r_1^2\right] = \frac{1}{r_2^2}\left[\vec{\mathbb{L}}_2^2 - (\frac{E_0}{c})^2 r_2^2\right] \Rightarrow \\ \Rightarrow \mathbb{L} = \mathbb{L}_1 = \mathbb{L}_2 = \frac{r_1 r_2}{c}\sqrt{\frac{E^2 - E_0^2}{r_2^2 - r_1^2}} = mcr_1r_2\sqrt{\frac{\gamma_1^2 - 1}{r_2^2 - r_1^2}} = \text{const.}, \Rightarrow r_1 r_2\sqrt{\frac{\gamma_1^2 - 1}{r_2^2 - r_1^2}} = \text{const.}\end{array}\right\} \tag{2.11.19}$$

Of course, if we have combinations of orbital (**L**) and spin moments (**S**), we will need to replace $\mathbb{L}$ with **L**+**S**. As we can see in [36], Anthony D. Osborne, & N. Vivian Pope effectively and analogically (could be unintentional, too) made a very significant extension of Sommerfeld concept to the macro-world of planets, stars, and galaxies, and it is evident that this way the new chapter of Cosmology and Astronomy is being initiated. ♣ **]**

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Elements of specific stable space-time structure with periodical motions (planetary systems, for instance) are mutually coupled by fields and forces integrating them into a stable macro system, and essential associated condition or consequence regarding such stability is the creation of standing waves of involved fields. Positions and paths of planets (inside such periodical-motions systems) are defined by stable or stationary energy-momentum conditions of the system in question, which are related to system minimal energy dissipation, or maximal mechanical quality factor conditions (for instance, found by solving relevant Euler-Lagrange-Hamilton equations). To give an idea how we could evolve this quantum-like conceptualization of Gravitation it would be beneficial to see the Appendix (at the end of this book) that is innovatively treating "**Bohr's model of hydrogen atom and particle-wave dualism".** Of course, some other time, ideas paved with (2.11.10) - (2.11.21) should be better elaborated, extended, and verified, *but significant and innovative brainstorming breakthrough is already made.* What we should, conceptually and imaginatively, visualize and upgrade here is that we are no more dealing only with time-stable and spatially isolated linear (and circular) planetary orbits and discrete planetary masses. Masses of planets in orbital motions are embedded in certain energy-momentum, time and spatial standing-waves distributions that are presenting material, mass-energy extensions, links, and bridges between all elements of such planetary systems. What we see as planetary masses and orbits (described by Kepler and Newton laws) are only space-time localized effective centers (or channels) of such energy-momentum agglomerations that are (in the broader space-time frame) structured as standing waves.

Based on the planetary macro-waves conceptualization which is presented from (2.11.12) until (2.11.19) we can also create kind of Schrödinger equation valid for such standing-waves situations of quantized mass (or energy-momentum) distributions (primarily related to planetary systems). Here we should consider relevant (equivalent) mass in its extended meaning as relativistic, velocity-dependent, spatially distributed and coupled with surrounding energy-momentum states and fields (familiar to conceptualization given in "**2.2. Generalized Coulomb-Newton Force Laws"**, equations (2.3) - (2.4-3)). This time, the important wave function Ψ is directly related to a planet or satellite motional energy on a self-closed path or orbit, or to a relevant radius of orbiting (like having spatial standing waves on a self-closed and oscillating circular string). Geometrically and analogically, this is modeling based on formulating closed (three dimensional or multi-dimensional) spatial structures where all relevant and mutually coupled motional elements with certain periodicity are becoming stationary and stable. The first association related to any standing waves formation is that this should also be a kind of resonance. Moreover, such periodical structures or states have an integer number of specific elementary wavelengths or an integer number of other relevant elementary domains, and such states can be qualified as quantized states. Really, there is nothing more significant to implement or profit from Quantum Theory here.

For creating a better idea about such "standing waves *packing",* it is useful to see equations under (2.9.5-9), *Chapter 4.1, T.4.2., Wavelength analogies in different frameworks,* and *Chapter 5, T.5.3., Analogical Parallelism between Different Aspects of Matter Waves*. If we insist on creating some clear, preliminary and conceptual visualization of planetary orbital motions with associated gravitational matter-waves structure (as spatially distributed energy-momentum states, enclosed in ***toroidal*** forms), this could be intuitively linked to the illustration on Fig. 2.6. and strongly related to future creative modeling and innovative solutions resulting from equations (2.11.20) to (2.11.23).

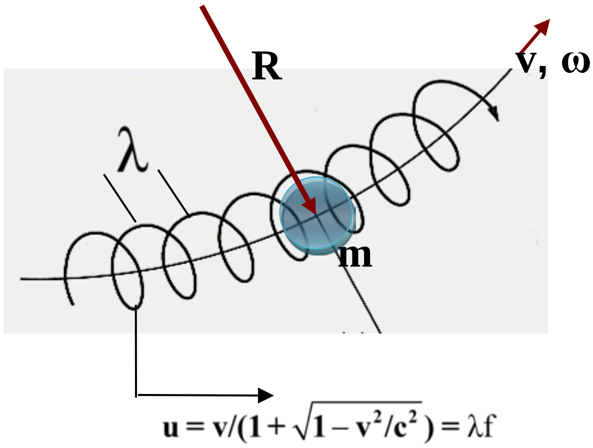


Fig.2.6. Gravitational matter waves and orbital planetary motion

Natural way of modeling such geometrical forms (standing waves) is causally related to Analytic Signal concept (see chapter 4.0. and [57]) and Schrödinger equation, which has its conceptual, historical, and analogical origins in generalization of d'Alembert, Classical Wave Equation, which is related to standing waves oscillations of an ordinary (self-closed) string. In addition, the same, Classical wave-equation (in specific analogical form of d'Alembert equation) has been known in different fields of Classical Mechanics, Fluid Mechanics, Acoustics and Maxwell Electromagnetic Theory long before being "renamed, modified and analogically applied" to waves phenomenology in micro Physics by Schrödinger and other Quantum Theory founders. Another striking analogy (showing that a mass in motion should have some kind of associated helix-spinning field or oscillating matter wave tail) is related to fluid flow vortices and vortex flow-meters, as speculated in ***chapter 4.0 and 4.1; -see equations (4.3-0), and (4.3-0)-a,b,c,d,e,f,g,h,I,j,k...***

This concept of associated helicoidally spinning field (around a path of linear motion) is causally related to the Analytic Signal modeling (as presented in chapter 4.0). Let us consider a specific linear movement of a particle, or an equivalent wave group (in the same state of linear motion). If we present this motion (meaning its power or force function, or relevant field function) with specific wave function $\Psi(t)$***, using the Analytic Signal model, we can create another, associated wave function*** $\hat{\Psi}(t)$***, where a couple of such functions (***$\Psi(t)$ ***and*** $\hat{\Psi}(t)$***) are generating the Complex Analytic Signal*** $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1 + jH)\ \Psi\ (t)$ ***(see much more in chapter 4.0). Now, based on such Analytic Signal modeling, we can determine de Broglie or matter-wave frequency, wavelength, amplitude, and phase functions. The healthy, and fundamental mathematics, well connected to the stable and naturally stable body of Physics (without artificial and postulated theoretical concepts), is***

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

always producing good and realistic mathematical predictions, meaning that both $\Psi(t)$ and $\hat{\Psi}(t)$ must be realistic, measurable wave functions of something that exists in our Physics and Universe. One of the examples for such coupled wave functions is the electromagnetic field that is combining electric and magnetic field functions in a similar way as realized in the Analytic Signal model. Here, as a case causally related to gravitation, we have the situation that any linear and spinning or helix wave motion should be on the same way coupled (creating an Analytic Signal).

[♣ COMMENTS & FREE-THINKING CORNER:

Anyway, in many cases, we can conclude that linear and helix or spinning and rotating motions (of masses) are mutually complementary and united (concerning matter-waves, or PWDM elaborated in this book). Such concepts could be, (imaginatively, creatively, and analogically) extrapolated from atoms to planetary systems and galactic formations. On some way, our universe is globally rotating and spinning, following helix-like paths of associated matter-waves. What we see as red or blue, Doppler shifts (of electromagnetic radiation) coming from a remote deep space, could be effects of such globally present, macro-rotating effects. As we know, the tangential velocity of the certain rotating mass, v_t is equal to the product of relevant orbital velocity, ω, and relevant radius R, $v_t = \omega R$. Hubble's law is maybe saying something similar, such as, $v = (v_t) = H_0 R$, where H_0 is Hubble constant, which could be certain metagalaxy, orbital (or helicoidally spinning associated matter-waves), tangential velocity. ♣]

<u>Citation took from the Internet; -Wikipedia, the free encyclopedia</u>:

*"**Hubble's law** or **Lemaître's law** is the name for the astronomical observation in physical cosmology that: (1) all objects observed in deep space (intergalactic space) are found to have a Doppler shift observable relative velocity to Earth, and to each other; and (2) that this Doppler-shift-measured velocity, of various galaxies receding from the Earth, is proportional to their distance from the Earth and all other interstellar bodies. In effect, the space-time volume of the observable universe is expanding, and Hubble's law is the direct physical observation of this process.[1] It is considered the first observational basis for the expanding space paradigm and today serves as one of the pieces of evidence most often cited in support of the Big Bang model. Although widely attributed to Edwin Hubble, the law was first derived from the General Relativity equations by Georges Lemaître in a 1927 article where he proposed that the Universe is expanding and suggested an estimated value of the rate of expansion, now called the **Hubble constant**.[2][3][4][5][6] Two years later Edwin Hubble confirmed the existence of that law and determined a more accurate value for the constant that now bears his name.[7] The recession velocity of the objects was inferred from their redshifts, many measured earlier by Vesto Slipher (1917) and related to velocity by him.[8]*

*The law is often expressed by the equation $v = H_0 D$, with H_0 the constant of proportionality (the **Hubble constant**) between the "proper distance" D to a galaxy (which can change over time, unlike the comoving distance) and its velocity v (i.e. the derivative of proper length with respect to cosmological time coordinate; see Uses of the appropriate distance for some discussion of the subtleties of this definition of 'velocity'). The SI unit of H_0 is s^{-1}, but it is most frequently quoted in (km/s)/Mpc, thus giving the speed in km/s of a galaxy 1 megaparsec (3.09×10^{19} km) away. The reciprocal of H_0 is the Hubble time.*

As of 3rd Oct 2012, the Hubble constant, as measured by NASA's Spitzer Telescope and reported in Science Daily, is 74.3 ± 2.1 (km/s)/Mpc"

The formulation of the Schrödinger equation is well-known and, in this book, additionally elaborated and generalized (later, in chapter 4.3). Anyway, from different publications we already have definite confirmation that Schrödinger equation is well applicable to solar systems quantizing (see [63], Arbab I. Arbab, and [67], Johan Hansson), since results of Schrödinger equations related to N. Bohr hydrogen atom are directly generating all results of planetary orbit parameters quantizing, as in (2.11.14), when we apply analogical replacement $\boxed{\frac{Ze^2}{4\pi\varepsilon_0} \rightarrow GmM}$. Since orbiting planets are respecting certain periodicity and "*macro matter-waves packing rules*", like standing, de Broglie matter-waves in a micro-universe (see explanations around equations (2.9.5-9), (2.11.5) - (2.11.9), (2.11.9-1) - (2.11.9-4), (2.9.5-1) - (2.9.5-5) and (2.11.12) - (2.11.14)), we are in the position to apply relevant, generalized Schrödinger-like equations, as equations (4.10), to planetary orbital motions. Of course, it is essential to address accurately all relevant parameters and analogical replacements.

[♣ COMMENTS & FREE-THINKING CORNER:

Let us first consider the specific chain of thinking and logical conclusions with several unavoidable facts and step-stones of modern Physics, such as:

1. *Schrödinger equation is something that works well in the world of microphysics (and modern Quantum Theory), where it is unavoidable and producing correct results. Exceptionally brilliant and the early triumph of Schrödinger equation (in spherical coordinates) was experienced with exploring N. Bohr, hydrogen atom model, when all experimentally known results, as well as results of old quantum and atom theory, have been calculated based on Schrödinger equation results. We could only argue about historical Schrödinger equation foundations and development, which has been like tricky patchwork, with mathematical trial and errors experiments, until Schrödinger constructed or intuitively fitted the right and useful, present form of his equation. However, since it is working well, and producing useful results, we forgot how it was created or postulated, without enough systematic and logical arguments.*

2. *In this book, (see chapter 4.3) it is anyway shown that there is another mathematical approach and modeling, which starts from Classical, universally valid wave equation and produces almost the same, but a logically consistent, more generally correct, more logical and intuitively clear family of equations compatible to Schrödinger equation. This is realized based on Analytic Signal and Hilbert Transform modeling, without using Probability and Statistics décor, and without almost arbitrary and complicated postulations.*

3. *Anyway, old, or innovated Schrödinger equations are working on the same or similar way and producing perfect (spectral) results related to N. Bohr, hydrogen atom model (just to start with). Consequently, we should conclude that something exciting and significant for Physics and our Universe should be linked to mentioned Schrödinger equations family.*

4. *Number of authors, including the author of this book, theoretically concluded, experimentally explained, and supported by astronomic measurements and observations, that there is the striking analogy between results of N. Bohr hydrogen atom model (regarding quantized electron energy, velocity, radius…), and similar results applicable to planetary or solar systems (see in this chapter equations (2.11.12) – (2.11.14) and table T.2.8.). Nobody presently claims that such analogy, and mutually comparable, and by measurements verifiable results are entirely correct, but what we can verify is very much indicative. Naturally, we need to admit that there is undoubtedly familiar, and monumentally simple, intrinsic, ontological, and experimental, unifying background (of micro and macrocosmic entities) here.*

5. *Since Schrödinger equation has its extremely significant place regarding N. Bohr, hydrogen atom model, and such atom model have striking analogies with planetary systems (with similar*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

periodical motions), the logical conclusion is that we should be able to explain quantizing within stable solar systems by exploring relevant (customized) Schrödinger equations. Of course, somewhat innovated Schrödinger equations (based on Analytic Signal Wave-function) should eventually be formulated in spherical coordinates (as in case of hydrogen atom model) to better address Gravitation. Results of gravitational, planetary system Schrödinger equation will enrich our understanding of Gravitation, far better compared to Newton and Relativity Theory concepts. ♣]

It is essential to underline that Schrödinger-like, analogically formulated wave equation, applicable to gravitational fields, and macro-mechanical motions within planetary systems has almost nothing to do with stochastic and probability concepts applied in contemporary Quantum Theory.

Let us briefly specify logical, a mathematical chain of initial and final forms and conclusions in the process of analog formulation of such wave equation. We can start by complying with generalized Schrödinger equation (4.10) from the Chapter 4.3, which will address deterministic (certainly non-stochastic) planetary and satellites orbital motions, including associated (deterministic and dimensional) wave functions in a field of gravitation, as for instance,

$$\left\{\begin{array}{l}
\left[\begin{array}{l}
\frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E} + \mathbf{E_0} - U_p)\overline{\Psi} = \mathbf{0}, \\
\frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi} - \mathbf{U_p}\,\overline{\Psi} = -(\tilde{E} + \mathbf{E_0})\,\overline{\Psi} = -\mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} - \mathbf{U_p}\,\overline{\Psi} = \frac{\hbar^2}{\tilde{E} + \mathbf{E_0} - U_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} - \mathbf{U_p}\,\overline{\Psi} = \\
= \mathbf{j}\hbar\mathbf{u}\nabla\overline{\Psi} - \mathbf{U_p}\,\overline{\Psi},\ \ (\frac{\mathbf{E_{total}} - \mathbf{U_p}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial t^2} = \mathbf{0}\,,\ \frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \mathbf{u}\nabla\overline{\Psi} = \mathbf{0}, \\
\nabla = \mathbf{i}\frac{\partial}{\partial \mathbf{x}} + \mathbf{j}\frac{\partial}{\partial \mathbf{y}} + \mathbf{k}\frac{\partial}{\partial \mathbf{z}} = \mathbf{grad},\ \Delta = \nabla^2 = \frac{\partial^2}{\partial \mathbf{x}^2} + \frac{\partial^2}{\partial \mathbf{y}^2} + \frac{\partial^2}{\partial \mathbf{z}^2}\,,\ \overline{\Psi} = \Psi + \mathbf{j}\hat{\Psi},\ \mathbf{j}^2 = -\mathbf{1}
\end{array}\right] \mathbf{(4.10-1)},\text{chapter 4.3} \\
\& \\
\left[\begin{array}{l}
\mathbf{v} = \omega_\mathbf{m}\mathbf{R} \cong \mathbf{2u} = 2\lambda_\mathbf{0}\mathbf{f_0} = \frac{2\pi\mathbf{r}}{\mathbf{T}} = 2\pi\mathbf{R}\mathbf{f_m} = \sqrt{\frac{\mathbf{GM}}{\mathbf{R}}} << \mathbf{c},\ (\mathbf{R} = \mathbf{R_0}\frac{\mathbf{1+e}}{\mathbf{1+e\cdot\cos\theta}}), \\
\omega_\mathbf{m} = 2\pi\mathbf{f_m} = \frac{4\pi\mathbf{f_0}}{\mathbf{n}} = \frac{2\pi}{\mathbf{T}} = \frac{\sqrt{\mathbf{GM}}}{\mathbf{R}^{3/2}} = \frac{\mathbf{v}}{\mathbf{R}},\ \mathbf{p} = \mathbf{mv} = \frac{\mathbf{H}}{\lambda_\mathbf{0}} = \frac{\mathbf{nH}}{2\pi\mathbf{R}},\ \mathbf{n} = \mathbf{1,2,3,...} \\
\mathrm{L} = \mathbf{n}\frac{\mathbf{H}}{2\pi} = \mathbf{pR} = \mathbf{mvR} = \mathbf{mR}^2\omega_\mathbf{m} = \mathbf{mR}^2\frac{2\pi}{\mathbf{T}} = 2\pi\mathbf{mR}^2\mathbf{f_m} = \sqrt{\mathbf{GMR}}\cdot\mathbf{m}, \\
\tilde{\mathbf{E}} = \mathbf{E_k} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{mv}^2 = \mathbf{Hf_0} = \frac{\mathbf{GMm}}{\mathbf{2R}},\ \mathbf{U_p} = -\frac{\mathbf{GMm}}{\mathbf{R}},\ \mathbf{E_k} - \mathbf{U_p} = \frac{\mathbf{3}}{\mathbf{2}}\frac{\mathbf{GMm}}{\mathbf{R}}
\end{array}\right] \begin{array}{l}\mathbf{(2.11.13)} \\ \mathbf{(2.11.14)}, \\ \mathbf{(2.11.15)}\end{array} \\
\& \\
\left[\begin{array}{l}
\mathbf{h} \leftrightarrow \mathbf{H} = \frac{2\pi\sqrt{\mathbf{GMR}}}{\mathbf{n}}\cdot\mathbf{m},\ \hbar = \frac{\mathbf{h}}{2\pi} \leftrightarrow \hbar_{\mathbf{gr.}} = \frac{\mathbf{H}}{2\pi} = \frac{\sqrt{\mathbf{GMR}}}{\mathbf{n}}\cdot\mathbf{m} = \frac{\mathbf{GMm}}{\mathbf{v_0}}, \\
\mathbf{m} \leftrightarrow \frac{\mathbf{mM}}{\mathbf{m+M}} = \mu \cong \mathbf{m} << \mathbf{M},\ \frac{\mathbf{Ze}^2}{4\pi\varepsilon_\mathbf{0}} \to \mathbf{GmM}.
\end{array}\right]
\end{array}\right\} \Rightarrow$$

$$\left[\begin{array}{l} \frac{(\frac{H}{2\pi})^2}{2m}\Delta\overline{\Psi} + (E_k + E_0 - U_p)\overline{\Psi} = 0,\ U_p = -\frac{GMm}{R},\ E_k = \frac{GMm}{2R},\ E_0 = mc^2, \\ \frac{(\frac{H}{2\pi})^2}{2m}\Delta\overline{\Psi} - U_p\overline{\Psi} = -(E_k + E_0)\overline{\Psi} = -j\hbar\frac{\partial\overline{\Psi}}{\partial t} - U_p\overline{\Psi} = \frac{\hbar^2}{E_k + E_0 - U_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} - U_p\overline{\Psi} = \\ = j\hbar u\nabla\overline{\Psi} - U_p\overline{\Psi},\ (\frac{E_{total} - U_p}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial t^2} = 0,\ \frac{\partial\overline{\Psi}}{\partial t} + u\nabla\overline{\Psi} = 0,\ u = v/2. \end{array}\right] \tag{2.11.20}$$

The solutions of (2.11.20) will show that planetary and satellite orbits (here populated by a wave function $\overline{\Psi}$) are closed circular or elliptic lines, but only approximately. Certain harmonic, very low frequency standing waves, having helical paths, or amplitude-modulation of relevant radius, with toroidal and helix, field-envelope should be measurable, when planets and satellites' orbits are very precisely monitored (because of rotation, spinning and mutual interactions among participants).

The closest, extraordinary and unique publications about existence and grounds of such (helix and toroidal) planetary motions are coming from Lucas Jr. Charles when he is explaining Chiral Symmetry of Spiraling Planetary Orbits (on Surface of a Toroid) about the Sun (see [54]). Unfortunately, publications and ideas of Mr. Lucas are not enough addressed in the mainstream of officially supported science, probably because he is too original and sometimes gravitating around arbitrary, religiously flavored environments, this way maybe creating some minor doubts regarding his scientific objectivity and ideological neutrality. Anyway, regardless of personal ideological preferences of Mr. Lucas, his conceptualization of spiraling planetary orbits on a surface of the toroid is amazingly seducing and significant contribution to understanding Gravitation (and familiar to ideas and concepts elaborated in this book).

It is evident that the future development of here-introduced macro-cosmological matter-waves concept will significantly enrich our understanding of Gravitation.

To get generally valid and entirely natural solutions $\overline{\Psi} = \overline{\Psi}(\mathbf{R},\theta,\varphi)$ of (2.11.20), involved operators ($\nabla, \nabla^2 = \Delta$) should be applied in spherical, polar coordinates ($\mathbf{R},\theta,\varphi$). In addition, elliptic planetary orbits should be taken into account (of course, after we upgrade all equations from (2.11.12) until (2.11.18), which are valid for ideal circular orbits, into new and equivalent expressions applicable for elliptic planetary orbits, and consequently, all of that will modify differential equations found in (2.11.20)). The same gravitational, Schrödinger equation, (2.11.20), in its natural, spherical coordinates will eventually look like the equation applicable on N. Bohr, the hydrogen atom model,

$$\frac{(\frac{H}{2\pi})^2}{2m}\left[\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\cdot\overline{\Psi} + U_p(R) = E_k\cdot\overline{\Psi}, \tag{2.11.21}$$
$$\overline{\Psi} = \overline{\Psi}(R,\theta,\phi) = X(R)\cdot Y(\theta,\phi),$$

where $\mathbf{X(R)}$ is expressible in terms of associated Laguerre functions, and $\mathbf{Y(\theta,\phi)}$ are the spherical harmonic functions.

To find solutions of (2.11.21) will not be easy, but to rely on analogies between a specific planetary system and N. Bohr, hydrogen atom model, and directly make results conversions, like in T.2.8., will be much more comfortable, since the validity of analogical replacements, $\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow \frac{GmM}{r^2},\ Ze \Leftrightarrow M, e \Leftrightarrow m \cong \frac{mM}{m+M} = \mu, Z \Leftrightarrow \frac{M}{m}, \frac{1}{4\pi\varepsilon_0} \Leftrightarrow G, h \Leftrightarrow H\right\} \Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \to GmM\right\}$ is shown as working very well. For instance, to get an (analogical) idea about possible spatial shapes of gravitational, matter-wave function from (2.11.21), we can see the picture given below, which is addressing hydrogen wave function (taken from Quantum Theory, standard literature):

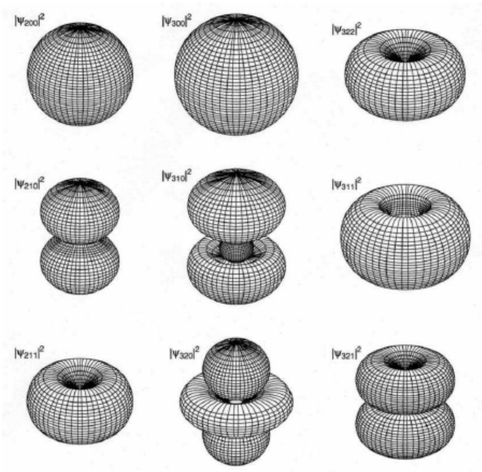


Fig.2.6. Surfaces of the constant $\left|\Psi\right|^2$ for the first few hydrogen wavefunctions

In fact, more correct explanation of the ideas found in (2.4) - (2.11.21) could be complicated compared to here-presented, but for the purpose of introducing new concepts about Wave and Quantum Gravitation, Particle-Wave Duality, force-field charges, and unification between linear and rotational elements of every motion, hire initiated, conceptual platform is already sufficiently clear. In order to understand the broader meaning of wave functions it is useful to see the chapter "4.3 Wave Function and Generalized Schrödinger Equation"; -equations: (4.33-1), (4.41-1) to (4.45), T.4.2 and T.4.3, as well as matter-waves conceptualization around equations (4.3) and (4.3-1) in the chapter 4.1.

Also, based on Parseval's identity (that is universally valid, and connecting time and frequency domains of any wave function), we can establish very realistic and deterministic meaning of the matter-waves, wave function. For more information, see chapter 4.0, and equations (4.0.4).

The wave function $\overline{\Psi} = \overline{\Psi}(\mathbf{R}, \theta, \varphi, \mathbf{t})$, instead of being certain probability, any oscillating amplitude, or specific displacement, or harmonically modulated orbital radius (like in (2.11.20) and (2.11.21)), could analogically get an extended meaning of spatially distributed, matter-wave power (see (2.11.22)). In such situations, (instead of Probability and Statistics concepts and postulations), number of mutually linked, universally valid conditions and relations would be naturally satisfied, as for instance,

$$\Psi^2 = \frac{dE}{dt} = \frac{dE_k}{dt} = \frac{d\tilde{E}}{dt} = v\frac{dp}{dt} = \omega_m \frac{dL}{dt} = vF = \omega_m \tau = \text{Power} = P,$$

$$\left\{\begin{array}{l} E_k = \tilde{E} = \frac{1}{2}mv^2 = mvu = pu = 2mu^2 = \frac{1}{4}mv_e^2 = \frac{GmM}{2R} = \frac{1}{2}\cdot\left(\frac{GmM}{R^2}\right)\cdot R = \frac{1}{2}\cdot F_{m-M}\cdot R = \\ = \frac{m}{2}\left(\frac{2\pi R}{T}\right)^2 = \frac{8m\pi^2R^2}{n^2}f_o^2 = 2m(\pi Rf_m)^2 = (2\pi mR^2f_m)\cdot(\pi f_m) = L\pi f_m = (\frac{2L\pi}{n})\cdot f_o = Hf_o = \\ = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty}a^2(t)dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \\ = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\overline{U}(\omega)\right|^2 d\omega = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\right]^2 d\omega = \int_0^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega = \\ = \int_{-\infty}^{+\infty}P(t)dt \ (=)\ [J] \end{array}\right\},$$

$$\left\{\begin{array}{l} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = H\frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1+\frac{uv}{c^2}}, \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{Hf}{p} = \frac{f}{f_s} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p},\ f_s = k/2\pi, \lambda = \frac{H}{p}, \\ \Rightarrow 0 \le 2u \le \sqrt{uv} \le v \le c \end{array}\right\}, \tag{2.11.22}$$

because, regarding orbital planetary motions we have harmonic and periodical wave-functions (and motions), which create stable, self-closed, spatial standing-waves, and resonant-like field states. The nature of fields and forces involved here is related to mechanical motions and gravitation. Such movements are also mixed with associated electromagnetic fields (at least because mutually identical mathematical forms of Newton and Coulomb's force laws are applicable). We should not exclude the possibility that Gravitation has its primary and essential origin in Electromagnetism (see such electromagnetic forces conceptualizations around equations from (2.4-6) to (2.4-10), in the same chapter; -see also [54]). Deterministic, ***square of the power-related macrocosmic wave-function*** from (2.11.21) and (2.11.22) is in reality a product between two of relevant, mutually linked, dynamic and motional-energy related values, such as relevant current and voltage, or force and velocity, etc. (see (4.0.82) from the chapter 4.0, with more of supporting background), as follows,

$$\Psi^2(t,R) = P(t,R) = \frac{d\tilde{E}}{dt}(=)\text{Active Power } (=)$$

$$(=)\left\{\begin{array}{lll} i(t)\cdot u(t) & (=) & [\text{Current}\cdot\text{Voltage}],\text{ or} \\ f(t)\cdot v(t) & (=) & [\text{Force}\cdot\text{Velocity}],\text{ or} \\ \tau(t)\cdot\omega(t) & (=) & [\text{Orb.}-\text{moment}\cdot\text{Angular velocity}],\text{or} \\ (\vec{E}\times\vec{H})\cdot\vec{S} & (=) & \left[\overrightarrow{\text{Pointyng Vector}}\right]\cdot\overrightarrow{\text{Surface}} \\ ----- & (=) & --------- \\ s_1(t)\cdot s_2(t) & (=) & [(\text{signal}-1)\cdot(\text{signal}-2)] \end{array}\right\}. \tag{4.0.82}$$

It is also important to mention that specific, new, unbounded, creative and intellectually flexible approach should still be implemented in a future fruitful merging between the wave-function environment from (2.11.22), (4.0.82), and wave equations (2.11.21) to formulate new wave and quantum gravity theory. What we have here, so far, are just early brainstorming and first intuitive steps.

The opinion of the author of this book about gravitational waves is that if such waves exist, we will find that this is certain cosmic, electrostrictive and/or magnetostrictive oscillatory and waving effect (like in cases of piezoelectric and magnetostrictive transducers), including associated electromagnetic waves, fields and forces, manifesting on particles distributions and motions (analogical to waves in fluids). What effectively exist (and what is measurable) in our astronomic environment are different electromagnetic waves, photons, streams of electrically charged and neutral particles, and surrounding electric and magnetic fields. Also, in our cosmic environment, we can find different mass-energy-momentum flow and streaming situations of different particles, waves, and fluids. Mentioned, primarily electromagnetic effects (being as original sources of vibrations) are producing secondary, temporally, and spatially evolving electromechanical effects, by contemporary physics specified as gravitational force and waves. The principal sources of gravitation are not static masses, but rather motional and oscillating masses with linear and orbital moments, and, all masses are specific packing formats of specifically structured and polarized electromagnetic entities (like electrons, protons, neutrons, positrons, photons… and their combinations). The fact is that our Universe is already united and stable, regardless if we (or our Physics) miss the proper global unification theory of all-natural forces and fields. Since our Universe is firm and united, consequently, the same natural force should oversee the micro and macro world of physics. The best and most logical candidate for such universal force is electromagnetic fields' related phenomenology (see more in Chapter 3.). Structural or spatial stability and organizing of our Universe is described by R. Boskovic universal natural force, [6], and such force, by its nature, should be single and unique (meaning cannot be gravitational plus electromagnetic plus weak and strong nuclear force).

2.3.3-5 Uncertainty and Entanglement in Gravitation

Until present, certain defendable legitimacy regarding planetary and gravitational wave functions and wave equations has been established. Consequently, whenever we have wave functions, we can analyze associated couples of mutually conjugated, space, time, and spectral domains. Uncertainty Relations, which are generally applicable to such situations (in mathematics), are mutually relating durations of mutually conjugated, original, and spectral domains, of involved wave functions. To better understand such, generally valid uncertainty relations, it is recommendable to read chapter 5. of this book (around relations (5.5)),

$$\begin{aligned}&TF > \frac{1}{2},\ T\Omega > \pi,\ \Omega = 2\pi F\\&T - \text{absolute time duration of the } \Psi \text{ function}\\&F - \text{absolute frequency duration of the } \Psi \text{ function.}\end{aligned} \qquad (5.5)$$

Obviously, in cases of planetary systems, Planck constant **h** has not its natural place there and new and analogous **H** >> **h** constant is becoming much more relevant. The number of uncertainty relations can be now formulated for gravitational and planetary wave-functions, on a similar way as practiced in Quantum Theory concerning constant **H** (for covering much more extensive background about wave functions and Uncertainty Relations; -see chapters 4.0 and 5). As shown in chapter 5, we will conclude that typically mechanical and motional entities, as practiced in present interpretations of gravitation, should be enriched with naturally associated electromagnetic set of relevant parameters (see equations from (5.2) until (5 .4.1)), if we like to have more complete picture about Gravitation. ***What makes Uncertainty Relations, equally applicable (on the same way), both in micro and macro world of Physics, is the fact that simple, geometric or spatial dimensions, durations and size of certain rest-mass are different and smaller when compared with mass-energy-momentum matter-wave packet associated to the same mas (including all associated electromagnetic items). Within Uncertainty Relations, we should naturally operate with matter-waves durations, both in relevant original and spectral domains.*** ***High Power Mechanical, ultrasonic or acoustical energy, moments, forces, oscillations and vibrations, or audio signals and music, can be created and transferred by applying different signal-modulating techniques on laser beams and dynamic plasma states, using laser and plasma states as carriers for lower frequency mechanical vibrations (or signals); - See relations from Chapter 10. under (10.2-2.4) and literature references from [133] until [139].***

Another, still fantastic, imaginative, and hypothetical vision, or prediction, concerning planetary wave functions, will be a necessity of "Gravitational Entanglement" (see more about entanglement in Chapter 4.3, equations (4.10-12) and in Chapter 10). If we search what could or should be such entanglement in Astronomy and Gravitation, we will come to concepts of globally coupled orbital and spinning moments of planetary and galactic formations, where all of such rotating and spinning states (belonging to members of specific planetary system) are mutually communicating, balancing and compensating every perturbation by infinite speed. Such orbital moments communication should be present locally or internally (inside of specific planetary system), and globally or externally concerning the larger astronomical environment. See literature under [36], Anthony D. Osborne, & N. Vivian Pope, where something similar or equivalent to Gravitational Entanglement is also promoted.

[♣ COMMENTS & FREE-THINKING CORNER:

2.3.3-6 Rudjer Boskovic and Nikola Tesla's theory of Gravitation

The most exciting and most profound brainstorming conceptualization of Gravitation, based on elaborations presented in this book, is to understand and develop it in a familiar framework already established by Rudjer Boskovic [6], as the "**Universal Natural Force**", including intuitive concepts of similar "**Dynamic Theory of Gravity**", as sporadically commented by Nikola Tesla, [97]. We can imaginatively and creatively (with much of positive intellectual freedom) see that forces and fields discussed by R. Boskovic and N. Tesla are already present within forces keeping atoms stable (like in the N. Bohr's atom model). We can also see from the results presented in this book (*see chapter "2.3.3. Macro-Cosmological Matter-Waves and Gravitation"; -equations from (2.11.10) until (2.11.22), and "T.2.8. N. Bohr hydrogen atom and planetary system analogies"*) that solar or planetary systems have big level of analogy with N. Bohr's atom model (and vice versa). An innovative extension of N. Bohr's atom model (see "8. BOHR's MODEL OF HYDROGEN ATOM AND PARTICLE-WAVE DUALISM") is elaborating that such structures (like hydrogen atom, or analogically conceptualized planetary systems; -see equations (8.64) until (8.74), and "8.3. Structure of the Field of Subatomic Forces") should be surrounded with a complex force-field $\mathbf{F}(r, \theta, \phi, t)$ that could be modeled as Rudjer Boskovic Universal Natural Force, and be also conceptually compatible to Nikola Tesla's Dynamic Force of Gravity, since N. Tesla anyway had R. Boskovic as the primary source of his ideas about gravitation. Here is convenient to rethink about extended foundations of gravitation in "2.2.1. WHAT THE GRAVITATION REALLY IS", at the beginning of this chapter, where is speculated that all atoms and masses (in our Universe) are

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

continuously and synchronously communicating electromagnetically, mechanically and electromechanically.

*We also know that electric charge and magnetic flux are naturally bipolar entities (able to create dipole structures). Something similar should be valid for linear and angular moments, what is already known and related to action and reaction forces, electromagnetic induction, and different inertial effects. In other words, since gravitational force is known only as an attractive force, to satisfy mentioned bipolarity, some real mass-energy-momentum flow should exist as a reaction-force complement to gravitation, what N. Tesla conceptualized as "**radiant energy**" and mass flow from all masses towards other masses, [97].*

We could speculate that, on some way, electric charges, magnetic fluxes, linear and angular moments are always mutually coupled, complementarily integrated or packed, presenting the most important source of natural fields and forces. This is already kind of General Field Unification Platform, which is in harmony with Rudjer Boskovic's "Universal Natural Force", [6], and Nikola Tesla's "Dynamic Force of Gravity", [97].

*Citation from PowerPedia, on Internet; -"Tesla's Dynamic Theory of Gravity: The **Dynamic Theory of Gravity** of Nikola Tesla explains the relation between gravitation and electromagnetic force as a unified field theory (a model over matter, the aether, and energy). It is a unified field theory to unify all the fundamental forces (such as the force between all masses) and particle responses into a single theoretical framework".*

Also, all fields and wave phenomena (presently known in Physics) should be considered as a natural evolution of elementary, microparticles and fluids' states towards diversity of macro momentum-energy or mass states, being a kind of "communicating, coupling and gluing medium" in a space between particles. Of course, waves are always oscillations of a certain medium, or fluidic and elastic matter states (where energy can fluctuate between its kinetic and potential forms). Consequently, in an absolute vacuum state, where electromagnetic waves, neutrinos and various cosmic radiation are propagating, it should exist some fluidic matter (still not well conceptualized in contemporary physics), as N. Tesla stated many times, [97]. In the same context, particles could be considered as specifically condensed (or solidified) energy states of self-sustaining, internally folded forms of rotating matter waves (where the process of such stabilizing is directly related to internal standing-waves formations). See much more of similar ideas in [117], Jean de Climont.

R. Boskovic's and N. Tesla universal and dynamic force $\mathbf{F}(r, \theta, \phi, t)$, in order to comply with Bohr's planetary atom model, and to analogical solar systems modeling (as elaborated earlier in this chapter), should have, at least two force components $\mathbf{F_1}(r, \theta, \phi, t) + \mathbf{F_2}(r, \theta, \phi, t)$, (one being potential, and the other solenoidal vector field, as already exercised in the chapter 8.), as for instance:

$$\mathbf{F}(r, \theta, \phi, t) = \mathbf{F_1}(r, \theta, \phi, t) + \mathbf{F_2}(r, \theta, \phi, t), \nabla \times \mathbf{F} \neq 0, \nabla \mathbf{F} \neq 0 \Rightarrow$$
$$\Rightarrow \begin{cases} \nabla \times \mathbf{F}_1 = 0, \nabla \mathbf{F}_1 \neq 0 \\ \nabla \times \mathbf{F}_2 \neq 0, \nabla \mathbf{F}_2 = 0 \end{cases} . \quad (2.11.23)$$

Inside an atom, mentioned forces and fields have electromagnetic nature. Since we already know that striking analogy between atoms and planetary systems exist, we could also make model of the field of gravitation on a similar way by exploring the possibility that gravitational force is an analogical extension of interatomic forces (2.11.23), within the framework of the Extended Bohr's atom model (as presented in chapter 8). This way, we will model gravitational forces as a composition (or superposition) of one solenoidal and one potential vector field (as in (2.11.23)), but on the way as R. Boskovic suggested, and N. Tesla commented (see more in the chapter 8.).

Gravitational attraction is implicitly suggesting that there is certain energy fluctuation and gravitational potential (which has certain associated speed) between mutually attracting masses (see supporting elaborations around equations (2.4-11) – (2.4-17)). Presence of such energy fluctuations and communications (related to standing waves phenomenology) would also influence or alternate the meaning of particle velocity (in a field of gravitation that has solenoidal and potential vector components).

In the chapter 8. of this book (8.3. Structure of the Field of Subatomic Forces), we can find familiar elaborations about Gravitation as matter-waves exchange effects between atoms and Universe. "The condition of the balance of the potential energy of all attractive and all repulsive forces (see (8.74)) within an atom may be added on by a hypothesis about the existence of permanent communication, by an interchange of electromagnetic quanta between stationary states of a nucleus and electron-waves-shell of an atom. This is happening synchronously and coincidently, in both directions, so that the internal field of an atom always captures such interchange, i.e., could not be noticed in the external atom space, if an atom is really neutral, self-standing, not connected to other atoms, and non-excited. Here we could profit from analogically extension of the same conceptualization, hypothetically saying that mentioned (bidirectional) electromagnetic, quantized exchanges between atom nucleus and electrons' shell are explicable by the force of gravitation, penetrating almost endlessly towards infinity of an outer, external atom space (outside of atoms, towards other atoms and cosmic formations). In other words, all atoms are on such way connected within our Universe, continuously radiating, and receiving electromagnetic waves.

Similarly, Nikola Tesla, [97], conceptualized existence of specific "radiant energy", or radiant fluid flow from all atoms towards the universe (and vice versa). Outside of neutral atoms and other masses, we have a dominant presence of forces we qualify as gravitational attraction. Every mechanical force should be a time derivative of certain momentum, F = dp/dt. Since action and reaction forces are always mutually compensating and synchronously present, we could imagine a continuous, steady flow of certain "radiant" fluid, where microelements of such fluid have linear moments, p = mv (this way to be able to conceptualize existence of gravitational force in comparison with A. Einstein elevator).

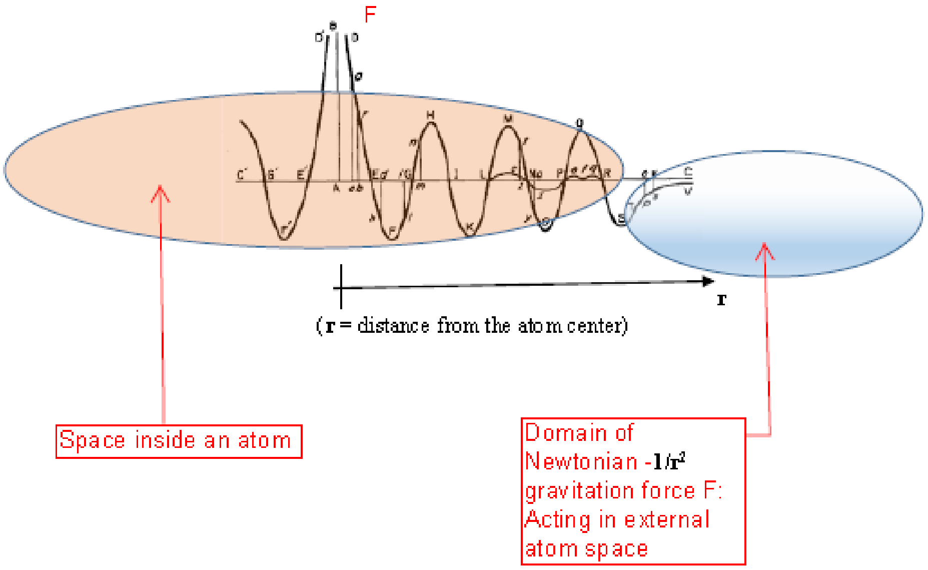
We also know that planetary or solar systems are analogically structured as atoms, respecting standing matter-waves resonant packing and couplings; -See "2.3.3. Macro-Cosmological Matter-Waves and Gravitation, 2.8. N. Bohr hydrogen atom and planetary system analogies".

External reaction forces in question (outside of atoms and masses) belong to attractive effects manifesting as Gravitation (being essentially and primarily of electromagnetic nature, since atoms are anyway communicating internally and externally by exchanging photons, including what belongs to cosmic radiation). Practically, all atoms, other particles, more significant astronomic objects, and our Universe are mutually communicating bi-directionally (or omni-directionally) by radiating electromagnetic energy, and by receiving an echo of "Electromagnetic-Gravitation" related forces.

The roots of such interpretation of an atomic or macro masses alternating force-field, which looks like standing waves resonant structure (of course, created after few derivations or integrations), are present in the works from Rudjer Boskovic, [6], about universal natural force ([6], Principles of the Natural Philosophy), as well as in certain papers published in «Herald of Serbian Royal Academy of Science» between 1924 and 1940 (J. Goldberg 1924; V. Žardecki, 1940). Nikola Tesla's, [97], Dynamic theory of Gravitation, is also very close to Rudjer Boskovic's unified natural force (see the picture below), and to a here-elaborated concept about extended Bohr's atom model, summarized with (8.74) in chapter 8. Another source of familiar ideas we can find in [73], Reginald T. Cahill, Dynamical 3-Space; -Emergent Gravity." R. Boskovic is practically explaining where repulsive (anti-gravitation) force elements are, since our generalized experience is that natural forces and charges should have positive and negative, or attractive and repulsive nature. Such effects of attractive and repulsive forces, analog to gravitation, we can produce and easily detect with ultrasonic resonators (where nodal zones are zones of only attractive forces, and anti-nodal zones are manifesting only repulsion).

$$\oiiint\limits_{\left\{ r\in[0,\infty],\ \theta\in[0,2\pi],\ \phi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \right\}} \mathbf{F}d\mathbf{R} = \mathbf{0} \,, \qquad (8.74)$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.



Rudjer Boskovic's Universal Natural Force Function

Nikola Tesla, [97], made several patents (and presented affirmative and successful experiments) showing that his "radiant fluid" or steady radiant mass flow exists, and has electromagnetic nature (being able to carry positive and negative electric charges and mechanical moments). Consequently, we can draw conclusions that all atoms and masses in our universe are mutually communicating, being familiar to Rudjer Boskovic Universal Natural Force, [6], as mutually coupled and tuned resonators, thanks to surrounding and coupling fluidic medium named as an ether (that is useful until we find another more convenient name, and better conceptualization). Properties of such ether (based on Nikola Tesla concepts) have both electromagnetic and mechanical nature (having extremely fine and small carriers of mechanical and electromagnetic moments and charges, as well as having dielectric and magnetic character).

2.4. How to unite Gravitation, Rotation, and Electromagnetism?

It should already be evident that all possible forces and fields in our Universe are anyway united, regardless whether we know how to formulate the Unified Field Theory. In given (philosophical) frontiers we can mention the best starting points for creating a new (analogous and hypothetical) field structure of rectilinear and rotational motions, which would (conveniently) follow Faraday-Maxwell electromagnetic field definition. For instance, Lorentz and Laplace forces are the explicit connection between the rectilinear motion of electrical charge (current) and magnetic field and can be transformed to analog forms of a specific interaction between participants being in rectilinear and rotational movement (using concepts of analogies already elaborated here). The Ampere-Maxwell's, Biot-Savart's, and Faraday's induction laws can serve to complete the previous situation mathematically, for a more precise description of "linear-rotational" fields (just by transforming mentioned laws into corresponding analog expressions). See larger background about electromagnetic and mechanical complexity and essential origins of Gravitation in Chapter 3.

Many attempts are already known in modern science regarding the formulation of the Maxwell-like theory of gravitation and explaining the origins of inertia. The traditional formulation of gravitation field takes mass as a (primary) source of gravity. However, in this book it is demonstrated that more essential (and dominant) sources of gravity-related phenomenology should be found in different interactions between moving objects, such as, between their linear and/or angular (or spinning) moments, which are coupled with certain electric and magnetic moments or dipoles, and between their motional and state of rest energies. (See also chapter 4.1 of this book for more supporting elements regarding associated de Broglie, matter waves). Revitalizing and updating Wilhelm Weber's force law (to cover electromagnetic and gravity related interactions, with combined linear and rotational motion elements) would be a very healthy platform towards establishing a new Maxwell-like theory of gravitation (see literature under [28] and [29]).

Most probably that many force/fields manifestations and components of constant, or accelerated movements, (such as Coriolis, centrifugal, centripetal, gyroscope-effect, pendulum oscillations, inertial and similar forces, Gravitomagnetic induction from General Relativity Theory, etc.) could conveniently be incorporated, interpreted and mutually united with here proposed concepts about gravitation. It is conceptually already clear what the author of this book is suggesting related to links between rotation, linear motion, and electromagnetism (see chapter 10; -equations (10.1.4) - (10.1.7)). See also equations (4.18), (4.22) - (4.29), (5.15) and (5.16).

After establishing a new platform for understanding the complementary nature of "linear-rotational" fields and motions (see chapters 4.0 to 4.3 of this book), we shall have an open way for creating a full set of "Gravity-Rotation" field equations, making them initially analog with Maxwell equations of an electromagnetic field. Later, we could modify and upgrade such equations up to the most meaningful and useful forms that will correspond to the reality of different natural fields and forces (see the development of equations (4.22) - (4.29)). Later (in Chapter 3), it will be shown that Maxwell Theory should also be slightly upgraded to become compatible for unification with the upgraded theory of Gravitation (see also literature [23] – [26]).

Anyway, to present a significant and new insight regarding gravitation, we would need to introduce unique and original concepts that do not show only redundant and analogical variations of already known field theories. Let us initiate one of such thoughts, as follows.

2.5. New Platforms for Understanding Gravitation

Gravitation can also be conceptualized by making analogies with mechanical or acoustical resonators. *Let us imagine that our macro-universe or cosmos effectively presents a kind of fluid-like substance with a different particle or mass agglomerations submersed (or hanging) in such substance. Mentioned particle agglomerations (in the frames of this conceptualization) would be different cosmic objects, planets, stars, galaxies, dust, atoms, plasma states etc. Let us now imagine that such composite cosmic fluid is being mechanically vibrated by certain constant frequency (from an external, presently unknown source of mechanical vibrations). In case of performing a real experiment (just to visualize the concept and make relevant analogies), in a vessel filled with liquid that is mixed with solid particles, by vibrating such vessel we will notice creation of three-dimensional standing waves structure, where submerged (and suspended) particles would make higher mass density, or mass agglomerations in nodal areas of standing waves. Such effects are known as acoustic and/or ultrasonic effects of levitation. Nodal areas, in this case, are zones where oscillating velocities are minimal (or zero), and oscillating forces and mass-density are maximal. If we intentionally introduce a small test particle somewhere in a vicinity of any of such nodal areas with high mass density (while vessel filled with liquid and other particles is resonating), we will notice that the test particle will be attracted by the closest nodal zone (or closest particle). Of course, here we are temporarily excluding cases of involvements of possible electromagnetic forces to make the situation quite simple in its first brainstorming steps. Similar attractive force (in the vicinity of a nodal zone) can be observed in the case of resonant, standing wave oscillations of half-wavelength solid resonators or multiple half-wavelength resonators, known in ultrasonic technology). If external vibrations that are driving mentioned resonators are suddenly switched-off, the attractive forces towards nodal areas will disappear. Now we could conceptualize our universe as an equivalent (or analogical) mechanical fluid-like system that is permanently in a state of* ***<u>very low frequency</u>*** *resonant and standing waves oscillations (see time-frequency relations (5.14-1) in Chapter 5, Uncertainty). Such standing-waves oscillations are forcing all astronomical objects to take only certain stable nodal positions (or orbits) of the easiest agglomerating areas, which are kind of its space-matrix texture (apart from other linear and rotational motions, involved). Placed around such astronomical objects (planets, stars, galaxies…), every test mass would experience only an attractive force (similar like in cases of gravitation). Later, the same initial concept can be upgraded by considering linear and rotational motions (of submerged particles, or astronomical objects) that are again forced to comply with agglomeration rules around global standing-waves nodal areas, complying with the framework of Euler-Lagrange-Hamilton mechanics (see similar concepts in [99] from Konstantin Meyl).* ***Understanding of mass, as conceptualized here, is indirectly considering that any mass is a storage or modus of matter-waves energy packing or agglomerating (and in the same time kind of "frozen, rotating energy states").*** *The problem here could be the fact that we know that between astronomical objects in our universe there is significant "empty-space of vacuum states", and our imaginative fluid-substance (which should mechanically resonate)* *would have problems regarding performing only mechanical vibrations, but such problems will be eliminated since acoustic, mechanical, electromechanical and electromagnetic vibrations are anyway mutually coupled. Again, we would need to understand the specific nature of such fluid-like substance that is a carrier of externally introduced mechanical vibrations on some new innovative way. Contemporary physics made efforts to show that ether-type fluids are not something that could be experimentally confirmed. In mechanics and acoustics, we already know that vacuum cannot be a carrier of mechanical vibrations, and for supporting here introduced concept of gravitation, we need to have kind of mechanical resonant and standing waves states of our universe. Most probably that specific electromagnetic, magnetostrictive and/or electrostrictive coupling nature should also be involved here (in a framework of coupled oscillators) to realize penetration of mechanical vibrations through vacuum and empty-space states (and certainly vacuum in our universe is not at all an empty space). Anyway, the situation regarding explaining gravitation, as initiated here, could be richer and different compared to present Newton, Kepler, and Einstein framework, since none of them is explaining why gravitation is only manifesting as an attractive force. In the same time, we know that standing-waves mechanical resonators are easily showing the existence of such (only) attractive forces in their nodal areas (creating acoustic levitation), and by analogy, we could make hypothetical predictions regarding what behind the force of gravitation should be. The remaining question to answer here would be, where and what the source of mentioned vibrations is, or what is the source of cosmic standing waves?* ***Since all constituents of our universe are mutually connected and interacting, as well as in permanent relative motions and we know that matter-waves are associated with mass motions, this should be an important element of the answer regarding origin of mentioned intrinsic vibrations and their standing waves (in the context of understanding the nature of gravitation).*** *Einstein's General Relativity Theory is already explaining gravitation from specific space and fields' geometry-related modifications and deformations, taking this as a fact, not speculating (as here) that specific spatial-matrix texture could be a consequence of complex resonating, standing waves formations. Since everything that exists in our Universe is anyway mutually cross-linked, coupled, and united (in some cases most probably without our full knowledge about such unity), any new theory about Gravitation should consider electromagnetic and other forces coupled with gravitation. Of course, the ordinary Newton (static) gravitation force is for many orders of magnitude weaker than all other forces (electromagnetic, nuclear…), compared on the same scale, making that we usually neglect interactions between gravitation and other fields. If the concept proposed here has enough practical and theoretical grounds, this would be a breakthrough in the novel and better understanding of gravitation (being also applicable to other forces like electromagnetic ones). Another contemporary fields-unification theory (which is going much deeper and wider in conceptualizing a multi-dimensional space with its elementary and vibrating building blocks that are taking forms of strings and membranes) that could be in some ways familiar to here introduced concepts is the Superstrings or M-theory (which is still evolving and searching for its best foundations). We should not forget that any new concept of gravitation should be simple, elegant, and well-integrated into remaining chapters of physics that are already working well, and some attempts in creating such modeling will be made later.*

Let us review original abstracts from different publications, showing new, emerging aspects of still evolving, and future theory of gravitation:

A) **Dynamical 3-Space. Emergent Gravity**; Reginald T. Cahill, School of Chemical and Physical Sciences, Flinders University, Adelaide 5001, Australia, E-mail: Reg.Cahill@inders.edu.au . Invited contribution to: **Should the Laws of Gravitation be reconsidered**? Héctor A. Munera, ed. (Montreal: Apeiron 2011), **[73].**

The laws of gravitation devised by Newton, and by Hilbert and Einstein, have failed many experimental and observational tests, namely the borehole g anomaly, at rotation curves for spiral galaxies, supermassive black hole mass spectrum, uniformly expanding universe, cosmic filaments, laboratory G measurements, galactic EM bending, precocious galaxy formation,... The response has been the introduction of the new epicycles: "dark matter", "dark energy", and others. To understand gravity, we must restart with the experimental discoveries by Galileo, and following a heuristic argument, we are led to a uniquely determined theory of a dynamical 3-space. That 3-space exists has been missing from the beginning of physics, although it was first directly detected by Michelson and Morley in 1887. Uniquely generalizing the quantum theory to include this dynamical 3-space, we deduce the response of quantum matter and show that it results in a new account of gravity and explains the above anomalies and others. The dynamical theory for this 3-space involves G, which determines the dissipation rate of space by matter, and α, which experiments, and observation reveal to be the fine structure constant. For the first time, we have a comprehensive account of space and matter and their interaction - gravity.

B) **The Nature of Space and Gravitation; Jacob Schaff.** Instituto de Fsica, Universidade Federal do Rio Grande do Sul (UFRGS), Porto Alegre, Brazil. Email: schaf@if.ufrgs.br. Received May 12, 2012; revised June 8, 2012; accepted July 1, 2012. doi: 10.4236/jmp.2012.38097. Published Online August 2012 (http://www.SciRP.org/journal/jmp), **[74].**

Many recent highly precise and unmistakable observational facts achieved thanks to the tightly synchronized clocks of the GPS, provide consistent evidence that the gravitational fields are created by velocity fields of real space itself, a vigorous and very stable quantum fluid like spatial medium, the same space that rules the propagation of light and the inertial motion of matter. It is shown that motion of this real space in the ordinary, three dimensions around the Earth, round the Sun and round the galactic centers throughout the universe, according to velocity fields strictly consistent with the local main astronomical motions, correctly induces the gravitational dynamics observed within these gravitational fields. In this space-dynamics, the celestial bodies, all firmly rest with respect to the real space, which, forth-rightly leads to the observed null results of the Michelson light anisotropy experiments, as well as to the absence of effects of the solar and galactic gravitational fields, on the rate of clocks moving with Earth, as recently discovered with the help of the GPS clocks. This space dynamic exempts us from explaining the circular orbital motions of the planets around the Sun; likewise, the rotation of Earth exempted people from disclosing the diurnal transit of the heavens in the days of Copernicus and Galileo because it is space itself that so moves. This space dynamic also eliminates the need for dark matter and dark energy to explain the galactic gravitational dynamics and the accelerated expansion of the universe, respectively. It also straightforwardly accounts regarding well-known and genuine physical effects for all the other observed effects, caused by the gravitational fields on the velocity of light and the rate of clocks, including all the new effects recently discovered with the help of the GPS. It moreover simulates the non-Euclidean metric underlying Einstein's space-time curvature. This space dynamics is the crucial innovation in the current world conception that definitively resolves all at once the troubles afflicting the current theories of space and gravitation.

C) **Deriving gravitation from electromagnetism**. Can. J. Phys. 70, 330- 340 (1992). A. K. T. ASSIS[1]. *Department of Cosmic Rays and Chronology, Institute of Physics, State University of Campinas, C. P. 6165, 13081 Campinas, Sao Paulo, Brazil.* Received November I, 1991. Can. J. Phys. 70.330 (1992), **[75].**

We present a generalized Weber force law for electromagnetism including terms of fourth and higher order in v/c. We show that these additional terms yield an attractive force between two neural dipoles in which the negative charges oscillate around the positions of equilibrium. This attractive force can be interpreted as the usual Newtonian gravitational force as it is of the correct order of magnitude, is along the line joining the dipoles, follows Newton's action and reaction law, and falls off as the inverse square of the distance.

D) The Electrodynamic Origin of the Force of Gravity, Part 1; ($F = Gm_1m_2/r^2$). **Charles W. Lucas, Jr.** 29045 Livingston Drive, Mechanicsville, MD 20659-3271, bill@commonsensescience.org, **[76].**

The force of gravity is shown to be a small average residual force due to the fourth order terms in v/c of the derived universal electrodynamic contact force between vibrating neutral electric dipoles consisting of atomic electrons vibrating concerning protons in the nucleus of atoms. The derived gravitational force has the familiar radial term plus a new non-radial term. From the radial term, the gravitational mass can be defined in terms of electrodynamic parameters. The non-radial term causes the orbits of the planets about the sun to spiral about a circular orbit giving the appearance of an elliptical orbit tilted concerning the equatorial plane of the sun and the quantization of the orbits as roughly described by Bode's law. The vibrational mechanism that causes the gravitational force is shown to decay over time, giving rise to numerous phenomena, including the expansion of the planets (including the earth) and moons in our solar system, the cosmic background radiation, Hubble's redshifts versus distance (due primarily to gravitational redshifting), Tifft's quantized redshifts (Bode's law on a universal scale), Tifft's measured rapid decay of the magnitude of redshifts over time, the Tulley-Fisher relationship for luminosity of spiral galaxies, the unexpected high velocities of the outer stars of spiral galaxies, and Roscoe's observed quantization of the luminosity and size (Bode's law) of 900 spiral galaxies. Arguments are given that this derived law of gravity is superior to Newton's Universal Law of Gravitation ($F = Gm_1m_2/r^2$) and Einstein's General Relativity Theory ($G_{\mu\nu} = -8\pi G/c^2 T_{\mu\nu}$).

The next citation (or copy) from ***[83], David L. Bergman, editor: Selected Correspondence on Common Sense Science #1. November 2013, Volume 16, number 4. Common Sense Science, P.O. Box 767306, Roswell, GA 30076-7306, E-mail: bergmandavid@comcast.net***

……………………..

Gravity Dilemma. I seem to remember that gravity cannot be shielded. An internet search agrees. Yet I believe electromagnetism can be shielded. If so, how can gravity be of electrodynamic origin?

Russel Moe,
Wildwood, Florida

Reply by Dave Bergman. More than once I have asked myself the same question. Eventually I resolved the dilemma in this way. First, recall that matter is composed of elementary particles—mostly electrons and protons.

Second, every electron and every proton has self-generated electromagnetic fields surrounding the charged particle and spreading outward into an increasing volume of space with field intensities that decrease in accordance with the inverse square law that applies also to gravity.

Third, these electromagnetic fields have an *oscillating component* and a *non-oscillating component*, corresponding respectively to radiation of energy (including **light** energy) and **gravity**.

Fourth, actual measurements show that shielding of an oscillating electromagnetic field ranges from *no shielding* to *partial shielding* to *high shielding* in correspondence with the **wavelength** of the oscillation [D. G. Fink, Editor-in-Chief, **Standard Handbook for Electrical Engineers, Tenth Edition,** McGraw-Hill Book Company, p. 29-23, fig. 29-40 (1957).]

♦♦♦♦♦♦♦♦♦♦♦

Reply by Dr. Lucas. Good question. The answer is that the experiments of William J. Hooper as described in his book **New Horizons in Electric, Magnetic and Gravitational Field Theory** identify that all three types of electric and magnetic fields have different empirical properties. For instance in Chapter 1 Hooper lists 14 empirical properties of E fields. In Table 1 he gives what these properties are for electrostatic E fields, E fields dependent on dA/dt, and E fields dependent on motion $V \times B$. In particular he notes in property 6 that the motionally caused E fields cannot be shielded.

There is an interesting story regarding this E field that cannot be shielded. When Hooper discovered this effect, he invented and patented a speedometer for airplanes. When a plane was flying with respect to the surface of the earth, Hooper's speedometer measured the $V \times B$ term due to the velocity of the plane with respect to magnetic field lines of the earth being crossed.

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

He arranged a demonstration with the military. They flew Hooper's speedometer and compared it readings with respect to other methods. The military testers confirmed that Hooper's speedometer worked very well. However, when Hooper applied for a government contract to supply the military with these speedometers, his application was rejected by the scientific reviewers, because they said that the metal hull of the military aircraft would shield the effect. According to Maxwell's theory of electrodynamics, all three forms of the electric field have the same identical properties. The scientific reviewers ignored the fact that the military testers had tested Hooper's speedometer inside a metal-hulled military airplane, and it worked inside that shielded environment.

In our derivation of an improved and more general version of the electrodynamic force law, we treat each of the three types or sources of electric fields separately as distinctly different types of E fields.

It is the term proportional to $R(R \times V)$ in the universal electrodynamic force law that gives rise to the force of gravity. It is equivalent to Hooper's motional E field.

…………………………………..

specifically to the helical charged particles. Thus, inertial mass was not fundamental, but a specific, calculable property of the unit charged particles due to their electro-magnetic structure.

Your approach differs in considering a net neutral, but finite size, structured combination of particles, interacting with all the other such dipole particles in the universe. Thus, the amount of inertia demonstrated by a particle's resistance to motion is not even a specific amount, calculable from the charge, speed of light c and physical dimensions, but depends on the amount and distribution of all the other particles in the universe.

……………………………………………………

There is a hierarchy of interactions in electrodynamics in order of decreasing strength as follows:

1. Charge to charge – Coulomb force
2. Charge to neutral electric dipole – inertial force
3. Neutral electric dipole to neutral electric dipole – gravitational force
4. Charge to neutral electric quadruple – dust or plasma aggregation & rotation
5. Neutral electric dipole to neutral electric quadruple – dust aggregation & rotation
6. Neutral electric quadruple to neutral electric quadruple – dust aggregation & rotation etc.
7. The charge to vibrating neutral electric dipole force is second order in the hierarchy.

The sentence "In the next section the interaction force that gives rise to the force of gravity will be considered." should have been "On the grand scale the interaction force that gives rise to the force of gravity is defined in terms of inertial mass as…."

> "Thus the interaction force that Einstein referred to above that gives rise to the force of inertia of any specific dipole pair is due to the vibration of that pair, and all the other vibrating, neutral, electric dipoles in the rest of the universe."

would perhaps be better stated

> "Thus the interaction force that Einstein referred to above that gives rise to the force of inertia of any specific dipole pair is due to the interaction of that dipole with all the other charges in the universe."

In a similar manner one could say that

> "The interaction force that gives rise to the force of gravity on any specific electric dipole pair is due to the interaction of that electric dipole with all the other electric dipoles in the universe."

The notion of mass is somewhat poorly defined in science. In the Standard Model of elementary particles and Einstein's theory of relativity, mass is an inherent property of a point-elementary-particle. Thus the specifics of the interactions within the structure of the particle are not taken into account. However, if one measures the *mass* of an atom, it is not the sum of the masses of the electrons, protons, and neutrons, but something less indicating that if the mass of a particle depends on internal structure, it changes within the environment of the atom. If one measures the total *charge* of an atom, it is exactly the sum of the charges of the electrons, protons, and neutrons in the atom. *Thus mass is not an inherent fixed quantity like charge!*

From the definition of the force of inertia $F = ma$, mass m is a measure of resistance to motion in some environment. In fluid dynamics the resistance to motion of a particle depends on the density of the fluid around it. Thus mass depends more on the environment than on fixed inherent local properties of the particle of mass. The total resistance to motion is not inherent to the particle, but depends on its environment.

In the work of Barnes and Bergman there is a feedback effect due to Lenz's Law on a charged elementary particle causing it to have a mass of inertia. From Mach's Principle we can see that there are two parts to this mass. The first is the local asymmetry term, and the second is the contribution on the grand scale from the rest of the universe. The first term will depend intimately on the internal structure of the particle. The second term will depend heavily on the symmetry of the universe.

There can be multiple types of contributions to inertial mass. In my work so far I have concentrated on the hierarchy of electrodynamic interactions, i.e.

1. charge to charge - Coulomb force
2. charge to electric dipole - force of inertia
3. electric dipole to electric dipole - force of gravity, etc.

Besides these supposedly primary contributions, there can be secondary and tertiary contributions just as in the case of the atom where there are fine structure and hyper-fine structure contributions. My electrodynamic force derivation of the force of inertia is able to explain the origin of the cosmic microwave background radiation and its spectrum, the unusual gyroscope experiments of Eric Laithwaite in defiance of Newton's force of inertia, the constant high velocity of the outer arms of spiral galaxies in defiance of General Relativity requiring the invention of dark matter, and the general expansion of the universe in defiance of General Relativity requiring the invention of dark energy.

…………………………………………………..

Reply by Charles Wm. (Bill) Lucas, Jr. See the URLs below for the "politically correct view" that Russell Humphreys relies upon.

http://en.wikipedia.org/wiki/Hafele%E2%80%93Keating_experiment

http://en.wikipedia.org/wiki/Time_dilation_of_moving_particles

http://en.wikipedia.org/wiki/Pound%E2%80%93Rebka_experiment

For time dilation of moving particles, Special and General Relativity theories are point-particle theories. No real elementary particles such as protons, neutron, muons, etc. are point-particles. All have both finite size and internal structure. Since in my work these elementary particles consist of multiple charge current loops, the elementary particles experience an electromagnetic feedback effect when they move that compresses the particle and increases its binding energy and effective mass. The increased binding energy causes the half-life of the muon to increase with velocity. Special Relativity theory mathematically predicts the same result, but for the wrong reasons due to its use of many idealizations such as the point-particle idealization and that space is homogeneous and isotropic that does not correspond to reality.

The gravitational redshift—a tenet extrapolated from Einstein's theory of general relativity—claims that clock rates change with gravitational potential, as a result of space-time being bent by objects of large mass. Ed Dowdye has shown that for starlight passing near the sun there is no bending of the path of the light due to general relativity theory. The only bending of starlight that occurs is when the light passes through the electrical plasma rim of the sun due to electromagnetic effects. At larger distances the predicted bendings of light due to general relativity theory are *not* observed.

With regard to the Pound-Rebka experiment at Harvard in 1959 to detect the redshift and blue-shift in light moving in a gravitational field due to clocks running at different rates at different places in a gravitational field, note that the Mossbauer effect was used. In 1958 Mossbauer had reported that all the atoms in a solid lattice absorb the recoil energy when a single atom in the lattice emits a gamma ray. The test is based on the following principle: When an electron in an atom transits from an excited state to a ground state, it emits a photon with a specific frequency and energy. When an electron in an atom of the same species in its ground state encounters a photon with that same frequency and energy, it will absorb that photon and transit to the excited state. If the photon's frequency and energy is different by even a little, the atom cannot absorb it (this is the basis of quantum theory). When the photon travels through a gravitational field, its frequency and therefore its energy will change due to the gravitational redshift. As a result, the receiving atom cannot absorb it. But if the emitting atom moves with just the right speed relative to the receiving atom the resulting Doppler shift cancels out the gravitational shift and the receiving atom can absorb the photon.

http://en.wikipedia.org/wiki/Doppler_shift

The "right" relative speed of the atoms is therefore a measure of the gravitational shift. The frequency of the photon "falling" towards the bottom of the tower is blue-shifted. Pound and Rebka countered the gravitational blue-shift by moving the emitter away from the receiver, thus generating a relativistic Doppler redshift. The energy associated with gravitational redshift over a distance of 22.5 meters is very small. The fractional change in energy is given by $\partial E/E = gh/c^2 = 2.5\times10^{-15}$. Therefore short wavelength high energy photons are required to detect such minute differences.

http://en.wikipedia.org/wiki/Electromagnetic_spectrum

When it transitions to its base state, the 14 keV gamma rays emitted by iron-57 proved to be sufficient for this experiment.

…………………………………………

The Doppler shift required to compensate for this recoil effect would be much larger (about 5 orders of magnitude) than the Doppler shift required to offset the gravitational redshift. But in 1958, Mössbauer reported that all atoms in a solid lattice absorb the recoil energy when a single atom in the lattice emits a gamma ray.

http://en.wikipedia.org/wiki/Rudolf_M%C3%B6%C3%9Fbauer

http://en.wikipedia.org/wiki/Lattice_model_(physics)

http://en.wikipedia.org/wiki/M%C3%B6ssbauer_effect

<http://en.wikipedia.org/wiki/Pound%E2%80%93Rebka_experiment#>

Therefore the emitting atom will move very little. However, the notion of a photon being emitted or absorbed on a single electron of an atom is not supported by the Mossbauer effect. The wave nature of light is supported by the Mossbauer effect where the crystal lattice acts as an antenna for emission and absorption of light.

In Dr. Lucas's electrodynamic theory of gravity, vibrating neutral electric dipoles are the source of the gravitational force. The movement of the vibrating neutral electric dipoles changes the strength of the gravitational field to match the red and blue shifts. Thus there is no role for Einstein's general relativity theory. This result is also consistent with the work of Ed Dowdye showing that there is no gravitational bending of starlight due to general relativity. Thus the so-called gravitational redshift of light supports only the electrodynamic theory of gravity, *not* General Relativity theory.

Charles W. Lucas, Jr.
Mechanicsville, Maryland

2.6. A short resume of possibilities for direct experimental and theoretical verification of innovated theory of Gravitation concerning Particle-Wave Duality and Matter Waves:

1. Vortex flow meter, Karman Vortex Streets, vortices-frequency, and Strouhal-Reynolds number, in linear and robust relationship to fluid flow velocity, can be explained using here-elaborated coupling between linear and spinning motions, including associated matter-waves and particle-wave duality concepts. A similar concept can be analogically extended to motions of planets within planetary systems. Modern engineering is using vortex flowmeters since a very long time, without real and fundamental explanation and insight why fluid flow velocity is directly and linearly proportional to vortices frequency (see Chapter 4.1, where vortex flow meter equation is developed and explained as the consequence of liner and spinning motions coupling).

2. *The work of matter-waves and associated gravitation related forces we can find in analyzing a **spin-stabilized satellite.** This is a satellite, which has the motion of one axis held (relatively) fixed by spinning the spacecraft around that axis, using the gyroscopic effect. The attitude of a satellite or any rigid body is its orientation in space. If such a body initially has a fixed orientation relative to inertial space, it will start to rotate, because it will always be subject to small torques. The most natural form of attitude stabilization is to give the rigid body an initial spin around an axis of minimum or maximum moment of inertia, meaning in the same direction where helical matter waves tend to be naturally created. The body will then have a stable rotation in inertial space. Rotation about the axis of minimum moment of inertia is at an energy maximum for a given angular momentum, whereas rotation about the axis of maximum moment of inertia is at a minimum energy level for a given angular momentum. In the presence of energy loss, as is the case in satellite dynamics, the spin axis will always drift towards the axis of maximum moment of inertia. For short-term stabilization, for example, during satellite insertion, it is also possible to spin-stabilize the satellite about the axis of minimum moment of inertia. However, for long-term stabilization of a spacecraft, spin stabilization about its axis of maximum moment of inertia must be used (read about Lyapunov stability concept). Here is the place to explain, harmonize and generalize mentioned items with **PWD**, matter waves conceptualization, and convenient mathematics (as vastly elaborated in this book).*

   We can also find supporting background to helical matter waves associated with modern-guns bullets motion. Rifling is the process of making helical grooves in the barrel of a gun or firearm, which imparts a spin to a projectile around its long axis. This spin serves to stabilize a projectile gyroscopically, improving its aerodynamic stability and accuracy. Bullet stability depends primarily on gyroscopic forces, the spin around the longitudinal axis of the bullet imparted by the twist of the rifling. Once the spinning bullet is pointed in the direction the shooter wants, it tends to travel in a straight line, until it is influenced by outside forces, such as gravity, wind, and impact with the target. Without spin, the bullet would tumble in flight. Modern rifles are only capable of such fantastic accuracy because the bullet is stable in flight (thanks to the gyroscopic effect). Even spherical projectiles must have a spin to achieve any acceptable accuracy. We could analogically reverse the cause and effect in the same situation (of spinning bullets propagation), and say that particle in inertial, stationary, and stable motion should have helical, spinning matter wave around its long axis of propagation, because such matter wave and moving particle are in mutually relative spinning motion (either one or the other is spinning). Such factual situation (of enormously increased bullets accuracy and stability) should be verifiable by mathematical relations that are used in matter waves' conceptualization.

3. In addition, since linear and helix or spinning and rotating motions (of masses) are mutually complementary and united (in relation to matter-waves, or **PWDC** elaborated in this book, Chapter 4.1), we could imaginatively, creatively and analogically extrapolate such concepts from atoms to planetary systems and galactic formations, and try to differently address Hubble's law. On some way, our universe is globally rotating and spinning, following helix-like paths of associated matter waves. What we observe as red or blue, Doppler shifts (of electromagnetic radiation) coming from remote, deep space, could be consequences of such globally present, macro-rotating effects of distant masses. As we know, the tangential velocity of the certain rotating mass, $\mathbf{v_t}$ is equal to the product of relevant orbital velocity ω, and relevant radius $\mathbf{R}$, $v_t = \omega R$. Hubble's law is maybe saying something similar, such as, $v = (v_t) = H_0 R$, where H_0 is Hubble constant, which could be certain metagalaxy, orbital (or helicoidally spinning associated matter waves) velocity. If an expansion of our Universe is on some ways partially mistaken and masked by, or related to such macro-rotation, this will open a window into amazing new Cosmology research areas.

4. Another similar phenomenon is related to helical liquid funneling (spiral spinning) when liquid is in a vessel that has an open hole, or a sink at the bottom. Outgoing liquid flow speed should be directly proportional to the spiral, funneling frequency of the liquid in the same vessel (this is the proposal for an experimental verification). The explanation should take into consideration that every linear motion is intrinsically linked to spinning in the same direction of movement (as elaborated in this book).

5. Macro matter-waves related situation (that is analogical to micro-world, de Broglie matter-waves) are also waves on a quiet water surface created by some moving object (a boat), where the water surface is visualizing matter-waves associated to a moving object. An average wavelength of such surface water-waves should be roughly equal to $\lambda = H/p = H/mv$ (or inversely proportional to the motional object speed, where (m, H) = constants). Of course, here we should be able to show the applicability of other matter-waves relations elaborated in this book, such as: $E_k = \tilde{E} = \frac{mv^2}{2} = \frac{pv}{2} = Hf, \Rightarrow u = \lambda f = \frac{v}{2}$. For instance, we can easily measure if the phase speed of surface water-waves, u, behind moving particle (or boat), is two times smaller than particle speed v.

6. Turbulences (including vortex turbulences) are also direct manifestations, or imprints, streaming and reverberations of matter-waves associated with masses motions in and around fluid environments. Fluids (in relative motion to the specific particle) should present sensitive sensor bodies (or spatial antennas and displays) for detecting and visualizing matter waves, vortices, spinning, and turbulences, including the possibility to detect astronomic macro matter-waves. Of course, cosmic macro matter waves could be detected when observing low-frequency acoustic fields' complexity inside vast lakes and ocean spaces. Present conceptualization related to Fluid dynamics and Navier-Stokes equations should also be enriched and optimized by considering here-elaborated matter-waves manifestations, as helically rotating fields' perturbations around and behind motional particles (see chapter 4.1), and the same should apply to enriching matter-waves concepts with elements of Fluid dynamics and Navier-Stokes equations. Also, we could extend familiar analogical associations to orbital planetary motions in solar systems (see Chapter 2. Gravitation; 2.3.3. Macro-Cosmological Matter-Waves and Gravitation).

7. Many publications are showing (and experimentally documenting) weight reduction when certain spinning discs, spinning magnets, and gyroscopes, combined with other rotating and oscillating motions are specifically coupled (see much more in [36]). Such, seemingly anti-gravity effects are in fact consequences of natural, linear and rotational motion couplings (including associated electromagnetic dipoles separation and polarization), when involved spinning discs and oscillatory systems are interacting with balanced natural motions, producing unbalanced effects of weight reduction (as long as we maintain such movements). Here, we should not forget that besides Newton linear motion forces, we also have coupled effects of rotational or torque forces. A certain specific combination of implemented torque components (by spinning disks or magnets) can be the forces acting against gravitation.

8. Universally valid and known effects of diffraction of light rays, particle beams, fluid beams, jets and similar phenomena, could also be explicable if we take into account that certain repulsive force (in relation to associated matter-waves and spinning) is developing between effective mass (or matter-waves) packets of "parallel flow elements". Such repulsive force should be causally related to the resonant half-wavelength of involved matter-waves, being the consequence of unity and couplings between linear and spinning motions. The force law that is addressing diffraction (or beams repulsion) should respect Newton-Coulomb $1/r^2$ force law (see [3]). If centers of active mass packets in described parallel motion are mutually separated by one half-wavelength, the repulsive force (or measured diffraction angle) should be maximal.

9. Theories related to "standing-waves" quantizing of planetary systems (as one elaborated in this book, in chapter 2.) that are directly analog to quantizing in Bohr atom model, are practically showing the triumph of here-elaborated matter waves and linear-rotational motions coupling concepts. In such modeling it is possible to integrate electromagnetic effects, like in early atom models (see [63] and [67]), showing existence of more complete unity between mechanical motions, gravitation and electromagnetic fields, since such rich quantization in planetary systems (like in atoms) cannot exist without the dominant presence of electromagnetic forces and fields. In such quantized systems of synchronized and periodic (planetary) motions with standing waves structure, it is possible to associate masses presence only to stable stationary orbits that are equivalent to spatial nodal zones (where orbital acceleration and density are maximal, and oscillating amplitudes minimal, like in resonant mechanical systems and standing-waves related acoustic and/or ultrasonic levitation). See more of familiar concepts in [99] from Konstantin Meyl.

10. Our sun and a countless number of stars can be regarded as a variety of blackbody objects (concerning blackbody radiation). Planck's blackbody radiation formula is mathematically fitted to experimentally measured situations, but the real, essential explanation of what is happening inside of a black body cavity is still missing. We can try to estimate what happens inside a black body cavity where we have complex, random motion of hot gas particles, random light emissions, absorptions, photons, and electrically charged particles collisions and scattering (including participation of particles with magnetic moments). We only know from Planck's formula the resulting (fitted and averaged) spectral distribution of outgoing light emission, in the case when we make a small hole on the surface of a black body, and let photons be radiated and measured in the external, free space of a black body. This external light radiation is characterized by free photons where each photon has the same phase and group velocity $v = u = c = \text{constant}$. This is not the case inside the black body cavity, since there are many mechanical and fields interactions between photons, gas particles, matter waves, and cavity walls, and there we have broad distributions of group and phase velocities of different energy-momentum entities,

$0 \leq 2u \leq \sqrt{uv} \leq v \leq c$. A significant number of wave packets (de Broglie matter wave groups with mutually-united or coupled mechanical and electromagnetic properties, with linear and spinning motion components), inside a blackbody cavity, permanently interact (among themselves, as well as with the cavity and gas particles), and we cannot consider them being freely propagating wave groups, or stable and synchronized standing matter-waves formations. It is logical (as the starting point in an analysis of such case) to imagine that mean particle or group velocity of such wave groups is (in average) directly proportional to the blackbody temperature, and when gas temperature (inside a black body radiator) is relatively low, than we should dominantly have motions with non-relativistic particle velocities ($v << c \Rightarrow v \cong 2u$). When a temperature is sufficiently (or remarkably) high, we should dominantly have the case of relativistic particle motions with high speeds ($v \approx c \Leftrightarrow v \approx u \approx c$). There is a big difference between free wave groups, like free photons in open space, and mutually interacting (de Broglie) matter-waves (inside of a limited space of a black body cavity). On the contrary, in most analyzes of similar situations in modern Quantum Mechanics, we do not find that such differentiation is explicitly underlined and adequately treated (mostly we see that de Broglie matter waves are treated similarly to free photons or to other free wave groups, or as virtual and artificial probability waves). Also, in mathematical development of Planck's blackbody radiation law, we can essentially find specific particularly suitable (oversimplified, mainly poor and unrealistic) modeling and curve-fitting situations, where phase velocities of a black body photons are always treated as the velocity of free (externally radiated) photons, or as $u = \lambda f = v = c = \text{Constant}$. This book is offering new elements to understand and develop blackbody radiation formula on a more natural way (see such elaborations in chapter 4.1 and chapter 9).

11. *Very much neglected, or still not well conceptualized approach in addressing mechanical systems and mechanical-circuits, is something that is very much known and practiced in electric-circuits and electromagnetic theory. Every electrical circuit, to be completely described and understood, should be treated as a closed circuit, or as a network with electric components, that has its front-end (input generator or electricity source) and its last-end or load. Also, all internal circuits (as network elements) should be treated and analyzed as closed current-flow circuits, both as being, either DC, or AC, or mixed currents circuits. Analogically valid is that all mechanical systems or mechanical and acoustical circuits should also have front and last ends (meaning sources and loads). Currents in mechanical systems (or circuits) are forces and angular moments, presenting temporal flow functions of linear and angular momenta. In the contemporary Physics (mechanics, acoustics, vibrations theory...) we are still too often presenting and analyzing open-ends circuits, either without front or last-ends or without both. Doing this way, we are not in a position to understand the real and complete nature of the specific mechanical or planetary (or astronomic) system, and its connections with other mechanical systems, and we do not see that such systems could also be connected on number of ways, realizing energy-momentum exchanges with different matter states and matter waves. Mechanical currents (meaning forces and angular moments) can have both DC and AC nature (using the analogy with electric currents; -see much more about electromechanical similarities in the first chapter of this book). Also, hypothetical and innovative proposals presented in this book are indicating that electric currents and effective mechanical orbital moments (within the specific motion of particles and masses) could often be mutually coupled and synchronized, followed by magnetic field effects, what should produce results of gravitational attraction. Only such conceptualization, analysis, and understanding of all mechanical systems could bring the proper understanding of what is happening in our Universe. If carefully and imaginatively analyzed, concepts of Nikola Tesla about*

electromagnetic phenomenology of radiative energy and "Dynamic Gravity Theory" are creatively illustrating similar, closed electromechanical circuits, matter-waves connections and interactions between all masses in our Universe. See more in the literature under [97] until [101], in the first chapter about Analogies, in chapter 4.1, around Fig. 4.1.6, and in chapters 8. and 9.

Now is also the right place to mention connections between here-introduced concepts of closed current circuits, couplings, analogies, and equivalency between linear and angular motions, and associated electric currents and magnetic field effects. Such links and couplings are conceptually presented on illustrations in chapter 4.1, on Fig.4.1, Fig.4.1.2, Fig.4.1.3, Fig.4.1.4, Fig.4.1.5, with equations under (4.3), and later.

Also, we could mention simple experiments about closed circuits of energy-momentum flow from prof. Eric Laithwaite, where he demonstrated unusual and extraordinary couplings between linear and rotational motions of spinning gyroscopes, and similar magnetic field effects (see more in [102]).

The existence of realistic matter wave's connections between masses is also elaborated in this chapter, concerning planetary macro matter waves, around equations starting from (2.11.14) until (2.11.14)-h.

2.7. Dark Matter, Dark Energy, Hidden Invisible Mass, and similar imaginative and virtual entities could be qualitatively, mathematically, and creatively explained by the fact that mass, moments and energy involved in "4-vectors of Energy-Momentum" invariant expressions from Relativity Theory (Minkowski space formalism) are presentable as complex or hyper-complex mathematical, Analytic Signal functions, having Real, Imaginary and Apparent parts. See more in chapter 10 of this book (10.1 **Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors concerning matter-waves and particle-wave duality**). Such background could support macrocosmic effects of resonant synchronization, entanglements, and the existence of Dark Matter & Energy, presenting mentioned "dark" items as imaginary or apparent mass components, since a mass of our Universe is "Mass in permanent motion", where linear and angular motions are specifically united, respecting **P**article **W**ave **D**uality **C**oncepts, as widely elaborated in this book.

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

[♣ COMMENTS & FREE-THINKING CORNER (still in preparation and brainstorming phase)**:**

2.3.3-1 Binary Systems, Kepler and Newton Laws and Matter Waves Hosting

The essential fact in the background of (2.11.10) - (2.11.14) is that gravitation is the central force. Its direction is always along a radius, either towards or away from a point, we are using as an origin or force center. The magnitude of such central force depends solely upon the distance from its origin, $\mathbf{r}$. We can present such forces as, $F_{m-M} = \frac{GmM}{r^2} = F(r),\ \vec{F}(r) = F(r)\cdot\frac{\vec{r}}{r}$. Central forces are interesting because we find them very often in physics. The gravitational and electrostatic forces are central forces (as well as forces between permanent magnets). Much of classical mechanics or physics can be placed in the framework of elaborated applications of Newton Laws. Let us start with the Second Newton Law, $\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$, and express the associated torque and angular momentum as, $\vec{\tau} = \vec{r}\times\vec{F} = \vec{r}\times m\vec{a}$, $\vec{L} = \vec{r}\times\vec{p}$. Since torque is the time derivative of angular momentum, let us find the torque for central forces (where $\vec{F}(r)$ is parallel with $\vec{r}$) as, $\left(\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \vec{r}\times m\vec{a} = \vec{r}\times\vec{F} = 0\right) \Rightarrow \vec{L} = \text{const.}$

Consequently, an orbital planetary motion has constant angular momentum because gravitational force is the central force. A little bit later we will see that this is the real origin of quantization in physics (equally applicable to Coulomb Law related analogical situations), as well as to micro-world of atoms and elementary particles where building blocks have constant angular moments or spin characteristics.

Let us now analyze the simplified case of gravitational attraction between two masses $m_1 = m$ and $m_2 = M$ from the point of view of Binary Systems relations in their center of mass coordinate system. The total separation between the centers of the two masses is $\vec{r} = \vec{r}_1 + \vec{r}_2$. We may define the center of a mass point placed between two objects through the equations,

$$m_1 r_1 = m_2 r_2,\ r = r_1 + r_2,\ r_1 = \frac{m_2}{m_1+m_2} r,\ r_2 = \frac{m_1}{m_1+m_2} r. \qquad (2.11.14\text{-}1)$$

From gravitational attraction between m_1 *and* m_2 *nothing will change if we imagine that* m_1 *and* m_2 *may be in a uniform rotational motion around their common center of mass since masses will in the same time experience mutually repulsive balancing centrifugal force (possible spinning* is not considered). Let us imagine that m_1 and m_2 are rotating (around their common center of mass) with certain angular speed $\omega = \frac{2\pi}{T}$, what can be described with another set of equations,

$$\omega = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v}{r},\ \frac{v_1}{v_2} = \frac{r_1}{r_2},\ p_1 = m_1 v_1 = m_2 v_2 = p_2 = p = p_r = m_r v_r,\ \vec{p}_1 + \vec{p}_2 = 0,$$
$$\vec{v}_i = \frac{d\vec{r}_i}{dt},\ \vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_r = \vec{v}_1 + \vec{v}_2,\ v_1 v_2 = \omega^2 r_1 r_2, v_i = \omega r_i,\ \frac{v_1 v_2}{v^2} = \frac{r_1 r_2}{r^2}. \qquad (2.11.14\text{-}2)$$

Here v_1 and v_2 are tangential velocities of m_1 and m_2. In cases of such circular, rotational motions, every mass is experiencing certain centrifugal (mutually opposed) force with a tendency to separate them, for example,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$F_c = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{dp}{dt} = \frac{dp_r}{dt} = \frac{m_r v_r^2}{r} = \frac{p_r v_r}{r} \Leftrightarrow m_1 v_1 = m_2 v_2 (= p = m_r v_r = p_r) \Rightarrow$$
$$\left\{ \begin{array}{c} \Rightarrow \text{after integration} \Rightarrow \\ v_r = \frac{dr}{dt} = \frac{p_0}{m_r} \cdot \frac{r}{r_0},\ p_r = m_r v_r = p_0 \cdot \frac{r}{r_0}, F_r = F_c = \frac{dp_r}{dt} = \frac{p_0}{r_0} v_r = p_0 \omega \cdot \frac{r}{r_0},\ [p_0, r_0] = \text{constants} \end{array} \right\}. \quad (2.11.14\text{-}3)$$

If the distance between two masses m_1 and m_2 is remaining unchanged (stable orbital motions), mutually opposed (or repulsive) centrifugal forces should be balanced with similar (central) attractive force between them, which is Newton force of gravitation F_g. Conceptualizing given case of a stable Binary System this way, we are developing and formulating Kepler's third law, as follows.

$$\left\{ \begin{array}{l} F_g = G\frac{m_1 m_2}{r^2} = F_c = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = m_1 v_1 \omega = m_2 v_2 \omega = m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = \frac{m_1 m_2}{m_1 + m_2} v_r \omega = \frac{m_1 m_2}{m_1 + m_2} r\omega^2 = m_r r\omega^2 = \frac{m_r v_r^2}{r} \\ m_r = \frac{m_1 m_2}{m_1 + m_2},\ v_r = \omega r = \frac{dr}{dt},\ v_1 = \frac{m_2}{m_1 + m_2} v_r = \omega r_1,\ v_2 = \frac{m_1}{m_1 + m_2} v_r = \omega r_2 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \omega = \sqrt{G\frac{m_1 + m_2}{r^3}} = \frac{2\pi}{T}, \Leftrightarrow \left(\frac{T}{2\pi}\right)^2 = \frac{r^3}{G(m_1 + m_2)}. \quad (2.11.14\text{-}4)$$

Another conclusion radiating from here is that natural tendency of masses (regarding stable Binary Systems, or multi-mass systems) is to create uniform or stationary rotational motions (around their common center of mass), this way balancing attractive Newton force with associated centrifugal force. If such rotation is not a visible case, at least mathematically and by respecting relevant conservation laws every Binary System could be equally presentable as a case of mutually coupled rotating bodies (including rotating disks, toroids…). The coupling force in question (for instance in cases of electromagnetically neutral bodies) is the gravitation.

We could also say that boundary or asymptotic tendency (or just mathematically equivalent state in the same center of mass coordinates) of Binary Systems is that initial masses m_1 and m_2 can be effectively replaced by one bigger central mass which is equal $m_c = m_1 + m_2$ and placed in their common center of mass position (being there in a state of rest). In addition to such central mass m_c, there is another, (mathematically generated) reduced mass $m_r = \frac{m_1 m_2}{m_1 + m_2}$, which is rotating around the central mass m_c. Such reduced mass m_r will have the total kinetic energy and orbital moment of masses m_1 and m_2. The distance between m_r and m_c (or relevant circle radius) is again the same as before $r = r_1 + r_2,\ r_1 = \frac{m_2}{m_1 + m_2} r,\ r_2 = \frac{m_1}{m_1 + m_2} r,\ m_1 r_1 = m_2 r_2 \Rightarrow p = m_1 v_1 = m_2 v_2$. Angular (mechanical rotating) velocity ω of the new Binary System m_r and m_c will stay the same as found previously for Binary System of masses m_1 and m_2 ($\omega = v_1 / r_1 = v_2 / r_2 = v_r / r = 2\pi f_m$). The attractive gravitational force between m_1 and m_2 will be the same as the attractive force between m_r and m_c, for instance:

$$F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2} = \frac{m_r v_r^2}{r} = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{m_1 v_1^2 + m_2 v_2^2}{r_1 + r_2} = m_r r\omega^2 = m_1 r_1 \omega^2 = m_2 r_2 \omega^2. \quad (2.11.14\text{-}5)$$

We can also find involved orbital moments of rotating masses m_1, m_2 and m_r, taking into account that the total orbital moment of a Binary System is conserved (constant).

$$\left\{\begin{bmatrix} \mathbf{L}_1 = p_1 r_1 = m_1 v_1 r_1 = m_1 r_1^2 \omega = \dfrac{m_1 v_1^2}{\omega} = \mathbf{J}_1 \omega, \\ \mathbf{L}_2 = p_2 r_2 = m_2 v_2 r_2 = m_2 r_2^2 \omega = \dfrac{m_2 v_2^2}{\omega} = \mathbf{J}_2 \omega, \end{bmatrix} \& \begin{bmatrix} \dfrac{\mathbf{L}_1}{\mathbf{L}_2} = \dfrac{\mathbf{J}_1}{\mathbf{J}_2} = \dfrac{r_1}{r_2} = \dfrac{v_1}{v_2}, \ p_1 = p_2 = p = p_r \\ \mathbf{L}_r = p_r r = m_r v_r r = m_r r^2 \omega = \dfrac{m_r v_r^2}{\omega} = \mathbf{J}_r \omega \end{bmatrix}\right\} \Rightarrow$$

$$\begin{aligned} &\vec{\mathbf{L}}_i = \vec{r}_i \times \vec{p}_i \Rightarrow \mathbf{L}_i = m_i v_i r_i = m_i r_i^2 \omega = \frac{m_i v_i^2}{\omega} = \mathbf{J}_i \omega = \frac{2}{\omega} E_{ki}, \\ &\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2 = (\mathbf{J}_1 + \mathbf{J}_2)\omega = \frac{2}{\omega}\left(\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}\right) = \frac{2}{\omega}(E_{k1} + E_{k2}) = \frac{2}{\omega} E_{orbital} = \frac{m_r v_r^2}{\omega} = \frac{\mathbf{J}_r \omega^2}{\omega} = \\ &= \mathbf{J}_r \omega = \mathbf{L}_r = (\mathbf{J}_1 + \mathbf{J}_2)\sqrt{G\frac{m_1 + m_2}{r^3}} = \text{const.}, \ \mathbf{J}_r = \mathbf{J}_1 + \mathbf{J}_2, \ v_r = v_1 + v_2 = \omega r, \\ &E_{ki} = \frac{1}{2} m_i v_i^2 = \frac{1}{2} p_i v_i = \frac{1}{2} m_r v_r^2 \frac{r_i}{r} = \frac{1}{2} \mathbf{J}_i \omega^2 = \frac{1}{2} \mathbf{L}_i \omega, \ E_{kr} = E_{ki} \frac{v_r}{v_i} = E_{k1} + E_{k2}, \\ &\frac{E_{k1}}{v_1} = \frac{E_{k2}}{v_2} = \frac{E_{kr}}{v_r} = \frac{E_{ki}}{v_i} = \frac{m_i v_i}{2} = \frac{m_r v_r}{2} = \frac{1}{2} p, \ v_i = \omega r_i \ . \end{aligned} \qquad (2.11.14\text{-}6)$$

Now we will be able to show that for (isolated) Binary Systems that are conserving total orbital moment, specific orbital (or kinetic, or motional) energy is in some way quantized, or given by similar expression like Planck's energy of a photon (except that new Planck-like H-constant will be much bigger compared to Planck constant of micro-world).

$$\begin{aligned} &E_{orbital} = E_{k1} + E_{k2} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathbf{J}_1 \omega^2 + \frac{1}{2} \mathbf{J}_2 \omega^2 = \frac{1}{2} (\mathbf{J}_1 + \mathbf{J}_2) \omega^2 = \frac{1}{2} (\mathbf{L}_1 + \mathbf{L}_2) \omega = \\ &= \frac{1}{2} m_r v_r^2 \frac{r_1}{r} + \frac{1}{2} m_r v_r^2 \frac{r_2}{r} = \frac{1}{2} m_r v_r^2 \left(\frac{r_1}{r} + \frac{r_2}{r}\right) = \frac{1}{2} m_r v_r^2 = \frac{1}{2} \mathbf{J}_r \omega^2 = E_r = \frac{1}{2}\left[(\mathbf{J}_1 + \mathbf{J}_2)\omega\right] \cdot \omega = \\ &= \frac{1}{2}\left[\text{const}\right] \cdot \omega = \text{Const} \cdot \omega = H \cdot f = H \cdot f_0, \ \omega = 2\pi f_m = \frac{2\pi}{T} = \frac{H}{\text{Const}} \cdot f_0, \ f = f_0 \neq f_m . \end{aligned} \qquad (2.11.14\text{-}7)$$

The next significant remark here (relevant for Binary Systems) is that ***to experience an attractive gravitational force, rotating bodies should rotate in the same direction (both having mutually collinear angular speed and angular moments vectors).*** If the rotation is not externally (or macroscopically) detectable, it should be in some ways internally (intrinsically) present in Binary Systems relations. Simply, gravitation without rotation cannot be explained. We can also find expressions for such inherently associated angular velocity and angular momentum of Binary Systems as,

$$\begin{aligned} &F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2} = F_c = \frac{m_r v_r^2}{r} = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = m_1 v_1 r_1 \frac{v_1}{r_1^2} = m_2 v_2 r_2 \frac{v_2}{r_2^2} = \mathbf{J}_1 \omega \frac{v_1}{r_1^2} = \mathbf{J}_2 \omega \frac{v_2}{r_2^2} = \mathbf{L}_1 \frac{v_1}{r_1^2} = \mathbf{L}_2 \frac{v_2}{r_2^2} = \mathbf{J}_r \omega \frac{v_r}{r^2} = \mathbf{L}_r \frac{v_r}{r^2}, \\ &F_g r = G\frac{m_1 m_2}{r^2} r = G\frac{m_1 m_2}{r} = G\frac{m_r m_c}{r} = F_c r = \frac{m_r v_r^2}{r} r = m_r v_r^2 = m_r r^2 \omega^2 = \mathbf{J}_r \omega^2 = \mathbf{L}_r \omega = 2E_r = 2E_{orbital} = 2Hf, \\ &(m_1 r_1 = m_2 r_2, \ m_1 v_1 = m_2 v_2 = m_r v_r = p) \Rightarrow F_t = \frac{dp}{dt} = F_\tau, \ \left(\vec{\tau} = \vec{r} \times \vec{F}_t = \frac{d\vec{\mathbf{L}}}{dt}, \ \vec{F}_\tau = \frac{\vec{\tau}}{r} = \frac{\vec{r} \times \vec{F}_t}{r} = \frac{1}{r}\frac{d\vec{\mathbf{L}}}{dt}\right), \\ &\mathbf{L}_r v_r = G m_1 m_2 = \frac{\mathbf{L}_1 \mathbf{L}_2}{\mathbf{L}_1 + \mathbf{L}_2} v_r \Leftrightarrow G = \frac{\mathbf{L}_1 \mathbf{L}_2}{\mathbf{L}_1 + \mathbf{L}_2} \frac{v_r}{m_1 m_2} = \mathbf{L}_r \frac{\omega r}{m_1 m_2} = \frac{\mathbf{J}_1 \mathbf{J}_2}{\mathbf{J}_1 + \mathbf{J}_2} \frac{\omega^2 r}{m_1 m_2} = \mathbf{J}_r \frac{\omega^2 r}{m_1 m_2}, \\ &\mathbf{J}_r = \frac{\mathbf{J}_1 \mathbf{J}_2}{\mathbf{J}_1 + \mathbf{J}_2} = \mathbf{J}_1 + \mathbf{J}_2 = \sqrt{\mathbf{J}_1 \cdot \mathbf{J}_2} = \frac{m_1 m_2}{m_1 + m_2} r^2 = m_r r^2, \\ &\mathbf{L}_r = \mathbf{J}_r \omega = \mathbf{L} = \frac{\mathbf{L}_1 \mathbf{L}_2}{\mathbf{L}_1 + \mathbf{L}_2} = \mathbf{L}_1 + \mathbf{L}_2 = \sqrt{\mathbf{L}_1 \cdot \mathbf{L}_2} = \sqrt{G\frac{m_1 m_2 \mathbf{J}_r}{r}} \Rightarrow \omega = \sqrt{G\frac{m_1 m_2}{\mathbf{J}_r r}} = \sqrt{G\frac{m_r m_c}{\mathbf{J}_r r}} \ . \end{aligned} \qquad (2.11.14\text{-}8)$$

Another conclusion to draw is that gravitational constant **G** is the measure of here elaborated intrinsic rotation or angular (mechanical revolving) speed of Binary Systems, leading to another alternative form of Kepler's third law as,

$$\frac{1}{\omega^2} = \frac{1}{(2\pi f_m)^2} = \left(\frac{T}{2\pi}\right)^2 = \frac{J_r r}{Gm_1m_2} = \frac{r^3}{G(m_1+m_2)}, \qquad (2.11.14\text{-}9)$$

f_m (=) Mechanical (planet or satellite) revolving or orbiting frequency.

Shall we have a repulsive gravitational force in cases when masses in Binary Systems are not rotating in the same direction (when important angular moments' vectors are mutually opposed or maybe not collinear) is one of the logical questions to ask here? Let us exercise what could be the answer on a similar question if mentioned masses are also self-spinning (having finite spin moments $\vec{L}_{s1}$ and $\vec{L}_{s2}$, $\vec{L}_i \to (\vec{L}_i + \vec{L}_{si})$), and how such spinning moments would influence the attractive force/s between them?

$$F_g = F_c = G\frac{m_1m_2}{r^2} = G\frac{m_rm_c}{r^2} = \left(\frac{\pi G}{c^4}\right)\frac{m_1c^2m_2c^2}{\pi r^2} = \left(\frac{G}{c^4}\right)\frac{m_rc^2m_cc^2}{r^2} = \left(\frac{\pi G}{c^4}\right)\frac{E_{t1}E_{t2}}{\pi r^2} = \left(\frac{\pi G}{c^4}\right)\frac{E_{tr}E_{tc}}{\pi r^2},$$

$$\left\{\begin{array}{l}(v_i << c) \Rightarrow E_{ti} = m_ic^2 = \gamma_i m_i c^2 \Rightarrow E_{ti} \cong m_ic^2 + \frac{1}{2}m_iv_i^2 = m_ic^2 + \frac{1}{2}J_i\omega^2 = m_ic^2 + \frac{1}{2}\vec{L}_i\vec{\omega} \\ \left(\vec{L}_i \to (\vec{L}_i + \vec{L}_{si}), \vec{L}_i = J_i\vec{\omega}, \vec{L}_{si} = J_i\vec{\omega}_{si}\right) \Rightarrow E_{ti} \cong m_ic^2 + \frac{1}{2}(\vec{L}_i + \vec{L}_{si})\vec{\omega}\end{array}\right\} \Rightarrow \qquad (2.11.14\text{-}10)$$

$$F_g \cong \left(\frac{G}{c^4}\right)\frac{\left[m_1c^2 + \frac{1}{2}(\vec{L}_1 + \vec{L}_{s1})\vec{\omega}\right]\left[m_2c^2 + \frac{1}{2}(\vec{L}_2 + \vec{L}_{s2})\vec{\omega}\right]}{r^2} = \left(\frac{G}{c^4}\right)\frac{\left[m_rc^2 + \frac{1}{2}(\vec{L}_r + \vec{L}_{sr})\vec{\omega}\right]\left[m_cc^2 + \frac{1}{2}(\vec{L}_c + \vec{L}_{sc})\vec{\omega}\right]}{r^2}.$$

If there is a stable ground in here hypothesized exercise about gravitational force, the presence of spin and orbital moments (of participants) could increase or decrease the total gravitational force between two bodies in a Binary System (depending on relative mutual positions of important orbital and spin moments). Most probably, such contributive spin-related members are too small compared to other involved energy-related members (in cases of planetary or solar systems), and it has been not easy to notice such possibility for addressing modifications of the old Newton Law. ***Here we should enrich the same situation by paying more attention to matter-waves nature of such binary interactions by additional elaborations around equations (2.4-11) to (2.4-17) from the same chapter.***

Apparently, in the absence of repulsive centrifugal forces, planets (or orbits) of specific Solar System would collapse and unite masses with their Sun if there are no orbital rotations. Since the repulsive (centrifugal) gravitational force (as formulated here) is something exclusively related to rotation, most probably that the hidden nature of Gravitation itself is on a similarly effective way intrinsically and inherently also associated with specific (equivalent) rotation inside of matter substance of gravitational masses.

Another step in exercising and hypothesizing the same situation (regarding decoding essence of Gravitation in Binary Systems relations) is to notice connections between different aspects of (involved) energy components and work of "matter vortices" characterized by orbital and spin moments which should have certain torque. There is a tiny imaginative step from here to start thinking how to conceptualize rest masses as some "frozen or self-stabilized matter vortices' states" (since dimensionally torque is measured by the same units as energy).

Until here we did not address any of relativistic aspects of motional masses, since by the nature of astronomic, gravitational Binary Systems (or planetary systems), we can consider that in majority of relevant cases relevant orbital velocities are much smaller compared to the speed of light, and in such cases it is clearly valid,

$$(v_{1,2} << c) \Rightarrow F_g = G\frac{m_1m_2}{r^2} = G\frac{m_rm_c}{r^2} \Leftrightarrow m_1m_2 = m_rm_c, m_r = \frac{m_1m_2}{m_1+m_2}, m_c = m_1 + m_2. \qquad (2.11.14\text{-}11)$$

Let us now imagine that some of Binary Systems (not necessarily of exclusively gravitational nature) could be orbital speed sensitive and let us analyze the consequences (again about the important center of mass).

$$\left\{ m_i \to \gamma_i m_i = \frac{m_i}{\sqrt{1-\frac{v_i^2}{c^2}}} = m_i^* \right\} \Rightarrow \left\{ \begin{array}{l} \left(m_r = \frac{m_1 m_2}{m_1+m_2} \right) \to \frac{\gamma_1 m_1 \gamma_2 m_2}{\gamma_1 m_1 + \gamma_2 m_2} = \frac{m_1^* m_2^*}{m_1^*+m_2^*} = m_r^* = \gamma_r m_r \\ (m_c = m_1 + m_2) \to \gamma_1 m_1 + \gamma_2 m_2 = m_1^* + m_2^* = m_c^* \\ (m_1 m_2 = m_r m_c) \to \gamma_1 m_1 \gamma_2 m_2 = m_1^* m_2^* \end{array} \right\} \Rightarrow \quad (2.11.14\text{-}12)$$

$$\Rightarrow \left\{ \begin{array}{l} F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2} = G\frac{m_1^* m_2^*}{r^2} = G\frac{m_r^* m_c^*}{r^2} \\ \gamma_r = \frac{\gamma_1 \gamma_2}{\gamma_1 \frac{m_1}{m_1+m_2} + \gamma_2 \frac{m_2}{m_1+m_2}} \end{array} \right\}.$$

To make a simple validity test, of here elaborated relativistic masses relations, it would be very indicative and almost sufficient to present the case when one of masses is enormously more significant compared to other,

$$(m_1 = m << m_2 = M) \Rightarrow \left\{ \begin{array}{l} m_r = \frac{m_1 m_2}{m_1+m_2} \cong m = m_1 \Rightarrow \gamma_r = \frac{\gamma_1 \gamma_2}{\gamma_1 \frac{m_1}{m_1+m_2} + \gamma_2 \frac{m_2}{m_1+m_2}} \cong \gamma_1 \\ E_{k1} + E_{k2} = E_{kr} = E_{ki}\frac{v_r}{v_i} = (\gamma_r - 1) m_r c^2 \cong (\gamma_1 - 1) m_1 c^2 = E_{k1} \end{array} \right\} \quad (2.11.14\text{-}13)$$

what already looks like the correct result (see also equations (2.4-11) - (2.4-18)).

Many possible consequences are starting from here. For instance, any stable planetary system (with one big solar mass M_s) and number of orbiting planets with masses $\{m_i\}_{i=1}^{n}$ can be ***decomposed and analyzed as an ensemble of simple Binary Systems*** with masses m_i orbiting around $(M_c - m_i)$, for example,

$$m_i \cdot (M_c - m_i) = m_{r-i} \cdot M_c,\ M_c = M_s + \sum_{i=1}^{n} m_i,\ m_{r-i} = \frac{m_i \cdot [M_c - m_i]}{M_c},\ \forall i \in (1, n), \quad (2.11.14\text{-}14)$$

where m_i and M_s are only an approximation of a Binary System masses when $M_c \cong M_s >> \sum_{i=1}^{n} m_i$.

Let us extrapolate two-body problem analysis to an equivalent n-body situation. For instance, imagine that n of astronomic objects (like planets, including one massive star) are mutually approaching, entering specific n-body interaction, and becoming a stable planetary or solar system (this time we will analyze such situation without considering impacts). Kinetic energy balance in such case will be:

$$\left\{ \begin{array}{l} 2-\text{body situation} \\ M_c = m_1 + m_2,\ m_r = \frac{m_1 \cdot m_2}{m_1 + m_2} = \frac{m_1 \cdot m_2}{M_c} \\ E_{k1} + E_{k2} = E_{kc} + E_{kr} \Leftrightarrow \\ \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} M_c v_c^2 + \frac{1}{2} m_r v_r^2 \end{array} \right\} \begin{array}{c} \Rightarrow \\ \text{by analogy} \end{array}$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left\{\begin{array}{l} n-\text{body situation} \\ M_c = \sum_{(i)} m_i \;\; (\text{including solar mass}) \\ \sum_{(i)} E_{ki} = E_{kc} + \sum_{(i)} E_{kr-i} \Leftrightarrow \\ \frac{1}{2}\sum_{(i)} m_i v_i^2 = \frac{1}{2}M_c v_c^2 + \frac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 = \frac{1}{2}M_c v_c^2 + \frac{1}{2}M_r v_r^2 \end{array}\right\} \Rightarrow \quad (2.11.14\text{-}14\text{-a})$$

$$\sum_{(i)} E_{ri} = \sum_{(i)} E_{ki} - E_{kc} = \frac{1}{2}\sum_{(i)} m_i (v_i^2 - v_c^2) = \frac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 .$$

From (2.11.14-14-a) we could easily establish the following, natural understanding of planetary systems with many planets orbiting around one big central mass. We can say that each planet has its reduced mass that is exactly equal to its ordinary mass $m_{r-i} = m_i$ (not modified) and that only relative velocities of particular masses are modified,

$$\begin{array}{l} \frac{1}{2}\sum_{(i)} m_i (v_i^2 - v_c^2) = \frac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 \Rightarrow \\ \sum_{(i)} m_{r-i} v_{r-i}^2 = \sum_{(i)} m_i v_{r-i}^2 = \sum_{(i)} m_i (v_i^2 - v_c^2) = M_r v_r^2 \Rightarrow \\ m_{r-i} = m_i \, , v_{r-i}^2 = v_i^2 - v_c^2 \Leftrightarrow v_i^2 = v_c^2 + v_{r-i}^2 \end{array} \quad (2.11.14\text{-}14\text{-b})$$

From (2.11.14-14-b), helix rotation of planets (observed from specific Laboratory System) is a natural conclusion (since $v_i^2 = v_c^2 + v_{r-i}^2$), but in the important center of mass system, we will only have orbiting (or rotation) of planets around the central mass.

We could on a similar way to exercise the situation of <u>*an ensemble of Binary Systems*</u> with masses m_i and $(M_c - m_i)$, for example,

$$\begin{array}{l} m_{r-i} = \frac{m_i \cdot [M_c - m_i]}{M_c} \\ \frac{1}{2}\sum_{(i)} m_i (v_i^2 - v_c^2) = \frac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 = \frac{1}{2}\sum_{(i)} \frac{m_i \cdot [M_c - m_i]}{M_c} v_{r-i}^2 = \frac{1}{2} M_r v_r^2 \Rightarrow v_i^2 - v_c^2 = \frac{M_c - m_i}{M_c} v_{r-i}^2 \end{array} \quad (2.11.14\text{-}14\text{-c})$$

Of course, similar elaborations can additionally be extended to other n-body problems. In the familiar mainstream of thinking, we can imagine that initial participants of n-body interaction have orbital and spinning moments and implement laws of linear and orbital moments' conservation to establish a much more powerful analyzing framework that will take into account mutual interactions of many-body systems. In cases when, also, participants have free and/or dipole types electromagnetic charges, the same situation is becoming richer for similar analyses. This will give us a chance to explore other non-Newtonian gravitation-related interactions between masses with spin and orbital moment's attributes, and electromagnetic charges (not to forget matter waves spinning associated with motions of masses).

2.3.3-2 Quantizing and Matter Waves Hosting

Circular orbits of stable Binary Systems (including most of stable solar or planetary systems), as conceptualized here, are presenting a uniform, stationary, periodical, and inertial motions. For inertial motions, we have seen in (2.9.1) and (2.9.2) that coincident validity and applicability of relevant linear and orbital momentum conservation is causally linked to standing matter waves formations. Consequently, stable Binary and Planetary Systems' Orbits as inertial motions, besides hosting orbiting masses could also host certain mutually synchronized standing matter waves formations, where synchronizing (or waves packing, or quantizing) criteria concerning the relevant center of mass coordinates system should be,

$$\begin{bmatrix} 2\pi r_i = n_i\lambda_i,\ \mathbf{L}_i = p_i r_i = m_i v_i r_i = m_i r_i^2\omega = \frac{m_i v_i^2}{\omega} = \mathbf{J}_i\omega = \frac{H}{\lambda_i} r_i,\ \lambda_i = \frac{2\pi r_i}{n_i} = \frac{H}{p_i}, H = \text{const.}, \\ \frac{\mathbf{L}_1}{\mathbf{L}_2} = \frac{\mathbf{J}_1}{\mathbf{J}_2} = \frac{r_1}{r_2} = \frac{v_1}{v_2} = \frac{n_1}{n_2} = \frac{E_{k1}}{E_{k2}},\ \frac{n_1}{r_1} = \frac{n_2}{r_2} = \frac{n_r}{r} = \frac{n_i}{r_i} = \frac{2\pi}{\lambda_i},\ n_i \in [1,2,3,...], \\ \omega = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v_i}{r_i} = \frac{v_1+v_2}{r_1+r_2} = \frac{v}{r},\ p_1 = m_1v_1 = m_2v_2 = p_2 = p = p_r,\ \ v_1v_2 = \omega^2 r_1 r_2 \end{bmatrix} \Rightarrow$$

$$\frac{H}{2\pi} = \mathbf{L}_1\frac{\lambda_1}{2\pi r_1} = \mathbf{L}_2\frac{\lambda_2}{2\pi r_2} = \mathbf{L}_r\frac{\lambda}{2\pi r} = (\mathbf{L}_1+\mathbf{L}_2)\frac{\lambda_1+\lambda_2}{2\pi(r_1+r_2)} =$$

$$= \frac{\mathbf{L}_1}{n_1} = \frac{\mathbf{L}_2}{n_2} = \frac{\mathbf{L}_1+\mathbf{L}_2}{n_1+n_2} = \frac{\mathbf{L}_r}{n_r} = \frac{\mathbf{L}_i}{n_i} = \frac{\mathbf{J}_i}{n_i}\omega = \frac{2}{n_i\omega}\frac{\mathbf{J}_i\omega^2}{2} = \frac{2}{n_i\omega}E_{ki} = \hbar_{gr.},$$

$$(v_i << c) \Rightarrow E_{ki} = \frac{n_i\omega}{2}\frac{H}{2\pi} = \frac{n_i v_i}{2r_i}\frac{H}{2\pi} \cong \frac{n_i 2u_i}{2r_i}\frac{H}{2\pi} = \frac{n_i u_i}{r_i}\frac{H}{2\pi} = H\frac{n_i\lambda_i f_i}{2\pi r_i} = Hf_i, \tag{2.11.14-15}$$

$$\omega r_i = v_i \cong 2u_i = 2\lambda_i f_i,\ \omega = \frac{2\pi}{T} = 2\pi f_m \cong 2\lambda_i\frac{f_i}{r_i} = 2\lambda\frac{f}{r} = \frac{4\pi}{n}f,\ f_m \cong \frac{\lambda}{\pi r}f = \frac{2}{n}f = \frac{2}{n_i}f_i,\ f = f_0,$$

$$E_{orbital} = E_{k1} + E_{k2} = H(f_1+f_2) = Hf = \frac{1}{2}F_g r = \frac{1}{2}G\frac{m_1m_2}{r^2}r = \frac{1}{2}G\frac{m_1m_2}{r} = \frac{1}{2}G\frac{m_rm_c}{r} = \frac{1}{2}F_c r = \frac{1}{2}\frac{m_rv_r^2}{r}r =$$

$$= \frac{1}{2}m_rv_r^2 = \frac{1}{2}m_rr^2\omega^2 = \frac{1}{2}\mathbf{J}_r\omega^2 = \frac{1}{2}\mathbf{L}_r\omega = E_r = \frac{H}{4\pi}(n_1+n_2)\omega = \frac{H}{2}(n_1+n_2)f_m = \frac{H}{2}nf_m,\ n = n_1+n_2 = n_r.$$

We could again attempt to characterize and quantify unity of orbital moments of specific stable Solar System ($\mathbf{L}_s, n_s$) with many orbiting planets ($\mathbf{L}_i, n_i$), considering the Sun as enormously more significant mass compared to any of related planets, $m_s >> m_i$) on a similar way, for instance,

$$\left[\frac{H}{2\pi} = \frac{\mathbf{L}_1}{n_1} = \frac{\mathbf{L}_2}{n_2} = \frac{\mathbf{L}_1+\mathbf{L}_2}{n_1+n_2} = \frac{\mathbf{L}_r}{n_r} = \frac{\mathbf{L}_i}{n_i}\right] \Rightarrow \frac{H}{2\pi} = \frac{\mathbf{L}_s}{n_s} = \frac{\mathbf{L}_i}{n_i} = \frac{\sum_{(i)}\mathbf{L}_i}{\sum_{(i)}n_i} = \frac{\mathbf{L}_s + \sum_{(i)}\mathbf{L}_i}{n_s + \sum_{(i)}n_i} = \hbar_{gr.}, \tag{2.11.14-16}$$

where $\mathbf{L}_s$ and n_s are characteristic parameters of the common Sun, $\mathbf{L}_i$ and n_i are related to each planet. This way the same Solar System can be decomposed on many simple binary systems (where each planet and the sun are presenting one elementary Binary System). Of course, such strategy should be additionally elaborated and united with masses decomposition criteria from (2.11.14-14).

There is still certain confusion and ambiguity in physics literature regarding relations between mechanical revolving (or orbital, rotating) frequency $f_m = \omega / 2\pi$ and associated, specific orbital, matter wave frequency $f = \omega_0 / 2\pi = f_0$, and resolutions of such discrepancies are being explained by *postulating correspondence principles* (what is not a real and very scientific explanation). The background of mentioned discrepancies is closely related to the nature of wave motions, to particle-wave duality and to specific relations between a group and phase velocity of the matter wave packet (which represents an energy-momentum wave model of a moving particle). For instance, the relation between group and phase velocity (where group velocity is in the same time real, measurable particle velocity) can be found as (see chapters 4.0 and 4.1),

$$v = v_g = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda},\ u = \lambda f,\ u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \begin{bmatrix} (v << c) \Rightarrow \omega r_i = v_i \cong 2u_i = 2\lambda_i f_i \\ (v \approx c) \Rightarrow \omega r_i = v_i \cong u_i = \lambda_i f_i \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \omega = \frac{2\pi}{T} = 2\pi f_m = \frac{v_i}{r_i} \cong \frac{2u_i}{r_i} \cong 2\lambda_i\frac{f_i}{r_i} = 2\lambda\frac{f}{r} = \frac{4\pi}{n}f,\ f_m \cong \frac{\lambda}{\pi r}f = \frac{2}{n}f = 2\frac{f_i}{n_i}, f_i = \frac{n_i}{n}f, (v << c) \\ \omega = \frac{2\pi}{T} = 2\pi f_m = \omega_m = \frac{v_i}{r_i} \cong \frac{u_i}{r_i} \cong \lambda_i\frac{f_i}{r_i} = \lambda\frac{f}{r} = \frac{2\pi}{n}f,\ f_m \cong \frac{\lambda}{2\pi r}f = \frac{1}{n}f = \frac{f_i}{n_i}, f_i = \frac{n_i}{n}f, (v \approx c) \end{bmatrix} \Rightarrow$$

$$\Rightarrow E_{ki} = \frac{n_i\omega}{2}\frac{H}{2\pi} = \frac{n_i v_i}{2r_i}\frac{H}{2\pi} = \frac{2n_i u_i}{2r_i}\frac{H}{2\pi} = H\frac{n_i\lambda_i f_i}{2\pi r_i} = Hf_i \Rightarrow \quad (2.11.14\text{-}17)$$

$$\Rightarrow E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf,\ f_1 + f_2 = f,\ n_1 + n_2 = n,\ n_i\lambda_i = 2\pi r_i,$$

where,

$$\begin{bmatrix} (v << c) \Rightarrow v \cong 2u = 2\lambda f = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} \Leftrightarrow u \cong -\lambda\frac{du}{d\lambda} \Leftrightarrow 2\frac{d\lambda}{\lambda} = -\frac{df}{f} \Rightarrow \ln\left|\frac{\lambda}{\lambda_0}\right|^2\left|\frac{f}{f_0}\right| = 0 \Rightarrow \left|\frac{\lambda}{\lambda_0}\right|^2\left|\frac{f}{f_0}\right| = 1 \\ \Leftrightarrow \lambda^2 f = \lambda_0^2 f_0, u\lambda = u_0\lambda_0,\ \lambda = \lambda_0\sqrt{\frac{f_0}{f}} = \frac{H}{p},\ u = u_0\frac{\lambda_0}{\lambda}, (f_0, \lambda_0) = const., p = \frac{Hf}{c}\sqrt{\frac{f_0}{f}} = \frac{nHf_m}{2c}\sqrt{\frac{f_0}{f}} \end{bmatrix}, \quad (2.11.14\text{-}18)$$

$$\begin{bmatrix} (v \approx c) \Rightarrow v \cong u = \lambda f = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} \Leftrightarrow \frac{du}{d\lambda} = \frac{fd\lambda + \lambda df}{d\lambda} \cong 0 \Leftrightarrow \frac{d\lambda}{\lambda} = -\frac{df}{f} \Rightarrow \ln\left|\frac{\lambda}{\lambda_0}\right|\left|\frac{f}{f_0}\right| = 0 \Rightarrow \\ \left|\frac{\lambda}{\lambda_0}\right|\left|\frac{f}{f_0}\right| = 1 \Leftrightarrow u = \lambda f = \lambda_0 f_0 = u_0 = c = const., (f_0, \lambda_0) = const.,\ \lambda = \lambda_0\frac{f_0}{f} = \frac{c}{f} = \frac{H}{p}, p = \frac{Hf}{c} = \frac{nHf_m}{c} \end{bmatrix}.$$

Until here we analyzed a Binary System composed of two rotating bodies (m_1 and m_2) around their common center of mass (where the total kinetic energy of both rotating participants is $E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = E_r$). Alternatively we can present the same situation as another (artificial and equivalent) Binary System where one of involved masses $m_c = m_1 + m_2$ is much bigger than other ($m_1 = m << m_2 = M \cong m_c$) and staying at rest in their common center of mass (having zero orbital, kinetic energy), and second (much smaller) mass $m_r = \frac{m_1 m_2}{m_1 + m_2} \cong m << m_c \cong M$ is rotating around much bigger mass m_c, having again the same total orbital energy as before ($E_{orbital} = E_{k1} + E_{k2} = E_{k1} = Hf_1 = Hf = Hf_0$).

If we imagine that the last phase of a Binary System evolution is its collapse towards the creation of a single spinning mass $m_c = m_1 + m_2$ (where mechanical spinning of m_c is characterized by ω_c), we will have (still in the center of the mass coordinate system),

$$E_{orbital} = E_{k1} + E_{k2} = \frac{1}{2}m_r v_r^2 = H(f_1 + f_2) = Hf = E_r = \frac{1}{2}J_r\omega^2 = \frac{1}{2}(J_1 + J_2)\omega^2 = \frac{1}{2}J_c\omega_c^2 \Rightarrow$$

$$(J_1 + J_2)\omega^2 = J_c\omega_c^2,\ \omega_c = 2\pi f_c = \omega\sqrt{\frac{J_1 + J_2}{J_c}} = \frac{2\pi}{T}\sqrt{\frac{J_1 + J_2}{J_c}} = 2\pi f_m\sqrt{\frac{J_1 + J_2}{J_c}} = \begin{Bmatrix} \frac{4\pi f_0}{n}\sqrt{\frac{J_1 + J_2}{J_c}}, v_r << c \\ \frac{2\pi f_0}{n}\sqrt{\frac{J_1 + J_2}{J_c}}, v_r \approx c \end{Bmatrix}. \quad (2.11.14\text{-}19)$$

Binary Systems (as conceptualized here) are planar motional systems, meaning that involved circular motions are in the same fixed plane, and this is the reason why quantizing or synchronizing, or standing-waves packing criteria is related only to one orbital quantum number. Here, we should not forget that involved mechanical rotations and spinning have much different angular velocities ω_m, ω_c, compared to associated (surrounding) matter waves angular velocities $\omega_0 = 2\pi f_0 = 2\pi f$. Of course, all of that is an idealization or approximation, since more real are multi-body systems, like planetary or solar systems (including micro-world and subatomic systems), where orbital single-plane circular motions are becoming multi-planar elliptic-paths motions (having quantized inclinations for relevant planetary orbits). Consequently, new quantizing or waves synchronizing rules are getting additional angular quantum numbers, like in semi-classical quantization of angular momentum (see [40], D. Da Roacha and L. Nottale). In mentioned Multi-component Systems (including Binary Systems), very appropriate quantizing and generalizing approach will be to apply, ***creatively and with intellectual flexibility***, Wilson-Bohr-Sommerfeld Action Integrals, combined with familiar theoretical concepts published by Anthony D. Osborne, & N. Vivian Pope (see [36]). Ironically, the early days Classical Quantum Physics related to N. Bohr Hydrogen Atom Model is much more a Quantum approach to macrocosmic, real planetary orbital motions, than anything that explains or conceptualize a real nature of hydrogen atom. Here (in relation with Binary Solar Systems) we are still not specifying what kind of matter waves we are talking about, but a solid candidate (besides others related to inertial effects, rotation, and gravitation) that cannot be excluded are electromagnetic fields and waves.

Quantization in Physics is merely a consequence of the existence of stable Binary and Multi-component Systems and energy-momentum communications between them (but we should not forget that other, transient and non-stable systems have a place in our universe). This is also the area where modern-day Quantum Theory started being complex and fuzzy, since for managing such situations (in the absence of real, clear, natural and obvious conceptualization), it was necessary to establish new, primarily mathematically operating theories and postulates, which have been deductively generating "second-hand", luckily useful results.

2.3.3-3 Standing-Waves Resonators and Gravitation

Another aspect of imaginable, stable standing-waves field structures in relation to gravitation is the fact that every two masses (of specific Binary System, including static masses that are mutually touching) can be presented as a kind of half-wave ($\lambda/2$) resonator, or a gravitation-dipole, where the distance between two of such masses is equal to $r = \lambda/2 = c_{gr}/2f_{gr}$. Here c_{gr} is the radial (central) gravitational-waves velocity acting along the distance r connecting centers of masses in question and f_{gr} is the relevant, resonant frequency of the associated standing wave (see (2.11.14-15) and (2.11.14-16)). This can mathematically be described as,

$$\left\{\begin{array}{l} r = \frac{\lambda}{2} = \frac{c_{gr}}{2f_{gr}} = r_1 + r_2 \,,\; r_1 = \frac{m_2}{m_1 + m_2} r \,,\; r_2 = \frac{m_1}{m_1 + m_2} r \,,\; m_1 r_1 = m_2 r_2 \,, \\ E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = \frac{1}{2} F_g r = \frac{1}{2} G \frac{m_1 m_2}{r^2} r = \frac{1}{2} G \frac{m_1 m_2}{r} = \frac{1}{2} G \frac{m_r m_c}{r} = \\ = \frac{1}{2} F_c r = \frac{1}{2} \frac{m_r v_r^2}{r} r = \frac{1}{2} m_r v_r^2 = \frac{1}{2} m_r r^2 \omega^2 = \frac{1}{2} \mathbb{J}_r \omega^2 = \frac{1}{2} \mathbb{L}_r \omega = E_r = \frac{H}{4\pi}(n_1 + n_2)\omega = \\ = \frac{H}{2}(n_1 + n_2) f_m = \frac{H}{2} n f_m \,,\; n = n_1 + n_2 = n_r \,,\; n f_m = 2f \,, \mathbb{L}_1 \rightarrow (\mathbb{L}_1 + L_{s1}),\; \mathbb{L}_2 \rightarrow (\mathbb{L}_2 + L_{s2}),\; n \rightarrow (n + n_s) \\ \frac{H}{2\pi} = \frac{\mathbb{L}_1}{n_1} = \frac{\mathbb{L}_2}{n_2} = \frac{\mathbb{L}_1 + \mathbb{L}_2}{n_1 + n_2} = \frac{\mathbb{L}_r}{n_r} = \frac{\mathbb{L}_i}{n_i} = \frac{\mathbb{L}_1 + \mathbb{L}_2}{n} = \frac{(\mathbb{L}_1 + L_{s1}) + (\mathbb{L}_2 + L_{s2})}{n + n_s} \end{array}\right\} \Rightarrow$$

$$E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = \frac{H}{2}(n_1 + n_2)f_m = \frac{H}{2}nf_m = \frac{1}{2}F_g r = \frac{G}{c_{gr}}m_1 m_2 f_{gr} = \frac{G}{c_{gr}}m_r m_c f_{gr} \Rightarrow$$
$$\Rightarrow H = H \cdot \frac{nf_m}{2f} = \frac{G}{c_{gr}}\frac{m_1 m_2 f_{gr}}{f} = \frac{G}{c_{gr}}\frac{m_r m_c f_{gr}}{f} = 2\frac{G}{c_{gr}}\frac{m_1 m_2 f_{gr}}{nf_m} = 2\frac{G}{c_{gr}}\frac{m_r m_c f_{gr}}{nf_m} =$$
$$= 2\frac{F_g}{c_{gr}}\frac{f_{gr}}{nf_m}r^2 = \frac{F_g}{c_{gr}}\frac{f_{gr}}{f}r^2 = 2\pi\frac{L_1 + L_2}{n} = \text{constant}, \ \frac{nf_m}{2f} = \frac{G}{c_{gr}H}\frac{m_1 m_2 f_{gr}}{f} = \frac{G}{c_{gr}H}\frac{m_r m_c f_{gr}}{f} = 1 \Rightarrow$$
$$\Rightarrow F_g = \frac{\pi c_{gr} nf_m}{nf_{gr}}\frac{(L_1 + L_2)}{r^2} = \frac{2\pi c_{gr} f}{nf_{gr}}\frac{(L_1 + L_2)}{r^2} = \frac{4\pi f}{n}\frac{(L_1 + L_2)}{r} = 2\pi f_m \frac{(L_1 + L_2)}{r} =$$
$$= \omega_m \frac{(L_1 + L_2)}{r} = v\frac{(L_1 + L_2)}{r^2} = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2},$$
$$\omega_m = \omega = \frac{2\pi}{T} = 2\pi f_m = \frac{v}{r} = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v_r}{r}, \ \frac{v_1}{v_2} = \frac{r_1}{r_2}, \ v(L_1 + L_2) = G\, m_1 m_2,$$
$$p_1 = m_1 v_1 = m_2 v_2 = p_2 = p = p_r = m_r v_r, \ \vec{p}_1 + \vec{p}_2 = \vec{0}, \ \vec{r} = \vec{r}_1 + \vec{r}_2,$$
$$\vec{v}_i = \frac{d\vec{r}_i}{dt}, \ \vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_r = \vec{v}_1 + \vec{v}_2, \ v_1 v_2 = \omega^2 r_1 r_2, v_i = \omega r_i, \ \frac{v_1 v_2}{v^2} = \frac{r_1 r_2}{r^2}. \qquad (2.11.14\text{-}20)$$

There are many challenging (still hypothetical) options regarding understanding the Gravitation starting from results found in (2.11.14-20). One of such possibilities, *offering the replacement for Newton Law* (*$F_g = v\frac{(L_1 + L_2)}{r^2} = G\frac{m_1 m_2}{r^2}$), is that gravitational force is directly dependent on the total resulting vector of angular and spin moments of Binary System participants.* Such angular moments ($\vec{L} = \vec{L}_1 + \vec{L}_2$) are externally visible (and measurable), and some of their components could be states related to spinning, or to another kind of hidden rotation of belonging subatomic entities (see also (2.2), (2.4-5), (2.5) and (2.11)). What is significant here is that all three vectors $\vec{r}, \vec{v}, \vec{L}$ are mutually orthogonal, meaning that relevant vectors' product will produce a vector of gravitational force $\vec{F}_g$ collinear with $\vec{r}$. Consequently, now we can be sure that the origin of gravitation is in an interaction between angular, orbital and/or spin moments of mutually attracting masses.

Half-wave resonator, as an intuitive concept for explaining gravitational attraction between two pulsating or oscillating masses (elaborated in (2.11.14-20)) can also be approximated and modeled as the situation when specific springs mutually connect two masses in question (Binary System). Such springs (obviously having non-linear spring coefficients k_1 and k_2), are effectively realizing Newton gravitational force, between masses in question and can be supported by the following (at least dimensionally correct and still hypothetical) relations,

$$\left\{\begin{matrix} F_g = k_1 r_1 = k_2 r_2 = G\frac{m_1 m_2}{r^2}, \ m_1 r_1 = m_2 r_2 = m_r r, \ m_r = \frac{m_1 m_2}{m_1 + m_2}, \\ f_{gr} = \frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}} = \frac{1}{2\pi}\sqrt{\frac{k_2}{m_2}} \ (=)[Hz], \\ r_1 = \frac{m_2}{m_1 + m_2}r = \frac{\lambda_1}{4}, \ r_2 = \frac{m_1}{m_1 + m_2}r = \frac{\lambda_2}{4}, \lambda_i = \frac{c_{gr-i}}{f_{gr}} = \frac{H}{p_{gr-i}}, \\ r = r_1 + r_2 = \frac{\lambda_1}{4} + \frac{\lambda_2}{4} = \frac{\lambda}{2} = \frac{c_{gr1}}{4f_{gr}} + \frac{c_{gr2}}{4f_{gr}} = \frac{c_{gr1} + c_{gr2}}{4f_{gr}} = \frac{c_{gr}}{2f_{gr}}, \\ F_g r = G\frac{m_1 m_2}{r} = G\frac{m_r m_c}{r} = k_1 r_1^2 + k_2 r_2^2 = 2Hf = nHf_m = \frac{2Gm_1 m_2}{c_{gr}}f_{gr}. \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} \frac{k_1}{k_2} = \frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{c_{gr2}}{c_{gr1}} = \frac{\lambda_2}{\lambda_1} \\ c_{gr-i} = \lambda_i f_{gr} = 4r_i f_{gr} = \frac{2r_i}{\pi}\sqrt{\frac{k_i}{m_i}} \end{matrix}\right\}$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER... ♣**]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\Rightarrow \left\{ \begin{array}{l} \lambda_1 = \frac{4m_2}{m_1+m_2} r = \frac{c_{gr-1}}{f_{gr}} = \frac{H}{p_{gr-1}} , \ \lambda_2 = \frac{4m_1}{m_1+m_2} r = \frac{c_{gr-2}}{f_{gr}} = \frac{H}{p_{gr-1}} , \\ H = \lambda_1 p_{gr-1} = \lambda_2 p_{gr-2} = \frac{4m_2}{m_1+m_2} p_{gr-1} r = \frac{4m_1}{m_1+m_2} p_{gr-2} r = \frac{c_{gr-1}}{f_{gr}} p_{gr-1} = \frac{c_{gr-2}}{f_{gr}} p_{gr-2} = \\ = H \cdot \frac{nf_m}{2f} = \frac{G}{c_{gr}} \frac{m_1 m_2 f_{gr}}{f} = \frac{G}{c_{gr}} \frac{m_r m_c f_{gr}}{f} = 2\frac{G}{c_{gr}} \frac{m_1 m_2 f_{gr}}{nf_m} = 2\frac{G}{c_{gr}} \frac{m_r m_c f_{gr}}{nf_m} = \\ = 2\frac{F_g}{c_{gr}} \frac{f_{gr}}{nf_m} r^2 = \frac{F_g}{c_{gr}} \frac{f_{gr}}{f} r^2 = 2\pi \frac{L_1 + L_2}{n} = \text{constant} \end{array} \right\}$$

$$\Rightarrow \left\{ \begin{array}{l} p_{gr-1} = m_1 \cdot \frac{G(m_1+m_2)}{4c_{gr} r} \frac{f_{gr}}{f} = m_1 \cdot \frac{G(m_1+m_2)}{2c_{gr} r} \frac{f_{gr}}{nf_m} = m_1 \cdot v_1^* , \\ p_{gr-2} = m_2 \cdot \frac{G(m_1+m_2)}{4c_{gr} r} \frac{f_{gr}}{f} = m_2 \cdot \frac{G(m_1+m_2)}{2c_{gr} r} \frac{f_{gr}}{nf_m} = m_2 \cdot v_2^* , \\ v_1^* = v_2^* = \frac{G(m_1+m_2)}{4c_{gr} r} \frac{f_{gr}}{f} = \frac{G(m_1+m_2)}{2c_{gr} r} \frac{f_{gr}}{nf_m} = v^* \end{array} \right\} \qquad (2.11.14\text{-}21)$$

What is interesting in (2.11.14-21) is that Binary Systems relations are conclusively showing that two masses, mutually exercising the Newton force of gravitation (as a Binary System), can be analyzed in a certain approximate way as two weakly coupled mass-spring oscillators (linked to their common center of mass), having the same resonant frequency on both sides. In order to achieve a global forces balance (like in cases of stable planetary systems, where attractive gravitational force is balanced by repulsive centrifugal force), attractive forces of such non-linear springs (between masses) should be compensated by equal repulsive forces of other two springs (connected in line with two masses in question in the mutually opposing directions). This way we can represent gravitational attraction between each of masses and the rest of the universe. This way (see Fig.2.5), we will be able to analyze (almost) independently, each of two mass-spring systems as an equivalent, macro $\lambda / 4$ resonator, as already practiced in (2.11.14-21).

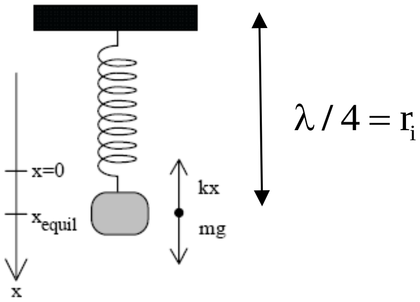


Fig.2.5. Simple Mass-Spring oscillator

The mass-spring oscillations (where mass m_i is oscillating with a certain amplitude Δr_i, Fig.2.5) can be mathematically presented by simple harmonic function $x = (\Delta r_i)\cos(\omega t + \varphi)$. In reality, we could imagine that (valid for both masses) distance $\mathbf{r_i}$ between a mass m_i and common center of both masses is pulsating (or harmonically oscillating) between two values: $r_i + \Delta r_i$ and $r_i - \Delta r_i$. This will (after applying few of mathematical steps valid for mass-spring systems, and applicable to particle-wave duality situations) extend the relation of proportionality between relevant elements of a Binary System in question (found in (2.11.14-21)) to,

$$\frac{k_1}{k_2}=\frac{m_1}{m_2}=\frac{r_2}{r_1}=\frac{c_{gr2}}{c_{gr1}}=\frac{\lambda_2}{\lambda_1}=\frac{(\Delta r_2)^2}{(\Delta r_1)^2}, \quad \left(r=r_1+r_2=\frac{\lambda_1}{4}+\frac{\lambda_2}{4}=\frac{c_{gr1}}{4f_{gr}}+\frac{c_{gr2}}{4f_{gr}}=\frac{\langle c_{gr}\rangle}{2f_{gr}}=\frac{c_{gr}}{2f_{gr}}\right). \tag{2.11.14-22}$$

If such (standing waves and resonant) oscillations exist between two astronomic objects, we should be able to detect them in some way. For instance if one of masses is our Sun and the other of masses is our planet Earth, the light beam coming from the Sun and detected on the Earth (by certain prism) should be ***wavelength-modulated*** producing that every specific color should have its bandwidth, directly proportional to the oscillatory speed amplitude $\omega\Delta r_i <<< c$ (like kind of Doppler effect). Such bandwidths can be measured (for many specific colors) on the Equator and somewhere far from Equator (as well as from some satellite observatory), and we should notice the differences between corresponding bandwidths. Since here we are talking about modulated and standing waves motions (between two masses), we can apply generally-valid relations between group and phase velocity, where: group velocity (of a relevant gravitational wave) is $v=c_{gr}=v_{gr}$, the phase velocity is $u=u_{gr}$, modulating planetary oscillating speed is $\Delta v=\omega\Delta r_i <<< c$, and mean group and phase velocities are $\bar{v}=\bar{c}_{gr}=\bar{v}_{gr}, \bar{u}=\bar{u}_{gr}$.

This would give us an idea of how to establish relations between relevant frequency and wavelengths bandwidths, as follows,

$$\left\{\begin{array}{l} v=u-\lambda\frac{du}{d\lambda}=-\lambda^2\frac{df}{d\lambda}=u+p\frac{du}{dp}=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=H\frac{df}{dp}=\frac{df}{df_s}=\frac{2u}{1+\frac{uv}{c^2}}, \\ u=\lambda f=\frac{\omega}{k}=\frac{\tilde{E}}{p}=\frac{Hf}{p}=\frac{f}{f_s}=\frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}=\frac{E_k}{p},\ f_s=k/2\pi, \\ \Rightarrow 0\le 2u\le\sqrt{uv}\le v\le c \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} d\tilde{E}=Hdf=mc^2d\gamma,\quad \frac{df}{f}=(\frac{dv}{v})\cdot\frac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} \\ (v\pm\Delta v)=(u\pm\Delta u)-(\lambda\mp\Delta\lambda)\frac{d(u\pm\Delta u)}{d(\lambda\mp\Delta\lambda)} \end{array}\right\}\Rightarrow\left\{\begin{array}{l} \frac{\Delta f}{f}=(\frac{\Delta v}{\bar{v}})\cdot\frac{1+\sqrt{1-\frac{\bar{v}^2}{c^2}}}{1-\frac{\bar{v}^2}{c^2}}\cong\frac{2\Delta v}{\bar{v}}=\frac{\Delta v}{\bar{u}}, \\ \frac{du}{\Delta u}=\frac{d\lambda}{\Delta\lambda}=2\frac{du}{\Delta v}=\frac{dv}{\Delta v}, \\ \bar{v}\cong 2\bar{u}=2\bar{\lambda}\cdot\bar{f},\ \Delta v\cong 2\Delta u, \\ 2\frac{\Delta u}{\bar{v}}=\frac{\Delta v}{\bar{v}}=\frac{\Delta v}{2\bar{u}}=\frac{\Delta f}{2\bar{f}}=\frac{\Delta v}{2\bar{\lambda}\cdot\bar{f}},\bar{\lambda}\cdot\Delta f=\Delta v \end{array}\right\}. \tag{2.11.14-22}$$

If we continue developing similar ideas about standing waves communications between masses, we should be able to explain "redshifts and blue-shifts" of the light spectra from deep and remote cosmic areas, captured by astronomic observatories on our planet.

No doubts that here we are faced with an oversimplified and accelerated mathematical and brainstorming conceptualization which is mostly useful as the first step towards familiarization with gravitational standing waves as an explanation of the nature of attractive gravitational force. Taking and proving-valid such approach will have consequences on a better understanding of origins of Gravitation.

2.3.3-4 Central Forces, Newton and Coulomb Laws

Next challenging question here is why central forces, like those that Newton and Coulomb's laws are describing, are inversely dependent from the square of the relevant distance, $F(r) = \frac{C}{r^2}, C = const.$? We can indirectly explain such situation ($F(r) = \frac{C}{r^2}$) by analyzing force components involved in orbital motions under a central force. Since in cases of central forces, relevant orbital momentum is constant $\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \vec{r} \times \vec{p} = \overrightarrow{const.}$, we can conclude that vector $\vec{L}$ is perpendicular to the plane defined by the vector $\vec{r}$ and the momentum $\vec{p}$. The fact that $\vec{L}$ remains constant is saying that relevant plane ($\vec{r}$, $\vec{p}$) also remains constant (or stable), and that every orbital motion (on such plane) under central force is a stable, planar, and two-dimensional motion (which can naturally host standing waves structures without big need to give probabilistic or stochastic meaning to any of such waves). This is very much like astronomic observations documenting that many solar systems are planar, facilitating involved mathematical processing, for example,

$$L = L(r,\theta) = mr^2 \frac{d\theta}{dt} = const., \vec{F}(r,\theta) = \vec{F}(r) + \vec{F}(\theta) = \vec{F}(r) = \overrightarrow{(m\frac{d^2r}{dt^2})} = m\vec{a}_r + m\vec{a}_\theta = \overline{\left[m\frac{d^2r}{dt^2} - mr(\frac{d\theta}{dt})^2 \right]},$$

$$a_r = \frac{d^2r}{dt^2} - r(\frac{d\theta}{dt})^2,\ a_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\cdot\frac{d\theta}{dt},\ mr\frac{d\theta}{dt} + 2m\frac{dr}{dt}\frac{d\theta}{dt} = 0 \Rightarrow \frac{d\theta}{dt} = \frac{L}{mr^2} = \frac{L}{m}\rho^2,\ \rho = \frac{1}{r},$$

$$\frac{d^2\rho}{d\theta^2} + \rho = -\frac{m}{L^2}\cdot\frac{1}{\rho^2}F(\frac{1}{\rho}),\ F(\frac{1}{\rho}) = F(r) = \frac{C}{r^2} = C\cdot\rho^2 \Rightarrow \frac{d^2\rho}{d\theta^2} + \rho = -\frac{mC}{L^2} \Rightarrow \rho = A\cos(\theta-\theta_0) - \frac{mC}{L^2},$$

$$r = \frac{1}{A\cos(\theta-\theta_0) - \frac{mC}{L^2}}.$$

The last equation is describing conic curves $r = r(\theta)$, such as ellipse, parabola, and hyperbola, depending on constants A, θ_0, m, C, $\vec{L}$. If we chose the reference coordinates where $\theta_0 = 0$, we will get for a planetary and satellite orbits $r = \frac{1}{A\cos\theta - \frac{mC}{L^2}}$ that is a conic section, which can also be transformed to $r = r_0 \frac{1+e}{1+e\cdot\cos\theta}$, where **e** presents eccentricity of the orbit. For **e** < **1**, the orbit is an ellipse, for **e** = **1**, the orbit is a parabola, for **e** > **1** the orbit is a hyperbola, and for **e** = **0** the orbit is a circle (covering all cases of planetary, asteroids and satellite orbits). ----------------------------- ♣]

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.