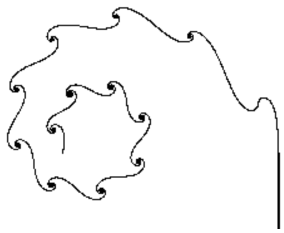
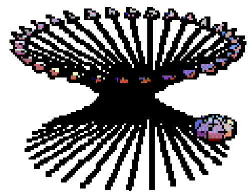
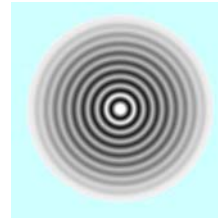
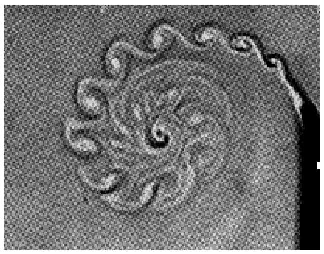
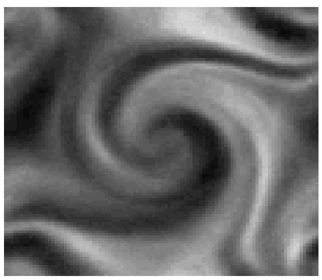




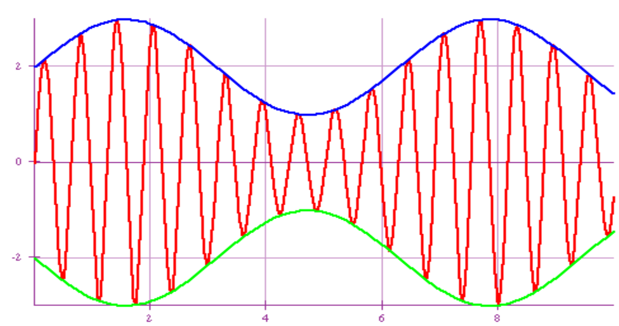

CERTAINTY of PARTICLE – WAVE UNITY

or

UNCERTAINTY of PARTICLE – WAVE DUALITY

Miodrag Prokic

Switzerland, 2006

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INNOVATIVE ASPECTS OF PARTICLE-WAVE DUALITY, GRAVITATION AND ELECTROMAGNETIC THEORY ADDRESSED FROM THE POINT OF VIEW OF ELECTRIC AND MECHANICAL ANALOGIES

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Abstract:

An overall and multi-level system of physics-related analogies is presented in this paper. Based on the presented analogies, several hypothetical predictions are made regarding innovative conceptual understanding of de Broglie matter-wave phenomenology (or particle-wave duality) in connection with different aspects of mass rotation and Faraday-Maxwell Electromagnetic Theory, generating elements for new conceptualization of Gravitation. The new aspects of gravitation are extended to (still hypothetical) phenomenology of mass and fields rotation and to particle-wave states of motional energy interactions, suggesting that gravitation field should have a complementary field (which has not been well understood until present, or still has not been considered or modeled as a complement to gravitation), in the similar fashion as electric and magnetic fields are mutually coupled and complementary fields, implicating that similar coupling and unity should exist between linear and rotational aspects of any other motion. It is also contemplated that the gravitation force could be related to formations of standing waves of resonating universe, acting towards nodal domains of such standing waves.

Then, the background regarding understanding probabilistic and other ontological roots of contemporary Orthodox Quantum Theory is also innovatively addressed (since Quantum Theory is presently the only widely accepted theory dealing with particle/s-wave/s duality). It is also shown why quantum theory mathematically works well, even incorporating certain conceptual problems (or having not enough common-sense and causally and tangibly clear concepts), here resulting in the formulation of much more general forms of Schrödinger and Dirac-like wave equations (than presently known), but almost without using any assumptions of probabilistic Quantum Theory (and mostly being on the level of elementary mathematics). The generalization of the Schrödinger equation is achieved first by using very powerful mathematic presentation of physics-related wave functions in the form of Analytic Signal function (based on Hilbert transformation), and by introducing and respecting specific relations between wave packet group and phase velocity (where both of them are never higher than the maximal speed of light), also excluding the rest energy and rest mass from the wave packet model (resulting that de Broglie particle-wave phenomenology should only be related to all states of motional energy). The meaning of stochastic and probabilistic quantum mechanical wave function is equally well explained void of most of its probability decor, showing that hidden sources of probability-like distributions are coming from particle-wave duality (or unity) phenomenology and from hidden effects of particles and fields rotation, conveniently modeled using the framework of Probability, Statistics and Signal Analysis (when the instantaneous time-space signal phase is not taken into account, or not known because of the limits of applied modeling and/or stochastic nature of analyzed phenomenology). It is also explained why present probabilistic wave function modeling works well.

The new field unification platform proposed here is initially (only as a starting point) based on exploiting united multi-level analogies and basic continuous symmetries between different natural couples of mutually original and spectral, or conjugated domains (such as Fourier-transform couples: time frequency, momentum position, electric-magnetic charges, angle-*orbital-momentum*, etc.), being followed by appropriate upgrading and integration into wider texture of known laws of Physics.

Uncertainty relations are also generalized and treated as the matching (or mismatching) interval relations between original and spectral signal duration/s (and between their elementary parts, similar as treated in cases of digital signal synthesis), while explaining and extending this concept much further than presently exercised in Quantum Theory. Consequently, the particle-wave duality concept, *here presented more as the particle-wave unity*, is extended to any situation where motional or time-dependent energy flow is involved (regardless of its origin), indicating that any change of motional (or driving) energy is immanently coupled with associated de Broglie matter waves, creating inertial-like forces in the form of matter waves (where such inertial forces could have gravitational, mechanical, electromagnetic, or some other nature, depending on the participants involved in the interaction. In fact, in this paper is underlined that also all kind of ordinary waves, known in Physics, are forms of de Broglie matter waves).

Using an oversimplified formulation we will come to the conclusion in this paper that de Broglie matter waves always have their stationary, stable (or "parking") places when rotating inside number of micro-domains of particle/s structure/s (or inside atoms and elementary particles), and become externally active (and measurable) in all situations when particles are changing their previous energies and/or states of motions. In other words, when a matter wave creates a kind of self-closed, stationary and standing waves rotating structure, becoming "self-localized" in a limited space, such matter wave "packaging" presents what we are usually naming as a particle (while different packaging formats of basic elementary particles are creating all other particles).

This paper (mostly formulated using the language of mathematics) intentionally avoids complicated mathematical analyses in order to facilitate the conceptual and common-sense understanding, and easy acceptance of the ideas given here, by a wide public of professionals and amateurs interested in the same field. Thus, the author hopes, the initial scholastic and dogmatic criticism of this paper could be avoided, in order to grant the priority to conceptual, common sense and intuitive understanding of the presented ideas, and to serve, as much as possible, as a challenging and brainstorming framework for starting many new projects and more rigorous analyses in the future. Certain chapters of this paper have origins from partially rewritten and updated (also translated) version of the author's B.S. diploma work (cited literature: [3]).

The paper is created and structured as an open-ended brainstorming and preliminary draft, where a lot of open-minded, creative, unbounded and positively colored good will should be implemented in order to read it and get its messages correctly. The paper is still unfinished and a lot of its parts should be mutually better united or connected (also some of them waiting to be translated in English), because the author started to write it sporadically and without planning (as a hobby and unprofessionally) from his early student days, not really finalizing it until present days. Even in such unfinished and not well synchronized state, the paper contains many significant messages and innovations that the author decided to prepare it in a form which would be available and understandable to the others involved in the familiar kind of thinking. For very strict and serious, critically oriented readers and officially recognized experts in the same field, this paper and its author would be an easy target, but for somebody who is creatively reading and catching the exciting and challenging concepts, this could be quite a different intellectual experience. Simply, such a large problematic cannot be properly elaborated by one man only during an average human life, and this is the main reason why the paper is given to the public in its present state. Author intends to make a step-by-step upgrading of it as long as the nature related to performing such activities would allow him. All suggestions and collaboration proposals are welcome.

CONTENTS

Appendix:

INTRODUCTION

The principal subject of this paper is related to the formulation of an updated theory of particle-wave duality ***(or particle-wave unity)***, but indirectly this paper also creates (or opens the window to) the new platform regarding understanding fields' unification theory.

Presently (meaning from the beginning of the 21 century) official science is still considering the biggest achievements of Physics belonging to: 1° **Mechanics** including **G**ravitation **T**heory (**GT**) and Einstein **R**elativity **T**heories (**RT**), 2° **Maxwell Electromagnetic Theory** (**EM**), and 3° **Quantum Theory** (**QT**), including all surrounding theories and mathematical tools in connection to them. Any new theory regarding particle-wave duality (including the one from this paper) should also be placed in the same space of **EM**, **GT**, **RT** and **QT** (and, of course, all new formulations and updates of such theory should start from already known works and results related to L. de Broglie wave hypothesis and Max Planck quantized energy concepts). *In this paper the <u>Renewed **P**article-**W**ave **D**uality</u> concept (**PWD**) is presented as the most important link (or necessary background) for creating Universal and Unified Field Theory. It is also shown that there couldn't be any **PWD** manifestation without presence of a certain two-body (or many-bodies) field interaction, meaning that **PWD** is a product and a consequence of field interactions (and space-time evolving forces) between moving objects, when certain state of motion is changing. If we could imagine a moving particle in an "absolutely empty space" there wouldn't be any **PWD** manifestation. Also without intrinsic rotation (and torsional field/s components) associated with all kinds of movements there wouldn't be any particle. In fact, here, the use of the term <u>particle-wave duality</u>, presents only the verbal link to the traditional way of describing the same phenomenology, and it is shown in this paper that the much more convenient formulation would be the <u>particle-wave unity</u> (it will be explained how and why particle's and wave properties are united).*

It is already known that Einstein (and many others) tried to make the unification of **EM** and **GT** in the frames of **RT**, without having significant success, most probably because there were at least two components missing that created the biggest and unsolvable problems to Einstein's efforts, as for instance:

1. **SRT** (Spec. Relativity Theory) is not addressing rotational and other accelerated movements, even though such motional elements inseparably exist in different forms, associated with any linear motion, and as intrinsic (structural) properties of all elementary particles and atoms (such as spin and orbital moment characteristics). In fact modern **RT** should mostly be a kind of theoretical consequence and mathematical product of a well-established **EM**, which still has space to be better (or more coherently and more generally) formulated. It could also happen that new **SRT** would be so radically modified and upgraded (compared to the old one), that it would really present new and much more generally valid theory.

2. **EM** mathematical foundations (present forms of Maxwell equations) should also have "rotational or torsion field" components (as well as electromagnetic waves in certain circumstances should have axial electric and magnetic field vector components) since **EM** fields and electric charges are omnipresent "inside and around" almost all (already rotating) constituents of our macro and micro universe, and in addition, it is a fact that **originally Maxwell equations are developed having in mind number of conceptual similarities and symmetries between electromagnetic and fluid-mechanics phenomenology**. Unfortunately, in the contemporary physics it is no more fashionable, acceptable or defendable to deal with a space texture, matter matrix or some other carrier-fluid (formerly named ether) where electromagnetic waves and fields are propagating, regardless of the fact that form of Maxwell equations analogically corresponds to certain fluid-mechanics equations (where we know what the carrier-fluid is). Consequently, Maxwell equations should have some other (maybe still unknown) similarities to the phenomenology known in fluid mechanics, and should evolve towards formulating more united and more mutually dependant electric and magnetic field entities, and solutions to such generalized Maxwell equations which should generate most of the important building blocks of a renewed **SRT**. In fact, an innovated **EM** should take the higher practical and theoretical position compared to **SRT**, which would then remain only as the consequence of such **EM**.

Briefly saying, both modern **SRT** and **EM** should be (firstly) upgraded or modified (or replaced by new theories) up to the level (conceptual, mathematical, theoretical and factual) when they would become much more mutually compatible for unification (otherwise, all efforts to unite them at present premature state will be too difficult, or artificial). Similar comments can be given regarding merging of **QT**, **RT** and **EM.** Anyway, we cannot easily unite the theories which are mutually still not compatible and comparable for unification, because they probably have (apart from all the good sides) some of artificial, unnatural, very particularly valid, missing, or wrong building elements, assumptions and/or concepts, regardless how successful each of these theories is in its own domain. Such theories in the present state of art are often very useful mostly for explaining already known experimental facts. Of course, there are modern field theories that are addressing such problems, as Superstrings or M-theory, but this paper would deal mostly with common grounds and establishments of **PWD** concepts, for if grounds are well made, further theoretical advances based on such grounds should have much bigger significance and applicability.

The new **p**article-**w**ave **d**uality concept (**PWD**) in this paper is presented highlighting the following step-stones:

A) First (as a relatively low level of new ideas initiation), a multilevel system of electromechanical and other physics-related analogies is established (in the first chapter) indicating and generating elements, expressions and hypothetical predictions regarding new **PWD** and fields' unification framework. The author presents and defends the philosophical platform that well-established, mutually equivalent, symmetrical or similar mathematical models (or mathematical structures) should describe the same reality (only seen from different theoretical perspective/s, from different space projection/s, or dimensionally presented using different set/s of coordinates, or different cross-sections). In fact, everything in nature or universe we are part of is united and cross-linked in number of ways, but we are still not able to formulate a good unifying theory. Starting deductively from the fact that **unity exist** (with or without our theoretical approach to it), we could ask ourselves what is the best and easiest strategy to make our sporadic and mutually separated knowledge bases and our particular physics theories mutually more united; -and the answer here is to use analogies, this way establishing important "filtering process" for testing new theories. Consequently, if we can show that today's "**probability framework"** of **QT**, or important concepts, assumptions and models regarding **SRT** and **RT** are just one among other (mutually equivalent or isomorphic) mathematical structures, equally or sufficiently well describing the **QT** and **RT** world, we could materialize some other (non-probabilistic, or non-relativistic) picture/s of **QT** world, as well as some other Space-Time and Relativity concepts, without applying the same set/s of initial assumptions as presently done. Let us formulate the same opinion differently: If we know that contemporary way of applying **QT**, **SRT** and **RT**, on the practical and mathematical level is producing correct results and correct predictions in cases of certain experimental situations (or well explaining already known facts), this is a kind of confirmation that mathematical frames of mentioned theories are maybe not mutually united and not the best, but particularly valid and sufficiently correct (in their own domains of definition), regardless how many initial assumptions and weak starting points such theories could have in their fundaments. Of course, this would stay valid and correct unless one day we find new experimental situation/s and facts that couldn't be explained using tools from **QT**, **SRT** and **RT**. It is also safe to say that mathematically we could have sufficiently correct final concepts (theories, results or formulas) in modeling certain Physics-related events, even if we started from partially wrong assumptions, and on the way of developing our mathematical models we start introducing some other, maybe again partially wrong, but interactively and iteratively correcting assumptions, canceling errors linked to previous assumptions, eventually creating sufficiently correct models and correct data fitting (after applying a few of such iterative, self-correcting steps), what is probably the case of some contemporary Physics theories. The biggest mistake in such situations is that eventually, when we see that our (final) theory is producing good results and correct predictions, we start glorifying (some or all of) our initial and in-process-made steps and mistake correcting assumptions as absolutely correct (what could be the situation with certain aspects of contemporary **QT** and **RT**). In order to test existence of such "structural and logical defects" of our fundamental theories, here is favored a method of using the multilevel analogy platform (including most generic, basic, continuous Symmetries), as the most general and most neutral comparison frame between different results, models and theories independently developed in Physics, by creating a mosaic of known and hypothetical mathematical expressions and equations that should be applicable in making mutual and non-contradictory cross-theoretical and interdisciplinary penetrations and predictions. If certain initial or crucial assumption exists only in **QT** and/or **RT** (or in another theory, but not in all of them), eventually making them well-operational, and we wouldn't be able to apply it in other theories and domains of Physics (where we already have some other well-operating theories), this would indicate that such an assumption is maybe wrong (or only locally valid, serving the purpose of correcting some other mistake/s, or filling missing conceptualization with whatever fits and works the best). Consequently, the message of this paper is that the power of multilevel analogical screening and cross-correlating of mutually compatible and coherent results of different theories, assumptions and results in Physics should be exploited much more than we presently use such strategies (combined, of course, with satisfying all conservation laws and/or basic continuous symmetries). Not doing something like that would mean that we are still not ready to face the possibility that some of our contemporary theories could be much more locally valid, artificial and short-living, contrary to how their founders and followers would like to present them (as eternally stable).

B) It is also shown (in the third chapter of this paper) that **EM** could be slightly upgraded (generating Lorentz-like **EM** field transformations, without use of any **SRT** methodology and assumptions), where the possibility of the existence of **EM** rotational field components becomes an explicit prediction. By formulating generalized and fully mutually symmetrical definitions and equations of electric and magnetic fields, charges, currents and voltages, we can additionally upgrade present Maxwell-Faraday **EM** theory, and extend the system of electromechanical analogies. It is sufficiently well shown that such updated **EM** theory (analogically and symmetrically extended to mechanics) would start generating or reinventing the foundations of **SRT**, without a big need to have any significant support in Einstein **RT** (what is easy to say now, but in the time when Einstein established **RT**, such options were not obvious).

C) Later on, it is made obvious that every linear (or rectilinear) particle/s motion is coupled with some kind of (complementary and conjugated) associated rotational motion (indicating that **SRT** should also be upgraded to cover different aspects of rotation...), and that such associated rotational component/s are linked to de Broglie matter-wave phenomenology and to reaction and/or inertial forces (being an extension of Newton action-reaction and inertia concept to **EM**, **SRT**, Torsion and other fields). ***Even in a purely linear motion of an ordinary particle (which has a rest mass) there is a unity and coupling of linear and rotational motions, and we usually do not see the rotational aspects of such unity because different aspects of rotation are internally hidden or captured by the particle rest mass.***

D) The appearance of **PWD** phenomenology is related to dynamic and transitory (waving and spinning) effects in all situations when interacting objects (particles, quasi-particles, waves...) are approaching each other, and separating from each other, and/or when the state of motional (or total) energy of certain system is changing. It is also precisely shown that only states of motional (and time variable) energies are creating de Broglie matter waves (or that states of relatively stable rest mass and rest energy do not belong to matter-wave phenomenology). Without considering elements of rotation (on different qualitative levels of particles and wave motions and inside their internal structures) as a complement to linear motion, it is not possible to explain formation of real (space-time stable) particles, atoms and their constituents. We already know that all elementary particles, subatomic particles, photons and other quasi-particles have certain intrinsic spin and/or orbital moment/s characteristics, and something similar appears to be also typical for planets, planetary systems and galaxies, since all of them rotate in some way/s relative to something. Quantum nature of the atom world should be a kind of energy exchanges, couplings and **natural-gearing** between resonance-alike interference and fitting manifestations (between de Broglie matter-waves), when different **PWD** objects (atom constituents, for instance) are mutually fitting (or packing) into certain unified, more complex and relatively stable object (which would eventually, after formation, have a stable rest mass). Even if we sometimes do not know what really rotates, oscillates or makes matter-waving, it should be clear that certain still unknown or hidden **PWD** entity should exist in the still unexplained background of relevant phenomena (because, categorically saying that everything in our universe, especially in a micro-world, follows the "channels and rivers" of mysterious and quantified probability waves, or probability distributions presented as waving forms, is a common-sense absurd, or fogy and metaphysical statement). ***It will be shown that de Broglie matter waves already exist (creating intrinsic, internal particles structure, in their states of relative rest) even before we can detect anything regarding PWD phenomenology, becoming externally, directly or indirectly, measurable in all situations when particles are changing previous states of motions.*** In fact, every presently known form of wave motion (such as electromagnetic waves, light, sound etc.) presents a particular form of de Broglie matter waves (but usually we do not treat them as de Broglie waves). *Also, not all motions, matter waves and field manifestations in our universe are quantum type phenomena, opposite to the position overwhelmingly supported by **QT**.*

E) In this paper, all complex theoretical (or mathematical) analyses regarding Fields Unification Theory and modern Topology are avoided. After formulating generalized forms of energy expressions and universal wave equations (almost without using any of methods practiced in Quantum Theory), and applying them deductively and backwards, towards all domains of physics, it will become obvious what solutions or directions for (topological, and other theoretical) upgrading of **EM**, **RT**, **GT** and **QT** will be (in order to create the same wave equations, but now going step-by-step from the opposite end). In this way, possible hidden conceptual mistakes of mentioned theories would become easily detectable and easy to correct.

F) Foundations of Orthodox Quantum Theory (**QT**) are conceptually, mathematically and intuitively demystified in this paper, showing why, how and when **QT** works well (regarding **PWD**), and showing that the same results (or even more rich analyses and conclusions) could be obtained using a bit different, more general and more deterministic mathematical modeling (placing stochastic philosophy and probabilistic modeling on their proper and much less ontological place than the case in the current **QT** is). *In this paper, all references regarding **QT** are related only to Schrödinger's wave mechanics, since updated **PWD** concept presented here is also developed around (mathematically) similar wave mechanics. The three remaining quantum theories (Heisenberg Matrix Mechanics, Dirac's Transformation Theory and Feynman's Sum-over histories formulation of **QT**) are not analyzed (or mentioned) in this paper (and the author of this paper does not have any competence for such an analyzes).* Obviously, since contemporary **QT** is experimentally and theoretically sufficiently well supported (mathematically producing correct predictions in number of situations), this paper is only explaining that its modeling and predictions are correct because the probabilistic **QT** is also compatible (in average) with all conservation laws of Physics (being particularly well generalized and formulated in dimensionless, normalized form, on the way similar or equivalent to the framework of Probability Theory). Since intrinsic rotational (and space-time phase) components of particles and waves motion aren't still universally recognized and established as (mathematically) operative elements for modeling universal natural forces and fields, the only solution applied in **QT**, in order to compensate this virtually missing (but very important) aspect of motion was found in "randomizing" results of micro-world interactions (omitting the immediate and continuous time-dependant aspects of motion), by searching for correct probability distributions, indicating where we could expect that certain event will "materialize in-average". Of course, **QT** didn't formulate its foundations using such formulations (as mentioned here), especially not by saying that probability-related strategy is compensating missing elements of certain physics-related conceptuality, and also compensating some unknown rotation and phase elements of particles and waves' motions. The proper key or code for creating physics-relevant and productive probability distributions (in the wave **QT**) was found (and in this paper maximally underlined, exploited and extended) in respecting and incorporating set of basic **PWD** concepts (originally established by L. De Broglie, M. Planck, Schrödinger, Heisenberg...) into already existing macro-mechanics (the meaning of the basic set of **PWD** concepts will be explained in the chapter, 4.1). The rest of the results **QT** just creatively borrowed or copied from mathematics (mostly from Statistics, Probability Theory and Signal Analysis), as well as from necessity to comply with obvious Symmetries valid in Physics, this way also improving, adjusting and correcting its own weak areas. In this paper an attempt is made to go beyond probabilistic **QT**, producing conceptually clearer and richer picture about **PWD** nature of our universe (than Orthodox Quantum Theory is currently presenting), by showing that an isomorphic, equivalent and more deterministic modeling of **QT** world is possible (and also universally applicable, not only to a micro world). We cannot neglect that existing **QT** mathematically works almost perfectly, and in this paper is strongly underlined that this should be considered mostly as a convenient modeling, merging and fitting of different, healthy and useful theoretical (generally valid) platforms and known facts, mixed with certain, sometimes non-proven, or not provable assumptions for filling some of still inexplicable spots. Anyway, the attempt of this paperwork is to show that **QT** should one day pass the transitory state of extreme complexity towards elegant and monumental simplicity.

Certain Physics theories, most of them *in a vicinity* of **QT**, established in 20-th century of modern human history on the planet Earth are in fact more of "**Patch-In** and **Fit-In**" theories than theories really and strongly integrated into the solid and stable body of the rest of Natural Sciences. It is also the author's opinion that the most of today's activities on the Field Unification Theory are presenting various geometry-related mathematical modeling possibilities, by creating or inventing different topological structures, symmetries, groups etc., while dogmatically and religiously staying inside of the old framework of the Orthodox Quantum Theory. It looks that majority of contemporary science people (in Physics) are almost copying, repeating and following each other's works, competing in publishing or presenting "looks like new ideas" (probably in some cases for the main purpose of maintaining their academic positions and chairs), which, in reality, dominantly and effectively belong to the same old-idea family, and which will be in most cases forgotten very short after publishing (for instance: modeling different geometry structures, and then making "mathematical experiments", starting from certain field vectors and their combinations, and testing what could be the consequences after satisfying certain equations or mathematical structures... and if they could fit such results into some aspects of real physics). In fact, what today's physics needs are new conceptual breakthroughs, refinements and unifications of all theories produced until the end of 20-th century (of the creatively organized technological history on the planet Earth).

Our (human) advantage is that we are in the position "*to see*" (intellectually detect, measure, test, model, conceptualize...) many of the facts regarding secrets and laws of Nature: we see (or measure) elements that are entering into certain reaction and we see (or measure) results after interaction happens. We can analyze an event (experiment) inductively, knowing its important starting points and facts, or deductively, knowing only results, and going backwards to discover elements that should create such results. If we apply valuable methodology and intuition, based on multilevel and well-established analogies and basic continuous symmetries, while respecting Conservation Laws and universal principles, using language of mathematics, and if we are able to perform fruitful and multilevel "intellectual cross-correlation process", creatively combining well-known experimental and other theoretical facts, we do not need to create countless number of artificial and arbitrary "mathematical experiments" in order to search for explanation of certain phenomenology, without seeing the global picture. And the most global picture regarding our existence and our Universe is that all presently known and maybe unknown fields and forces of Nature are intrinsically and naturally united, regardless the fact that in some cases we still do not know how they are united. Of course, whatever we create in process of our Physics-unifying efforts should be well integrated in the background of the present Physics knowledge, mathematically generalized, and experimentally confirmed as the final step of the creation of every new theory. The big weakness of modern official natural science interpretation is the tendency to show that it is already constructed and optimized as very convincing, well-established, very stable, very powerful and almost final knowledge (regarding **EM**, **RT**, **GT** and **QT**), and that only small improvements and insignificant upgrades could be in front of us. Here, one of objectives is to show that a lot of very significant, unifying and fundamental reconstruction work is in front of us regarding contemporary science interpretation.

In this paper, most of the new ideas, proposals and hypotheses are established until the level of presenting conceptually, mathematically and intuitively clear (common sense) physics-related picture, or easy understandable and sufficiently complete, preliminary project definition (often over-simplified), in order to serve as the starting (or challenging) platform/s for some future, more systematic and more professional work (since the author of this paper is only a *passionate and curious amateur* regarding all subjects discussed here). *In fact, the author's position is that one day, sooner or later, somebody should propose a set of new ideas (regarding Physics reconstruction business), formulated more or less professionally, but at least showing clearly new directions, opening new horizons, and going out of the well-established old circles.*

The multilevel analogy platform as here favored strategy is chosen (by the author) because of a very simple and obvious reason that could be explained as follows: Our contemporary Physics is composed of several complex and voluminous theories or *chapters*; -each of them formulated as almost self-sufficient and self-consistent theory, working well in its backyard, and each of them has been developed and mathematically modeled (often independently from other physics chapters), during relatively long time, involving contributions of almost countless number of research workers and scientists. In modern days it is really impossible to be a universal expert in all *chapters (or fields)* of Physics. In order to contribute something to modern Physics (or to criticize something), we should devote our life and career only to a certain very specific subject, often not seeing the global picture and knowing where that subject belongs. Otherwise, it would be very easy to make mistakes, and it could happen that some parts of scientific establishment of leading professors, scientists and some of their non-critical and well-obeying (basically existentially dependent) followers and students (behind each Physics *chapter*) would suppress, accuse or eliminate any of non-professionally or too originally formulated contributions or critics (since a minority of conservative science establishment in power could say that their scientific property already functions well, that they published a lot of books and papers to support their teachings that are in full agreements with earlier publications of their teachers and founders, that they have stable and respectful professional positions to practice such teaching/s, recognized officially by society in power etc.). We also know that any bigger waving against any kind of stationary mainstream has never been easily accepted in human history, life, ideology, politics and science (even this is the natural law of inertia regarding stationary and uniform motions). In order to open new scientific perspectives and new ways of thinking, without creating big intellectual energy dissipating séances, the easy, common and painless way

(proposed here) is to apply "***analogical screening, cleaning and rearrangements***" in physics. Later we will be able to apply more efficiently other heavy weapons of modern physics, such as conclusions, predictions and generalizations based on Symmetries.
Observing the same problematic (Physics) from another (philosophical) platform, we already know that we are an integral part of a naturally united universe, and consequently all Physics chapters (if correctly formulated) should intrinsically present well-united, synchronized and complementary descriptions of the same reality, and we also know that this is still not the case with some (or many) "departments" in our contemporary Physics.

What could be the most common and binding skeleton to search for Physics unification? Author of this paper is proposing (only as a starting point) systematical formulation and utilization of ***mutually compatible and coincidently applicable multilevel analogies and basic, continuous symmetries*** found in/and between different chapters or domains of Physics. Taking such analogies, as an argument to introduce or propose new ideas, hypothetical statements and unification platforms, is largely neutral, simple and painless strategy to capture large and interdisciplinary set of phenomena (which will be analyzed) and to avoid possible resistance and sanctions of some minor parts of officially recognized establishment and masters of particular Physics *departments*, since everybody knows in advance that analogical predictions are impersonal, sometimes being initially suspicious, and only good as indicative starting-points (but also in too many cases being shown as correct or very useful if properly established and applied). This way, we could always say that by following certain analogy platform (and understanding that the Nature is always better and more united than we presently know) we would get certain interdisciplinary predictions, possibly (or probably) applicable to several *chapters* of Physics previously not well integrated (and of course, we are not personally *"culpable"* if later would be shown that our predictions are for some other reason/s not fully correct). Applying and developing systems of multilevel analogies, iteratively, step-by-step, and testing them experimentally and theoretically, we shall be in every new step closer to more-correctly unified Physics. This is exactly the greater part of the strategy applied in this paper ***(to make the Physics More Clear, Logical and Analogical).*** We also know that theories based on establishing different Symmetries could bring even more general unification and research platform in Physics (**since all conservation laws are "married" to a certain kind of continuous symmetries**), but before we apply any strategy based on Symmetries, we should have a "*healthy and sufficiently large"* set of elements and good mathematical modeling frame for playing with Symmetries (or saying the same differently, combining Multilevel Analogies and continuous Symmetries with properly established mathematical framework should be the best mutually complementary research tool). The theory of Symmetries is already well developed in mathematics and applied a lot in contemporary physics, often covering very abstract spaces where it is sometimes too difficult to make simple and clear conceptualizations and correlations between certain mathematically described symmetries and real-world events. ***We should also ask ourselves if there is a certain hierarchy among the number of different symmetries we are able to construct mathematically, because only certain symmetries are stable, always applicable and universally valid, like continuous symmetries (regarding the world of Physics), and others have only certain level of applicability, probability, durability and/or stability.*** Since we are most probably living in a multidimensional universe, where we are still not able to conceptualize well the other, non-perceptible or still non-detectable dimensions, we are obviously making mistakes or experiencing problems in constructing symmetries (even without knowing that such problems could exist). This is also one of the reasons why in this paper multilevel analogies are taken as the "guiding and bottom-line channels and platforms" for filtering and building every other higher level conceptualization. ***Of course, creating analogies and making predictions and conclusions based on analogies is also nothing new regarding any aspect of humans' activity (maybe also valid for many other species). There are already numbers of analogies we have been using in different scientific and other disciplines. Here, we will analyze only analogies relevant for physics and among number of them it will be selected the most relevant and the most coherent set of multilevel analogies that are universally applicable from different points of view (including basic continuous symmetries that are compatible and coincidently applicable to such multilevel analogies), serving to increase the power of conceptualization, unification and predictions in Physics. If we do not have such simple "guiding channels" in formatting and addressing our knowledge about the Nature, we could be (intellectually and creatively) lost among countless number of abstract options that have been generated by contemporary concepts of Symmetries (basically over-dissipating our intellectual energy and being not focused in our fundamental scientific activities).***

Present-days' scientific (and quasi-scientific) theater is also crowded with a lot of new concepts, proposals, theories, and critical opinions about existing official science, coming from a number of official mainstream science dissidents (most of them still non-recognized or fully ignored and marginalized, but some of them having officially acceptable background and good scientific references). Most probably the author of this paper also belongs to a part of such a crowded population. What looks like a kind of a common characteristic of mentioned science dissidents is that many of them are promoting their theories and concepts based on criticizing certain illogical, internally contradictory, ad-hock, postulates, explaining rather well and step-by-step how certain big scientific personality made (a) mistake/s, and how such mistake/s could be corrected. In fact, such an approach is in many cases fully or partially wrong, regardless of its base in proper thinking. The reason for this is that we know that present physics was formulated through connecting piece by piece of mosaic-like knowledge elements during relatively long time, and in some cases people did not have good, better or any answer regarding explanation of certain confusing phenomena. Some of more curious, more ambitious and faster going (maybe even crazier) scientific personalities simply hybridized certain of their exotic ideas and concepts with remaining scientific body of relevance, sometimes without having enough arguments and without fully respecting the scientific logic and existing positive backgrounds, because implementing such new concepts was simply replacing the missing factual links making theory in question operational and from certain point of view explicable. Later such original scientific personalities (at lest few of them), got rewarded and recognized, since nobody else proposed better solution, and instead of being science dissidents they marked the history of physics engraving their names in officially accepted books. The fact is that some of them introduced good, correct and long lasting concepts, and some of them introduced ad-hock postulates and whatever works sufficiently good in given circumstances, and almost nobody (in the frames of the relevant historical period) was able to separate who is who and how much they are on good or bad side. Later, science has been advancing, introducing new of similar fitting and correcting ideas and concepts (often without correcting or erasing old or erroneous creations), and after applying and combining many of such self-correcting and iteratively improving steps, and by completing certain symmetries that showed obvious, we got present-day physics that is operating sufficiently well (regarding how we are using it and what we are presently expecting from it). This way we also got the situation that some early and maybe suspicious science step-stones have been implicitly accepted as fully correct (because the final creation has been working well), and therefore almost nobody (except certain science dissidents) is asking if the early foundations were really and fully correct. Now, the mainstream (or official science people) is quietly stating that since we already have something what is relatively well-operational, it looks unnecessary and time-wasting (even implicitly forbidden) to start revising early foundations which are almost no more of big use (what is not quite correct). This could easily pass as a diplomatic refusal for opening new channels of research. *The author of this paper abandoned the way of systematic searching for mistakes, defects and flaws in modern theories-foundations, because the much more effective and rational way is to start from what is already working well and what is generally, analogically and symmetrically applicable by comparing different contemporary physics-related theories. Later we would be better armed to readdress some old physics chapters and their foundations, if it shows appropriate. Knowing that Nature or Universe is anyway united (even without our clear and complete knowledge about such a unity), is giving much weight and power to predictions and conclusions made using analogies and symmetries. Of course, the very important condition is to establish the best possible and physics-realistic framework of analogies and symmetries, and the objective in writing this paper has been to find and formulate such framework.*

<u>Personal reflections of the author:</u>

The biggest problem in present-days science reality regarding amateurs and hobbyist involved (somehow) in scientific or quasi-scientific activities is not in personal and conflicting relations and misunderstandings between such “innovative” and not recognized debutants (often too enthusiastic, partially naïve and not sufficiently supported by their personal and general Physics knowledge) and some specific science leaders, officially recognized and admired in different domains of Physics. They anyway do not speak too much the same language, but it should be obvious that in both groups we can find creative and non-creative, advanced and conservative, intelligent and less intelligent, curious and non-curious, and differently educated elements. Problem is much more of a kind of natural law of universal inertia and resistance to everything what makes sudden changes and big waves (equally valid in Physics, Biology, Economy, Sociology, Politics, Ideology… intrinsically integrated in human nature), and easy victims in such situations are the most courageous, or most advanced, or most naïve science children (if we exclude obvious and bottom-line, trivial cases). This natural inertia and resistance (against big changes of officially established mainstream) is well masked and supported by multi-layer regulative and cosmetics, such as: rules how to write scientific papers and how to present new ideas, mathematics and general language conventions, ultimate requests to know and to respect all official references, publishing and formal limitations, recognition and approval procedures, particular interests of certain groups and professional departments, economical interest of the society or its leaders, habitual diplomacy, inactivity and prudence in such matters, necessity to have technical means to participate in such activities, etc., most of them not well known, not available or not interesting to courageous and impatient debutants and amateurs in science, since such people are primarily and dominantly obsessed and guided only by their (often crazy, not well-supported, poorly presented) ideas and visions (not well understanding, or understanding but not accepting, why anything else of administrative and formal nature, is more important than their innovative thinking). And the big secret of all science advances is that only free-thinking and unconventional minds (not too much saturated with current mainstream knowledge, assumptions and conventions) are able to bring significant novelties (usually working against the officially recognized mainstream of science and society establishment/s). The worst kind of official science establishment sabotage could happen in cases when somebody is trying to promote his new, original, advanced, more common-sense and conceptually better theory, in comparison and competition with already (very long time) existing and officially accepted theories. *Usually, an old, existing theory is already able to explain correctly number of experimental facts (on its old fashioned and sometimes complicated way, or in some cases like Ptolemy's elaborated theory that explained "**how the planet Earth is really the center of Universe, and all other celestial objects are rotating around it**"). On a practical level (only regarding calculated planetary paths compared to observations from the planet Earth), Ptolemy's theory has been perfectly mathematically operational, but in reality it is conceptually very wrong, and as we know Earth is not the center of Universe… Contrary, a new theory (which should be in competition with some already existing, well-established and maybe even in reality a "Ptolemy-type" theory, but we still do not know this), in its first steps, does just the same, producing the same or similar results as theories already known (and very little of new results). However, it could be more general, presenting the facts on a more elegant, simpler, more logical and more scientifically acceptable way (introducing new concepts and seeing reality in question from different points of view), although not professional enough.* Some minor parts of officially recognized scientific community (intentionally excluding others) are usually dismissing such new theories saying (using very elaborated rhetoric and long-time, well-perfected inquisitor style) that such incomers do not bring anything new in science, only explain the facts already known and well explained long time ago (in some old books, or ***bibles***), using old theories, that there is no more practical need to address such items, etc. It could look like absolutely useless, forbidden and heretic to introduce new theories dealing with the same subjects of old theories, or if not explicitly formulated like here (because of usual diplomacy and politeness in such situations), the resistance of "officiallyzed" scientific authorities appears on some other (better hidden) highly negative way against new voices, that could become really painful and long process if somebody is persistent in promoting his new and competing theory. The opinion of the author of this paper is that this is partially the case with modern **Quantum Theory** and **R**elativity **T**heory, both of them operating mathematically well, producing sufficiently correct results (in their domains), but being rich of internal conceptual complexity and some of hard-to-accept initial assumptions, and being mutually still not sufficiently compatible. If a new, more common-sense and conceptually promising (but still unaccepted and hypothetical) theory were given almost one century of development time of thousands of enthusiastic and creative research workers (like the case is with **QT** and **RT**), we could only imagine the consequences of scientific results comparison between such new and old theories. If we were against giving a chance to the new visions and concepts in science, it would be too difficult to promote new theories and to bring significant scientific contributions (that are usually coming much later, after initialization).

The experienced and officially recognized science veterans are often making formally perfect theories and "science bibles", mathematically well-fitted with the texture of measurements and relevant historical backgrounds, being already known in their fields of work, this way usually creating (almost) dead-end streets for others, or for future advances (since in reality they are mostly making the best fitting/s, or interpolation, or result approximation on the limited set of already known points, because naturally everybody (of creative human beings) has also a tendency to finalize and verify his work by making most general formulations that are practically not leaving a lot of space for future developments, or a space for creative or *naïve painters* and inexperienced incomers).

Also giving licenses and certificates to specifically trained, chosen and/or educated groups, regarding having exclusivity and priority to produce and publish new scientific results (politically and conceptually correct), and systematically neglecting others (who have much different ideas) is in some cases not giving expected results and significant scientific advances; -very much in agreement with F. Nietzsche's philosophical statements that only extreme personalities, clearly separated from the mainstream, are producing extraordinary and exceptional results (either positive or negative, depending from which side we qualify them).

The very common and very negative reality of today's scientific theatre could also be the fact that some of officially title-recognized scientific people are in permanent need to publish (where the bottom line motivation is maintaining some kind of power or economy-related existence). Sooner or later they are mastering the "politically and diplomatically perfect language" of saying and writing good sounding scientific-alike, papers, comments and statements, combining available real and correct science facts with ingredients of semi-reality, virtual reality and semi-truth facts, using supporting chain of similar semi-false references coming from certain small number of similar, title-recognized colleagues. In most of the cases they even do not see or recognize that they are part of such a process. Later, some of these people could take important positions in a system which is in charge of creating acceptance criteria and filtering ideas, results and contributions to the others who still did not master a "politically perfect language" of multilevel combinations and manipulations regarding scientific paperwork (and even they could have preferences to enrich their operating environment with similar and well obeying colleagues), and instead of establishing challenging and competitive science, merged with real and unbounded creativity, the self-degrading and conservative system would be unintentionally established. Of course, this is the worst possible imaginable scenario, or negative vision that should absolutely be avoided in practice if we really would like to make important scientific advances and progress.

The author of this paper is a kind of passionate and (in this field) self-educated amateur (with all of the above mentioned weaknesses of courageous and naïve players), trying to promote his vision of new Physics (mostly regarding Particle-Wave Duality of matter; and, of course, on his own, intuitive, simplified and mostly brainstorming, common-sense way). Regardless to all disadvantages of being courageous, passionate and self-educated amateur, the author of this paper has been thinking that his science related concepts should be somehow documented (even unprofessionally, by writing this paper), in a hidden hope that somebody more experienced in Physics, and more open-minded could, one day, transform such rough ideas into much more healthy and richer property of the valuable Physics (compared to the state of the art of our contemporary Physics).

The leading ideas of the author is that Physics should be formulated to be easy understandable, simple, clear, internally coherent theory, based on experimental and mathematical facts, as much as possible (without unverifiable assumptions), in order to explain profound secrets of Nature (and this is not always the case of modern Physics). Only outsiders, debutants and amateurs faced with Physics facts and existing theoretical explanations can say how much simple, clear, common-sense and elementary theory they find in available literature (since professionals are well armed and bounded by what and how they learned to become officially recognized professionals, and they are usually loosing the ability to ask simple questions, and to see presentation-related problems). To put it differently, "children and debutants" or amateurs, are able to ask good, unusual, unbounded (or crazy) questions, and to initiate new and extraordinary insights. The biggest complexity of problems in modern Physics-house is in its fundaments and starting assumptions, and we are usually analyzing only what is the visible part of the house (or floating tip of the iceberg), only adding new "decorations" and new levels of complexity to its structure, avoiding touching the basements, because one of the famous and untouchable predecessors invested there his authority and his life-carrier, and left a lot of powerful followers and guardians of his published heritage, designing and maintaining the Physics-theater almost like a background of long-lasting religious and ideological dogmas, where the highest authority is a certain "bible", which is written once in the past, made to stay valid forever, and it is not allowed to be changed, and everybody who starts introducing ideas belonging to the world outside of the frames of existing bibles has no rights because his arguments are not in agreement with bible's prescriptions and teachings. In other words, some of the officially accepted "*bibles"* have hidden tendency of promoting themselves as unique and exclusive sources of all acceptable arguments and frameworks for discussing, criticizing, dismissing and punishing any other teachings that are touching the space inside and outside of them. Consequently, it could happen that somebody who takes a freedom to modify a bible (or write a new one) should be as soon as possible in a way punished or eliminated, or totally discredited and classified as a trivial and foolish case *(of course, all of that kind of doings in the name of general well-being would be realized using much more elaborated and more sophisticated methods and language, than over-simplified formulations found here, still being applicable in all other domains of human life).*

We could also safely say that until the end of the 20-th century of modern human history, the human society (in all aspects of life, including science) was guided and mastered mostly by the rigid "**power of authority**", where **authority** means power of strong personalities and their operating bodies, and power associated to certain "bibles", politics and ideologies established by strong personalities and groups. It has been always somebody who could say YES or NO, regardless of arguments and logic (even in natural sciences), but officially everything was packed in the frames of some kind of legalized correctness. Basically, the biggest obstacle for innovative free-thinkers in that period of human history on the planet Earth was the complexity of barriers, effectively disabling them to efficiently communicate, publish, propagate, or spread their unconventional ideas (and eventually punishing or humiliating them economically, existentially, and on number of different perfidy ways, some of them being very cruel, if they would try to oppose officially accepted concepts).

Fortunately, with the development of high power computing and advanced, global communicational means (one of them Internet), we are now slowly and gradually getting free of old power managements, and entering the period where "**power of arguments and creativity**" would (and should) be more and more dominant managing framework. In the present period of human history, almost everybody is getting able to find official or unofficial channels and means to spread his ideas, regardless the opinion/s and positions of powerful authorities. It is really interesting and challenging to analyze the evolving conflict between the two different power managements, being an integral part of it, since our present period of life is still a transitional one (and maybe it would or should stay like that forever).

Allover this paper are scattered small comments placed inside the squared brackets, such as: ***[♣ <u>COMMENTS & FREE-THINKING CORNER…</u> ♣]****. The idea here has been to establish intuitive and brainstorming, not-confirmed and free-thinking corners for making fast comments, and presenting challenging ideas, that could be some other time developed towards something much more meaningful and more properly integrated into Physics. Without addressing such situations, many of potentially valuable ideas and concepts (or just good proposals) would simply disappear. This is also a way to document the logical and thinking background regarding generating new ideas, concepts and hypotheses (to present how an author was really thinking in process of formulating his contribution…). Now we could have the situation that last judgments and explanations regarding how somebody of founders of certain theory was thinking (and what he/she had in mind) is given by present "untouchable rulers" of state of the art theories that are officially accepted, which in some cases could basically modify such explanations (mostly unconsciously) in order to additionally support their own platforms, indirectly saying that they are direct successors and the best students of their famous founder/s and maybe even greater than their origins (the situation being something like a hierarchy of priests who are interpreting god's messages, in well-established religious structures).*

Author of this paper would also like to say that this paper has been evolving, changing, and updating during very long period of time (in rear moments when author was able to find a little bit of his free-thinking and mentally relaxing time; -during more than 25 years), and that there is a significant redundancy, scattering and discontinuity regarding presenting basically similar or familiar concepts allover this paper (written or simply attached to old chapters in different time periods). Since, this paper presents mostly an amateurship type, still time evolving effort of the author to roughly and briefly summarize number of his ideas, it should be clear that a lot of additional work, editing and smoothing must be invested to make the real, well-organized and scientifically looking paper. Regardless of such weak sides, the author is profoundly convinced that the potential power and significance of here presented ideas, concepts and brainstorming thinking would survive all negative aspects scattered allover this paper, and hopes that future readers wouldn't over-dissipate their mental power only on criticizing such negative elements, instead of taking the most positive and creative approach that could result in developing new and more valuable scientific property based on their own thinking and on certain of ideas, concepts and proposals taken from this paper. ***Of course, the real and fully convinced members of the Orthodox Community of present Quantum Theory establishment are not advised to spend any of their precious time on such paperwork. Mostly Quantum dissidents and suspicious non-believers could find some pleasure reading this paper.***

Since the author realized relatively late that he cannot himself finish this paper, taking into account present age of the author, the author decided to publish the paper as it is (basically as a brainstorming open-ended draft and challenging task list), without asking anybody for authorization, editing and approvals, and in parallel continuing its updating as long as the nature would allow it.

Miodrag Prokic

May 2006, Switzerland, Le Locle

1. PRESENTATION OF BASIC ELECTROMECHANICAL ANALOGIES

Let us first establish or summarize the basic set of electromechanical analogies that will be used, later on, to formulate some of the most important ideas and messages in this paper. Comparing different configurations of electric circuits in Fig.1.1 a),b),c),d), consisting of electrical elements (resistance (**R**), inductance (**L**), and capacitance (**C**)), to different configurations of mechanical circuits, consisting of mechanical elements (mechanical resistance or damper (**R_m**), spring (**s**), and mass (**m**)), it is possible to determine six levels of different electro-mechanical analogies (see **[1],** pages 9-18**, [2]** and **[3]**). Here, we shall not present or analyze the way of getting these analogies (since this can be found in literature), but, as a general conclusion, we shall state that all of the known analogies have been created after noticing similarity of the forms of corresponding differential equations (between dual or similar electric and mechanical networks), which are describing currents, voltages, forces and velocities in those electric or mechanical networks. It is clear that we are thus creating mathematical analogies (which present the **first** criteria in creating analogies). The mathematical conclusion is that all the possible six analogy situations (**[1]** and **[2]**) are mutually equal, or equally useful.

We shall now introduce the second analogy criteria (**which is not mathematical**) to give more weight and power to certain analogies (from the existing six).

Let us say that beside mathematical analogies we would like, for equivalent electric and mechanical circuits, to have the same circuit configuration (topography) of their elements (if we look at how they are mutually connected in corresponding circuits or where they are placed). By introducing this **second** analogy criterion, the previous set of six analogies (which is systematically analyzed in **[1], [2]** and **[3]**) is reduced to the set of only two analogy situations (Fig.1.1 a), b), c), d)). The latest **mathematical and topography** analogy can be represented by **velocity-to-voltage and force-to-current analogy (known in literature as the Mobility type of analogy)**.

Fig. 1.1c

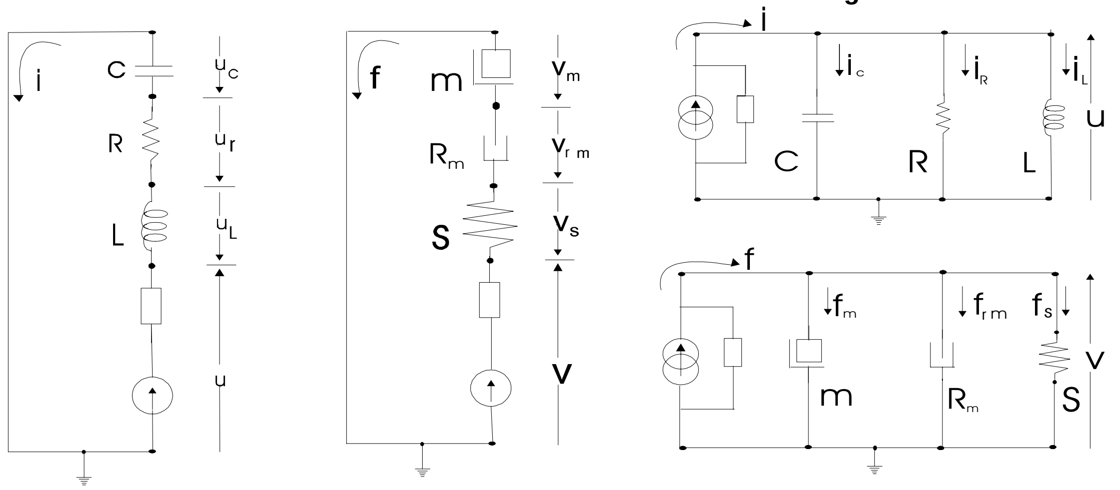


Fig.1.1a **Fig. 1.1b** **Fig. 1.1d**

Fig.1.1. Equivalent electric and mechanical circuits

The content that results from this double level analogy platform is shown in T.1.1. In this paper it will be shown that the analogies from T.1.1 are presenting the most important, most predictive, most practical and most natural (electro mechanical) analogy platform in physics (and later, the same Mobility type analogy platform from T.1.1 will be widely extended; -see T.1.2 until T.1.7).

T.1.1

Electric parameter / [unit]	Mechanical parameter / [unit]
Voltage (=) u (=) [V = volt]	**Velocity (=) v (=) [m/s]**
Current (=) i = dq/dt (=) [A = ampere]	**Force (=) f = dp/dt (=) [N = Kg m/s^2 = newton]**
Resistance (=) R (=) [Ω = ohm]	**Mech. Resistance (=) R_m (=) [m /N s = s/Kg]**
Inductance (=) L (=) [H = Henry]	**Spring Stiffness (=) S (=) [m/N = s^2/Kg]**
Capacitance (=) C (=) [F = farad]	**Mass (=) m (=) [Kg]**
Charge (=) q = Cu (=) [C = coulomb]	**Momentum (=) p = mv (=) [Kg m/s]**
Magn. Flux Φ (=) [Wb = V s = Weber]	**Displacement (=) x (=) [m]**

In order to establish perfect (1:1) analogy and symmetry between elements of electrical and mechanical circuits, as given in T.1.1 (**L ⇔ S** and **R ⇔ R_m**), it was necessary to redefine the unit of spring stiffness and unit of mechanical resistance or damping. In literature we find for spring stiffness **S (=) [N/m]**, and here we use **S (=) [m/N]**, and for mechanical resistance in today's literature is very usual to find **R_m (=) [Ns/m]**, and here we use **R_m (=) [m/Ns]**. The consequences of these parameter redefinitions are that usual (traditional) meaning of **Mechanical Impedance** = **Force/Velocity** should also be redefined into **"New Mechanical Impedance"** = **Velocity/Force** (presently known in Mechanics as **Mobility**), in order to fully support the **VOLTAGE-VELOCITY** and **CURRENT-FORCE** analogies. Similar redefinitions will be also applied to mechanical resistance (or friction constant) and spring stiffness valid for rotation (see all equations from (1.1) until (1.9)).

Needless to say, all the other analogies are equally useful (but only at the mathematical, formal level of analogies), and if we consider certain (additional) rules, correct use of any of these analogies will lead to the same result (but often in a more complicated, more difficult and not so natural way).

Let us come back to the equivalent models given in Fig.1.1 and introduce rotational motion, as yet another level of study for additional creation of analogies. Let us imagine two more (mechanical), analogous, oscillating circuit models added to this situation, presenting the case of rotation (in series and parallel configuration of oscillatory rotating mass having a moment of inertia J, angular velocity ω, angular momentum $L = J\ \omega$, torque τ = dL/dt, rotational friction $\mathbf{R_R}$, and spring stiffness for rotational oscillations $\mathbf{S_R}$). In this situation (analogous to cases shown in Fig.1.1 a),b),c),d)) we can establish all the relations between the parameters from T.1.1, adding to them similar relations concerning the rotational oscillatory motion (here not presented with any additional figure, but easy to visualize).

Generalized Ohm's Laws and Generalized Kirchoff's Laws are directly applicable to the circuit situation in Fig.1.1 (see [**2**]**, Vol. 2**).

Kirchoff's Voltage Law states: **"The sum of all the voltages in a loop is equal to zero."** Based on the situation in Fig.1.1 a), we shall have:

$$\sum \mathbf{u_i} = 0,\ \mathbf{u} = \mathbf{u_L} + \mathbf{u_R} + \mathbf{u_C} = \mathbf{L}\frac{\mathbf{di}}{\mathbf{dt}} + \mathbf{Ri} + \frac{1}{\mathbf{C}}\int \mathbf{idt} = \mathbf{L}\frac{\mathbf{d}^2\mathbf{q}}{\mathbf{dt}^2} + \mathbf{R}\frac{\mathbf{dq}}{\mathbf{dt}} + \frac{\mathbf{q}}{\mathbf{C}}, \quad (\mathbf{i} = \mathbf{i_L} = \mathbf{i_R} = \mathbf{i_C} = \frac{\mathbf{dq}}{\mathbf{dt}}), \tag{1.1}$$

For mechanical circuits, the analogy to Kirchoff's Voltage Law will state: **"The sum of all the velocities in a loop is equal to zero."** Based on the situation in Fig.1.1 b), we shall have:

$$\sum \mathbf{v_i} = 0,\ \mathbf{v} = \mathbf{v_S} + \mathbf{v_{Rm}} + \mathbf{v_m} = \mathbf{s}\frac{\mathbf{df}}{\mathbf{dt}} + \mathbf{R_m f} + \frac{1}{\mathbf{m}}\int \mathbf{fdt} = \mathbf{s}\frac{\mathbf{d}^2\mathbf{p}}{\mathbf{dt}^2} + \mathbf{R_m}\frac{\mathbf{dp}}{\mathbf{dt}} + \frac{\mathbf{p}}{\mathbf{m}}, \quad (\mathbf{f} = \mathbf{f_S} = \mathbf{f_{Rm}} = \mathbf{f_m} = \frac{\mathbf{dp}}{\mathbf{dt}}). \tag{1.2}$$

For a series rotational element circuit (similar to Fig.1.1 b)), the analogy to Kirchoff's Voltage Law will state: **"The sum of all the angular velocities in a loop is equal to zero"**,

$$\sum \omega_i = 0,\quad \omega = \omega_{SR} + \omega_{RR} + \omega_J = s_R\frac{d\tau}{dt} + R_R\tau + \frac{1}{J}\int \tau dt = s_R\frac{d^2L}{dt^2} + R_R\frac{dL}{dt} + \frac{L}{J},\ (\tau = \tau_{SR} = \tau_{RR} = \tau_J = \frac{dL}{dt}). \tag{1.3}$$

Kirchoff's Current Law states: **"The sum of all the currents into a junction is equal to the sum of the currents out of the junction".** Given the situation in Fig.1.1 c), we shall have:

$$\sum \mathbf{i_{inp.}} = \sum \mathbf{i_{outp.}},\ \mathbf{i} = \mathbf{i_C} + \mathbf{i_R} + \mathbf{i_L} = \mathbf{C}\frac{\mathbf{du}}{\mathbf{dt}} + \frac{\mathbf{u}}{\mathbf{R}} + \frac{1}{\mathbf{L}}\int \mathbf{udt} = \mathbf{C}\frac{\mathbf{d}^2\Phi}{\mathbf{dt}^2} + \frac{1}{\mathbf{R}}\frac{\mathbf{d}\Phi}{\mathbf{dt}} + \frac{\Phi}{\mathbf{L}},\ (\mathbf{u} = \mathbf{u_C} = \mathbf{u_R} = \mathbf{u_L} = \frac{\mathbf{d}\Phi}{\mathbf{dt}}). \tag{1.4}$$

For mechanical circuits, analogy to Kirchoff's Current Law will state: **"The sum of all the input forces is equal to the sum of the output forces"**. Looking at the situation in Fig.1.1 d), we shall have:

$$\sum \mathbf{f_{inp.}} = \sum \mathbf{f_{outp.}},\ \mathbf{f} = \mathbf{f_m} + \mathbf{f_{Rm}} + \mathbf{f_S} = \mathbf{m}\frac{\mathbf{dv}}{\mathbf{dt}} + \frac{\mathbf{v}}{\mathbf{R_m}} + \frac{1}{\mathbf{s}}\int \mathbf{vdt} = \mathbf{m}\frac{\mathbf{d}^2\mathbf{x}}{\mathbf{dt}^2} + \frac{1}{\mathbf{R_m}}\frac{\mathbf{dx}}{\mathbf{dt}} + \frac{\mathbf{x}}{\mathbf{s}},\ (\mathbf{v} = \mathbf{v_m} = \mathbf{v_{Rm}} = \mathbf{v_S} = \frac{\mathbf{dx}}{\mathbf{dt}}). \tag{1.5}$$

For a parallel rotational element circuit (similar to Fig.1.1 d)), the analogy to Kirchoff's Current Law will state: **"The sum of all the input torque is equal to the sum of the output torque"**,

$$\sum \tau_{inp.} = \sum \tau_{outp.}\ , \quad \tau = \tau_J + \tau_{RR} + \tau_{SR} = \mathbf{J}\frac{d\omega}{dt} + \frac{\omega}{R_R} + \frac{1}{s_R}\int \omega dt = \\ = \mathbf{J}\frac{d^2\alpha}{dt^2} + \frac{1}{R_R}\frac{d\alpha}{dt} + \frac{\alpha}{s_R}\ , \quad (\omega = \omega_J = \omega_{RR} = \omega_{SR} = \frac{d\alpha}{dt})\ . \tag{1.6}$$

To avoid possible confusion in understanding the previous system of analogies, we must make a difference between the electric potential and voltage, which is a difference of potentials between the two points ($\mathbf{u} = \varphi_2 - \varphi_1 = \Delta\varphi$). The same is valid for velocities concerning a certain reference level and for the velocity « on » a certain mechanical element, which is equal to the difference of two referential velocities.

A next important difference between the electric and mechanical analogue elements is in the fact that all electric elements from T.1.1 are scalars, and some of mechanical elements are vectors (**v, p, f**).

On the other hand, if we now start from the mechanical conservation law of momentum, we shall have:

$$\sum \mathbf{p}_{inp.} = \sum \mathbf{p}_{outp.} \Rightarrow \frac{\mathbf{d}}{\mathbf{dt}}\sum \mathbf{p}_{inp.} = \frac{\mathbf{d}}{\mathbf{dt}}\sum \mathbf{p}_{outp.} \Leftrightarrow \sum \mathbf{f}_{inp.} = \sum \mathbf{f}_{outp.}\ . \tag{1.7}$$

Clearly, momentum conservation (1.7) directly corresponds (or leads) to "Kirchoff's Force/Current Laws" (1.5). Following the pattern of conclusion in (1.7), we can return to the electric system and develop the total electric charge conservation law:

$$\sum \mathbf{i}_{inp.} = \sum \mathbf{i}_{outp.} \Rightarrow \frac{\mathbf{d}}{\mathbf{dt}}\sum \mathbf{i}_{inp.} = \frac{\mathbf{d}}{\mathbf{dt}}\sum \mathbf{i}_{outp.} \Leftrightarrow \sum \mathbf{q}_{inp.} = \sum \mathbf{q}_{outp.}\ . \tag{1.8}$$

If we now take the mechanical conservation law of angular momentum, we shall have:

$$\sum \mathbf{L}_{inp.} = \sum \mathbf{L}_{outp.} \Rightarrow \frac{d}{dt}\sum \mathbf{L}_{inp.} = \frac{d}{dt}\sum \mathbf{L}_{outp.} \Leftrightarrow \sum \tau_{inp.} = \sum \tau_{outp.}\ . \tag{1.9}$$

Obviously, angular momentum conservation (1.9) directly corresponds (or leads) to "Kirchoff's Force/Current Laws" for rotational motion (1.6), or, in other words, this is just a torque conservation law.

In the previous way, step-by-step, we introduced the **third** level of electro-mechanical analogies, connecting them to the well-known and proven conservation laws in physics ((1.1) to (1.9)). This task has to be completed by taking into account all other, known conservation laws in physics in such a way as to obtain parallelism between electric and mechanical formulas, as before. By using this method, we could (probably) establish some new (but up to now not explicitly expressed) conservation laws.

Now we can summarize all the additionally presented analogies (comparing the equations from (1.1) to (1.9)), and extend the table of analogies T.1.1 to T.1.2 (by adding the elements originating from "rotational" circuit situations, (see **[3]**)).

T.1.2

Electric parameter / [unit]	Mechanical parameter / [unit]
Voltage (=) u (=) [V = volt]	**Velocity (=) v (=) [m/s]**
	Angular Velocity (=) ω (=) [rad/s]
Current (=) i = dq/dt (=) [A = ampere]	**Force (=) f = dp/dt (=) [N = Kg m/s^2 = newton]**
	Torque (=) τ = dL/dt (=) [Kg m^2/s^2]
Resistance (=) R (=) [Ω = ohm]	**Mech. Resistance (=) R_m (=) [m /N s = s/Kg]**
	Mech. tors. Resistance (=) R_R (=) [s/Kg m^2]
Inductance (=) L (=) [H = henry]	**Spring Stiffness (=) S (=) [m/N = s^2/Kg]**
	Torsion. Spring Stiffness (=) S_R (=) [s^2/Kgm^2]
Capacitance (=) C (=) [F = farad]	**Mass (=) m (=) [Kg]**
	Moment of Inertia (=) J (=) [Kg m^2]
Charge (=) q = Cu (=) [C = coulomb]	**Momentum (=) p = mv (=) [Kg m/s]**
	Angular Momentum (=) L = J ω (=) [Kg m^2/s]
Magn. Flux Φ (=) [Wb = V s = weber]	**Displacement (=) x (=) [m]**
	Angle (=) α (=) [rad] (=) [1]

Along with T.1.2, let us **<u>dimensionally</u>** compare relevant expressions for rectilinear motion in a gravitational field, the situation in electromagnetic fields, and the situation related to the rotation of masses, T.1.3, T.1.4 and T.1.5. For the common and generic names of the corresponding columns (in the following tables), we shall take the names and symbols of relevant electro-magnetic parameters (see [**2**], **Vol. 1**).

T.1.3	[W] = [ENERGIES]	[Q] = [CHARGES]	[C] =[CAPACITANCES]
Electro-Magnetic Field	[Cu^2] (=) [qu^1] [Li^2] (=) [Φi^1]	[Cu^1] (=) [qu^0] (=) [q]	[Cu^0] (=) [qu^{-1}] (=) [C]
Gravitation	[mv^2] (=) [pv^1]	[mv^1] (=) [pv^0] (=) [p]	[mv^0] (=) [pv^{-1}] (=) [m]
Rotation	[J ω^2] (=) [L ω^1]	[Jω^1] (=) [Lω^0] (=) [L]	[Jω^0] (=) [Lω^{-1}] (=) [J]

T.1.4	[U] = [VOLTAGES]	[I] = [CURRENTS]	[Z] = [IMPEDANCES] (=[mobility] in mechanics)
Electro-Magnetic Field	[u] (=) [dΦ/dt]	[i] (=) [dq/dt]	[Z_e] (=) [u] / [i]
Gravitation	[v] (=) [dx/dt]	[f] (=) [dp/dt]	[Z_m] (=) [v] / [f]
Rotation	[ω] (=) [dα/dt]	[τ] (=) [dL/dt]	[Z_R] (=) [ω] / [τ]

T.1.5	[L] = [INDUCTANCES]	[R] = [RESISTANCES]	[Φ] =[DISPLACEMENTS]
Electro-Magnetic Field	[L]	[R]	[Φ] (=) [Li]
Gravitation	[S]	[R_m]	[x] (=) [Sf]
Rotation	[S_R]	[R_R]	[α] (=) [S_R τ]

*[♣ **<u>COMMENTS & FREE-THINKING CORNER:</u>** Let us make one digression towards the Special Relativity Theory. In T.1.2, we find that mass **m** is analogue to electric capacitance **C** (**m_0** - rest mass), meaning that this analogy can possibly be extended in the following way:*

$$\mathbf{m} = \frac{\mathbf{m}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \Rightarrow \mathbf{C} = \frac{\mathbf{C}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \tag{1.10}$$

*The analogy (1.10) can be strongly supported by the following example. Let us imagine the plane electrodes capacitance, where the surface of a single electrode is **S**, and the distance between the electrodes is **d**. In that case, electric capacitance by definition is:*

$$\mathbf{C} = \frac{\mathbf{q}}{\mathbf{u}} = \varepsilon_0 \frac{\mathbf{S}}{\mathbf{d}}. \tag{1.11}$$

*Let us suppose that the electric capacitance is moving by velocity **v** in the direction of its electrode distance (perpendicular to the electrode surface). In this case, the distance between the electrodes is described by the relativistic contraction formula:*

$$\mathbf{d} = \mathbf{d_o}\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}. \tag{1.12}$$

Introducing (1.12) in (1.11), we get the form for capacitance that is equal to (1.10):

$$\mathbf{C} = \frac{\mathbf{q}}{\mathbf{u}} = \frac{\varepsilon_0 \frac{\mathbf{S}}{\mathbf{d}_0}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} = \frac{\mathbf{C}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} = \frac{\frac{\mathbf{q}_0}{\mathbf{u}_0}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}. \tag{1.13}$$

From (1.13) we can get the relativistic formula related to the electric charge (on the capacitance electrodes):

$$\mathbf{q} = \mathbf{q}_0 \frac{\frac{\mathbf{u}}{\mathbf{u}_0}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}. \tag{1.14}$$

Because we know that electrical charge is not dependent on its velocity, (1.14) will be transformed in:

$$\mathbf{q} = \mathbf{q}_0 \Rightarrow \mathbf{u} = \mathbf{u_o}\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}. \tag{1.15}$$

*In (1.15), voltage **u** is equal to the potential difference between the capacitance electrodes,*

$$\mathbf{u} = \varphi_2 - \varphi_1 = \Delta\varphi = \mathbf{u_o}\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}} = (\Delta\varphi_{\mathbf{o}})\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}. \tag{1.16}$$

Also, velocities «on» mechanical elements are equal to the difference of their corresponding referential velocities (if we want to treat the previous analogies correctly).

Another interesting aspect of analogies can be developed if we compare the relativistic formulas for additions of the referential velocities with electrical voltages and potentials (but this might be somewhat premature and complicated at this stage).

Following the same analogy and the same supporting description as in (1.10) - (1.16), it is possible to conclude that the moment of inertia (for rotating particle in rectilinear motion), most probably, could be presented as (see T.1.2):

$$\mathbf{m} = \frac{\mathbf{m}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \Rightarrow \mathbf{J} = \frac{\mathbf{L}}{\omega} = \frac{\mathbf{J}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \left\{ \Rightarrow \mathbf{L} = \frac{\mathbf{J}_0\omega}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \right\}. \tag{1.17}$$

*The analogical prediction of the formula (1.17) is almost obvious as a correct result, since by definition of the moment of inertia, this is (dimensionally) the product of a certain mass and certain surface. Of course, it is clear that a rotating mass, which has the moment of inertia **J** (described by formula (1.17)) should be moving along its axis of rotation, having a linear (axial) velocity v. In the same case, combined, non-relativistic, motional (kinetic) energy can be expressed as (v << **c**):*

$$\mathbf{E_k} = \frac{1}{2}\mathbf{m}\mathbf{v}^2 + \frac{1}{2}\mathbf{J}\omega^2, \tag{1.18}$$

which is usually applicable for observations in a center of gravity system. In relativistic situations (using (1.17)), we can generalize (1.18) into (1.19), hypothetically, assuming the applicability of used analogies:

$$\begin{aligned} &\mathrm{E_k} = (\mathrm{m} - \mathrm{m}_0)\mathrm{c}^2 + (\mathbf{J} - \mathbf{J}_0)\omega_c{}^2 = \\ &= \frac{\mathbf{pv}}{1+\sqrt{1-(\frac{\mathrm{v}}{\mathrm{c}})^2}} + \frac{\mathbf{L}\omega}{1+\sqrt{1-(\frac{\mathrm{v}}{\mathrm{c}})^2}} \\ &(\Rightarrow \mathrm{pv}+\mathrm{L}\omega = \mathrm{E_k}\left[1+\sqrt{1-(\frac{\mathrm{v}}{\mathrm{c}})^2}\right], \ (\frac{\omega}{\omega_c})^2 = (\frac{\mathbf{v}}{\mathbf{c}})^2(1-\frac{\mathbf{v}^2}{\mathbf{c}^2})^{-1/2} \ ). \end{aligned} \tag{1.19}$$

From (1.19) we could try to go back to analogous electromagnetic equivalents, using again (1.13) and T.1.2 - T.1.5, (with an objective to create corresponding and meaningful expressions, in the frame of the system of analogies presented here, if possible).

*It would be interesting to make a comparison between electric charge **q**, and momentum **p** (its analogue match in Mechanics of rectilinear motion). As we already know, electric charge is invariant (always stays the same, constant amount), regardless of its velocity, or independent from velocity. By analogy, the same is valid for Linear Momentum (or quantity of motion), **p** = **m**v. If we (regardless of whether this is correct or not) want to make them fully analogous, than it would be:*

$$\begin{aligned} &\left\{ \begin{array}{l} \{\mathrm{q} = \mathrm{Cu} = \mathrm{inv.}\} \Rightarrow \{\mathrm{p} = \mathrm{mv^*} = \mathrm{inv.}\},\ \mathrm{m} = \mathrm{m}_0/\sqrt{1-\mathrm{v}^2/\mathrm{c}^2}, \\ \mathrm{C} = \mathrm{C}_0/\sqrt{1-\mathrm{v}^2/\mathrm{c}^2},\ \left\{\mathrm{u} = \mathrm{u}_0\sqrt{1-\mathrm{v}^2/\mathrm{c}^2}\right\} \Rightarrow \left\{\mathrm{v^*} = \mathrm{v^*}_0\sqrt{1-\mathrm{v}^2/\mathrm{c}^2}\right\} \end{array} \right\} \Rightarrow \\ &\Rightarrow \mathrm{p} = \mathrm{mv^*} = (\mathrm{m}_0/\sqrt{1-\mathrm{v}^2/\mathrm{c}^2}) \cdot (\mathrm{v^*}_0\sqrt{1-\mathrm{v}^2/\mathrm{c}^2}) = \mathrm{m}_0\mathrm{v^*}_0 = \mathrm{inv.}. \end{aligned} \tag{1.20}$$

*A similar concept (based on analogies) should be extendible to Angular Momentum **L** = **J**ω (for rotational motion). At present, nothing confirms the possibility that momentum **p** and **L** could be independent (invariant) regarding velocity, meaning that we still miss some important elements (regarding analogy between electric and mechanical charges) to make (1.20) possible. The only possibility "to save" (1.20) is to imagine somehow that we are taking about internal, intrinsic, stable and constant particle momentum **p** and **L**.*

Since many contributions and discussions regarding boundary areas and basic assumptions of Einstein Relativity Theory are currently on rise, it would be better to leave aside (for certain time) all premature and analogy based conclusions (as for instance (1.10)-(1.20)) that are in direct connection with the Relativity Theory. ♣]

A careful analysis of the previously established analogies, T.1.1 - T.1.5, shows that all the electric and mechanical parameters can be very conveniently classified into two groups according to their definition or nature: **Spatial/Geometry parameters, and Action parameters**, T.1.6.

T.1.6

Spatial/Geometry parameters	**Linear Motion Gravitation**	**Electromagnetism**	**Rotation**	***Spatial/Geometry parameters***
Velocities and voltages	$v = dx/dt$	$u = d\Phi/dt$	$\omega = d\alpha/dt$	Velocities and voltages
Displacements	$x = sf$	$\Phi = Li$	$\alpha = s_R\ \tau$	Displacements
Reactances (+)	s	L	s_R	Reactances (+)
Resistances and Impedances	(*) $R_m = (v / f)_{Real}$ $Z_m = (v / f)_{Complex}$	(*) $R = (u / i)_{Real}$ $Z = (u / i)_{Complex}$	(*) $R_R = (\omega / \tau)_{Real}\ Z_R = (\omega / \tau)_{Complex}$	Resistances and Impedances
Reactances (-)	m	C	J	Reactances (-)
Charges	$p = mv$	$q = Cu$	$L = J\omega$	Charges
Forces and Currents	$f = dp/dt$	$i = dq/dt$	$\tau = dL/dt$	Forces and Currents
Action Parameters	**Linear Motion Gravitation**	**Electromagnetism**	**Rotation**	***Action Parameters***

(*) $R = (u / i)_{Real} \Leftrightarrow u(t)$ and $i(t)$ in phase (=) Real, Active Impedance or Resistance,
$Z = (u / i)_{Complex} \Leftrightarrow u(t)$ and $i(t)$ out of phase (=) Complex Impedance (=) Resistance + Reactance.

As we can see, in the table T.1.6 we do not have an explicit separation regarding parameters relative only to electric and only to magnetic fields. In the next chapters of this paper it will be shown that such parameter separation (by analogy criteria) can be realized in the following way (see table T.1.7):

T.1.7

Spatial/Geometry parameters	**Electric field**	**Magnetic field**	***Spatial/Geometry parameters***
Velocities and voltages	$u = d\Phi/dt$	$i = dq/dt$	Velocities and voltages
Displacements	$\Phi = Li$	$q = Cu$	Displacements
Reactances (+)	L	C	Reactances (+)
Resistances and Impedances	(*) $R_{el.} = (u / i)_{Real}$ $Z_{el.} = (u / i)_{Complex}$	(*) $R_{mag.} = (i / u)_{Real}$ $Z_{mag.} = (i / u)_{Complex}$	Resistances and Impedances
Reactances (-)	C	L	Reactances (-)
Charges	$q = Cu$	$\Phi = Li$	Charges
Forces and Currents	$i = dq/dt$	$u = d\Phi/dt$	Forces and Currents
Action Parameters	**Electric field**	**Magnetic field**	***Action Parameters***

The classification given in T.1.6 and T.1.7 represents an almost complete and basic picture of electromechanical analogies, structure and order of physics parameters known at present. Based on the above-systematized analogies, several hypothetical statements and predictions will be formulated later, regarding new aspects of Gravitation, Faraday-Maxwell Electromagnetic Theory and Quantum Mechanics (see (2.1)-(3.5), (4.1)-(4.4), (5.15) and (5.16)). At the same time, a specific unification platform will be proposed (in later chapters of the same paper), based on generic **Symmetries**, T.1.8, that will underline possible and most probable unification areas for Gravitation, Electromagnetism

and Quantum Mechanics. The concept of basic **Symmetries** is additionally integrating and connecting all analogies given in T.1.3 – T.1.7.
In 1905, a mathematician named Amalie Nether proved the following theorem (regarding universal laws of Symmetries):

-For every continuous symmetry of the laws of physics, there must exist a conservation law.

-For every conservation law, there should exist a continuous symmetry.

Let us only summarize already-known conservation laws and basic symmetries in Physics, by creating the table T.1.7.1 (without entering into a more profound argumentation, since in later chapters of this paper we will again discuss basic continuous symmetries, introducing much wider background).

T.1.7.1 Symmetries of the Laws of Physics (mutually conjugate variables)

Original Domains ↔ (spatial, geometry or static parameters)	**↔ Spectral Domains** (motional, dynamic parameters)
Time = t	**Energy = E (or frequency = f , or mass = m)**
Time Translational Symmetry	**Law of Energy Conservation**
Displacement = $\mathbf{x} = \mathbf{S}\dot{\mathbf{p}} = \mathbf{SF}, (\mathbf{F} = \mathbf{Force})$	**Momentum =** $\mathbf{p} = \mathbf{m}\dot{\mathbf{x}} = \mathbf{mv}$
Space Translational Symmetry	**Law of Conservation of Momentum**
Angle = $\alpha = S_R\dot{L} = S_R\tau$	**Angular momentum =** $L = J\dot{\alpha} = J\omega$
Rotational Symmetry	**Law of Conservation of Angular Momentum**
Electric Charge = $\mathbf{q}_{el.} = \Phi_{el.} = \mathbf{C}\dot{\mathbf{q}}_{mag.} = \mathbf{Ci}_{mag.}$	**Magn. Charge =** $\mathbf{q}_{mag.} = \Phi_{mag.} = \mathbf{L}\dot{\mathbf{q}}_{el.} = \mathbf{Li}_{el.}$
Law of Total Electric Charge Conservation	**The Electric Charge-reversal Symmetry**
The Magnetic Charge-reversal Symmetry	**"Total Magnetic Charge" Conservation**
(Magnetic monopole or charge is not a free and self-standing entity)	

In the T.1.7.1, mutually conjugate variables are formally presenting Fourier integral transformations (in both directions) between all values from one side of the table to the other side of the table, showing deeper connections and symmetry between conditionally spatial or static parameters of our universe (left side of the table T.1.7.1), and their motional or dynamic conjugated couples (right side of the table T.1.7.1). Obviously (we can see from T.1.7.1) that couples of conjugate, Original-Spectral domains, can be extended on both sides, by creating T.1.8, using simple analogies already present in T.1.3 – T.1.7.1. ***In fact, the proper merging between Analogies and basic or generic Symmetries (related to most important conservation laws, generally applicable: as given in T.1.8) would be the strongest predictive and unifying platform of future Physics*** (very much applied allover this paper)**.**

T.1.8 Generic Symmetries and Analogies of the Laws of Physics

Velocity Voltage analogies	Original Domains $\leftrightarrow$ (spatial, static parameters)	$\leftrightarrow$ Spectral Domains (motional, dynamic parameters)	Force Current analogies
	Time = t	**Energy = E**	
	Time Translational Symmetry	Law of Energy Conservation	
$v = \frac{dx}{dt} = \dot{x}$	**Displacement = x**	**Momentum = p**	$F = \frac{dp}{dt} = \dot{p} = m\dot{v}$
	Space Translational Symmetry	Law of Conservation of Momentum	
$\omega = \frac{d\alpha}{dt} = \dot{\alpha}$	**Angle =** α	**Angular momentum =** L	$\tau = \frac{dL}{dt} = \dot{L} = J\dot{\omega}$
	Rotational Symmetry	Law of Conservation of Angular Momentum	
$u = \frac{d\Phi_{mag.}}{dt} = \dot{\Phi}_{mag.}$	**Electric Charge =** $q_{el.} = \Phi_{el.} = C\dot{q}_{mag.} = Ci_{mag.}$	**Magnetic Charge =** $q_{mag.} = \Phi_{mag.} = L\dot{q}_{el.} = Li_{el.}$	$i = \frac{d\Phi_{el.}}{dt} = \dot{\Phi}_{el.}$
	Law of Total Electric Charge Conservation	**The Electric Charge-reversal Symmetry**	
	The Magnetic Charge-reversal Symmetry	**"Total Magnetic Charge" Conservation**	

(Comment: static and permanent magnetic monopoles do not exist)

Of course, the concept of Symmetries could be extended much more than given in T.1.8, but in this paper we will consider having the strongest predictive platform when using unified analogies and continuous symmetries, as given in T.1.8.

[♣ <u>COMMENTS & FREE-THINKING CORNER:</u> Developing specific (or new) theory in physics (separately from other fields) is usually not an easy job, and can sometimes be an arbitrary and meaningless process, if we do not have a general picture of the natural place and (most probable) basic structure of the new theory in question. The author's position, and the main idea of this paper, is to show that multi-level analogies and symmetries, if correctly established, create a very common, simple and very much acceptable (conceptual and theoretical) platform for further generalizations in physics, and present an easy guide to new scientific discoveries (while respecting all conservation laws). Also, in the process of creating analogies we could be in a position to notice possible irregularities, missing links, or weak points in already known and well accepted theories, or to transfer positive achievements known (only) in one field to its analogous domain/s. Often analogies are helping to initiate and motivate a creative and intuitive thinking, supporting valuable brainstorming ideas.

Another feeling of the author of this paper is that some of contemporary Physics' theories and models are originally established and promoted by their founders, which were very strong and authoritative personalities, and later being dogmatized by the army of their faithful (but often non sufficiently critical) followers, and professionally dependant assistants, or in some aspects also existentially dependent co-workers and students, becoming successors and defenders of the theory founders, and natural barriers for penetration of new and different theories.

In order to painlessly introduce new ideas and concepts into such environment of old and "well established theories", the simplest and commonly acceptable strategy, proposed by the author of this paper, is to test them creating here explained multilevel analogy symmetries platform (and to see which elements of the larger Physics picture are missing or contradicting to other elements).

Modern science realized its biggest achievements (in 20th century) based on the concept that nature prefers symmetry in a physics theory and that symmetry is the key to constructing physics laws without disastrous anomalies and divergences (see [10] and [11]). The objective of this paper is to show that any symmetry concept should be supported and united with wide (and multilevel) background analogy platform in order to become the strongest framework in describing and mastering natural phenomena.
One can argue that analogy platform itself is a very week theoretical and practical platform (for making relevant conclusions and predictions) in comparison with modern Topology, but this is absolutely not correct since we are obviously living in an already united universe, where all forces and fields are coincidentally and harmonically present in every sequence of our existence (internally and externally). Most probably that our particular field theories are sometimes too primitive, too simple or incorrectly formulated, that we cannot see (or describe) the areas of their unification (from the point of view of modern Topology). If we just conclude that (today's) Maxwell Theory and Gravitation cannot be easily united because of problems with some second-rank and curvature tensors, this is maybe temporarily (and conditionally) correct, but it also indicates that most probably some important topology components are missing both in Maxwell and Gravitation Theory, and that we should continue our search for better mathematical models and for missing field components (as for instance, to find them by including rotational, or torsion field components and motions in Maxwell Theory, Relativity Theory and Gravitation). Really, to realize something like that, we do not have better starting points than using well-established and multilevel analogies, and later to test them experimentally and theoretically using modern Topology Theory, Symmetries, Group Theory... ♣]

2. NEW ASPECTS OF GRAVITATION

In order to introduce new conceptual platform regarding understanding origins of Gravitation let us construct certain kind of suitable "**intellectual cross-correlation**" process using multi-level analogies and phenomenological similarities between different domains in physics which are in some relation to Gravitation. According to (1.1)-(1.9), T.1.6 and T.2.1 (but not exclusively) we will make several starting platform statements, and intuitive and hypothetical remarks, as for instance:

1. *Motion of electrically charged particles in electrostatic field is in certain aspects analogous to a rectilinear motion of electrically neutral particles in a gravitational field (at least mathematically presentable with certain mutually analog expressions, such as given in T.2.1, based on the established analogies in the first chapter).*
2. *Linear motion of electrically charged particle in a magnetic field will be transformed to rotating helix-path motion, this way being similar to combined linear and rotational motion (of electrically neutral particles) in the field of gravitation (at least by analogy in relevant mathematical forms regarding moments and energies; -see T.2.1).*
3. *Electric and magnetic fields are mutually complementary (and conjugated), or intrinsically coupled fields with many known, mutual interactions; -the two fields also create an electro-magnetic field in the form of electromagnetic waves.*
4. *Coulomb and Newton force laws (first between charged, and second between electrically neutral particles, or masses) are mathematically identical, but applicable in two different fields: Coulomb force in Electric field, and Newton force in the field of Gravitation. Force law between two permanent magnets can also be given by the similar Coulomb or Newton force expression. Certainly, there must be some more profound analogy and connection between all of them (than only mathematical one), and presently missing link for completing this analogy is most probably related to rotation.*
5. *Rotation is an omnipresent natural phenomenon, as for instance: rotation of planets around their suns, rotation of moons around their planets, rotation of solar systems around galaxy centers, spinning, angular and magnetic momentum characteristics of all subatomic micro particles and quasi-particles, etc. The origin of rotation in Nature cannot be just a hazard, random movement distribution, or it does not happen only by pure chance, but it is statistics and probability quantifiable phenomena. In fact, every motion (or mathematical function describing that motion) that can be characterized by certain spectral distribution, frequency, wavelength, oscillating process, waving etc. should be linked to certain visible or hidden rotation phenomena (in its original or transformation domain, conveniently presented). In addition, every linear motion in reality belongs to a certain case of curvilinear motion. Pure straight line, constant parameters, uniform motion is only a mathematical idealization (or laboratory approximation) and does not exist in the world of physics (because of omnipresent coupled-fields and because of energy propagation in some of many forms of matter waves).*
6. *It could be that the field of Gravitation is the complementary field to a specific field (so far unknown, or better to say omnipresent and only partially known under different phenomenology, but in a contemporary physics theoretically still unrecognized as such key-field identity), effectively created by the rotation of masses. Gravitation and its conjugated (or complementary) field couple most probably have certain field-force interactions, and they are likely to create a complex "gravitational-rotational" field (presently still unknown, as formulated here, and similar to the complementarity's relations between electric and magnetic fields). Various phenomena of inertia and Gravito-magnetic induction should be in certain relation with above proposed platform (and obviously, the appropriate terminology is still missing here).*
7. *Presently, Newton's Law of Gravitation is only addressing the attractive force between more or less static rest masses (there is nothing velocities and moments dependant in Newton's law).*

We also know that all masses in our universe are in mutually relative motions (the same mass could be in a relative state of rest regarding certain system of reference and in many other states of relative motions regarding many other reference systems). Since all masses are always in certain kind of motion (relative to something), the real law of Gravitation (new one, upgraded) should also have dynamic motional members such as linear and orbital moments, velocities etc., and this is the conceptual platform that will be addressed in this chapter. The leading idea in exploring new (hypothetical) aspects of gravitation and electromagnetism in this paper is (initially) based on the extended Mobility type analogy chart (see chapter 1, T.1.1 and T.1.2). There we can clearly notice that some mosaic-like analogy positions in the chart are either empty, or undefined (see later T.2.1), or not in the full agreement with present formulations of the gravitation force, field and energy expressions. In order to complete empty places of such analogy chart, using again conclusion process based on analogies, we would be in a position to formulate new (and hypothetical) force and field relations (as given in T.2.2). This way, we are practically generating new ideas, mostly based on our confidence in extended Mobility-type analogies (since "Mobility system" of analogies satisfies three particular analogy levels: Mathematical level, Topology level, Similarity of different force expressions and Energy-Momentum/s conservation laws). How far and profound we could go in this process (while producing correct predictions) is still an unanswered question. At least we can notice where already established system of analogies is not fully coherent with some of other analogies (and analogical predictions), and in order to correct or harmonize such intriguing spots, we shall be in a position to make new hypothetical predictions (again based on analogies).

8. We could also imagine that Gravitation presents a "residual reminiscence force link" to asymptotically decaying electromagnetic field phenomena. This manifests when one body with macro-magnetic and electric charge properties (of its internal constituents) starts loosing (or losing) its externally measurable electromagnetic properties by randomizing its internal structure regarding space distribution of internal magnetic and electric dipoles and moments. Thus, it becomes internally, electrically and magnetically, self-compensated and charges neutralized. Elementary matter domains (atoms, different electric and magnetic dipoles etc., mostly composed of charged elementary particles) present forms of self-sustaining and auto-stabilized, internally closed, micro rotating states of motional energy. Furthermore, by the equivalence between mass and energy, it is obvious that such diversity of mutually coupled rotating forms has its effective masses. Since mass and Gravitation are mutually linked (Newton law), and mass is a "modus of energy packaging" ($E = mc^2$), this implicates that Gravitation should be coupled to other field forms, especially to electromagnetic field, because electrons, protons and neutrons are creating atoms and other larger masses. Neutron also separates on an electron and a proton, including all other known relations and interactions between photons, electrons and other elementary particles (all of that supporting the idea about intrinsic coupling between Gravitation and electromagnetic entities). Since electrons (and other charged particles) always have their intrinsic magnetic and orbital moments mutually coupled, where gyromagnetic ratio is constant, we should conclude that mechanical rotation and electromagnetic properties (of elementary matter constituents) are also mutually dependent. Moreover, most of the phenomena known in one field (mechanics of particles motions) should be presentable or (at least mathematically) convertible to the equivalent states in electromagnetism (see later literature [**31**], chapter 4., and T 4.0).

By continuing the same process (as found in the first chapter) regarding creating new analogies and basic symmetries (like given in the first chapter by T.1.8), we can start combining and comparing the expressions for energies and forces in different domains of physics, including the cases of mass rotation (see **[3]**). At the same time we shall remain in a full accordance with already established analogies presented in T.1.2 - T.1.7. The data for the following step are given in T.2.1. Table T.2.1 is filled with the known (and mutually analog) formulas, related to the energy and corresponding forces (in electrostatic field, linear motion, magnetic field, and rotational motion). Empty boxes with a question mark (**?**) are left for the hypothetical and imaginative conclusion process, which will "reconstruct" them based on Mobility type analogies (taken from T.1.1 to T.1.6).

T.2.1	Electric Field	Gravitation and Linear Motion	Magnetic Field	Rotation
Energy	$\frac{1}{2}\mathbf{Cu}^2 = \frac{1}{2}\frac{\mathbf{q}^2}{\mathbf{C}}$	$\frac{1}{2}\mathbf{mv}^2 = \frac{1}{2}\frac{\mathbf{p}^2}{\mathbf{m}} = (= \frac{1}{2}\mathbf{Sf}^2)$	$\frac{1}{2}\mathbf{Li}^2 = \frac{1}{2}\frac{\mathbf{\Phi}^2}{\mathbf{L}}$	$\frac{1}{2}\mathbf{J}\omega^2 = \frac{1}{2}\frac{\mathbf{L}^2}{\mathbf{J}} = (= \frac{1}{2}S_R\tau^2)$
Energy Density	$\frac{1}{2}\varepsilon\mathbf{E}^2$	?	$\frac{1}{2}\mu\mathbf{H}^2$	?
Coulomb Force types	$\frac{1}{4\pi\varepsilon}\frac{\mathbf{q}_1\mathbf{q}_2}{\mathbf{r}^2}$	?	?	?
Newton attractive Force	?	$-\mathbf{G}\frac{\mathbf{m}_1\mathbf{m}_2}{\mathbf{r}^2}$	?	?

After (conditionally) accepting the previous brainstorming and hypothetical conclusions and/or statements (points 1. to 8.), in order to additionally support them, we can fill the table T.2.1 with missing formulas, by using electro-mechanical analogies from T.1.2 (thus providing and exercising a more complete symmetry of corresponding and analogous formulas, by producing a new table, T.2.2).

T.2.2	Electric Field	Gravitation & Linear Motion	Magnetic Field	Rotation
Energy	$\frac{1}{2}\mathbf{Cu}^2 = \frac{1}{2}\frac{\mathbf{q}^2}{\mathbf{C}} \Downarrow$	$\frac{1}{2}\mathbf{mv}^2 = \frac{1}{2}\frac{\mathbf{p}^2}{\mathbf{m}} \Downarrow$	$\frac{1}{2}\mathrm{Li}^2 = \frac{1}{2}\frac{\Phi^2}{\mathrm{L}} \Downarrow$	$\frac{1}{2}\mathrm{J}\omega^2 = \frac{1}{2}\frac{\mathrm{L}^2}{\mathrm{J}} \Downarrow$
Energy Density	$\frac{1}{2}\varepsilon E^2 \Rightarrow$	$(\frac{1}{2}\mathbf{g}_G \mathfrak{G}^2)*$	$\Leftarrow \frac{1}{2}\mu H^2 \Rightarrow$	$(\frac{1}{2}\mathbf{g}_R \mathbf{R}^2)*$
Coulomb Force types	$\mathbf{F}_{ed} = \frac{\mathbf{1}}{\mathbf{4}\pi\varepsilon}\frac{\mathbf{q}_1\mathbf{q}_2}{\mathbf{r}^2} \Rightarrow$	$(F_{gd} = \frac{1}{4\pi g_G}\frac{p_1p_2}{r^2})*$	$(F_{md} = \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2})*$	$(F_{Rd} = \frac{1}{4\pi g_R}\frac{L_1L_2}{r^2})*$
Newton Force types	$(\mathbf{F}_{es} = (?)\frac{\mathbf{C}_1\mathbf{C}_2}{\mathbf{r}^2})*$	$F_{gs} = -G\frac{m_1m_2}{r^2}$ $\Leftarrow \quad \Rightarrow$	$(\mathbf{F}_{ms} = (?)\frac{\mathbf{L}_1\mathbf{L}_2}{\mathbf{r}^2})*$	$(F_{Rs} = (?)\frac{J_1J_2}{r^2})*$

(…)* -Hypothetical formulas; $\mathfrak{G}$ -Gravitation field, g_G -"Gravitational-permeability"; $\mathfrak{R}$ -Field of Rotation, g_R -"Rotational-permeability"; Indexing: ed = electro-dynamic, gd = gravito-dynamic, md = magneto-dynamic, Rd = rotational-dynamic, es = electro-static, gs = gravito-static, ms = magneto-static, Rs = rotational-static (most of them are here invented, temporary formulations for the purpose of making analogical expressions)

All energy and force/s expressions in T.2.2 are intentionally chosen or modified (or reinvented) to have mutually very similar mathematical forms regarding relevant constants (such as: $\frac{1}{4\pi\varepsilon}, \frac{1}{4\pi\mu}, \frac{1}{4\pi g_G}, \frac{1}{4\pi g_R}$). Of course, we could also transform constant of gravitational force as, $\frac{1}{4\pi g_R}$ etc., all of that just in order to obtain maximal symmetry and parallelism of mutually analog and corresponding force and energy expressions. What could be particularly interesting in the introduced strategy is to find different mutual cross-platform links of relevant constants, such as speed of light expressed as the function of electric and magnetic permeability, $c = \frac{1}{\sqrt{\mu\ \varepsilon}}$. This implicates existence of much more profound unity of different natural fields and forces (and something similar could also be found applicable to relevant analogical constants from mechanics and gravitation).

At this time it is better to postpone arbitrary discussions about the terminology and meaning of new formulas, constants and symbols introduced in T.2.2, because it is clear (from the obvious analogies) how certain formulas are created and what meaning they could have. In any case, the importance of the previous process would be mostly in sparking possible new and original ideas about connections and symmetries that could exist, but maybe we do not know about them yet. In addition, maybe we are explaining the same events in quite a different way/s, or we are not used to seeing the same facts from the point of view previously initiated. All formulas (in T.2.2) with the white background, marked by an asterisk (…)*, are newly created and hypothetical. Of course, on this level, there is still no commitment as to which hypothetical formula in T.2.2 has some real value, or a chance to be transformed towards something with Physics-related meaning (which is the task to be realized some other time). Based on the data in T.1.6, T.2.1 and T.2.2, we also see that (by analogy) magnetic flux Φ belongs to space-geometry parameters, similar to rectilinear displacement or angle. At the very least, we can conclude that (at this time) a full analogy (in all directions) is not satisfied for magnetic field phenomena (cf. T.1.6 and T.2.2) in the same way as it is satisfied for all other electromechanical parameters and entities. Later on, we shall attempt to show how to establish a more complete symmetry in electromagnetism; - see (3.1) to (3.5). It should be mentioned that it is experimentally known that attractive and repulsive forces between two permanent magnets (between the opposite or same magnet poles) are also satisfying the same Coulomb-Newton force-law expression analog to electrical charges situations.

[♣ <u>COMMENTS & FREE-THINKING CORNER:</u> *By analogy (from T.1.2 and T.1.6), as the electric capacitance* ***C*** *is a kind of a static reservoir or storage for the electric-charge elements,* ***q****, the same is (hypothetically) valid for the mass* ***m****, which should be a static storage of (somehow internally packed and stabilized) "momentum elements"* ***p****. In addition, a moment of inertia* ***J*** *could be understood as a "static storage" for (somehow internally packed) "angular momentum elements"* ***L****. Above statements and terminology are only temporarily established and conditionally good for the purpose of intriguing and analogical ideas generation, until some more convenient formulations are found. In fact, nothing very original and new has been claimed here, since we already know that mater (or mass) elements are atoms with all their complexity of internally synchronized and well packed motional states.* ***♣]***

2.1. Some Hypothetical Force Laws

If we give a little bit more freedom to hypothetical thinking, again based on analogies, (see **[3]**), from the table T.2.2 we could exercise that gravitational type of attractive force between the two moving masses is not only given by original Newton's formula, but rather by (2.1), as superposition of two forces. One which represents static case of Newtonian attraction between two masses, prefix "stat. = s", and the other, for the time being very hypothetical, which represents a dynamic interaction between corresponding moments, prefix "dyn. = d", which could be repulsive or attractive:

$$F_g = F_{gdyn.} + F_{gstat.} = + \frac{1}{4\pi g_G} \cdot \frac{p_1 p_2}{r^2} - G \cdot \frac{m_1 m_2}{r^2} . \qquad (2.1)$$

Similar thinking (and new hypothetical invention) can be applied to the situation with rotating masses (taken from T.2.2), in which case there could exist some kind of (attractive and/or repulsive or non-central) force between them, analogically expressed as in (2.1), as static and dynamic force members addition (with corresponding angular and inertia moments):

$$F_R = F_{Rdyn.} + F_{Rstat.} = + \frac{1}{4\pi g_R} \cdot \frac{\mathbf{L}_1 \mathbf{L}_2}{r^2} - G_R \cdot \frac{\mathbf{J}_1 \mathbf{J}_2}{r^2} , \qquad (2.2)$$

In addition, if we have presence of combined rectilinear and rotational elements of certain complex motion of two objects, we should appropriately combine (2.1) and (2.2), in order to describe the Newtonian resulting force between them. *We also need to address the proper "vectorial" forms of (2.1) and (2.2) in order to conceptualize the 3-dimensional picture regarding such forces* (but presently this is not the primary objective).

Of course, it is an open question if formulas such as (2.1) and (2.2) can be experimentally proven and how (since the most realistic approach should be that all elements of linear and rotational motion/s are coincidently present and mutually coupled in the motion of the same object. Consequently, (2.1) and (2.2) should also be united in one form that is more general). In fact, the unifying imperative produces that static members in (2.1) and (2.2) should be mutually identical, $G \times \frac{m_1 m_2}{r^2} = G_R \times \frac{\mathbf{J}_1 \mathbf{J}_2}{r^2} = F_{stat.} \Rightarrow$ $G \times m^2 = G_R \times \mathbf{J}^2$, and that force elements responsible for dynamic interaction between two moving objects would have both linear and orbital moments, $\frac{1}{4\pi g_G} \cdot \frac{p_1 p_2}{r^2} + \frac{1}{4\pi g_R} \cdot \frac{\mathbf{L}_1 \mathbf{L}_2}{r^2} = F_{dyn.}$, this way generalizing the force law between two moving objects as: $F_{1\text{-}2} = F_{dyn.} + F_{stat.}$.

The essential question here would be to ask if we really have any simple and clear candidate example of repulsive and other unusual gravitation-related forces, where we could start establishing a new experimentally and scientifically verifiable theory.

A) It could be that effects of micro-particles, photons, electrically charged or neutral particles and matter wave diffractions (when beam of such particles or quasi-particles is passing trough a small diffraction hole/s) should have something to do with here discussed repulsive forces. Since diffraction basically means that a rectilinear stream (or flow) of something material (with energy-mass content) is getting divergently dispersed, and dispersion could also be conceptualized as the two or many-body interaction of repulsive forces caused by intrinsic elements of particle rotation (the same proposal is introduced 1975 in [**3**] *for the first time). Particle-Wave dualistic aspects of mentioned diffraction events are making this situation relatively complex. We could argue whether wave or particle related effects are dominant, since by applying wave concepts we are able to explain diffraction patterns and pictures on a screen behind diffraction hole/s. Still here is also a place to ask questions about appearance of certain kind of particles repulsion. Then we may present such situations as equivalent beams of particles streaming, where certain kind of particles or quasi-particles are guided by the interaction field to pass diffraction holes (which is created between them and diffraction holes). In order to be bottom-line clear (or to try to visualize and maximally simplify this situation), we could say that the diffraction hole here is serving as a kind of a "bottleneck". This channels the particles in coincident mutually parallel (or mutually synchronized and coordinated) motion and they are forced to take mutually closer, higher density paths. Under such conditions hypothetical repulsive gravitation forces (such as forces (2.1) and (2.2)) are starting to work, dispersing (or diffracting) the particles after passing the hole (since such situation is followed by associated and intrinsic effects of rotating fields, basically appearing and becoming dominant in a space-time vicinity of any two-body interactions).*

B) The other candidates for searching for non-central, repulsive and/or torsional gravitation-force-related components could be investigated in relation to gyroscope behaviors, Coriolis forces and centrifugal or centripetal forces.

C) Most probably that (2.2) should mean that fast rotating objects would mutually create additional and measurable gravitation-related forces which are making significant difference compared to ordinary Newton-Coulomb static forces between them.

D) <u>It is also very important to notice that common in all here mentioned phenomena candidates (regarding different aspects of gravitation-related forces) is that all of them are in some ways combining linear and rotational motions.</u> Later (see chapter four) we will show that particle-wave duality and mater waves are always a consequence of fields coupling between linear and rotational motions.

E) Continuing the same process, we could also say that static and dynamic forces between different objects should also have elements of electromagnetic nature (again Coulomb force laws). Even further, we may find a way to show that electromagnetic force elements are effectively producing or influencing all other gravitation and rotation related forces and field effects (being in the conceptual background as the primary source/s of all mentioned forces). The similar concept of combining electrostatic and dynamic force elements between two moving and electrically charged particles, was long time ago introduced by Wilhelm Weber's force law (see literature [28], [29]).

[♣ <u>COMMENTS & FREE-THINKING CORNER:</u>

Just to give an idea about possible experimental evaluation, we can apply (2.1) and (2.2) to an alternative explanation of elementary micro-particles and quasi-particle diffraction, as well as to analyze different interaction phenomena between micro-particles with immanent rotation or spin characteristics, in the atom world (see **[3]**). For instance, the Pauli Exclusion Principle (<u>"Only two electrons which possess mutually opposite spin can occupy any given quantum, stationary orbit", in the conceptual frames of Bohr's planetary atom model</u>) should, most probably, be in some relation with (2.2). Spinning electrons (or electron wave packets with torsional field components) are creating mutually coupled magnetic and mechanical angular moments making that electromagnetic (or electrodynamic) Coulomb forces and attraction or repulsion between their magnet poles are coincidently present. All such (mutually coupled) forces with direct or indirect energy-mass rotation origins are balancing with all other electromagnetic interactions, giving the possibility of analyzing and proving interactions between rotating objects (formulated by force law (2.2)). It is almost obvious that a number of electrons in certain stationary atom orbit should be 2, thus creating a couple of two elementary magnets with mutually opposite spins, enabling an attraction between two opposite magnet poles. It is also known that protons and neutrons have the same spin as electrons, and obey the Pauli Principle (inside the atom nucleus). If a new type/s of (at present hypothetical) force/s, (2.2) exists between rotating objects, it would be sufficient to make just a small creative and hypothetical step towards using such force/s in explaining the nature of nuclear forces (also closely related to de Broglie matter waves' phenomenology (see (4.18)). **Since magnetic and orbital moments of elementary particles are mutually strongly related (constant gyromagnetic ratio/s), it could eventually happen that we will find that in the background all such "new and hypothetical" forces are very well known electromagnetic forces.**

The most exciting idea, coming from (2.1) and (2.2), would be the possibility of controlling or producing a kind of dynamic antigravity force, in a similar form of the repulsive force known between the same magnetic poles of two magnets (exploiting effects of mass rotation and various angular momentum interactions). **♣]**

Similar to (2.1) and (2.2) we could also exercise creating the mixed force formulas for electric and magnetic fields (combining static and dynamic case), using force formulas from T.2.2 (see also (3.1) - (3.5) and (4.18)). However, this would be too simplified mathematical modeling process, since we already have much more complex and more general Maxwell electromagnetic theory for describing electric and magnetic fields and forces. Moreover, there is a recent extension of Maxwell's Electromagnetic Theory and a different approach to Relativity Theory made by revitalizing and upgrading of Wilhelm Weber's force law, which is presenting the natural unification of fundamental laws of classical electrodynamics, such as: Gauss's laws, Coulomb's law, Ampere's generalized law, Faraday's law, and Lenz's law (see literature [28], [29].

Mass in the steady state of relative rest presents (macroscopically) a neutral electrical (and in most of the cases neutral magnetic) balance between its positive and negative electric constituents. Contrary, mass in motion (because of accelerations, effects of inertia and dynamic transitory states) would tend to create and/or increase internal electrical and magnetic dipoles and associated linear and orbital moments, since there is a big mass difference between electrons and protons (1836 times). Just a simple example of tidal waves and motion of big water masses on our planet caused by the gravitation force interaction between the Earth's and Moon's relative motion shows that what we usually approximate as the static and steady state (of our planetary mass) is not completely static in all of its aspects. This could also be the supporting conceptual background regarding explaining force laws (2.1) and (2.2). Of course, here given statements and ideas are only the starting and brainstorming points that should initiate a more creative and serious modeling process in order to eventually formulate some new and valuable force laws.

Another, original and interesting method of generating velocity dependent, static and dynamic, force-field formulas (starting from expressions for "Newton-Coulomb" force types) is presented in [4]. The method presented in [4] can easily be applied to all force formulas from T.2.2, as well as to all possible combinations similar to (2.1) and (2.2). This will reinforce here presented hypothetical thinking (since the author of [4] presented convincing experimental evidence of general validity of such force formulas). For instance, in [4], we can find for all velocity dependant "Newton-Coulomb" types of fields and forces the following expression: $\mathbf{E}_{moving} = \mathbf{E}_{dyn.} = \mathbf{E(v)} = \mathbf{E}_{stat.}(1 - \frac{\mathbf{v}}{\mathbf{c}}\cos\vartheta)$, *where:* $E(v)$ *is the intensity of certain "Newton-Coulomb" field/force type;* $E_{stat.}$ *corresponds to field/force formulas from T.2.2 and to force expressions like (2.1) and (2.2); v is the speed of the (relevant) field charge/s;* c *is the speed of light; and* ϑ *is the angle between the direction of the charge motion (the speed v) and the direction of field intensity propagation.*

2.2. Generalized Coulomb-Newton Force Laws

There is an easy way to transform or upgrade any of "Newton-Coulomb" force into several more of similar looking (presently still hypothetical) force laws. Let us start from Newton attractive force between two masses, $\mathbf{m_1}$ and $\mathbf{m_2}$, ($\mathbf{m = m_0} / \sqrt{\mathbf{1 - v^2 / c^2}}$, $\mathbf{m_0} = \mathbf{const.}$), and replace the two masses with their relativistic, total energy equivalents, $\mathbf{E_{t1}} = \mathbf{m_1 c^2}$, $\mathbf{E_{t2}} = \mathbf{m_2 c^2}$, ($\mathbf{E_t = E_0 + E_k} = \mathbf{m_0 c^2} / \sqrt{\mathbf{1 - v^2 / c^2}}$, $\mathbf{E_k} = \mathbf{(m - m_0)c^2} = \mathbf{pv} / (\mathbf{1} + \sqrt{\mathbf{1 - v^2 / c^2}}$), applicable in cases when masses are moving (or have any other kind of associated kinetic energy):

Newton force law between two (almost static) masses presents the classical understanding of gravitation. Since we know that every object in motion, wave, field, photon etc. (that has certain energy content), can easily be represented as having an equivalent mass content ($E = mc^2$), and vice versa, we would be able to apply Newton law/s almost everywhere (of course not to take it literally):

$$\begin{aligned} \mathbf{F_g} &= -\mathbf{G} \cdot \frac{\mathbf{m_1 \cdot m_2}}{\mathbf{r^2}} = -\frac{\mathbf{G}}{\mathbf{c^4}} \cdot \frac{\mathbf{(m_1 c^2) \cdot (m_2 c^2)}}{\mathbf{r^2}} = \mathbf{G}^* \cdot \frac{\mathbf{E_{t1} \cdot E_{t2}}}{\mathbf{r^2}}, (\mathbf{G^*} = -\frac{\mathbf{G}}{\mathbf{c^4}} = \mathbf{const.}) \\ \mathbf{F_g} &= \mathbf{G}^* \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\mathbf{E_{k1} + E_{01}}) \cdot (\mathbf{E_{k2} + E_{02}}) = \mathbf{G^*} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\mathbf{E_{k1}E_{k2} + E_{k1}E_{02} + E_{01}E_{k2} + E_{01}E_{02}}). \end{aligned} \tag{2.3}$$

It is immediately clear that in the force law (2.3) there are only relations (products) between motional (kinetic or dynamic) energies, $\mathbf{E_{k1} = E_{1dyn.}}$ In addition, $\mathbf{E_{k2} = E_{2dyn.}}$, and state-of-rest energies, $\mathbf{E_{01} = E_{1stat.}}$, and $\mathbf{E_{02} = E_{2stat.}}$, of the interacting objects. Since total energy members, $\mathbf{E_{t1} = E_{k1} + E_{01} = E_{1dyn.} + E_{1stat.}}$, $\mathbf{E_{t2} = E_{k2} + E_{02} = E_{2dyn.} + E_{2stat.}}$, could have different origins, composed from many elements (electromagnetic, gravitational, rotation, kinetic and potential energies, etc.), we can generalize "Newton-Coulomb" force laws (between two complex energy states, here marked with **"i ↔ 1"** and **"j ↔ 2")** creating the following expression, $\mathbf{F_g} = \mathbf{F_{1,2}}$:

$$\mathbf{F_{1,2}} = \mathbf{G^{*}} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} [(\sum_{i} \mathbf{E_{k1i}})(\sum_{j} \mathbf{E_{k2j}}) + (\sum_{i} \mathbf{E_{k1i}})(\sum_{j} \mathbf{E_{02j}}) +$$

$$+(\sum_{i} \mathbf{E_{01i}})(\sum_{j} \mathbf{E_{k2j}}) + (\sum_{i} \mathbf{E_{01i}})(\sum_{j} \mathbf{E_{02j}})] = \tag{2.4}$$

$$= \sum_{i,j} \mathbf{F_{(1-2)dyn.}} + \sum_{i,j} \mathbf{F_{(1)dyn.(2)stat.}} + \sum_{i,j} \mathbf{F_{(1)stat(2)dyn.}} + \sum_{i,j} \mathbf{F_{(1-2)stat.}} \ .$$

It should be underlined that in (2.4) there is only one purely static force member involving only rest masses interaction or interaction between static, stable and constant field charges (if mass could be characterized as the real, unique and fully representative gravitation field-charge):

$$\sum_{i,j} \mathbf{F_{(1-2)stat.}} = \mathbf{G^{*}} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \mathbf{E_{01i}})(\sum_{j} \mathbf{E_{02j}}) = -\ \mathbf{G} \cdot \frac{\mathbf{m_{01} \cdot m_{02}}}{\mathbf{r^2}} \ .$$

Here favored concept is that static or rest mass itself is just a part of the gravitation related picture, and that the other significant part (of other force components between two moving objects, still hypothetical) should be related to their linear and orbital moments, or dynamic, motional energy parameters, like we can find in (2.1), (2.2), and (2.4),

$$\sum_{i,j} \mathbf{F_{(1-2)dyn.}} = \mathbf{G^{*}} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \mathbf{E_{k1i}})(\sum_{j} \mathbf{E_{k2j}}) \ ,$$

$$\sum_{i,j} \mathbf{F_{(1)dyn.(2)stat.}} = \mathbf{G^{*}} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \mathbf{E_{k1i}})(\sum_{j} \mathbf{E_{02j}}) \ ,$$

$$\sum_{i,j} \mathbf{F_{(1)stat(2)dyn.}} = \mathbf{G^{*}} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \mathbf{E_{01i}})(\sum_{j} \mathbf{E_{k2j}}) \ ,$$

having in mind that the total motional energy (of both interacting objects) could have linear or rectilinear kinetic energy component $\mathbf{E_k} = \mathbf{pv/(1 + \sqrt{1 - v^2/c^2})}$ and certain amount of rotational motion energy (in case if any of interacting objects is somehow rotating or spinning).

We also know that, an internal rest mass structure has its intrinsic elements (atoms and subatomic constituents) composed of electromagnetically balanced charged states. These are in permanent motion, oscillating and rotating in their stationary energy states, being somehow auto-stabilized like self-closed standing waves, resonant structures, <u>or mutually coupled elementary magnets</u>. The common property of rest masses in our universe is that all of them have certain latent (internal), rest state energy (equal to $\mathbf{m_0 c^2}$). A certain force between them would still exist, even if they are (macroscopically, or externally) fully electrically and magnetically neutral (mutually compensated or neutralized). This is the force of Gravitation. Of course, when rest masses are in relative motion interacting with other energy states and other masses, the force of Gravitation would become only more complex, like in (2.4). In reality, there are only relative motions and relative rest states of all masses and energy states in our universe. The challenge here would be to find the profound relations and connections between electromagnetic and gravitation forces, since particularly, all of them (when isolated) are obeying the same form of Coulomb-Newton's law, like shown in T.2.2 (also having other similarities and differences).

It should also be noticed that what we are presently considering as original Newton law of gravitation force is only the most static and often the weakest part of the general force expression (2.4): $\sum_{i,j}\mathbf{F}_{(1-2)stat.} = \mathbf{G}^* \cdot \frac{\mathbf{1}}{\mathbf{r}^2}(\sum_{i}\mathbf{E}_{01i})(\sum_{j}\mathbf{E}_{02j}) = -\mathbf{G}\cdot\frac{\mathbf{m}_{01}\cdot\mathbf{m}_{02}}{\mathbf{r}^2}$. Obviously that (2.4) can be modeled or mathematically transformed to contain different force expressions similar to (2.1) and (2.2), or also to include linear and orbital moments, electromagnetic and other force and energy elements. Here, we can pose the question what real, elementary and essential sources, or charges of different fields and forces are. We can also try to find or model them deductively, starting from (2.4), taking the energy as the most common quantifying property for all of them (avoiding to start from particular masses and other field charges, as presently practiced in Physics). Of course, we know (or maybe we only think that we exactly know) that mass is the source of gravity, similar as the source of electric field is an electrical charge. Based on extended understanding of Newton-Coulomb force law (2.4), we would most probably find or re-invent some more of essential force charges (most of that matter being still hypothetical and based on analogical thinking). If our present knowledge regarding such situations is fully correct, the same, already known facts regarding field sources and charges should be re-confirmable from different conceptual and symmetry-analogy related platform/s, being mutually compatible for generalization/s, and being uniquely treated regarding all possible field charges, starting from (2.4), by going backwards to elementary relations between different field charges. As we know, this is still not the case in contemporary Physics. For instance, if we know that objects $\mathbf{m_1}$ and $\mathbf{m_2}$, from (2.3) are not only neutral masses, but also have certain (non-compensated) electrical charges and certain magnetic properties (like permanent magnets); we can extend the Newton-Coulomb force law (2.4) by adding two more force members. However, we need to remember that associated electromagnetic forces in many cases could be enormously stronger than any attraction caused by gravitation, and that in such cases the force member belonging to static mass attraction, $-\mathbf{G}\frac{\mathbf{m}_{01}\mathbf{m}_{02}}{\mathbf{r}^2}$, could be negligible, especially in cases when masses are sufficiently small). At the same time we still do not have any quantitative argument to say how big other (hypothetical) gravity related force members (found in expressions (2.1), (2.2) and (2.4)) could be, compared to electromagnetic forces (and maybe once it would be found that all of such “new force members” are essentially belonging to electromagnetic forces, or directly originating from them).

Since both, Newton and Coulomb force laws have the same mathematical form, for developing more general (for the time being only qualitative) understanding of force expressions between two objects “charged electrically, magnetically and gravitationally”; -we could easily transform (2.4 into):

$$\mathbf{F_{1,2}} = \mathbf{F_g} \Rightarrow \mathbf{F_{1,2}}\,\mathbf{(combined,\ multiple\ charges)} = \mathbf{F_g} + \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} + \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2} + \ldots .$$

$$\begin{aligned} F_{1,2} &= \sum_{i,j}F_{(1\text{-}2)stat} + \sum_{i,j}F_{(1\text{-}2)dyn.} + \sum_{i,j}F_{(1)dyn.(2)stat} + \sum_{i,j}F_{(1)stat(2)dyn} = \\ &= \left[-G\frac{m_1m_2}{r^2} + \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} + \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2} + \ldots\right] + \\ &+ \sum_{i,j}F_{(1\text{-}2)dyn.} + \sum_{i,j}F_{(1)dyn.(2)stat} + \sum_{i,j}F_{(1)stat(2)dyn} \end{aligned} \tag{2.4-1}$$

In fact, new, combined multiple charge force (2.4.1) will be directly and explicitly updated only in its static (corpuscular or “particle-nature related”) force member/s,

$$\sum_{i,j} F_{(1\text{-}2)stat} = -G\frac{m_1 m_2}{r^2} + \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} + \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2} + \ldots = \frac{\Omega_1\Omega_2}{r^2},$$
$$\Omega_{1,2}^2 = -Gm_{1,2}^2 + \frac{1}{4\pi\varepsilon}q_{1,2}^2 + \frac{1}{4\pi\mu}\Phi_{1,2}^2 + \ldots = \sum_{(i)}\Omega_{(1,2)\text{-}i}^2 , \tag{2.4-2}$$

and indirectly (or implicitly), all other force members from (2.4) and (2.4.1), where dynamic and motional energy components are involved, would keep their previous general form, $\sum_{i,j}\mathbf{F}_{(1-2)dyn.} + \sum_{i,j}\mathbf{F}_{(1)dyn.(2)stat} + \sum_{i,j}\mathbf{F}_{(1)stat(2)dyn}$. However, in reality, being modified for new motional energy contributions coming from interactions between electrical, magnetic and all other presently known or maybe unknown charges (if any of such still unknown charges really exist). The reason for having such situation is not only that Newton and Coulomb laws have the same mathematical forms, but also in the fact that every electrical or magnetic entity have its equivalent mass (or energy content). This means that logic applied in developing (2.4-1) can be extended to any Coulomb-Newton force situation between different field charges. At the same time, qualitatively and conceptually, (2.4-1) and (2.4-2) present a kind of simple field unification platform (at least one of good starting points for unification). Phenomenologically, different field charges (mass, electrical charge …) are only looking mutually different and distinct to us, but maybe in reality we only have different “packaging and energy atomizing formats” of the same universal field. The well known particle-wave **duality** (or better to say **unity**) concepts and their origins, known from Quantum Theory (first introduced by L. De Broglie, Max Planck, A. Einstein and N. Bohr), should also be (qualitatively) recognizable in (2.4-1) and (2.4-2), as for instance,

$$\begin{aligned}
&F_{1,2} = \sum_{i,j} F_{(1\text{-}2)stat} + \sum_{i,j} F_{(1\text{-}2)dyn.} + \sum_{i,j} F_{(1)dyn.(2)stat} + \sum_{i,j} F_{(1)stat(2)dyn} = \\
&= F(\text{Particle/s-related}) + F(\text{Waves \& Fields-related}) + \\
&+ F(\text{mixed-particles-waves}), \\
&F(\text{Particle/s-related}) = \sum_{i,j} F_{(1\text{-}2)stat} = \frac{\Omega_1\Omega_2}{r^2}, \\
&F(\text{Waves \& Fields-related}) = \sum_{i,j} F_{(1\text{-}2)dyn.}, \\
&F(\text{mixed-particles-waves}) = \sum_{i,j} F_{(1)dyn.(2)stat} + \sum_{i,j} F_{(1)stat(2)dyn}. \\
&\Omega_{1,2} (=) \text{ here introduced symbol for generalized universal field charge/s}
\end{aligned} \tag{2.4-3}$$

There is another $\frac{1}{r^2}$ force law between equidistant, parallel-paths moving electrical charges (q_1 , q_2) where magnetic force between them cannot be directly explained by original Coulomb’s law (like in cases of static electric charges), as for instance,

$$F_{1,2} = \frac{\mu}{4\pi}\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2} = K\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2},\ K = \text{const.}. \tag{2.4-4}$$

If we apply already known analogies, where electrical charge q is analog to linear, $p = mv$ or orbital, $L = J\omega$ momentum, or to a magnetic flux Φ, we will be able to transform (2.4-4) into couple more of interesting (hypothetical) $\frac{1}{r^2}$ force expressions, where instead of different field charges we will have products of corresponding motional, dynamic or field energy members, like in (2.3),

$$F_{1,2} = K\frac{(q_1v_1)\cdot(q_2v_2)}{r^2} \Rightarrow F_{1,2} = \begin{Bmatrix} K_p\frac{(p_1v_1)\cdot(p_2v_2)}{r^2} \\ K_L\frac{(L_1\omega_1)\cdot(L_2\omega_2)}{r^2} \\ K_\Phi\frac{(\Phi_1u_1)\cdot(\Phi_2u_2)}{r^2} \end{Bmatrix} = \begin{Bmatrix} K_p'\frac{(E_{k1})\cdot(E_{k2})}{r^2} \\ K_L'\frac{(E_{r1})\cdot(E_{r2})}{r^2} \\ K_\Phi'\frac{(E_{m1})\cdot(E_{m2})}{r^2} \end{Bmatrix}, \qquad (2.4\text{-}5)$$

$(K, K_p, K_L, K_\Phi, K_p', K_L', K_\Phi') = \text{Constants}.$

In fact, all of here presented analogical $\frac{1}{r^2}$-force related brainstorming (from (2.1) to (2.4-5)) is mostly indicative and challenging exercise, showing that generalized $\frac{1}{r^2}$-Coulomb-Newton forces should have both static and dynamic force members (like in (2.1) and (2.2)), and that in all of such cases energy content of mutually interacting participants has a dominant influence, like in (2.4). In addition, no need to underline that everything what exist in our universe is in certain kind of (mutually coupled) relative motion/s to its environment, and that internal structure of all particles has its intrinsic fields and wave nature.

[♣ <u>COMMENTS & FREE-THINKING CORNER:</u> What is still missing in generalized Newton-Coulomb force laws (2.4-1)-(2.4-5) are torsional (rotational or angular) force components, relevant in cases of rotating masses, and/or when masses have electric or magnetic charges. Electromagnetic parts of (2.4-1) – (2.4-5) are intrinsically carrying torsional force, fields, and wave components, and all rest masses are composed of atoms, which intrinsically have their electromagnetic constituents, and certain kind of rotating motional-energy content inside. The other missing aspects of Coulomb-Newton Gravitation-related forces could be an explicit presence of the "$\frac{\mathbf{d}}{\mathbf{dt}}$<u>- related force components</u>". We know that dynamic force acting on a particle in linear motion is defined as $\mathbf{F} = \frac{\mathbf{dp}}{\mathbf{dt}}$, that particle in rotation could experience a torque $\tau = \frac{dL}{dt}$, that electric and magnetic forces are not only forces between static electric charges or permanent magnets (all having directly or indirectly detectable $\frac{\mathbf{d}}{\mathbf{dt}}$-components). Consequently, new insight into forces of Gravitation should also include certain "$\frac{\mathbf{d}}{\mathbf{dt}}$<u>-related force components</u>".

Here we could go a step ahead creating a new dynamic conceptualization of Newton-Coulomb force laws. **It just looks that Newton-Coulomb force laws are mathematical forms where only attractive and/or repulsive forces between two static (or constant) charges are addressed. This is because we are usually considering mass or electric charge, or permanent magnet flux… as something fixed and static. In reality, if we start analyzing what is internally creating an electric charge or mass, or a magnet…, we will find different and very dynamic, also resonantly or standing-waves quantized, stationary and rotating complex field structures behind all of them. Mentioned entities are compositions of molecules, atoms and elementary particles, conveniently packed into structures we would externally consider as a stable charge, mass, magnet etc.** ***Most of such internal matter constituents and elementary particles already have intrinsic spin and orbital moment/s attributes, beside other charge attributes, and here should be the area to search for essential sources of gravitation. ♣]***

The position of the author of this paper is that **Particle-Wave duality enigma of modern Physics should be related to the complex nature of (already known and maybe in some cases still unknown) fields and forces between interacting objects. Here linear motion and associated rotation are mutually coupled or conjugated pair of motional entities (behaving similar as when an electric and magnetic field couple is creating electromagnetic waves; - See also equations (4.1), (4.2), (4.3), (4.33.1), (4.5-1)–(4.5-3), T.4.2 and T.4.3, supporting the unity of linear motion and rotation).**

The intention of the author of this paper is also to show (presently only qualitatively and on the conceptual level of understanding) that, regarding gravity and rotation-related phenomenology, the questions about their essential field sources and charges are still not completely and coherently answered. This is unlike in electromagnetic theory, regardless of the fact that General Relativity Theory, from the Topology platform, in most of cases is correctly describing all spatial deformations and consequences caused by presence of gravitational sources. It is also clear that present simple forms of Newton-Coulomb force laws are applicable only in a limited framework.

Here we are mostly trying to show (on an easiest possible way; -by addressing Newton-Coulomb force laws with new hypothetical proposals) that space between two interacting objects in mutually relative motion, should have multi-component field structure, like presented in (2.4-3) and earlier. In addition, ***that certain dynamic field and force components between interacting objects (closely involved in particle-wave duality phenomenology) are still not well conceptualized or maybe not known in the contemporary physics, probably being for orders of magnitude stronger than ordinary Newton attractive force between static masses.*** **Relations from (2.4) until (2.4-3) are also far from being complete and generally applicable, but just to give a(n) brainstorming idea about possible complexity of fields and forces between two interacting objects are qualitatively good enough (at least to show directions where we could search for new interactions and new ideas about field theory upgrading).**

Regardless of what would be found/established/invented as the most representative set of field charges (the same ones known presently, or new, replaced by some other, more general and mutually more coherent charge entities), the strongest and most general conceptual platform introduced by (2.3)-(2.4-3) and elsewhere in this paper is based on the energy content of matter domains. Energy content should be the starting position for any new charge/field/force definition or redefinition.

[♣ <u>COMMENTS & FREE-THINKING CORNER:</u> Also, the enormously big gaps and differences between Classical Mechanics, Quantum Theory and Relativistic Theory should be mostly related to one-sided, conceptually incomplete Particle-Wave Duality presentations and to its present mathematical modeling in Physics. Simply, in mentioned theories, some of particles and/or fields binding components, present in (2.4-3), are missing, or not being adequately taken into account (here also accountable only hypothetically). Number of particle-wave interactions, diffractions, interference and scattering phenomena, are still incompletely analyzed in modern Physics, replacing missing force components by "theory-saving" probabilistic modeling (putting in question the real place of causality and objectivity in natural sciences). Of course, we need to admit that Quantum Theory works very well in its frames, by performing its mathematical and "life-saving job" very successfully. Quantum Theory is also successful in convincing generations of scientist that "white could be black", and that what we see is only a certain level of probability of what it could be. Quantum Theory is also elaborating that we or somebody else could coincidently see (intellectually, mathematically and conceptually) the same situation differently, sometimes resembling a little bit to certain ideological and religious teachings found in the known part of human history. Also it should be mentioned that no new theory regarding particle-wave duality would be able to completely neglect and undermine most of concepts and models presently used in Quantum Theory, since a lot of successful fittings, modeling and arrangements (mostly on mathematical level and based on satisfying and completing obvious Symmetries) is already made there. So well operating mathematical structures (like known in Quantum Theory) could again be used (until certain limits and inside of new frameworks) in formulating new concepts and new theories.

Author of this paper will probably not solve such problems of modern Physics, but to show where the problems are and how we could address them differently than presently made in physics, would already be the significant first step. ***♣]***

2.3. How to start accounting rotation

The next (for the time being also hypothetical and still not completely formulated) opinion of the author is that mass is creating the field of gravity, because it effectively and dominantly presents an energy "<u>packaging format</u>". In reality mass is composed of atoms with countless number of internal elements that are rotating, having orbital, magnetic and linear moments, spin and other motional manifestations (or effects and states equivalent to some kind of rotation). In fact, we can safely say that also certain level of hidden motional energy is permanently blocked inside of atoms and micro particles, being distributed inside a space defined by external shape of macro particles. The ***<u>resulting action</u>*** of such randomly distributed and mutually coupled electromagnetic and mechanical moments (or spinning elements) should present the most important source/s of gravitation (see also chapter 4.3; -equations (4.41-1) until (4.45)).

Intuitively advancing (in order to conceptually support above-given idea), we could start with an oversimplified *<u>analogical and dimensional</u>* "manipulation" of the formula for a centripetal force applied on a rotating particle. In this way, let us create several of mutually equivalent, only <u>dimensionally comparable</u> expressions (2.5), and explore the concept that gravitation could present an external manifestation of a kind of resulting centripetal and orbital moments' related force of countless number of internal, mutually coupled, energy carrying and rotating micro-mass domains. These are, when observed externally, creating a property we consider as a total rest mass, which is the origin of Gravitation, as for instance:

$$F_c = \frac{mv^2}{r} (=) \left[\frac{E_k}{r}\right] (=) \left[\frac{L\omega}{r}\right] (=) \left[\frac{\tau}{r}\right] (=) \left[\frac{Lv}{r^2}\right] (=) \left[m\frac{dv}{dt}\right] (=) \left[\frac{Kg \times m}{s^2}\right] (=) [\text{Force}] \ ,$$

$$\underline{[E_k](=)[mv^2]\ (=)\ \left[\frac{Kg \times m^2}{s^2}\right]} (=) [\text{Motional energy of internal micro-domains}] \ ,$$

$$[L](=)[J\omega](=)[mr^2\omega](=)\left[\frac{Kg \times m^2}{s}\right](=)[\text{Orbital moments of internal domains}] \ , \qquad (2.5)$$

$$\underline{[\tau](=)\left[\frac{dL}{dt}\right](=)\left[\frac{Kg \times m^2}{s^2}\right]}(=)[\text{Torque}](=)[\text{Energy}](=)[J] \ ,$$

$$[v](=)[\omega r](=)\left[\frac{m}{s}\right](=)[\text{velocity}], \ [\omega](=)\left[\frac{1}{s}\right](=)[\text{angular velocity}] \ .$$

By making only dimensional comparisons inside of (2.5), we would be able to spark new ideas regarding what kind of physics-related parameters or values (such as: $m, J, p, L, \tau, v, \omega, E_k ...$) could be involved in the expression/s for a centripetal force and forces related to orbital moments and their derivatives. This way we are introducing an oversimplified, and conceptually clear enough starting platform (that should also stay compatible with (2.1)-(2.4-3)) for formulating new and still hypothetical force laws, analogically originating from "Newton-Coulomb" force laws. Briefly, the objective here is to show probable existence of missing force-link/s directly related to fields created by rotation that are complementarily coupled to linear motions. This is very much similar to coupling between an electric and magnetic field in electromagnetism.

The intrinsic coupling between electric and magnetic fields and material properties (known in electromagnetism) shows, for instance, that electric charges (besides respecting Coulomb-Newton force law) can perform in electric and magnetic fields almost any kind of linear and rotational movements (as a consequence of interactions based on different forms of attractive and repulsive forces).

We also know how to associate properties of mass and different moments to electromagnetic waves. Since mass is an important ingredient of Gravitation, one of the first confirmations of Relativity theory (at least qualitatively and in the limits of the measurements errors) was in astronomy, confirming that big masses are bending light beams (or attracting photons, respecting the Newton Law of Gravitation). In fact, this primarily confirms that photons (as purely electromagnetic entities, without having rest mass) have an equivalent dynamic-mass that behave similarly to (static) rest mass regarding forces of gravitation. ***What we know as an ordinary static mass (where rest mass exist) should only be a specific condensate or agglomerate (or packaging format) of certain amount of energy (thanks to its internal couplings between electric and magnetic intrinsic entities).***

Consequently, it would not be extremely surprising (see [23] – [26]) if one day we conclude or find that all various natural forces and charges (we are presently considering as mutually distinctive, or very much different and specific) have its deepest origins only in electromagnetic charges, and their "packaging-formats" and interactions. These are manifested in different states of motions (applicable, at least, to a macroscopic world, and to interactions between electrons, protons, neutrons and their antiparticles).

In the next several examples, see below **A**, **B**, **C** to **F**, we will hypothesize and exercise this idea and try to find what could or would be consequences if we start combining available options (see also equations in the chapter 5, from (5.4.1) 7until (5.4.10)).

- A -

Let us first imagine that certain kind of elementary matter-waves, or energy states, which coincidently have coupled elements of linear and rotational motions, are presenting essential building blocks of everything what is creating a mass. For instance, let us start with a mater-wave state Ψ that is in linear motion combined with a kind of rotation, which is effectively presenting moving quantity of linear motion with an angular mechanical momentum, being relatively stable time-space formation. The idea here is to show that a number of such elementary states, which are in a form of atomized energy domains (say wave packets), are effectively, after certain kind of superposition and integration, creating an entity we externally identify as a stable particle that would have its rest mass. We could start making such concept mathematically operational by introducing the elementary mater-wave function Ψ on the following way (see also analogies from the first chapter and from the chapter 4.3 see equations starting with (4.9.0)):

$$\left[\begin{array}{l} \left\{\begin{array}{l} E_k = E_{k-\text{lienear}} + E_{k-\text{rotational}} = \frac{1}{2}mv^2 + \frac{1}{2}\mathbf{J}\omega^2 = \\ = \frac{1}{2}pv + \frac{1}{2}\mathbf{L}\omega = \frac{p^2}{2m} + \frac{\mathbf{L}^2}{2\mathbf{J}}, p = mv,\ \mathbf{L} = \mathbf{J}\omega \end{array}\right\} \text{and} \left\{\begin{array}{l} m \\ v \\ p \\ vdp \end{array}\right\} (\text{analog to}) \left\{\begin{array}{l} \mathbf{J} \\ \omega \\ \mathbf{L} \\ \omega d\mathbf{L} \end{array}\right\} \Rightarrow \\ \Rightarrow dE_k = \underline{vdp + \omega d\mathbf{L}} = \Psi^2 dt \end{array}\right],$$

or,

$$\left[\begin{array}{l} \left\{\begin{array}{l} E = \sqrt{E_0^2 + p^2c^2} = E_0 + E_k = \gamma mc^2, \\ E_0 = mc^2 = \text{const.} \\ p = \gamma mv,\ \gamma = (1 - v^2/c^2)^{-0.5} \end{array}\right\} \text{and} \left\{\begin{array}{l} m \\ v \\ p \\ vdp \end{array}\right\} (\text{analog to}) \left\{\begin{array}{l} \mathbf{J} \\ \omega \\ \mathbf{L} \\ \omega d\mathbf{L} \end{array}\right\} \Rightarrow \\ \Rightarrow dE = dE_k = c^2 d(\gamma m) = \underline{vdp + \omega d\mathbf{L}} = \Psi^2 dt, \end{array}\right] \Rightarrow$$

$$\Psi^2 = \frac{dE}{dt} = \underline{v\frac{dp}{dt} + \omega\frac{d\mathbf{L}}{dt}} = \frac{dx}{dt}\frac{dp}{dt} + \frac{d\alpha}{dt}\frac{d\mathbf{L}}{dt} = \underline{vF + \omega\tau} = \text{Power} = [W] \qquad (2.5.1)$$

The idea behind (2.5.1) is to show that any particle motion (regarding its total energy content and its internal building blocs) is composed of mutually coupled linear motion components, $\mathbf{vdp}$ and rotational motion components, $\omega d\mathbf{L}$ and that both of them are intrinsically involved in creating a total particle mass, where <u>rotating elements are creating its rest mass</u>.

- B -

Motional or kinetic particle energy could be treated as usually, having any positive value (velocity dependent), if this energy is measured externally, in the space where a particle is in motion. If we attempt to solve the relativistic equation that is connecting all energy aspects of a single particle, we would find that one of solutions for kinetic energy could be the negative energy amount that corresponds to the particle rest mass energy.

$$\left.\begin{cases} E^2 = E_0^2 + p^2c^2 = (E_0 + E_k)^2 = E_0^2 + 2E_0E_k + E_k^2, \\ E_0 = mc^2,\ E = \gamma mc^2,\ E_k = E - E_0 = (\gamma - 1)mc^2,\ \gamma = (1 - v^2/c^2)^{-1/2} \end{cases}\right\} \Rightarrow$$

$$E_k^2 + 2E_0E_k - p^2c^2 = 0 \Rightarrow E_k = -E_0 \pm \sqrt{E_0^2 + p^2c^2} = -E_0 \pm E \Rightarrow$$

$$E_k = \begin{Bmatrix} +E_k \\ -E_0 \end{Bmatrix} = \begin{Bmatrix} (\gamma - 1)mc^2 \\ -mc^2 \end{Bmatrix} (=) \tag{2.5.1-1}$$

$$(=) \begin{Bmatrix} \text{motional particle energy in external space } = (\gamma - 1)mc^2 \\ \text{motional particle energy internaly captured by rest mass } = -mc^2 \end{Bmatrix}$$

Such result, (2.5.1-1), could look illogical and could be neglected or considered as unrealistic. If we take into account that internal particle structure (that is creating its rest mass) is also composed of motional energy components, well packed, self-stabilized and internally closed, we could consider valid another conceptual approach which is saying that negative motional energy belongs to the ordinary motional energy that is "frozen", packed or captured by the particle rest mass,

$$dE = dE_k = c^2 d(\gamma m) = vdp + \omega d\mathbb{L} \Rightarrow \int_{[\omega]} \omega d\mathbb{L} = -E_0 = -mc^2. \tag{2.5.1-2}$$

The process of stable rest mass creation should just be a part of a certain integration process, based on solving equations (2.5.1) and (2.5.1-2). The energy component $\omega d\mathbb{L}$ would stay internally blocked or captured and "frozen" by the particle structure, in most of the cases not being visible as anything what rotates externally, and basically creating constant particle rest mass that mathematically disappears in process of creating differential equations, because derivatives of constants are zeros. Saying the same differently, here is made an effort to show that rest mass represents an accumulator or reservoir of concentrated, self-stabilized and well-integrated, internal "rotational energy elements" (understanding that in this case the best terminology and complete conceptualization is still missing, but this would not affect the attempt to express the idea). When rest mass starts moving and interacting with other objects "externally" (for instance making linear motion, or participating in some events of scattering and impacts with other particles or quasiparticles), its internally packed "rotational energy, vortex content" is somehow "unfolding". Such unfolding process is becoming directly involved in de Broglie, matter wave creation, starting interacting with near objects (being indirectly measurable by its consequences such as Compton Effect, Photoelectric Effect, Diffraction of elementary particles etc.; See also chapter 4. of this paper). **The most direct externally detectable signs regarding documenting existence of such hidden (internally packed) rotation-related behaviors should be spin and orbital moment attributes of all elementary particles.**

- C -

To be bottom-line simple and clear, let us consider that a particle with mass $\mathbf{m}$ is moving in its laboratory system with velocity $\mathbf{v}$, performing externally visible rectilinear motion (with no measurable elements of rotation: middle column of T.2.3). Even here, we will attempt to prove that such mass in motion still presents the united state of rotational and rectilinear motions, where certain "sophisticated" elements of internal rotation are hidden (or packed) inside of the rest mass structure and externally not visible, but energy-vise present in the total particle energy. If the same moving particle is also rotating (spinning), this time having externally measurable rotation, its total motional energy would get one more member, as given in T.2.3 (right column).

T.2.3 **Motional particle with non-zero rest mass (in a Lab. System)**	**Particle which is only in rectilinear motion**	**Particle in rectilinear motion combined with externally measurable rotation**
Total particle energy $dE_t = dE = dE_k = vdp$	$E_t = E_0 + E_k =$ $= mc^2 + \int_{[v]} vdp$ $E_0 = E(\mathbf{L}_0) = mc^2$ $E_k = (\gamma - 1)mc^2 = \int_{[v]} vd\,p$ $E_t^2 = E_0^2 + p^2c^2$ $\left\{ v << c \Rightarrow E_k = \frac{1}{2}mv^2 \right\}$	$E_t = E_0 + E_k =$ $= mc^2 + \int_{[v]} vdp + \int_{[\omega]} \omega d\mathbf{L}$ $E_0 = E(\mathbf{L}_0) = mc^2$ $E_k = \int_{[v]} vdp + \int_{[\omega]} \omega d\mathbf{L}$ $E_t^2 = (E_0 + \int_{[\omega]} \omega d\mathbf{L})^2 + p^2c^2$ $\left\{ \begin{matrix} v << c, \ \omega = low \Rightarrow \\ E_k = \frac{1}{2}mv^2 + \frac{1}{2}\mathbf{J}\omega^2 \end{matrix} \right\}$
Linear momentum	$p = \gamma mv$	$p = \gamma mv$
Orbital momentum	$\omega = 0,\ \mathbf{L} = \mathbf{L}_0 + \mathbf{J}\omega = \mathbf{L}_0$ $\mathbf{L}_0 = const.$	$\omega \neq 0,\ \vec{\mathbf{L}} = \vec{\mathbf{L}}_0 + \mathbf{J}\vec{\omega}$

Of course, relations in T.2.3 should be in agreement with linear and orbital momentum conservation laws, and with the law of total energy conservation, regardless what kind of transformations moving particle is passing. Before being proven fully or partially right (and under which conceptual framework), there is still a high level of speculative and hypothetical meaning of energy transformation from rotation to a rest mass content, introduced in T.2.3, what could be equivalent to a picture that **rest mass state presents a kind of energy vortex-sink** (regarding rotational motion energy states: $E_t^2 = (E_0^*)^2 + p^2c^2 = (E_0 + \int_{[\omega]} \omega d\mathbf{L})^2 + p^2c^2 = \gamma mc^2$). In the chapter 4.1, we will elaborate similar idea a little bit more, by conceptualizing any particle or energy state motion as a case of two-body or two-state interaction. There the first state is effectively interacting with its overall vicinity (or with the rest of the universe), giving the chance (at least theoretically, or conceptually), to imagine the motion of certain equivalent center-of-mass (which corresponds to such "two-body" situation). Later, by introducing an effective center-of-mass reference system, we would be able to find that an energy part of such "binary system" presents rotating reduced mass around its effective center mass. Interesting consequence of such modeling is that any linear (or curvilinear) motion is just a case of certain rotational motion where relevant radius $\mathbf{r}$ could be sufficiently large that we would not notice elements of rotation. In other words, there is no purely linear motion. In such cases as a relevant differential energy balance of such curvilinear motion we will have $dE_t = dE = dE_k = vdp = \omega dL = c^2 d(\gamma m)$, $v = \omega r$, $\vec{r} \times d\vec{p} = d\vec{L}$.

- D -

Let us continue hypothesizing in the frames of the same idea, which is stating that linear and rotational motions are mutually complementary and united. If we start from the relativistic expression for a total moving particle energy, $\mathbf{E}_t^2 = \mathbf{E}_0^2 + \mathbf{p}^2\mathbf{c}^2$, we would be able to notice that such energy has one static or constant energy member $\mathbf{E_0}$, and another dynamic or motional energy member $\mathbf{pc}$. Because of "intrinsic unity" of linear and rotational motions, we could simply say that,

$$\mathbf{pc = Dynamic\ or\ motional\ energy\ part = E_{linear-motion} + E_{rotational-motion}}$$

or,

$$\left\{\begin{array}{l} \mathbf{E}_t^2 = \mathbf{E}_0^2 + \mathbf{p}^2\mathbf{c}^2 = \textbf{(Static or constant energy part)}^2 + \textbf{(Dynamic or motional energy part)}^2 \\ \mathbf{E_0} = \textbf{Static or constant energy part} \\ \mathbf{pc} = \textbf{Dynamic or motional energy part} = \mathbf{E}_{\text{linear-motion}} + \mathbf{E}_{\text{rotational-motion}} \end{array}\right\}$$

$$\Rightarrow \mathbf{E}_{\text{rotational-motion}} = \mathbf{pc} - \mathbf{E}_{\text{linear-motion}} = \mathbf{pc} - \frac{\mathbf{pv}}{\mathbf{1} + \sqrt{\mathbf{1} - (\frac{\mathbf{v}}{\mathbf{c}})^2}} = \mathbf{pc}\left(\mathbf{1} - \frac{\frac{\mathbf{v}}{\mathbf{c}}}{\mathbf{1} + \sqrt{\mathbf{1} - (\frac{\mathbf{v}}{\mathbf{c}})^2}}\right). \quad (2.5.1\text{-}3)$$

What we are getting as the result here simply implies that if rectilinear and rotational motions are mutually coupled, the energy of rotating motion should be limited and analytically dependent on parameters of its linear motion couple.

- E -

If we accept that internal elementary particle structure is a kind of complex and multiple, self-closed and rotating wave formation, we cannot claim that (in the same time) there is also another finite and initial rest mass inside of it. In other words, what we see and measure as a particle rest mass, externally, should be the product of specific internal wave energy packaging (making impression externally that this is a stable and solid particle). Consequently, none of the variables in differential relations in (2.5.1) can be treated as constant,

$$d(\gamma m) = md\gamma + \gamma dm,\ d(\gamma mv) = mvd\gamma + \gamma vdm + \gamma mdv = \frac{1}{c^2}dE, \quad (2.5.2)$$

However, after integration, we could have the constant rest mass as a product.

- F -

The second message that could be extracted starting from (2.5.1) and later, is that resulting force (or field) acting on any mass element in motion (not only between elementary particles) should have one linear and one angular component, $\mathbf{F} = \mathbf{F}_{\mathbf{linear}}$ and $\tau = \mathbf{F}_{\mathbf{angular}}$. Moreover, the **original Newton force definition, as the time derivation of linear momentum, should be generalized to have both, angular and linear force component/s**, as for instance,

$$\frac{dE}{dt} = v\frac{dp}{dt} + \omega\frac{dL}{dt} = vF + \omega\tau = \frac{1}{dt}\cdot\left[dx\cdot\frac{dp}{dt}\right] + \frac{1}{dt}\cdot\left[d\alpha\cdot\frac{dL}{dt}\right] =$$
$$= \frac{1}{dt}\cdot\left[\begin{pmatrix}\text{Energy realized}\\ \text{by linear}\\ \text{force}\\ \text{component}\end{pmatrix} = vFdt\right] + \frac{1}{dt}\cdot\left[\begin{pmatrix}\text{Energy realized}\\ \text{by angular}\\ \text{force}\\ \text{component}\end{pmatrix} = \omega\tau dt\right] = c^2\frac{d(\gamma m)}{dt} = \Psi^2 \tag{2.5.3}$$

Probably that here introduced concept should be much better elaborated, but the principal message is already given: **Rotation and linear motion are always united and their force components (F and τ) are mutually interacting respecting rules of Vector-Algebra** (of course after properly applying necessary dimensional arrangements)**.**

Let us, for a moment, just forget all questions regarding specific fields and forces, which are involved in realizing certain motion, and start from the general platform, assuming that any object (particle, wave etc.) has certain energy and certain effective mass. The most common to all of matter motions would be that any of them should have one linear and one angular force component. First, linear force component acting along its axial path, (displacement Δx), and second sweeping an angle on its circular path (angle segment $\Delta\alpha$), like indicated in (2.5.3), including certain constant (or static, or relative rest) energy level $\mathbf{E_0}$:

$$E_{tot.} = F_{linear}\cdot\Delta x \ + \ F_{angular}\cdot\Delta\alpha \ + \ E_0 = E_t \tag{2.6}$$

Let us now (mathematically) rearrange the expression for the linear force definition (regarding the force acting on a moving particle in linear motion, based on Newton force definition: $\mathbf{F} = \mathbf{F}_{\mathbf{linear}} = \mathbf{dp/dt}$), replacing mass by its relativistic energy equivalent, on the similar way as it was already realized in developing force expressions (2.4)-(2.4-3).

$$\mathbf{F} = \frac{\mathbf{dp}}{\mathbf{dt}} = \frac{\mathbf{d(\gamma mv)}}{\mathbf{dt}} = \frac{\mathbf{d(\frac{E_t}{c^2}v)}}{\mathbf{dt}} = \frac{\mathbf{d(\frac{E_0+E_k}{c^2}v)}}{\mathbf{dt}} = \underline{\frac{\mathbf{E_t}}{\mathbf{c^2}}\frac{\mathbf{dv}}{\mathbf{dt}} + \frac{\mathbf{v}}{\mathbf{c^2}}\frac{\mathbf{dE_t}}{\mathbf{dt}}} =$$
$$= \frac{\mathbf{E_0}}{\mathbf{c^2}}\frac{\mathbf{dv}}{\mathbf{dt}} + \frac{\mathbf{E_k}}{\mathbf{c^2}}\frac{\mathbf{dv}}{\mathbf{dt}} + \frac{\mathbf{v}}{\mathbf{c^2}}\frac{\mathbf{dE_0}}{\mathbf{dt}} + \frac{\mathbf{v}}{\mathbf{c^2}}\frac{\mathbf{dE_k}}{\mathbf{dt}} = \underline{\frac{\mathbf{E_0}}{\mathbf{c^2}}\frac{\mathbf{dv}}{\mathbf{dt}} + \frac{\mathbf{E_k}}{\mathbf{c^2}}\frac{\mathbf{dv}}{\mathbf{dt}} + \frac{\mathbf{v}}{\mathbf{c^2}}\frac{\mathbf{dE_k}}{\mathbf{dt}}} = \mathbf{F_{linear}}\,(=)\left[\frac{\mathbf{Kg\cdot m}}{\mathbf{s^2}}\right]. \tag{2.7}$$

By analogy with (2.7), we could "reinvent" the expression for the missing angular force definition (known as torque),

$$\tau = \frac{d\mathbf{L}}{dt} = \frac{E_t}{\omega_c^2}\frac{d\omega}{dt} + \frac{\omega}{\omega_c^2}\frac{dE_t}{dt} = \underline{\frac{E_0}{\omega_c^2}\frac{d\omega}{dt} + \frac{E_k}{\omega_c^2}\frac{d\omega}{dt} + \frac{\omega}{\omega_c^2}\frac{dE_k}{dt}} = F_{angular} = \left[\frac{Kg\cdot m^2}{s^2}\right], \qquad (2.8)$$

$$\omega_c (=) [c/R] (=) [1/s].$$

Since the total energy, (2.6), or total resulting force, should have both, linear and rotational elements, we can simply combine (2.7) and (2.8),

$$\left.\begin{array}{l} E_{tot.} = E_t = \iiint\limits_{[x,\alpha]} (vdp + \omega d\mathbf{L}) = F_{linear}\cdot\Delta x \ + \ F_{angular}\cdot\Delta\alpha \ + \ E_0 = \\ = (\frac{E_0}{c^2}\frac{dv}{dt} + \frac{E_k}{c^2}\frac{dv}{dt} + \frac{v}{c^2}\frac{dE_k}{dt})_{linear}\cdot\Delta x + \\ +(\frac{E_0}{\omega_c^2}\frac{d\omega}{dt} + \frac{E_k}{\omega_c^2}\frac{d\omega}{dt} + \frac{\omega}{\omega_c^2}\frac{dE_k}{dt})_{angular}\cdot\Delta\alpha \ + \ E_0 = \sqrt{E_0^2 + p^2c^2} \\ E_k = E_{k-linear} + E_{k-angular} = \ F_{linear}\cdot\Delta x \ + \ F_{angular}\cdot\Delta\alpha = \int\limits_{[\Delta t]} \Psi^2(t)dt. \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \overline{E}_t = E_0 \pm I\cdot pc = E_t\cdot e^{\pm I\theta} = \sqrt{E_0^2 + p^2c^2}\cdot e^{\pm I\cdot \operatorname{arctg}\frac{pc}{E_0}} = \\ = \gamma mc^2\cdot e^{\pm I\theta} = \gamma\overline{m}c^2,\ \overline{m} = m\cdot e^{\pm I\theta}, \\ \theta = \operatorname{arctg}\frac{pc}{E_0} = \operatorname{arctg}(\gamma\frac{v}{c}) = \operatorname{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}}, \\ \overline{\Psi}^2 = \frac{\partial\overline{E}}{\partial t} = \pm Ic\frac{\partial p}{\partial t},\ I^2 = -1. \end{array}\right. \qquad (2.9)$$

Following the same patterns of thinking, here we are arriving closer to the possibility of **reformulating the universal law of inertia regarding uniform, stationary and relative motions (considering as a general case mutually coupled linear and rotational motions):** ***"All objects, particles or energy states in our universe have intrinsic elements of mutually coupled linear and rotational motions. For changing an inertial or uniform motion state of an object, at least one of linear (*** $F = \frac{dp}{dt} = \dot{p} = F_{linear}$ ***) and/or orbital (or torque,*** $\tau = \frac{dL}{dt} = \dot{L} = F_{angular}$ ***) force components should be involved in modifying its previous state of motion." Saying the same differently we should consider that every mass (particle or mass energy equivalent*** E/c^2***) always has the following mutually-correlated attributes (in relation to certain system of reference):*** $\left\{[m, J], [p, \dot{p}], [L, \dot{L}]\right\}$***. What we are presently considering as an inertial or uniform state of motion should also be redefined as certain stationary or stable state of*** $\left\{[m, J], [p, \dot{p}], [L, \dot{L}]\right\}$***.***

Here we did not say something very new and too original. It is the fact that everything in our universe, from galaxies to elementary particles is mutually in a state or relative motion/s to its environment, either rotating or spinning, or manifesting like being effectively related to certain kind of rotation, or having linear and rotating motional elements. Here mentioned intrinsic and mutually coupled motional elements are similar to the kind of coupling between electric and magnetic fields, when an electromagnetic wave is being created. The similar kind of coupling is also described in the chapter 4.0 with relations between an original wave function and its Analytic Signal, Hilbert-transform couple. Later, we will discover that in cases of seemingly linear particle motions (where rotation is not externally detectable), there is still a presence of coupling between rotation and linear motion, where rotating energy elements are captured by the internal rest mass structure.

We could here attempt to generalize the meaning of the field concept. Let us start from the gravitation as the most common force in our universe. The field of gravitation (in this case acceleration) is defined as a gravitation force from certain mass divided by the mass in question, $\mathbf{E_g} = \mathbf{F_g/m}$ = acceleration. If instead of mass we take the total particle energy related to the same mass, we will have, $E_g = \frac{F_g}{m} = \frac{F_g}{mc^2}c^2 = \frac{F_g}{E_t}c^2 = acceleration$.

If we consider (as a new field definition, applicable to any other field) that every field can be found as a product between certain specific force (in the vicinity of a specific charge, or object) and $\frac{c^2}{E_t}$. Applying such strategy on (2.5.3), we will get,

$$\begin{aligned}&\frac{c^2}{E_t}\cdot\frac{dE}{dt} = (\frac{c^2}{E_t}F)\cdot v + (\frac{c^2}{E_t}\tau)\cdot\omega = v\frac{dp}{dt}\cdot\frac{c^2}{E_t} + \omega\frac{dL}{dt}\cdot\frac{c^2}{E_t} = \\ &= \frac{c^2}{E_t}\cdot\frac{1}{dt}\cdot\left[dx\cdot\frac{dp}{dt}\right] + \frac{c^2}{E_t}\cdot\frac{1}{dt}\cdot\left[d\alpha\cdot\frac{dL}{dt}\right] = \Psi^2\cdot\frac{c^2}{E_t}(=)\left[\frac{m^2}{s^3}\right]\end{aligned} \tag{2.9.1}$$

Now we will be able to define two types of generalized fields, as for instance,

$$\left(\frac{c^2}{E_t}F\right)(=)\left[\dot{v}\right](=)\left[\frac{m}{s^2}\right], \text{Linear acceleration field (Like field of gravitation)}$$
$$\left(\frac{c^2}{E_t}\tau\right)(=)\left[\dot{\omega}R^2\right](=)\left[\frac{m^2}{s^2}\right], \text{Angular acceleration field (Field associated to rotation).} \tag{2.9.2}$$

Of course, redefined fields (2.9.2) would open process of writing new chapters of field theories where analogically we can apply the same idea to any other field.

What we empirically know regarding fields and forces is that we have static (only space parameters dependent) forces like gravitation, electrostatic and magnetic forces between permanent electric charges or magnets, and all other dynamic or time-space dependent forces (produced by our engineering activity, or as different transient and mass flow effects).

From what we know as static force manifestations, we could safely say that such forces are belonging to certain (more or less) stable standing-waves spatial structure/s, or creating such structures. The nodal areas of such standing waves static forces, are kinds of mass-energy-charges agglomerating zones or zones with increased spatial mass-energy density (well captured by concepts covered by Euler-Lagrange-Hamilton modeling). The absolute values of such static forces are reaching maximal values in mentioned nodal areas (belonging in many cases to $1/r^2$-dependent forces). Globally conceptualizing forces this way, we could say that all macroscopic matter in our universe (related to particles, masses, planets etc.) is an integral part of certain standing-waves spatial matrix. For conceptually supporting standing waves (of any kind), we surely need to have certain external, stable and resonant-type source of vibrations (because standing waves belong to resonant effects). In line with the same meaning, our existence (including our universe), should be related to sets of specific, stable, multidimensional resonant states (of distributed masses and different charges) and to still not well-known external sources of relevant oscillations. Of course, we are also mastering technologies of creating different resonant and standing waves events, where we know what the external source of relevant oscillations is.

Most of communications and motions known in our universe should be related to activity of different non-static, time-space dependent forces, acting with and between stabilized standing-waves energy agglomerations.

Understanding forces and matter structure that way, we can easily realize that $1/r^2$-dependent and other static forces are only temporary valid sequences of our forces understanding (or misunderstandings) knowledge and that we should find some much more generally valid theory. Something similar should also be valid for all other forces (with more of associated complexity), which are measurable tendencies towards mass and/or energy agglomerations that are influencing all motions. Certain force could be explicable as only static, or only dynamic, or mixed nature force, depending from which reference we are observing its manifestation. If we are literally submersed into a field of static force/s (like actual case with Gravitation), it is normal that we will in our early scientific steps make only space parameters dependant force modeling (Newton, Coulomb). The next step in front of us is to go beyond such static, stable, and standing waves structures.

[♣ COMMENTS & FREE-THINKING CORNER:

What, in addition to already elaborated relations between linear and rotational motions, could be a little bit intriguing (in (2.5) and later) is that dimensionally, the same unit (Joule) measures energy and torque,

$$\left[\frac{\mathrm{Kg}\cdot\mathrm{m}^2}{\mathrm{s}^2}\right] (=) [\tau] (=) [\mathrm{Torque}] (=) [\mathrm{Energy}] (=) [\mathrm{J}] (=) \text{ Joule}. \tag{2.10}$$

The bottom line fact regarding orbital or angular force or torque is that it is eventually behaving like ordinary, Newton or linear motion force, since it can cause motion of something on its way (in the direction defined by torque vector). Any torque, having dimension of energy, can easily be presented (at least dimensionally) as certain equivalent linear motion force $\mathbf{F}^{*}_{\mathbf{linear}}$ (collinear to a torque-vector τ) acting along linear path Δs in front of it, as for instance,

$$\tau = F_{\mathrm{angular}} = F^{*}_{\mathrm{linear}} \times \Delta s \ (=) [\mathrm{N} \times \mathrm{m}]. \tag{2.10.1}$$

Since dimensionally a torque presents an energy amount, and since we know the relation between rest mass and its internal energy content $\mathbf{E} = \mathbf{mc^2}$, we could imaginatively say that every rest mass presents an equivalent to its internally captured (and resulting, or RMS) torque $\tau_{\mathrm{int.}} = mc^2$. Thus, we associate such "frozen and stabilized torque" to a number of randomly distributed mass micro-domains, all of them having certain micro-torque amount. In fact, (following the same concept) we would also be able to claim that a total, internally captured, and center mass related, rotating domains torque, packed inside of a certain mass in a relative state of rest, is equal to zero as a resulting vector. However, it is not equal to zero locally, when only limited number of non-mutually canceling elementary torque domains is considered. This is also meaning that total internal torque (of non-spinning particle) should be composed at least of two mutually opposed and equal torque vectors (because external resulting torque equals zero). Reasoning that way it is more logical to consider $\tau_{\mathrm{int.}} = \frac{1}{2} mc^2$. **This way, we are starting to introduce the concept that real sources of gravitation related forces are internal torque elements dynamically packed inside of the entity we identify as a mass.**

Let us try to conceptualize the total rest mass (of a certain non-spinning, electrically and magnetically neutral macro-particle) $\mathbf{m}$ as the condensed energy state that is resulting from mutual interactions of number of micro mass domains being rotating states. These are in this way always mutually canceling or neutralizing the particle resulting orbital moment, and resulting angular velocity (when observing the particle from its external space). In other words, externally (in a Laboratory system), the same macro particle looks like being in a state of rest (standstill) and not rotating, but internally it is like having continuum of micro-rotating, mutually coupled and mutually neutralizing, energy carrying states ("macro-vectorial" sum of all internal orbital moments is equal zero). The same mass is (externally) obviously the source of gravitation force, described by Newton Law of Gravitation.

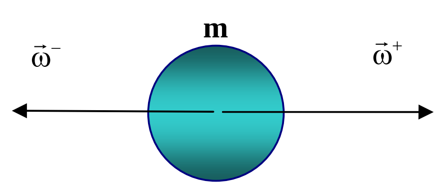


Fig. 2.1. Rest mass with internally compensated orbital moments

We could also imagine that resulting, center of mass angular velocity of such rest mass should be equal to zero, and that it would always have at least two mutually opposite vectorial components in any direction, regardless of which side we see it. Applying derivations on such center of mass angular velocity, we will be able to come to an expression where Orbital Moments (torque components) will be explicitly present (see below).

$$\vec{\omega} = \vec{\omega}^+ + \vec{\omega}^- = \frac{\sum_{(i)} \mathbf{J}_i \vec{\omega}_i}{\sum_{(i)} \mathbf{J}_i} = \frac{\sum_{(i)} \vec{\mathbf{L}}_i}{\sum_{(i)} \mathbf{J}_i} = 0 \Leftrightarrow \frac{1}{\mathbf{J}} \iiint_{[V]} d\vec{\mathbf{L}} = 0$$

$$\dot{\vec{\omega}} = \dot{\vec{\omega}}^+ + \dot{\vec{\omega}}^- = \frac{\sum_{(i)} \mathbf{J}_i \dot{\vec{\omega}}_i}{\sum_{(i)} \mathbf{J}_i} = \frac{\sum_{(i)} \dot{\vec{\mathbf{L}}}_i}{\sum_{(i)} \mathbf{J}_i} = \frac{\sum_{(i)} \vec{\tau}_i}{\sum_{(i)} \mathbf{J}_i} = \frac{\vec{\tau}^+}{\sum_{(i)} \mathbf{J}_i} + \frac{\vec{\tau}^-}{\sum_{(i)} \mathbf{J}_i} = 0 \tag{2.11}$$

$$\vec{\tau} = \frac{d\vec{\mathbf{L}}}{dt},\ \dot{\vec{\omega}} \sum_{(i)} \mathbf{J}_i = \dot{\vec{\omega}}\ \mathbf{J} = \vec{\tau}^+ + \vec{\tau}^- = 0 \Rightarrow$$

$$\Rightarrow mc^2 = 2\left|\tau^+\right| = 2\left|\tau^-\right| = 2\left|\tau\right| \Leftrightarrow m = \frac{2\left|\tau\right|}{c^2} = \frac{1}{c^2} \iiint_{[V]} \vec{\omega} d\vec{\mathbf{L}}$$

Since the idea, here has been that absolute value of orbital momentum τ presents in the same time a kind of an energy content of the particle, we know that any neutral macro-particle should have at least two of such (mutually opposite) torque components (in any direction, in order that they are mutually cancelled as vectors). Thus, we are in a position to redefine the particle total mass using a new rotation related concepts. Of course, the ideas presented here could also be the starting point for developing new concepts regarding Gravitation, where Newtonian attraction between masses would be explicable by attraction between fields created by hidden orbital moments (which are internal mass constituents), like already initiated in (2.2).

Continuing to develop the concept regarding linear and rotational motions coupling, it is gradually appearing that de Broglie matter waves and angular forces (2.8) should somehow be mutually related, what will be more elaborated later (see chapter 4.1).

Obviously, (2.9) presents an oversimplified development procedure in order to get the qualitative picture about unity of linear and angular motional force components (at least being dimensionally correct). ***It is also becoming clear that Newton force definition should absolutely be generalized to account rotation related forces.***

In reality, the same energy state could have many mutually coupled levels of its (internally and externally), energetically atomized structure, where each level has its own linear and rotational couple of motional components. These are symbolically visualized on the Fig. 2.2 with four of such levels (see also T.4.2 and chapter 6, MULTIDIMENSIONALITY, where an attempt is made to formulate similar concepts mathematically).

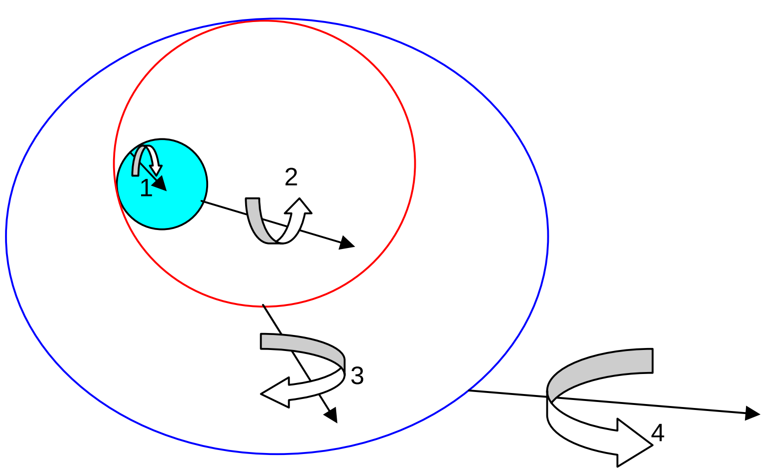


Fig. 2.2. Symbolic visualization of multilevel linear-rotational motional couples

If we would like additionally to address the unity of linear (or rectilinear) and rotational motions from another analogy and symmetries platform, a good starting (also brainstorming) point would be to take the three of Kepler's laws of planetary motion (around the sun), and try to understand them from an innovative conceptual level. Present understanding of Kepler's laws is mostly in connection with the law of conservation of angular momentum and Newton gravitation and force law/s. In addition, we should try to find all the possible analogies and similarities between Kepler's laws and analogically corresponding or symmetrical electromagnetic phenomenology. What is interesting here is to understand the process when part of distributed and internally compensated, resulting orbital momentum is being transformed, sunk or "injected" into a certain rest mass. This way we would **test**, enrich and support the framework introduced by generalized force energy expressions all over this chapter, and especially starting from (2.3) to (2.4.3) and from (2.5) to (2.9). Here we could also refresh and challenge the early and old concepts (presently considered invalid) about gravitation, related to imaginative and elusive fluid named ether, which should be the carrier of gravitation, by asking ourselves again if something like that in some new and revitalized form could exist.

2.3.1. Rotation and stable rest-mass creation

Let us imagine that certain particle (also thin disk, or thin walls toroidal object) is rotating around the fixed point **O** (see the picture Fig.2.3 below, case A), with radius of rotation r. The same particle is then passing trough certain process or transformation becoming only the spinning particle ($r \to 0$), and now performing only rotation (or spinning) around its own axis around the same fixed point **O** (see the picture Fig.2.3 below, case B). It is clear that all energy and momentum conservation laws should be satisfied between all phases of motional states transformation. Eventually, somehow, the same particle is transformed into an equivalent "standstill" mass (see the picture Fig.2.3 below, case C). Here we are trying to speculate how motional energy of any kind could be transformed or "packed" into a rest mass, and where the role of rotation in such process could be. In order to describe such hypothetical process, we will apply analogical formulation of energy expressions between linear and rotational motion (see T.2.4 and T.2.5), presently without entering into an analysis if, when and why such analogical expressions are valid.

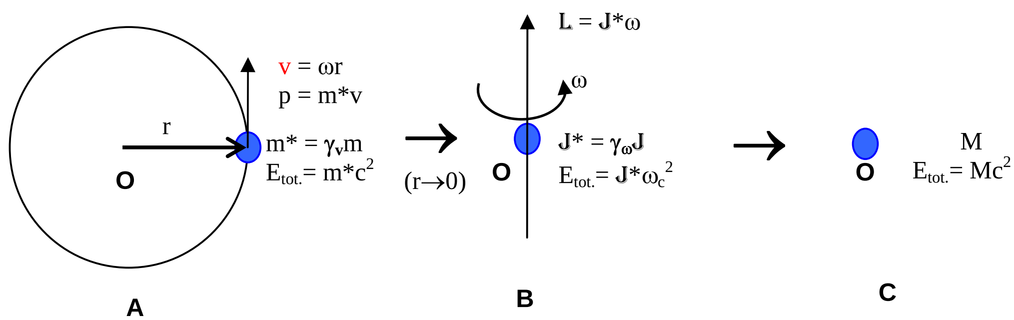


Fig.2.3. Hypothetical evolution of a rotating mass towards standstill mass

Case A: Particle m is only rotating around some externally fixed point (not spinning around its own axis).
Case B: Particle m is only spinning around its own axis and not making any other motion.
Case C: Particle m "energy-wise" transformed into a standstill or rest mass M.
In all cases, involved particle/s could also have a form of a thin disk, or thin walls toroidal object.

T.2.4. Motional or kinetic energy expressions formulated based on analogies:

Case → Value ↓	**A** **Linear motion**	**B** **Spinning**	**C** **Standstill**
Linear speed: v	$v = v_A$	$\leq c$	0
Angular speed: ω	$\omega = \omega_A$	$\omega = \omega_B$	0
Initial Rest Mass: m	$m = m_A$	m_B	m_C
Motional mass: m*	$m_A^* = m^* = \gamma_v m_A$	$m_B^* = m^*$	0
Total mass: M	$M = m_A^*$	$M = m_B^*$	$M = m_A^* = m_B^* = m^*$
Linear Moment: p	$p = p_A = mv, p^* = p_A^* = m^* v$	Not applicable , $(p = 0)$	Not applicable , $(p = 0)$
Static Moment of Inertia: I	$\mathbf{J}_A$	$\mathbf{J}_B$	$\mathbf{J}_C$
Motional Moment of Inertia: I*	$\mathbf{J}_A^*$	$\mathbf{J}_B^*$	0
Angular moment: L	$\mathbf{L}_A = \mathbf{J}_A^* \cdot \omega_A = \mathbf{L}$	$\mathbf{L}_B = \mathbf{J}_B^* \cdot \omega_B = \mathbf{L}_A = \mathbf{L}$	= 0, externally $= \mathbf{L}$, internally
Total Energy: $E_{tot.}$	$E_{tot.A} = m_A^* \cdot c^2 = \mathbf{J}_A^* \cdot \omega_{cA}^2 =$ $= \sqrt{(mc^2)^2 + p^2c^2}$	$E_{tot.B} = m_B^* \cdot c^2 = \mathbf{J}_B^* \cdot \omega_{cB}^2$	$E_{tot.A} = E_{tot.B} = E_{tot.C} =$ $= M \cdot c^2$
Motional Energy: E_m , E_k	$E_m = E_k = (m_A^* - m) \cdot c^2 =$ $= (\gamma_v - 1)mc^2 =$ $= (\mathbf{J}_A^* - \mathbf{J}_A) \cdot \omega_{cA}^2 =$ $= (\gamma_\omega - 1)\mathbf{J}_A \omega_{cA}^2 =$ $= \frac{pv}{1 + \sqrt{1 - v^2/c^2}}$	$E_m = (m_B^* - m) \cdot c^2 =$ $= (\gamma_v - 1)mc^2 =$ $= (\mathbf{J}_B^* - \mathbf{J}_B) \cdot \omega_{cB}^2 =$ $= (\gamma_\omega - 1)\mathbf{J}_B \omega_{cB}^2 =$ $= \frac{\mathbf{L}\omega}{1 + \sqrt{1 - \omega^2/\omega_c^2}}$	= 0 , externally $= M \cdot c^2$, internally
Rest Energy: E_0	$m_A \cdot c^2 = \mathbf{J}_A \omega_{cA}^2$	$m_B \cdot c^2 = \mathbf{J}_B \omega_{cB}^2$	$M \cdot c^2$
Lorentz factor for linear motion: γ_v	$\gamma_v = (1 - v^2/c^2)^{-0.5} = \gamma$	Not applicable	Not applicable
Analogically formulated "Lorentz factor" for rotational motion: γ_ω	$\gamma_\omega = (1 - \omega_A^2/\omega_{cA}^2)^{-0.5}$	$\gamma_\omega = (1 - \omega_B^2/\omega_{cB}^2)^{-0.5}$	Not applicable
	γ_ω - Applicable only when mass is spinning around its own axis (regardless of its linear motion), $v = \omega r \Leftrightarrow c = \omega_c r, \frac{\omega}{\omega_c} = \frac{v}{c}, \omega_c = c\sqrt{\frac{m}{\mathbf{J}}} = c\sqrt{\frac{m^*}{\mathbf{J}^*}} = c\frac{\omega}{v}$.		

Let us elaborate more the same situation from Fig.2.3 regarding motional energy transformation towards rest mass (with internally captured and packed motional energy: A → B → C), by extending kinetic energy expressions from the table T.2.4 into equivalent energy expressions as given in T.2.5.

T.2.5. Motional or kinetic energy expressions formulated based on analogies:

	Linear motion: Case A	Spinning: Case B
Kinetic Energy	$E_k = (m^* - m)\cdot c^2 = (\gamma_v - 1)mc^2 =$ $= \frac{\gamma_v mv^2}{1+1/\gamma_v} = \frac{\gamma_v (mr^2)\omega^2}{1+1/\gamma_v} = \frac{\alpha\cdot\mathbf{J}\cdot\omega^2}{1+1/\gamma_v}$ $= \frac{\mathbf{J}^*\cdot\omega^2}{1+1/\gamma_v} = \frac{p\cdot v}{1+1/\gamma_v} = \frac{\gamma_v (mr^2\omega)\omega}{1+1/\gamma_v}$ $= \frac{\mathbf{L}\cdot\omega}{1+1/\gamma_v},$ $dE_k = vdp = \omega d\mathbf{L} = c^2 dm^* = dE_{tot.}$ $\mathbf{J}^* = \gamma_v (mr^2) = \alpha\cdot\mathbf{J} = m^* r^2 =$ $= \frac{p\cdot v}{\omega^2} = \frac{p\cdot r}{\omega} = \frac{\mathbf{L}}{\omega},$ $m^* = \gamma_v m,\ v = \omega r,\ p = \gamma_v mv = m^* v,$ $L = \mathbf{J}^*\omega = \alpha\cdot\mathbf{J}\cdot\omega = p\cdot r, \alpha > 0,$ $\alpha = \frac{\gamma_v mv^2}{\mathbf{J}\cdot\omega^2} = \frac{p\cdot v}{\mathbf{J}\cdot\omega^2} = \frac{\mathbf{J}^*}{\mathbf{J}} = \frac{p\cdot r}{\mathbf{J}\cdot\omega} = \frac{\mathbf{L}}{\mathbf{J}\cdot\omega}$	$E_k = \frac{\alpha\cdot\mathbf{J}\cdot\omega^2}{1+1/\gamma_v} = \frac{\mathbf{J}^*\cdot\omega^2}{1+1/\gamma_v} =$ $= \frac{\mathbf{L}\cdot\omega}{1+1/\gamma_v}$ $dE_k = \omega d\mathbf{L} = c^2 dm^* = dE_{tot.}$
	Case C: Here, externally visible rotation should somehow be transformed into "internally captured" rotation, $r \to 0$, and mass is becoming standstill.	
	$Lim(\frac{\mathbf{J}^*\cdot\omega^2}{1+1/\gamma_v})_{r\to 0} = E_{tot.} = E_k = \frac{\mathbf{J}^*\cdot\omega^2}{2} = \frac{\mathbf{L}\cdot\omega}{2} = m^* c^2 = Mc^2,$ $r \to 0 \Rightarrow v = \omega r \to 0, \gamma_v \to 1.$	

In all other situations of combined motions where certain mass is performing linear motion and spinning around its own axis (for instance, combining case A and case B), motional energy will be

$$dE_k = vdp + \omega d\mathbf{L} = c^2 dm^* = dE_{tot.}. \tag{2.11.1}$$

In case if "rotating particle" is a photon, its motional energy will be, based on data from T.2.5:

$$\tilde{E} = E_k = Lim(\frac{p\cdot v}{1+1/\gamma_v})_{v\to c} = Lim(\frac{\mathbf{L}\cdot\omega}{1+1/\gamma_v})_{v\to c} = p\cdot c = \mathbf{L}\cdot\omega \tag{2.11.2}$$

Obviously, if we would like to get Planck's expression for photon energy $\tilde{E} = hf$, from (2.11.2), $E_k = \mathbf{L}\cdot\omega$, the following conditions should be satisfied:

$$\begin{aligned} &E_k = \tilde{E} = p\cdot c = \mathbf{L}\cdot\omega = hf \\ &\omega = 2\pi f,\ \mathbf{L} = h/2\pi,\ p = hf/c = mc,\ m = hf/c^2 \\ &pr = (hf/c)r = \mathbf{L} = h/2\pi \Rightarrow \\ &\Rightarrow r = c/2\pi f,\ 2\pi r = c/f = \lambda = h/p = h/mc. \end{aligned} \tag{2.11.3}$$

Since we already know (from analyses of Compton and Photoelectric Effect) that photon's energy is equal to $\tilde{E} = hf$, and from Maxwell electromagnetic theory that photon's coupled electric and magnetic field vectors are rotating along its path of propagation, our picture about photon, as a particular energy-packet, can get some additional conceptual grounds. Such chain of conclusions is certainly over-simplified, but still good to make new conclusions based on analogies. From large experimental and theoretical knowledge base accumulated in present Quantum Theory we also know that other elementary energy quanta, which are not necessarily photons (and maybe not forms of electromagnetic energy), also have energies that can be expressed by $\tilde{E} = hf$, meaning (by analogy) that intrinsic rotation is also involved there as in case of photons. In addition, experimentally is known how high-energy photon can be fully transformed into an electron-positron couple, or how mechanical contact of an electron and positron (or some other particle and its anti-particle) is annihilating initial participants and producing, for instance, two high-energy photons. The intention here is to exercise the idea that elementary "seeds and grains" of all macro objects, or everything what has a rest mass, are particularly packed and coupled rotating elementary matter waveforms (and eventually maybe we would find only photons as real most-elementary matter waveforms; - see also T.4.0, chapter 4.1). Specific energy packaging of different matter waves, directly related to rotation, is creating particles that are self-standing and have non-zero rest masses, and here is the meaning of the concept that every mass should be an energy-packaging format. Presently, we consider mass as being the source or direct cause of gravitation. We are also starting to get familiar (since long time) with a concept that internal mass content are rotating matter waves. In addition, we know that all masses in our universe are in mutually relative motions (having linear and angular moments). All of that is pointing to the conclusion that Newton law of gravitation should be upgraded with some dynamic (velocities and moments dependent) members, and that real sources of gravitation are certain waves and fields between masses as forms of energy agglomerations.

Process of fusion between the rest mass and its spinning or rotating energy-mass equivalent ($E_{spinning} / c^2$) can be conceptualized even simpler if we imagine that certain standstill (rest) mass m is passing the process of its energy transformation from the level **A** trough the level **B**, and ending with the level **C**, where relevant energy levels are defined by the following table:

T.2.6. Process of rotational energy "injection" into a rest mass

	A	B	C
Levels/States of mass transformation	<u>Initial Standstill, Rest Mass</u>	<u>Only Spinning Mass</u> Mass from **A** starts spinning	Final, new, <u>Equivalent Rest Mass</u> after the spinning mass is being transformed into a standstill mass
Rest Mass	m	m	$M = m + E_{spinning} / c^2$
Total Mass	m	$M = m + E_{spinning} / c^2$	$M = m + E_{spinning} / c^2$
Rest Energy	mc^2	mc^2	Mc^2
Total Energy	mc^2	Mc^2	Mc^2
Motional Energy	0	$E_{spinning}$	0

Of course, here (in T.2.6) we are not saying how, when and under which circumstances rotating energy would be captured by its rest mass carrier. What is implicitly stated here (regardless of terminology that is still conditional) is that <u>*under certain circumstances rotating energy can be transformed, captured or packed in a form of stable rest mass, and that every rest mass is an equivalent "packaging" form of certain amount of "frozen rotating or spinning energy"*</u>.

The generalized case of every particle in motion (which has non-zero rest mass m), based on situations from T.2.4, T.2.5 and T.2.6, is that particle total energy should always have a combination of static (rest), rotating and liner (rectilinear or curvilinear) motion energy members, as for instance,

$$E_{tot} = (m + E_{spinning} / c^2)c^2 + \frac{pv}{1+\sqrt{1-v^2/c^2}} = Mc^2 + E_{k-lin} = \sqrt{(Mc^2)^2 + p^2c^2} = \gamma Mc^2,$$

$$M = m + E_{spinning} / c^2, \; p = \gamma Mv, \; E_{k-lin} = E_k = \frac{pv}{1+\sqrt{1-v^2/c^2}}, \qquad (2.11.4)$$

$$dE = dE_{tot} = dE_k = vdp = \omega dL.$$

Even the rest mass m, from (2.11.4), is also presenting a “frozen rotating energy state” which was created or stabilized in some earlier phase of initial particle creation (what could implicate a preexistence of succession of such states). The same ideas will be more elaborated later in the chapter 4.1 (see *T.4.3 and (4.5-1)-(4.5-4)*).

From (2.11.4) we can also extract the roots of particle-wave duality, *if we accept that matter-wave energy (at least for the micro-world of atoms and its constituents) is equal to relevant particle kinetic or motional energy*, $\tilde{E} = E_k = E_{tot} - Mc^2 = hf$, *and that it can also be presented analogically as the photon energy in (2.11.3)*, as for instance,

$$\begin{aligned} & p = \frac{1}{c}\sqrt{E_{tot}^2 - (Mc^2)^2} = \frac{1+\sqrt{1-v^2/c^2}}{v}(E_{tot} - Mc^2), \\ & \frac{v}{1+\sqrt{1-v^2/c^2}} = c\sqrt{\frac{E_{tot} - Mc^2}{E_{tot} + Mc^2}} = u = \lambda f = v_{phase}, \\ & \lambda = \frac{c}{f}\sqrt{\frac{E_{tot} - Mc^2}{E_{tot} + Mc^2}} = \frac{\left[\frac{(E_{tot} - Mc^2)}{f}\right]}{p} = \frac{\tilde{E}/f}{p} = \frac{h}{p}, \\ & \tilde{E} = hf = E_k = E_{tot} - Mc^2 = E_{tot} - mc^2 + E_{spinning}. \end{aligned} \tag{2.11.5}$$

The philosophical or conceptual consequence of results from (2.11.5) is that particle-wave duality and mater waves’ nature is directly related to internal rotating energy content of a particle in motion, and that mater waves should be an “*external unfolding manifestation*” of already existing “*internally-folded*” rotating matter waves (what will be elaborated much more in the chapters 4.1, 4.2 and 4.3). In other words, masses in motion are creating a space-time web or matrix, capturing and sensing each other, and showing or experiencing manifestations of waves, forces and fields, where Gravitation is one of such fields.

The wave energy (or waves velocities) in cases of different waves could be on a different way dependent on relevant waves’ frequency, proportional either to f or f^2*, or also to* f/v or f^2/v^2 *(like shown in chapter 4.1, T.4.1). The micro-world of sufficiently isolated atoms, subatomic particles and states, and photons is dominantly respecting Planck’s-Einstein-de-Broglie energy-wavelength formulations, where* $\tilde{E} = hf,\ \lambda = h/p,\ u = \lambda f = \omega/k,\ v = d\omega/dk$*, and other wave phenomena from a Marco-objects world could have different wave energy to frequency relations. The idea here is to initiate thinking that for macro objects, like planets and other big objects, also exists certain relevant and characteristic wavelength, analog to de Broglie matter-waves wavelength, but no more proportional to Planck’s constant. The problem in defining de Broglie type of wavelength for macro objects is that such wavelength (*$\lambda = h/p$*) is meaningless and extremely small, and since macro objects in motion should also create associated waves, like any other micro-world object, the macro-wavelength in question should be differently formulated.*

To show more directly that rotation is an essential (ontological) source responsible for initial particles creation, we can again start from the relativistic particle expression that is connecting particle’s total energy $E = E_{tot} = \gamma mc^2$, its linear motion momentum $p = \gamma mv = \gamma_v mv$, and its rest mass $m = E_0/c^2$,

$$\begin{aligned} & \left[E_{tot}^2 = E_0^2 + p^2c^2\right]/c^2 \Rightarrow \\ & \left(\frac{E_{tot}}{c}\right)^2 = \left(\frac{E_0}{c}\right)^2 + p^2. \end{aligned} \tag{2.11.6}$$

If the particle is in the same time performing kind of circular (rotational) motion around certain fixed point (without spinning around its own axis), where r_0 is the distance between that fixed point and moving particle in question, we can consider that the particle also has certain orbital momentum $\vec{L} = \vec{r}_0 \times \vec{p}$, $L = r_0 \cdot p$, $dE = \omega \cdot dL = v \cdot dp = dE_k = dE_{tot}$. Now, we can multiply both sides of (2.11.6) with radius of rotation r_0 and get kind of relativistic relation where relevant orbital moments are involved,

$$\begin{aligned} \left(\frac{E_{tot}}{c}\right)^2 &= \left(\frac{E_0}{c}\right)^2 + p^2 \\ \left(r_0 \cdot \frac{E_{tot}}{c}\right)^2 &= \left(r_0 \cdot \frac{E_0}{c}\right)^2 + (r_0 \cdot p)^2 \\ L_{tot}^2 &= L_0^2 + L^2 \end{aligned} \tag{2.11.7}$$

From the analogical point of view, to facilitate comparison between (2.11.6) and (2.11.7), it is reasonable to introduce (2.11.8) for underlining analogies of here involved linear and orbital moments, such as,

$$\left\{\begin{array}{l} \left(\frac{E_{tot}}{c}\right)^2 = \left(\frac{E_0}{c}\right)^2 + p^2 \\ p_{tot}^2 = p_0^2 + p^2 \\ p_{tot} = \frac{E_{tot}}{c} = \gamma_v mc \\ p_0 = \frac{E_0}{c} = mc \\ p = \gamma_v mv \\ \\ \gamma_v = (1 - v^2/c^2)^{-0.5} \\ 0 \le v \le c \end{array}\right\} \overset{\text{analog to}}{\rightleftarrows} \left\{\begin{array}{l} \left(\frac{L_{tot}}{r_0}\right)^2 = \left(\frac{L_0}{r_0}\right)^2 + \left(\frac{L}{r_0}\right)^2 \\ L_{tot}^2 = L_0^2 + L^2 \\ L_{tot} = r_0 \cdot p_{tot} = r_0 \cdot \frac{E_{tot}}{c} = r_0 \cdot \gamma_v mc = \gamma_\omega J \omega_c \\ L_0 = r_0 \cdot p_0 = r_0 \cdot \frac{E_0}{c} = r_0 \cdot mc = J \omega_c \\ L = r_0 \cdot p = r_0 \cdot \gamma_v mv = \gamma_\omega J \omega \\ \\ \gamma_\omega = (1 - \omega^2/\omega_c^2)^{-0.5} \\ 0 \le \omega \le \omega_c \end{array}\right\}. \tag{2.11.8}$$

From (2.11.8) is obvious that the rest mass $m = E_0/c^2$ should have the origin directly related to certain kind of rotation (or to L, L_0, J, ω, ω_c) because,

$$\left\{\left(\frac{E_{tot}}{c}\right)^2 = \left(\frac{E_0}{c}\right)^2 + p^2\right\} \equiv \left\{\left(\frac{L_{tot}}{r_0}\right)^2 = \left(\frac{L_0}{r_0}\right)^2 + \left(\frac{L}{r_0}\right)^2\right\} \Rightarrow$$

$$\left\{\begin{array}{l} \frac{E_{tot}}{c} = \frac{L_{tot}}{r_0} = p_{tot} = \gamma_v mc = \frac{\gamma_\omega J \omega_c}{r_0}, \\ \frac{E_0}{c} = \frac{L_0}{r_0} = p_0 = mc = \frac{J \omega_c}{r_0}, \\ p = \frac{L}{r_0} = \gamma_v mv = \frac{\gamma_\omega J \omega}{r_0}, \\ dE = \omega \cdot dL = v \cdot dp = dE_k = dE_{tot} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} m = \dfrac{L_{tot}}{r_0\gamma_v c} = \dfrac{L}{r_0\gamma_v v} = \dfrac{\gamma_\omega J\,\omega_c}{r_0\gamma_v c} = \dfrac{\gamma_\omega J\,\omega}{r_0\gamma_v v} = \dfrac{L_0}{r_0 c} = \dfrac{J\,\omega_c}{r_0 c} = \\ = \dfrac{E_{tot}}{\gamma_v c^2} = \dfrac{p_{tot}}{\gamma_v c} = \dfrac{p}{\gamma_v v} = \dfrac{E_0}{c^2} = \dfrac{p_0}{c} \\ \dfrac{\omega}{\omega_c} = \dfrac{v}{c} = \dfrac{L}{L_{tot}} = \dfrac{cp}{E_{tot}} = \dfrac{p}{\gamma_v p_0} \end{cases}. \qquad (2.11.9)$$

The next consequence of such conceptualization is that all kind of motions in our universe are curvilinear, or combinations of linear and rotational, or rectilinear and torsional motions (see later T.4.3 and (4.5-1)-(4.5-4) in the chapter 4.1). If this were not the case, we would not have stable particles with non-zero rest masses. Also, based on (2.11.1) to (2.11.5) and (2.11.7), we know that internal particle structure (its rest mass) has intrinsic elements of rotation, which are directly coupled to all kind of externally rotating motions. That kind of coupling of internal and external elements of rotation should be the source of Gravitation. Here also could be the background of the hypothetical formula for gravitational force given by expression (2.2).

2.3.2. Macro-Cosmological Mater-Waves and Gravitation

In case of micro-universe of atoms and elementary particles, de Broglie matter waves are manageable using the following relations: $\tilde{E} = hf = E_k,\ \ \lambda = h/p,\ u = \lambda f = \omega/k = E_k/p,\ \ v = d\omega/dk = dE_k/dp$. Let us now try to construct or exercise what could be the macro-universe equivalent to de Broglie matter-waves concept. The idea here is to show that planets and similar macro objects are also respecting certain periodicity and "macro matter-waves packaging rules", like de Broglie matter waves in a micro universe. The best for exercising such brainstorming is to start from the Kepler's third law (of planetary orbital motions), which is also applicable to all satellite motions around certain planet or big mass. Let us temporarily focus our attention only to pure circular rotations, where radius of rotation is r, in order to use simpler mathematical expressions. Kepler's third law is showing that period T of planetary (or satellite) orbiting around a mass M (or its sun) is given by (2.11.10),

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \quad . \qquad (2.11.10)$$

The maximal orbital or escape speed v_e (when planet or satellite would start escaping its closed circular orbit) can easily be found as (2.11.11),

$$v_e = \sqrt{\frac{2GM}{r}} . \qquad (2.11.11)$$

We will now attempt to show that (like in case of de Broglie matter waves applied on Bohr's hydrogen, planetary atom model) the circular planetary (or satellite) orbit, or its perimeter, is related to some kind of (associated), orbital standing waves, $2\pi r = n\lambda_o$, $n = 1, 2, 3...$. Here is appropriate to say that planetary rotation around its sun has a period T and frequency of such (mechanical) rotation that is equal $f_m = 1/T$. f_m is not necessarily the frequency of the associated (and still hypothetical) macro matter-wave f_o. For underlining possible difference between mechanical revolving frequency f_m and orbital macro matter-wave frequency f_o, we will assume that $f_m \neq f_o$. Since the framework of this exercise is implicitly accepting that relevant planetary or satellite velocities are much lower compared to the light speed ($v << c$), we could safely say that certain planetary or group velocity (or its orbital velocity) should be two times higher than its phase velocity, $v = 2u$. Now we can find mentioned orbital frequency, wavelength group and phase velocity as (2.11.12),

$$\left. \begin{cases} 2\pi r = n\lambda_o,\ T = \dfrac{1}{f_m} = \dfrac{2\pi}{\omega_m},\ v = \dfrac{2\pi r}{T} = 2\pi r f_m = \omega_m r \cong 2u,\ n = 1, 2, 3, ... \\ u = \lambda_o f_o \cong \dfrac{1}{2} v = \dfrac{\pi r}{T} = \pi r f_m,\ T^2 = \left(\dfrac{4\pi^2}{GM}\right) r^3 = \dfrac{1}{f_m^2}, v_e = \sqrt{\dfrac{2GM}{r}} \end{cases} \right\} \Rightarrow$$

$$\lambda_o = \frac{2\pi r}{n},\ f_o = \frac{u}{\lambda_o} = n\frac{u}{2\pi r} = n\frac{f_m}{2} = \frac{n}{2T} = \frac{n\sqrt{GM}}{4\pi r^{3/2}}, f_m = \frac{2f_o}{n} = \frac{1}{T} = \frac{\sqrt{GM}}{2\pi r^{3/2}}, \tag{2.11.12}$$

$$u = \frac{1}{2}\sqrt{\frac{GM}{r}} = \frac{1}{2\sqrt{2}} v_e,\ v = 2u = \sqrt{\frac{GM}{r}} = \frac{1}{\sqrt{2}} v_e, \forall n = 1, 2, 3... .$$

Based on the group or planet's velocity $v = \dfrac{2\pi r}{T}$, (2.11.12), the wave energy or kinetic energy of a rotating planet, which has mass m, can be expressed as:

$$\begin{aligned} &\tilde{E} = E_k = \frac{1}{2} m v^2 = mvu = pu = 2mu^2 = \frac{1}{4} m v_e^2 = \frac{GmM}{2r} = \frac{1}{2}\cdot\left(\frac{GmM}{r^2}\right)\cdot r = \frac{1}{2}\cdot F_{m-M}\cdot r = \\ &= \frac{m}{2}\left(\frac{2\pi r}{T}\right)^2 = \frac{8m\pi^2 r^2}{n^2} f_o^2 = 2m(\pi r f_m)^2 = (2\pi m r^2 f_m)\cdot(\pi f_m) = L\pi f_m = (\frac{2L\pi}{n})\cdot f_o = Hf_o, \\ &L = \frac{nH}{2\pi} = 2\pi m r^2 f_m,\ p = mv = \frac{\tilde{E}}{u} = \frac{Hf_o}{u} = \frac{H}{\lambda_o} = n\frac{H}{2\pi r} = m\sqrt{\frac{GM}{r}}, \\ &\lambda_o = \frac{H}{p} = \frac{2\pi r}{n},\ H = \frac{2L\pi}{n} = \text{const.}. \end{aligned} \tag{2.11.13}$$

The magnitude of the angular momentum L of a rotating planet (or satellite) in (2.11.13) is,

$$L = pr = mvr = mr^2\omega_m = mr^2\frac{2\pi}{T} = 2\pi m r^2 f_m = \frac{nH}{2\pi} = \sqrt{GM}\cdot m r^{1/2} = \text{const.} \tag{2.11.14}$$

It is becoming obvious that planetary macro-wave energy, $E_k = Hf_o$ from (2.11.13) is in some ways analog to Planck's wave-quantum energy (of a photon $\tilde{E} = hf$, for instance), where new "macro-world Planck-like constant" H is,

$$H = H(m, r, n) = \frac{2\pi}{n} L = \frac{2\pi\sqrt{GM}}{n}\cdot m r^{1/2} = \frac{\tilde{E}}{f_o} = \tilde{E}T = \text{const.} >> h. \tag{2.11.15}$$

The difference between Planck's constant h and analog constant of astronomic macro waves H is that h is universally valid constant, and H could be different for every planetary or satellite orbit…(to be verified). In fact, the most challenging would be if we could find conditions when H would also become the universally valid constant (by associating to every planetary orbit different integer **n**, similar to Bode's law). For instance, if the ratio between any two of H constants, applied for planets (with circular orbits) from the same solar system, could be equal to one, than we will have more supporting arguments to say that H is the universally valid constant.

$$\frac{H_1}{H_2} = \frac{H(m_1, r_1, n_1)}{H(m_2, r_2, n_2)} = \frac{n_2}{n_1}\frac{m_1}{m_2}\sqrt{\frac{r_1}{r_2}} \ \ (=1,?!),\ (n_1, n_2) \in [1,2,3...) \tag{2.11.16}$$

In addition, if certain wave-like periodicity (in planetary motions) exists, it should be in a close relation to relevant angular momentum. Such periodicity could also be presentable as integer number n_α of angular segments $\alpha = \frac{2\pi}{n_\alpha}$, capturing the angle of a full circle that is equal $n_\alpha \alpha = 2\pi = n\lambda_o / r,\ n_\alpha = 1, 2, 3...$

$$\begin{aligned}
&L = n\frac{H}{2\pi} = \frac{n}{n_\alpha}\cdot\frac{H}{\alpha} = \frac{H}{\lambda_o}r = \frac{2\pi\sqrt{GM}}{n\lambda_o}\cdot mr^{3/2} = \sqrt{GM}\cdot mr^{1/2},\\
&\alpha = \frac{2\pi}{n_\alpha} = \frac{n}{n_\alpha}\cdot\frac{H}{L} = \frac{n}{n_\alpha}\cdot\frac{\lambda_o}{r} = \frac{2\pi\sqrt{GM}}{n_\alpha L}\cdot mr^{1/2},\ (n, n_\alpha) \in [1,2,3,...).
\end{aligned} \tag{2.11.17}$$

(See later Wavelength analogies in different frameworks, T.4.2, from the chapter 4.1.).

Anyway, our macro-universe is known to behave as a big and very precise astronomic clock, where periodical motions are its intrinsic property. It will be just a matter of finding or fitting proper integers $(n, n_\alpha) \in [1,2,3,...)$ into above given (or similar) macro matter-waves relations, in order to support here presented concept. Of course, the situation analyzed here is presently addressing only purely circular planetary orbits (for having mathematical simplicity), and in later analyses we would need to take into account elliptic planetary orbits (this way, most probably generating additional quantum numbers, or integers like n, n_α). The natural development of such quantized model of planetary systems will be in some ways similar to the evolution of Bohr's planetary atom model towards Sommerfeld's atom model (related to the period before the wave quantum mechanics and Schrödinger equations started to be dominant theoretical approach). In order to generalize the same concept for any closed planetary orbit, we would need to apply Wilson-Sommerfeld action integrals. Wilson-Sommerfeld action integrals (see [9]), related to any periodical motion on a self-closed stationary orbit C_n, applied over one period of the motion, present the kind of general quantifying rule (for all self-closed standing waves, which are energy carrying structures, having constant angular momentum) that was successfully used in supporting N. Bohr's Planetary Atom Model. In cases of planets rotation, Wilson-Sommerfeld action integrals are producing,

$$\begin{aligned}
&\left\{\oint_{C_n} Ld\alpha = n_\alpha H,\right\} \Leftrightarrow \left\{\oint_{C_n} pr\frac{dr}{r} = \oint_{C_n} pdr = n_\alpha H\right\},\ 0 \le \alpha \le 2\pi,\\
&\left\{\oint_{C_n} p_r dr = n_r H\right\} \Leftrightarrow \left\{\oint_{C_n} pdr = n_\alpha H\right\} \Rightarrow (n_\alpha = n_r = n) \in [1,2,3,...)\\
&\Rightarrow \oint_{C_n} Ld\alpha = \oint_{C_n} pdr = nH.
\end{aligned} \tag{2.11.18}$$

In addition to (2.11.18), for a certain stable planetary system (with number of planets) it should also be valid that its effective "center of inertia angular velocity" $\vec{\omega}$ is constant (including spinning moments of planets, moons and asteroids),

$$\left\{\begin{array}{l}
\vec{\omega} = \dfrac{\sum\limits_{(i)} J_i\vec{\omega}_i}{\sum\limits_{(i)} J_i} = \dfrac{\sum\limits_{(i)} \vec{L}_i}{\sum\limits_{(i)} J_i} = const.\\
\oint_{C_n} Ld\alpha = \oint_{C_n} pdr = nH
\end{array}\right\} \Rightarrow \left\{\begin{array}{l}
\vec{\omega}\sum\limits_{(i)} J_i = \sum\limits_{(i)} \vec{L}_i = const.\\
\sum\limits_{(i)} \oint_{C_n} \vec{L}_i d\alpha = \sum\limits_{(i)} n_i H_i = const.
\end{array}\right\}. \tag{2.11.19}$$

Elements of certain stable space-time structure with periodical motions (planetary systems, for instance) are mutually coupled by fields and forces integrating them into a stable macro system, and important condition for such stability is the creation of standing waves of involved fields. Positions and paths of planets (inside

such systems) are defined by stable or stationary conditions of the system in question, which are related to minimal energy dissipation, or maximal mechanical quality factor conditions (for instance, found by solving relevant Euler-Lagrange-Hamilton equations). In order to give an idea how we could evolve this quantum-like conceptualization of Gravitation it would be very useful to see the Appendix (at the end of this book) that is innovatively treating "**Bohr's model of hydrogen atom and particle-wave dualism".**
Based on the planetary macro waves conceptualization which is presented from (2.11.12) until (2.11.19) we can easily create kind of Schrödinger equation valid for such situations. Since establishment of Schrödinger equations is elaborated later, in the chapter 4.3, here we will only briefly formulate one of the final forms of such equation (by analogy with (4.10)), which will address planetary and satellites motions,

$$\left\{\begin{array}{l} \dfrac{\hbar^2}{\tilde{m}}(\dfrac{u}{v})\Delta\overline{\Psi}+(\tilde{E}-U_p)\overline{\Psi}=(\dfrac{\tilde{E}-U_p}{\hbar})^2\cdot\overline{\Psi}+\dfrac{\partial^2\overline{\Psi}}{\partial t^2}=\dfrac{\partial\overline{\Psi}}{\partial t}+u\nabla\overline{\Psi}=0, \\ \hline \Psi^2=\dfrac{dE}{dt}=\dfrac{dE_k}{dt}=v\dfrac{dp}{dt}+\omega_m\dfrac{dL}{dt}=\text{Power},\ \overline{\Psi}=\Psi+j\hat{\Psi},\ j^2=-1, \\ v=\omega_m r\cong 2u=2\lambda_o f_o=\dfrac{2\pi r}{T}=2\pi r f_m=\sqrt{\dfrac{GM}{r}}\ll c,\ \tilde{m}\leftrightarrow m\cong\dfrac{mM}{m+M}\ll M, \\ \omega_m=2\pi f_m=\dfrac{4\pi f_o}{n}=\dfrac{2\pi}{T}=\dfrac{\sqrt{GM}}{r^{3/2}}=\dfrac{v}{r},\ p=mv=\dfrac{H}{\lambda_o}=\dfrac{nH}{2\pi r},\ n=1,2,3,... \\ L=\dfrac{nH}{2\pi}=pr=mvr=mr^2\omega_m=mr^2\dfrac{2\pi}{T}=2\pi mr^2 f_m=\sqrt{GM}\cdot mr^{1/2}, \\ \tilde{E}=E_k=\dfrac{1}{2}mv^2=Hf_0=\dfrac{GMm}{2r},U_p=-\dfrac{GMm}{r},E_k-U_p=\dfrac{3}{2}\dfrac{GMm}{r}, \\ h\leftrightarrow H=\dfrac{2\pi\sqrt{GM}}{n}\cdot mr^{1/2},\ \hbar=\dfrac{h}{2\pi}\leftrightarrow\dfrac{H}{2\pi}=\dfrac{\sqrt{GM}}{n}\cdot mr^{1/2} \end{array}\right\}\Rightarrow$$

$$\frac{(\frac{H}{2\pi})^2}{2m}\Delta\overline{\Psi}+(E_k-U_p)\overline{\Psi}=(2\pi\frac{E_k-U_p}{H})^2\cdot\overline{\Psi}+\frac{\partial^2\overline{\Psi}}{\partial t^2}=\frac{\partial\overline{\Psi}}{\partial t}+\frac{v}{2}\nabla\overline{\Psi}=0\,. \tag{2.11.20}$$

It is obvious that a future development of here introduced macro-cosmological mater-waves concept will significantly enrich our understanding of Gravitation. In fact, full and correct explanation of the ideas found in (2.4)-(2.11.20) could be much more complex than here presented, but for the purpose of introducing the new concept of force-field charges and uniting linear and rotational elements of every motion, given message is already sufficiently clear. See also the chapter "4.3 Wave Function and Generalized Schrödinger Equation"; -equations: (4.33-1), (4.41-1) to (4.45), T.4.2 and T.4.3). ***♣]***

2.4. How to unite Gravitation, Rotation and Electromagnetism

The most challenging ideas and possibilities *radiating* from T.2.2 and (1.19) - (2.9) could be intuitively addressed to the new field concept (its definition or redefinition) related to rectilinear and rotational motion/s. We usually relate or define gravitational field to the specific space deformation around certain mass. However, from T.2.2, from analogous expressions for energy density, we can see that gravitational and rotational fields can be differently introduced (in two ways): in static case (prefix "stat."), as certain space deformation proportional to $\mathbf{m}$ and $\mathbf{J}$; and in dynamic case (prefix "dyn."), as the space deformation proportional to $\mathbf{p}$ and $\mathbf{L}$. *We can even create vector addition of both of them (when shows relevant), combining (2.1) and (2.2) with (2.4))*.

Under new field/s formulation, we can accept (only in the initial phase) to follow the frames of Faraday-Maxwell electromagnetic field concept. Based on it, we should be able to formulate the corresponding and analogous structure for (so far hypothetical) field/s associated to rectilinear and rotational motions. Maxwell Theory is well tested and proven valid from many possible aspects and reality of Nature and it should be that all possible forces and fields are anyway united, regardless whether we know how to formulate the Unified Field Theory.

In given (philosophical) frontiers we can just mention the best starting points for creating a new (analogous and hypothetical) field structure of rectilinear and rotational motions, which will (conveniently) follow Faraday-Maxwell electromagnetic field definition. For instance, Lorentz and Laplace forces are the explicit connection between rectilinear motion of electrical charge/current and magnetic field, and can be transformed to analogue forms of certain interaction between (only) rectilinear and rotational motion (using previously mentioned analogies). The Ampere-Maxwell's, Biot-Savart's, and Faraday's induction laws can serve to complete mathematically the previous situation, for more precise description of "rectilinear rotational" field/s (just by transforming mentioned laws into corresponding analogue expressions).

A number of attempts are already known in modern science regarding formulation of Maxwell-like theory of gravitation and explaining the origin/s of inertia. Traditional formulation of gravitation field takes mass as a (primary) source of gravity. However, in this paper it is demonstrated that more essential (and most probably, energetically dominant) source/s of gravity-related phenomenology should be found in dynamical interactions between moving objects (between their linear and/or angular moments and between their motional and state of rest energies; -see also chapter 4. of this paper for more supporting elements). Revitalizing and updating Wilhelm Weber's force law (to cover electromagnetic and gravity related interactions, with combined linear and rotational motion elements) would be a very healthy platform towards establishing new Maxwell-like theory of gravitation (see literature [28]. [29]).

Most probably that many force/fields manifestations and components of constant, or accelerated movements, (such as Coriolis, centrifugal, centripetal, gyroscope effect, inertial and similar forces, Gravitomagnetic induction from General Relativity Theory etc.) could conveniently be incorporated, interpreted and mutually united with here proposed concepts. It is conceptually already very clear what the author of this paper is proposing related to links between rotation, linear motion and electromagnetism. See also equations (4.18), (4.22)-(4.29), (5.15) and (5.16)).

After establishing new platform for understanding the complementary nature of "linear-rotational" field/s and motions (see chapters 4.0 to 4.3 of this paper), we shall have an open way for creating full set of "Gravity-Rotation" field equations, making them initially analogue with Maxwell equations of electromagnetic field. Later on, we could modify and/or upgrade such equations up to the most meaningful and useful forms that will correspond to complex reality of different natural fields and forces (see development of equations (4.22)-(4.29)). Later (chapter 3), it will be shown that Maxwell Theory should also be slightly upgraded in order to become compatible for unification with upgraded theory of Gravitation (see also literature [23] – [26]).

Anyway, in order to present a significant and new insight regarding gravitation we would need to introduce very new and original concepts that do not present only analogical variations of already known field theories. Let us formulate one of such concepts, as follows.

2.5. New Platforms for Understanding Gravitation

Gravitation could also be conceptualized by making analogies with mechanical or acoustical resonators*. Let us imagine that our macro-universe or cosmos effectively presents a kind of fluid-like substance with different particle or mass agglomerations submersed (or hanging) in such substance. Mentioned particle agglomerations (in the frames of this conceptualization) would be different cosmic objects, planets, stars, galaxies, dust, atoms etc. Let us now imagine that such composite cosmic fluid is being mechanically vibrated by certain constant frequency (from an external, presently unknown or hypothetical source of mechanical vibrations). In case of performing a real experiment (just to visualize the concept and make relevant analogies), in a vessel filed with liquid that is mixed with solid particles, by vibrating such vessel we will notice creation of three-dimensional standing wave/s structure, where submersed particles would make higher mass density agglomerations in nodal areas of standing waves. Nodal areas in this case are zones where oscillating velocities are minimal (or zero) and oscillating forces are maximal. If we intentionally introduce a small test particle somewhere in a vicinity of any of such nodal areas with elevated mass density (while vessel filed with liquid and other particles is resonating), we will notice that the test particle will be attracted by the closest nodal zone (or closest particle). Of course, here we are temporarily excluding cases of involvements of possible electromagnetic forces in order to make the situation very simple in its first brainstorming steps. Similar attractive force (in a vicinity of a nodal zone) could be observed in the case of resonant, standing wave oscillations of half-wavelength solid resonators or multiple half-wavelength resonators, known in ultrasonic technology). If external vibrations that are driving mentioned resonators are suddenly switched off, the attractive force/s towards nodal areas will disappear. Now we could conceptualize our universe as an equivalent mechanical fluid-like system that is permanently in a state of* ***<u>very low frequency</u>*** *resonant and standing wave oscillations, which are forcing all astronomical objects to take only certain stable nodal positions of the easiest agglomerating areas, which are kind of its space-matrix texture (apart from other linear and rotational motions involved). Placed around such astronomical objects (planets, starts, galaxies…), every test mass would experience only an attractive force (very much similar like in cases of gravitation). Later, the same initial concept can be upgraded by considering linear and rotational motions (of submersed particles, or astronomical objects) that are again forced to comply with agglomeration rules around global standing waves nodal areas, complying with the framework of Euler-Lagrange-Hamilton mechanics.* ***Understanding of mass, conceptualized here, is indirectly considering that any mass is a storage or modus of energy packaging and/or agglomerating (and in the same time kind of "frozen" rotating energy state).*** *The problem here could be the fact that we know that between astronomical objects in our universe there is significant "empty space of vacuum states", and our imaginative fluid-substance (which should mechanically resonate) would have problems regarding performing mechanical vibrations. Again, we would need to understand the specific nature of such fluid-like substance that is a carrier of externally introduced mechanical vibrations on some new innovative way, since contemporary physics made many efforts to show that ether-type fluids are still not something what could be experimentally confirmed. In mechanics and acoustics,* *we already know that vacuum cannot be a carrier of mechanical vibrations, and for supporting here introduced concept of gravitation, we really need to have kind of mechanical resonant and standing waves states of our universe. Most probably that some kind of electromagnetic, magnetostrictive or electrostrictive coupling nature should also be involved here in order to realize penetration of mechanical vibrations trough vacuum and empty space states (and maybe vacuum in our universe is not at all an empty space). Anyway, the situation regarding explaining gravitation, as initiated here, could be richer and different compared to old Newton, Kepler and Einstein framework, since none of them is explaining why gravitation is only manifesting as an attractive force, and standing waves mechanical resonators are easily showing existence of such forces in their nodal areas. Einstein's General Relativity Theory is already explaining gravitation from the point of view of specific space and fields' geometry-related modifications and deformations, taking this as a fact, not speculating (as here) that specific space-matrix texture could be a consequence of complex resonating, standing wave formations. Since everything what exists in our Universe is anyway mutually cross-linked, coupled and united (in some cases most probably without our best and full knowledge about it), any new theory about Gravitation should take into account electromagnetic and other forces coupled with gravitation. Of course, the ordinary (Newton, static) gravitation force is for many orders of magnitude weaker than all other forces (electromagnetic, nuclear…), compared on the same scale, making that we usually neglect interactions between gravitation and other fields. If the concept proposed here has enough realistic grounds, this would be a breakthrough in novel and better understanding of gravitation (maybe also applicable to other forces like electromagnetic ones). Another contemporary field unification theory (which is going much deeper and wider in conceptualizing a multi-dimensional space with its elementary and vibrating building blocks that are taking forms of strings and membranes) that is in some ways familiar to here introduced concept is the Superstrings or M-theory. The remaining question to answer here would be <u>where and what the source of mentioned vibrations is</u>?* ***Since all constituents of our universe are mutually in permanent relative motions, and we know that matter-waves are associated to mass motions, this should be an important element of the answer regarding origin of mentioned intrinsic vibrations and their standing waves (in the context of understanding the nature of gravitation).*** *We should not forget that any new concept of gravitation should be simple, elegant and well integrated into remaining chapters of physics that are already working well, and some attempts in creating such modeling will be made later.*

3. POSSIBILITIES FOR GENERALIZATION OF FARADAY-MAXWELL THEORY

The author's initial position in this chapter is that present days Maxwell-Faraday Electromagnetic Theory has very realistic, stable and profound foundations in Physics. Because of that, it should be fully and naturally integrated with Einstein Relativity Theory (more than presently known), as much as Relativity Theory is presently correct. If the Relativity Theory is not sufficiently correct or complete in some of its aspects, we should refer back to Maxwell-Faraday's Theory as the conceptually richer and more natural field theory to serve as a model and source-theory for updating. The principal, and author's guiding idea in this chapter is to suggest possible new merging domains between Maxwell Electromagnetic Theory and Relativity Theory. This is going to implicate that today's **Q**uantum and **R**elativistic **E**lectrodynamics, **QRE**, regardless how well experimentally and theoretically proven as being correct, is not presenting the final, the best and the most natural merging and upgrading concept between them, neither Electromagnetic and Relativity Theory themselves are in their final self-standing frames that are ready for higher level unification. In other words, well-operating mathematical modeling of present **QRE** is successfully constructed satisfying known and relevant experimental facts. It is well fitted in the environment of surrounding theoretical background (which is related to contemporary Electromagnetic, Quantum and Relativity Theory), but simplicity, full natural and logical unity, and conceptual elegancy are still missing there (theory is simply too bulky and too complex to be in its final frames). Just to give a good example about similar situations: a Ptolemy geocentric planetary motion theory was also working well in practice, but conceptually has been completely wrong.

We are already familiar to the terminology often used in a contemporary physics papers and books, such as relativistic, non-relativistic, classical-mechanical, quantum-mechanical, probabilistic or statistic… point/s of view. When we start thinking a bit more seriously and logically, it is becoming clear that such divisions in present physics are only the product of our insufficient knowledge, incomplete modeling and missing general or unifying concepts regarding different Physics chapters. Attributes like relativistic, non-relativistic, classical-mechanical, quantum-mechanical and similar ones, one day should be replaced by certain new and united theory that is equally well addressing all of them on the similar and consistent way. Consequently, it is also clear that some of theories, or all of them, previously mentioned, could be either partially wrong, or incomplete, or only particularly valid, and that a new and well-unifying theory is still missing.

At this time, it is also the author's position that Maxwell-Faraday theory presents still the best available framework and naturally convenient model for constructing other field theories in physics (again, starting with analogies and symmetries as an ideas generating platform, and later implementing necessary modeling and additional upgrading in order to satisfy known experimental facts, and general laws of physics). Here, we shall only underline and spark certain aspects of possible and still hypothetical extensions of Maxwell-Faraday Theory that will support the main ideas of this paper (as for instance, the intrinsic connection and complementary nature between rectilinear motion and rotation). In addition, a contribution in establishing more complete system of electromechanical analogies then already elaborated or known in Physics will be formulated.

3.1. Modification of Maxwell Equations

The (non-relativistic) expressions of Lorentz electric and magnetic forces (in homogenous and isotropic Galilean space), in the presence of combined magnetic and/or electric field/s or induction/s, makes possible to establish the following (very much symmetrical) forms of electric and magnetic fields (**E** and **H**), and induction/s (**D** and **B**), as the functions of velocity **v,** (see **[4]**),

$$\begin{aligned}
&\vec{E}(v) = \vec{E}_0 \pm \vec{v}\times\vec{B}_0 = \vec{E}_{stat.} \pm \vec{E}_{dyn.},\ \vec{E}_{stat.} = \vec{E}_0,\ \vec{E}_{dyn.} = \vec{v}\times\vec{B}_0, \\
&\vec{H}(v) = \vec{H}_0 \mp \vec{v}\times\vec{D}_0 = \vec{H}_{stat.} \mp \vec{H}_{dyn.}, \vec{H}_{stat.} = \vec{H}_0,\ \vec{H}_{dyn.} = \vec{v}\times\vec{D}_0, \\
&\left\langle \vec{B}_0 = \mu\vec{H}_0,\ \vec{D}_0 = \varepsilon\vec{E}_0, \vec{B}(v) = \mu\vec{H}(v),\ \vec{D}(v) = \varepsilon\vec{E}(v) \right\rangle,
\end{aligned} \tag{3.1}$$

Where index "**0**" indicates that certain field also exists in a static case, when **v** = 0 (or in a state of relative rest), $\langle v = 0\rangle \Rightarrow \langle E(v) = E(0) = E_0 = E_{stat.},\ H(v) = H(0) = H_0 = H_{stat.}\rangle$. The field members indexed by **dyn.** (=) Dynamic, are representing fields created as a consequence of motion (being velocity dependent).

Mutually complementary structure (and full mathematical symmetry and analogy) between equations of electric and magnetic fields could be expressed (at this time still hypothetically) by transforming (or extending) the present explicit system of almost virtually independent equations (3.1) into new, implicit, mutually dependent system of equations (3.2), since naturally electric and magnetic fields are fully united and coupled, and they should be fully mutually dependent,

$$\begin{aligned}
&\vec{E}(\vec{v}) = \vec{E}_0 \pm \vec{v}\times\vec{B}\left[\vec{E}(\vec{v}),\vec{B}_0\right] = \vec{E}\left[\vec{H}(\vec{v}),\vec{E}_0\right] = \frac{1}{\varepsilon}\vec{D}\left[\vec{H}(\vec{v}),\vec{D}_0\right] = \vec{E}_{stat.} \pm \vec{E}_{dyn.}, \\
&\vec{H}(\vec{v}) = \vec{H}_0 \mp \vec{v}\times\vec{D}\left[\vec{H}(\vec{v}),\vec{D}_0\right] = \vec{H}\left[\vec{E}(\vec{v}),\vec{H}_0\right] = \frac{1}{\mu}\vec{B}\left[\vec{E}(\vec{v}),\vec{B}_0\right] = \vec{H}_{stat.} \mp \vec{H}_{dyn.}.
\end{aligned} \tag{3.2}$$

Now we could analyze different solutions of (3.2) introducing various conditions regarding the nature of material media, and making suitable connections between electric field and electric induction, and magnetic field and magnetic induction (always satisfying the well known laws, boundary conditions and equations from Faraday-Maxwell Theory). The goal is to create an updated (a bit more general and more symmetric) system of Maxwell equations of self-interacting electromagnetic fields (in their integral and local forms), starting from implicit relations given in (3.2). *Since we did not precisely say what kind/s of implicit functions are involved in (3.2), there will be a lot of (adjusting and fitting) freedom to make correct mathematical modeling later on.*

It is almost obvious that we can thus achieve a much higher (mathematical) level of symmetry between intrinsically mutually interacting electric and magnetic fields, and later analyze some new (still not explained or hypothetical) phenomena in the same domain.

Generally valid, in real material media (not necessarily in a homogenous and isotropic, free space), electric and magnetic induction vectors from (3.2) should also have the following symmetrical and mutually dependant forms:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 (\vec{E} + \vec{E}_{int.}),\ \vec{E}_{int.} = \frac{\vec{P}}{\varepsilon_0},\ (\text{or } \vec{D} = \|\varepsilon\|\ \vec{E}),$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \vec{H}_{int.}),\ \vec{H}_{int} = \vec{M},\ (\text{or } \vec{B} = \|\mu\|\ \vec{H}),$$

$$\vec{D}\left[\vec{H}(\vec{v}), \vec{D}_0\right] = \|\varepsilon\| \cdot \vec{E}(\vec{v}) = \|\varepsilon\| \cdot \vec{E}\left[\vec{H}(\vec{v}), \vec{E}_0\right] = \vec{D} + \vec{D}_{int} = \vec{D}_{stat.} \pm \vec{D}_{dyn.}$$

$$\vec{B}\left[\vec{E}(\vec{v}), \vec{B}_0\right] = \|\mu\| \cdot \vec{H}(\vec{v}) = \|\varepsilon\| \cdot \vec{H}\left[\vec{E}(\vec{v}), \vec{H}_0\right] = \vec{B} + \vec{B}_{int} = \vec{B}_{stat.} \mp \vec{B}_{dyn.} \tag{3.3}$$

$$\vec{E}_{int.} = \frac{1}{\|\varepsilon\|} \vec{D}_{int} = \text{polarization electric field inside of media} = \vec{E}_{stat.}, (v = 0)$$

$$\vec{H}_{int.} = \frac{1}{\|\mu\|} \vec{B}_{int} = \text{polarization magnetic field inside of media} = \vec{H}_{stat.}, (v = 0)$$

$\|\varepsilon\|$ = **dielectric constant as a tensor,**

$\|\mu\|$ = **magnetic permeability as a tensor.**

The main strategy in (3.2) and (3.3) is to keep mutual analogy and symmetry in all mathematical expressions of electric and magnetic fields (related to the same phenomena), and always formulate them as a system of mutually dependent equations (and later to solve such equations). How this unifying concept would be made practical and operational is another question, but at least with (3.2) and (3.3) we are making first steps, explicitly formulating what the important framework would be. In **[4]** we can also find an original modeling process, regarding Maxwell equations, that can support, and/or complement previously introduced electromagnetic field concept. How else we could make equations of magnetic and electric field formally symmetrical and mutually analog, is already known and well presented in literature about the same problematic (see for instance the book: “The Electromagnetic Field, from Albert Shadowitz, Dover Publications, 1988), and if that has been the only objective, it would be just a simple mathematical problem to solve. We already know that any kind of electrical charge would create electrical field (or electrical induction) in its vicinity, and we also know that electrical charge/s in any kind of visible or hidden motion (relative to something) would create certain electrical current, which is immediately creating magnetic field around the electrically conductive zone; -so there is no wonder that Maxwell-Faraday equations and formulas of electric and magnetic field can (and should) be made fully coupled, symmetrical, fully mutually dependent and analogical. Since this is still not the case, here is the source of the challenging motivation to start certain reformulation and upgrading of present electromagnetic theory.

In this paper another objective is much more important and could be simplified in formulating the following question: What are the direct or indirect consequences of such electromagnetic compatibility and united reality (between an electric and magnetic field) when addressing phenomenology in the world of electromagnetically neutral masses like atoms and other macro particles? The answer on such question in this paper is still an evolving process guided by noticing and following certain level of analogies between linear or rectilinear motions and electric field phenomena, and analogies between mechanical rotation and rotation of electric charges, which is in fact effectively or equivalently (of course mathematically) replaced by closed loops of electric currents. Since electric and magnetic fields are mutually coupled and complementing, it is intuitively and implicitly clear that something similar could exist between linear motions and rotations of neutral particles. To prove something like that the best and obvious strategy would be, first to make a total formal symmetry and analogy between all field expressions of magnetic and electric fields (just to have a simpler, more intuitive and more indicative starting platform). Then by analogy apply the same to linear and rotational motions and find similar elements of fields' complementarities between them (this time we would have analogies between certain kinds of mechanical and mutually coupled gravity-related fields where one field component would belong to linear motion and the other, more hypothetical, to certain kind of rotation). Briefly summarizing, the idea here is to formulate all possible analogies and symmetries between **magnetic field and rotation**, and later between **electric field and linear motion**. Furthermore, based on already known coupling relations between electric and magnetic field the idea is to be able to formulate analogically possible "coupling relations" between linear motion and rotation. Of course, eventually we would need to make a new and well operating modeling or higher level symmetry between electromagnetic and linear-rotational field concepts.

The next important objective (or project) would be to find what kind of mechanics and gravity-related items (presently formulated independently, and out of the frames of Maxwell-Faraday electromagnetic theory) could be "analogically equivalent" or symmetrical to (3.2) and (3.3), in order to extend the concept of present Maxwell electromagnetic field equations to other fields known in Physics. This should not be too much surprising and hypothetical idea since whatever we know as a relatively stable (electrically neutral) rest mass matter, is actually composed of atoms, and atoms are composed of electrons, protons and neutrons (in reality all of them being, directly or indirectly, motional electrical charges, and/or magnetic moments` carriers, cross-linked with their linear and orbital moments, *having mutually coupled electric and magnetic fields and very much stable gyromagnetic relations*).

The other important objective of here presented strategy is to show that mutually and implicitly dependent system of electromagnetic field equations (3.2) is naturally generating field vector solutions that are compatible or in agreement with Lorenz transformations (what is basically already known fact in physics, but here a little bit differently elaborated). **This is making Special Relativity Theory intrinsically incorporated in the structure of electromagnetic reality (or making that Renewed Relativity Theory would be only a simple consequence of certain intrinsic relations of electric and magnetic fields; what is going to eliminate the need to apply Lorenz transformations how they are presently applied).** This way, since we would also establish parallelism between electromagnetic and gravity-mechanics-related fields, we would be able to extend similar relativity-related conclusions to motions of neutral masses (again showing that Relativity Theory has its roots and origins in Maxwell-Faraday electromagnetic theory merged with relevant conservation laws). We will now continue by having in mind here-elaborated strategy and objectives.

3.2. Generalized view about Currents, Voltages and Charges

Currents and voltages (or potential differences) are presenting essential engineering and phenomenological values in our electromagnetic theory and practice (being easy measurable). Let us introduce, and establish, unified and generalized (mutually symmetrical and analog) definitions of electric and magnetic currents $\mathrm{i}_{\text{electr.}}$, $\mathrm{i}_{\text{mag.}}$, voltages $\mathrm{u}_{\text{electr.}}$, $\mathrm{u}_{\text{mag.}}$, and charges $\mathrm{q}_{\text{electr.}}, \mathrm{q}_{\text{mag.}}$, using formulations of electric and magnetic induction from (3.3), as follows:

$$\begin{aligned}
&\mathrm{i}_{\text{electr.}} = \mathrm{u}_{\text{mag.}} = -\frac{\mathrm{d}\Phi_{\text{electr.}}}{\mathrm{dt}} = i_{el.stat.} + i_{el.dyn.} = \mathrm{i}_{\text{electr.}}(x,y,z,t)\ ,\\
&\Phi_{\text{electr.}} = \iint_S \vec{\mathrm{D}}\mathrm{d}\vec{\mathrm{S}} = \mathrm{q}_{\text{electr.}} = \mathrm{q}_{\text{el.stat.}} + \mathrm{q}_{\text{el.dyn.}} = \Phi_{\text{electr.}}(x,y,z,t),\\
&\mathrm{i}_{\text{mag.}} = \mathrm{u}_{\text{electr.}} = -\frac{\mathrm{d}\Phi_{\text{mag.}}}{\mathrm{dt}} = i_{mag.stat.} + i_{mag.dyn.} = \mathrm{i}_{\text{mag.}}(x,y,z,t)\ ,\\
&\Phi_{\text{mag.}} = \iint_S \vec{\mathrm{B}}\mathrm{d}\vec{\mathrm{S}} = \mathrm{q}_{\text{mag.}} = \mathrm{q}_{\text{mag.stat.}} + \mathrm{q}_{\text{mag.dyn.}} = \Phi_{\text{mag.}}(x,y,z,t).
\end{aligned} \tag{3.4}$$

The power $\mathrm{P(t)}$, energy $\tilde{\mathrm{E}}$, and equivalent mass $\tilde{\mathrm{m}}$, of certain electromagnetic field or electromagnetic wave group, in relation to (3.4), can be expressed as:

$$\begin{aligned}
&\mathrm{P(t)}\ = \mathrm{i}_{\text{electr.}} \cdot \mathrm{u}_{\text{electr.}} = \mathrm{u}_{\text{mag.}} \cdot \mathrm{i}_{\text{mag.}} = \frac{\mathrm{d}\Phi_{\text{electr.}}}{\mathrm{dt}} \cdot \frac{\mathrm{d}\Phi_{\text{mag.}}}{\mathrm{dt}} = \oiint_S \vec{\Gamma} \cdot d\vec{S} = \Psi^2(t),\\
&\text{where: } \vec{\Gamma} = \vec{\Gamma}(\vec{\mathrm{v}}) = \vec{\mathrm{E}}(\vec{\mathrm{v}}) \times \vec{\mathrm{H}}(\vec{\mathrm{v}}) = \text{Poynting's vector}\ ,\\
&\Psi^2(t) = (\textit{square of electromagnetic}\ \text{wave function}) = \mathrm{P(t)} = \text{power},\\
&\tilde{\mathrm{E}} = \int \mathrm{P(t)dt} = \int \Psi^2(\mathrm{t})\mathrm{dt}\ = \text{energy}\ \Rightarrow \tilde{m} = \frac{\tilde{\mathrm{E}}}{\mathrm{c}^2} = \textit{effective mass}.
\end{aligned} \tag{3.5}$$

In both (3.4) and (3.5), $\Phi_{electr.}$ and $\Phi_{mag.}$ present electrical and magnetic field fluxes.

In order to avoid temporarily all questions and possible dilemmas about the real nature (and ways of existence) of different forms of electric and magnetic charges, we can start from equations (3.4) and (3.5), and replace all corresponding electric and magnetic induction/s, or field members, using the expressions from equations (3.2) and (3.3), going backwards (deductively), in order to determine (or reinvent) all basic electromagnetic entities. This way we shall be able to generalize and unify the meanings of static and dynamic, or transient electrical and magnetic units (charges, currents... see T.5.2, (5.15) and (5.16)), just by following the usual paths of Maxwell-Faraday Electromagnetic Theory (expressing all Maxwell equations in their integral and local forms, using field forms from (3.2) and (3.3), which is the task to be realized in another paper). This would make Maxwell Theory a bit more general and more symmetric (regarding all expressions for electric and magnetic fields), as proposed in this paper.

*[♣ **COMMENTS & FREE-THINKING CORNER:** Since matter is anyhow immanently united (locally and non-locally: see for instance John Stewart Bell's Interconnectedness Theorem known in Quantum Mechanics since 1964; -CERN, Geneva, Switzerland) in coincidental existence of all of its aspects, forms, charges, fields and their multilevel and multidimensional interactions, there is no doubt that whatever exists in certain (well-known and proven valid) domain of physics should have analogous or closely related manifestations in some other domains. For instance, parallelism and analogy between Maxwell's Electromagnetic Theory and Gravitation should exist at a much deeper level than presently known (because basic matter or mass constituents are atoms, composed of electrons, protons and neutrons, and free neutron is naturally separating on an electron and proton, meaning that all of them and ourselves are the part of the electromagnetic world). Also, the Ampere-Maxwell's, Biot-Savart's, and Faraday's induction laws could be very inspiring models, from the point of view of analogous thinking, for more precise description of "linear-rotational" fields (just by conveniently transforming mentioned electromagnetic laws into corresponding analogue expressions of rectilinear and rotational motion; -which is the task to be realized in another paper).*

It looks to the author of this paper that presently the very significant extension of Maxwell's Electromagnetic Theory (including creating different approach to Relativity Theory) is going to be made by relatively recent revitalizing and upgrading of Wilhelm Weber's force law, which presents the natural unification of fundamental laws of classical electrodynamics, such as: Gauss's laws, Coulomb's law, Ampere's generalized law, Faraday's law, and Lentz's law (see literature [28] - [29]), and which should be appropriately integrated with the universal field-concept equations from the second chapter of this paper (see (2.3) to (2.9)). ♣]

3.3. New, Relativistic-like Formulation of Maxwell Equations

Let us now take the case of homogenous and isotropic media (free space and vacuum) and find the simplest particular solutions for electric and magnetic fields, starting from the system of equations (3.2),

$$\begin{aligned}
&\vec{\mathbf{E}}(\vec{\mathbf{v}}) = \vec{\mathbf{E}}\left[\vec{\mathbf{H}}(\vec{\mathbf{v}}), \vec{\mathbf{E}}_0\right] = \frac{1}{\varepsilon_0}\vec{\mathbf{D}}\left[\vec{\mathbf{H}}(\vec{\mathbf{v}}), \vec{\mathbf{D}}_0\right],\ \vec{\mathbf{H}}(\vec{\mathbf{v}}) = \vec{\mathbf{H}}\left[\vec{\mathbf{E}}(\vec{\mathbf{v}}), \vec{\mathbf{H}}_0\right] = \frac{1}{\mu_0}\vec{\mathbf{B}}\left[\vec{\mathbf{E}}(\vec{\mathbf{v}}), \vec{\mathbf{B}}_0\right] \\
&\vec{\mathbf{E}}(\vec{\mathbf{v}}) = \vec{\mathbf{E}}_0 \pm \vec{\mathbf{v}} \times \vec{\mathbf{B}}\left[\vec{\mathbf{E}}(\vec{\mathbf{v}}), \vec{\mathbf{B}}_0\right] = \vec{\mathbf{E}}_0 \pm \mu_0 \vec{\mathbf{v}} \times \vec{\mathbf{H}}\left[\vec{\mathbf{E}}(\vec{\mathbf{v}}), \vec{\mathbf{H}}_0\right] = \vec{\mathbf{E}}_0 \pm \mu_0 \vec{\mathbf{v}} \times \vec{\mathbf{H}}(\mathbf{v}), \\
&\vec{\mathbf{H}}(\vec{\mathbf{v}}) = \vec{\mathbf{H}}_0 \mp \vec{\mathbf{v}} \times \vec{\mathbf{D}}\left[\vec{\mathbf{H}}(\vec{\mathbf{v}}), \vec{\mathbf{D}}_0\right] = \vec{\mathbf{H}}_0 \mp \varepsilon_0 \vec{\mathbf{v}} \times \vec{\mathbf{E}}\left[\vec{\mathbf{H}}(\vec{\mathbf{v}}), \vec{\mathbf{E}}_0\right] = \vec{\mathbf{H}}_0 \mp \varepsilon_0 \vec{\mathbf{v}} \times \vec{\mathbf{E}}(\mathbf{v}).
\end{aligned} \tag{3.6}$$

One of possible (particular) solutions of (3.6) can be found as:

$$\begin{aligned}
&\vec{E}(\vec{v}) = \frac{\vec{E}_0 - \varepsilon_0\mu_0\vec{v}\cdot(\vec{v}\cdot\vec{E}_0)}{1-\varepsilon_0\mu_0 v^2} \mp \frac{\mu_0\vec{v}\times\vec{H}_0}{1-\varepsilon_0\mu_0 v^2} = \frac{\vec{E}_0 - \frac{\vec{v}}{c^2}\cdot(\vec{v}\cdot\vec{E}_0)}{1-\frac{v^2}{c^2}} \mp \frac{\mu_0\vec{v}\times\vec{H}_0}{1-\frac{v^2}{c^2}}\ , \\
&\vec{H}(\vec{v}) = \frac{\vec{H}_0 - \varepsilon_0\mu_0\vec{v}\cdot(\vec{v}\cdot\vec{H}_0)}{1-\varepsilon_0\mu_0 v^2} \pm \frac{\varepsilon_0\vec{v}\times\vec{E}_0}{1-\varepsilon_0\mu_0 v^2} = \frac{\vec{H}_0 - \frac{\vec{v}}{c^2}\cdot(\vec{v}\cdot\vec{H}_0)}{1-\frac{v^2}{c^2}} \pm \frac{\varepsilon_0\vec{v}\times\vec{E}_0}{1-\frac{v^2}{c^2}}\ .
\end{aligned} \tag{3.7}$$

From (3.7) we could "experience the picture" of ***mutually complementing***, and repetitively (or perpetually) ***self-generating or regenerating*** electric and magnetic fields and charges (when we apply Maxwell equations to (3.7): $\nabla\mathbf{x}\vec{\mathbf{H}} = \vec{\mathbf{J}} + \varepsilon\partial\vec{\mathbf{E}}/\partial\mathbf{t}$, $\nabla\mathbf{x}\vec{\mathbf{E}} = -\mu\partial\vec{\mathbf{H}}/\partial\mathbf{t}$, $\nabla\vec{\mathbf{H}} = \mathbf{0}$, $\nabla\vec{\mathbf{E}} = \rho/\varepsilon$**).**

***[♣ COMMENTS & FREE-THINKING CORNER:** It is also interesting to notice the validity of the following relations (in connection with understanding spontaneous field vectors bending and rotation):*

$$\left\{\begin{array}{l}\vec{\mathbf{v}}\cdot\vec{\mathbf{E}}(\vec{\mathbf{v}}) = \vec{\mathbf{v}}\cdot\vec{\mathbf{E}}_0 \Leftrightarrow \mathbf{v}\cdot\mathbf{E}(\mathbf{v})\cdot\cos(\vec{\mathbf{v}},\vec{\mathbf{E}}) = \mathbf{v}\cdot\mathbf{E}_0\cdot\cos(\vec{\mathbf{v}},\vec{\mathbf{E}}_0), \\ \vec{\mathbf{v}}\cdot\vec{\mathbf{H}}(\vec{\mathbf{v}}) = \vec{\mathbf{v}}\cdot\vec{\mathbf{H}}_0 \Leftrightarrow \mathbf{v}\cdot\mathbf{H}(\mathbf{v})\cdot\cos(\vec{\mathbf{v}},\vec{\mathbf{H}}) = \mathbf{v}\cdot\mathbf{H}_0\cdot\cos(\vec{\mathbf{v}},\vec{\mathbf{H}}_0)\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l}\mathbf{E}(\mathbf{v})/\mathbf{E}_0 = \cos(\vec{\mathbf{v}},\vec{\mathbf{E}}_0)/\cos(\vec{\mathbf{v}},\vec{\mathbf{E}}) \\ \mathbf{H}(\mathbf{v})/\mathbf{H}_0 = \cos(\vec{\mathbf{v}},\vec{\mathbf{H}}_0)/\cos(\vec{\mathbf{v}},\vec{\mathbf{H}})\end{array}\right\},$$

$$\left\{\begin{array}{l}\vec{v}\cdot\left[\vec{E}(v)-\vec{E}_0\right] = \vec{v}\cdot\Delta\vec{E} = 0 \\ \vec{v}\cdot\left[\vec{H}(v)-\vec{H}_0\right] = \vec{v}\cdot\Delta\vec{H} = 0\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\vec{v} = 0,\ \text{or}\ \ \Delta\vec{E} = 0,\ \ \text{or}\ \ \cos(\vec{v}\ ,\ \Delta\vec{E}) = 0 \\ \vec{v} = 0,\ \text{or}\ \ \Delta\vec{H} = 0,\ \ \text{or}\ \ \cos(\vec{v}\ ,\ \Delta\vec{H}) = 0\end{array}\right\} \tag{3.8}$$

Which are equally applicable to (3.1), (3.2), (3.6) and (3.7). ♣]

The next step would be to attempt to find generally valid solutions for (3.2), in some form similar to (3.7), and to apply definitions and expressions given in (3.4) and (3.5) to such solutions. After obtaining the most general forms of electric and magnetic currents and charges, we shall be able to go back and *re-establish* more symmetrical forms of all Maxwell equations, compared to the present situation. By applying electromechanical analogies we shall also be able to go backwards to new understanding of many natural forces, charges and their mechanical and electromagnetic moments.

The interesting situation regarding (3.7) is that today's Maxwell Theory (very much artificially, but also very correctly) formulated in a so-called Relativistic Electrodynamic form is also generating somewhat similar results (regarding Lorentz transformations of field vectors). Lorentz electromagnetic field transformations known in present-days Relativistic Electrodynamics are most probably mathematically fully correct (since all relevant experimental and theoretical facts are telling that Relativistic Electrodynamics is functioning perfectly), but apparently different, compared to (3.7), and also not too far from (3.7), because at least both of them are having/using/implementing the Lorentz factor $(1-\varepsilon_0\mu_0\mathbf{v}^2) = (1-\frac{\mathbf{v}^2}{\mathbf{c}^2})$. The consequences of such situation could be that either (3.2), or some aspects of today's Relativistic Electrodynamics, or expressions of Lorentz electric and magnetic forces found in (3.7) and earlier are in certain aspects incorrect or incomplete, or miss-formulated and/or maybe placed in a wrong conceptual framework, or that some of them are only approximately valid (including that Relativity Theory itself could also have similar weak areas). The "new" electromagnetic forces, not at all described by Maxwell electromagnetic theory, are already known and becoming an increasing area of scientific research (see **[30]**), being explicitly and experimentally verifiable, showing undoubtedly that present-days Maxwell equations are sooner or later going to be updated (most probably by certain conceptual unification between Maxwell and Wilhelm Weber's ideas; -see **[28]** and **[29]**, including new field concepts introduced in the second chapter of this paper by equations (2.3) – (2.9)).

[♣ <u>COMMENTS & FREE-THINKING CORNER:</u> *Obviously, it is very interesting that in (3.7) we can find characteristic Lorentz factor, usual for coordinate transformations in Special Relativity Theory (**SRT**),* $\gamma^2 = 1/(1-\varepsilon_0\mu_0\mathbf{v}^2) = 1/(1-\mathbf{v}^2/\mathbf{c}^2)$*, without applying anything what "smells" on **SRT**. The author of this paper is not in a position to offer some more precise and final (or better) comment/s about such situations. Based only on the brainstorming, common-sense intuition we could ask ourselves if today's (Lorentz) relativistic transformations of electric and magnetic field vectors (or applied anywhere else in physics) are really so much essential, fully correct, complete, and generally applicable framework to all kinds of particles, field and wave motions (or in our contemporary physics books we still have some particularly valid (well-fitted) cases of **"relativistic"** field transformations, **"explaining"** well **"mostly already known"** experimental facts, initially discovered* ***without using any of relativistic backgrounds****). **♣]***

Let us only mention possible ideas that will test general validity of today's Maxwell Relativistic equations. Magnetic field phenomenology is always in connection with its complementary electric field phenomenology, being mutually coupled and united in a form of electric and/or magnetic charges and/or currents circulation. For instance, magnetic field of an electromagnet can be created inside and around a ferromagnetic core, if the core is placed inside of a solenoid with circulating electrical current. Circulation of electrons in a solenoid around magnetic core presents a kind of rotation (or electric current). Any rotation also presents non-uniform accelerated motion, and **SRT** is not addressing such situations. If we now take as another example a permanent magnet, we can measure its magnetic field, without seeing a real solenoid and circulation of its electrons, but we know that this also should be the case of a bit hidden rotation inside of spinning electrical (current) elements of internal magnetic domains (meaning that again some kind of rotation on atomic scale is creating a permanent magnet field). In the case of propagation of electromagnetic waves (or fields) in an open and free space (or vacuum) we see neither core material nor solenoid, but again we can detect the presence of electromagnetic waves (or photons, composed of mutually coupled and rotating electric and magnetic field vectors). A kind of really well-hidden rotation (or vortex phenomena) of some strange "carrier-fluid flow" should exist in certain material form, and help creating the magnetic field component, associated to moving electric entity. However, it looks that we are still not able to detect direct presence of such strange space-time texture. Luckily, it also looks that we do not need to fully understand such kind of "carrier-fluid" for the purpose of modeling electromagnetic waves, since Maxwell Theory mathematically explains well the creation and propagation of electromagnetic field components, without big need to say explicitly what is their "carrier-fluid". <u>The analogy explaining light or electromagnetic wave's propagation could be conceptualized by comparing them with alternating currents and voltages in capacitors (in dielectrics), or in complex impedances electric-loads.</u>

*[♣ **<u>COMMENTS & FREE-THINKING CORNER:</u>** Relativistic Lorentz transformations (of **SRT**) applied on such (visibly or invisibly) rotating (and mutually complementary and coupled) states, which also perform rectilinear motion, should be (mathematically) more complex than just using simple Lorentz transformations applied only on rectilinear motions (as formally prescribed in **SRT**). Most probably that some of missing or limiting aspects of traditionally formulated Maxwell Theory and **SRT** (such as: not counted effects of accelerated motions and rotation, incomplete mathematical formulation of Faraday's Law of Magnetic Induction, incomplete symmetry in formulating basic Maxwell equations and/or equations of electromagnetic field…), are artificially (mathematically) compensated by formulating Quantum and Relativistic Electrodynamics, in order to satisfy and explain results of (already known) experimental observations. <u>In reality, if traditional, basic Maxwell Electrodynamics (formulated in Galilean space) was more appropriately formulated, we should be able to get relativistic field transformations (known in present days Relativistic Electrodynamics, or some other solutions similar to (3.7)), without any artificial math-hybridization with **SRT**, or without using or knowing **SRT** (see number of papers indicating such options **[23] – [26]**).</u> ♣**]***

Another connection between Maxwell Theory and Gravitation comes from the well-known fact that quantum of electromagnetic radiation, a photon, has its equivalent, dynamical mass, $\mathbf{hf/c^2}$, momentum, $\mathbf{hf/c}$, and spin (see analyses of Compton and Photoelectric effect, for instance). In addition, gravitation field has influence on the equivalent photon mass like on any other mass. Rectilinear motion and rotation again make a complementary couple (beside the possible presence of a rotating couple of electric and magnetic field unified in a photon formation). The creation of an electron-positron couple from high energy γ - photon (both of them naturally spinning after creation) is yet another experimental example (among others) confirming that photon (or wave energy) can be transformed into real and rotating particles. For instance, from (3.5) we can find effective or equivalent (static and dynamic) mass and momentum of electromagnetic radiation, in the function of electric and magnetic field vectors, $\tilde{\mathbf{m}} = \tilde{\mathbf{E}}/\mathbf{c}^2 = \mathbf{m}_{\mathrm{stat.}} + \mathbf{m}_{\mathrm{dyn.}}$, going backwards to (3.4) and (3.3) and applying all Maxwell equations. It is already known that A. Einstein (beside many others) tried during almost half of his life to apply Faraday-Maxwell Theory as the guiding model for describing Gravitation and Unified Field Theory, and his efforts ended without success, most probably because SRT uses some unnatural assumptions and it is not taking into account visible or hidden effects of rotation and other

accelerated motions, as previously mentioned. Here, the similar idea is proposed (as a kind of analogical thinking regarding unification between Gravity and Electromagnetic field). However, before we shall be able to start to realize it, first we should somehow establish the active presence of the "field of rotation" that is (or should be) complementary to Gravitation (following the analogy between mutual complementary nature of electric and magnetic fields). Then, using also a bit extended and upgraded Maxwell equations as a guiding model should be much easier to realize above-mentioned objectives. Basically, here favored strategy is to try to introduce (new, more realistic and more relevant) adjustments and modifications, both in Gravitation and Electromagnetic Theory, and make them more mutually compatible for unification (possibly to show that Gravitation also has its essential origins in Electromagnetic fields and forces, as suggested in [23] – [26]). Here, the research area for mentioned field unification is also linked to inertial and reaction forces, "fields of rotation" and de Broglie matter waves phenomenology (see (4.18), (4.19), (4.22)-(4.29), (5.15) and (5.16)), and it will be more closely analyzed in the following chapters of this paper.

3.4. The Important Extension of Electromechanical Analogies

Since in (3.4) we introduced static and dynamic currents, charges and fluxes (both for electric and magnetic field), the system of analogies given in T.1.3, T.1.4 and T.1.5 can be extended, separating electric and magnetic parameters, to equivalent (dimensional) analogies given in T.3.1, T.3.2 and T.3.3:

T.3.1	[E] = [ENERGIES]= = $\int\{[U]\,[I]\}dt = \int[P]dt$	[Q] = [CHARGES] = = [C][U]	[C] = [REACTANCES] = = [Q]/[U]
Electric Field	$[C_{el.}u_{el.}^{2}]$ (=) $[q_{el.}u_{el.}^{1}]$ $[L_{el.}i_{mag.}^{2}]$ (=) $[\Phi_{el.}i_{mag.}^{1}]$	$[C_{el.}u_{el.}^{1}]$ (=) $[q_{el.}u_{el.}^{0}]$ (=) $[q_{el.}]$ (=) $[\Phi_{el}]$	$[C_{el.}u_{el.}^{0}]$ (=) $[q_{el.}u_{el.}^{-1}]$ (=) $[C_{el.}]$ (=) $[L_{el.}]$ (=) [C]
Magnetic Field	$[C_{mag.}u_{mag.}^{2}]$ (=) $[q_{mag.}u_{mag.}^{1}]$ $[L_{mag.}i_{el.}^{2}]$(=) $[\Phi_{mag.}i_{el.}]$	$[C_{mag.}u_{mag.}^{1}]$ (=) $[q_{mag.}u_{mag.}^{0}]$ (=) $[q_{mag.}]$ (=) $[\Phi_{mag}]$	$[C_{mag.}u_{mag.}^{0}]$ (=) $[q_{mag.}u_{mag.}^{1}]$ (=) $[C_{mag.}]$ (=) $[L_{mag.}]$ (=) [L]
Gravitation	$[mv^{2}]$ (=) $[pv^{1}]$	$[mv^{1}]$ (=) $[pv^{0}]$ (=) [p]	$[mv^{0}]$ (=) $[pv^{-1}]$ (=) [m]
Rotation	$[\mathbb{J}\,\omega^{2}]$ (=) $[\mathbb{L}\,\omega^{1}]$	$[\mathbb{J}\omega^{1}]$ (=) $[\mathbb{L}\omega^{0}]$ (=) $[\mathbb{L}]$	$[\mathbb{J}\omega^{0}]$ (=) $[\mathbb{L}\omega^{-1}]$ (=) $[\mathbb{J}]$

T.3.2	[U] = [VOLTAGES] = = d[X]/dt	[I] = [CURRENTS]= = d[Q]/dt	[Z] =[IMPEDANCES]=[U]/[I] (= [mobility] in mechanics)
Electric Field	$u_{el.} = d\Phi_{mag.}/dt = i_{mag.}$	$i_{el.} = dq_{el.}/dt = u_{mag.}$	$Z_{el} = u_{el.} / i_{el.} = i_{mag.} / u_{mag.} = Y_{mag.}$
Magnetic Field	$u_{mag.} = d\Phi_{el.}/dt = i_{el.}$	$i_{mag.} = dq_{mag.}/dt = u_{el.}$	$Z_{mag.} = u_{mag.} / i_{mag.} = i_{el.} / u_{el.} = Y_{el.}$
Gravitation	$v = dx/dt$	$f = dp/dt$	$Z_m = v / f$
Rotation	$\omega = d\alpha/dt$	$\tau = d\mathbb{L}/dt$	$Z_R = \omega / \tau$

T.3.3	[R] = [RESISTANCES]= = $[Z]_{real}$	[X] = [DISPLACEMENTS] = = $\int[U]\,dt$	[P] = [POWER] = d[E]/dt = = [U][I]
Electric Field	$R_{el.}$	$\Phi_{mag} = L_{mag}i_{el} = q_{mag}$	$u_{el.}i_{el.}$
Magnetic Field	$R_{mag.}$	$\Phi_{el.} = L_{el.}\,i_{mag.} = q_{el.}$	$u_{el.}i_{el.}$
Gravitation	R_m	$x = Sf$	vf
Rotation	R_R	$\alpha = S_R\tau$	$\omega\tau$

From T.3.2 we can easily notice that the product between electric and magnetic impedances is dimensionally equal to 1. If we apply the same analogy, searching for mechanical impedance (or mobility) of a linear motion, and mechanical impedance of the rotation that belongs to the same motion, where product of both of them would also be equal to 1, we shall obtain:

$$\left\{Z_{el.} \times Z_{mag} = \frac{u}{i} \times \frac{i}{u} = 1\right\} \Rightarrow \left\{Z_m \times Z_R = \frac{v}{f} \times \frac{\omega}{\tau} \neq 1\right\} . \qquad (3.9)$$

Obviously, one of mechanical impedances (for linear and/or for rotational motion) should be appropriately corrected in order to make the product between two of them equal to 1. Let as introduce correcting function $\alpha = \alpha(v, f, \omega, \tau)$ that will make this product equal to 1,

$$\left\{Z_{el.} \times Z_{mag} = \frac{u}{i} \times \frac{i}{u} = 1\right\} \Leftrightarrow \left\{Z'_m \times Z'_R = \alpha(v, f, \omega, \tau) \times \frac{v}{f} \times \frac{\omega}{\tau} = 1\right\} . \qquad (3.10)$$

Since mobility (or new mechanical impedance, as named in this paper) is already known in Mechanics as $Z_m = Z'_m = \frac{v}{f} (=) \left[\frac{s}{Kg}\right]$ and in a full agreement with extended electromechanical Mobility-system of analogies, we can only correct the meaning (or definition) of the rotational movement impedance, on the following way,

$$\{Z'_m \times Z'_R = Z_m Z_R = 1\} \Rightarrow Z_m = Z'_m = \frac{v}{f},\ Z_R = \alpha(v,\ f,\ \omega,\ \tau) \times \frac{\omega}{\tau}\ (= Z'_R)\,. \quad (3.11)$$

At this point we still do not know what should be the expression for correcting function $\alpha = \alpha(v, f, \omega, \tau)$, and we can only find that dimensionally this should be the square of a linear momentum,

$$[\alpha](=)\left[(\frac{Kg \times m}{s})^2\right](=)\left[(mv)^2\right](=)\left[p^2\right](=)\left[m\omega L\right](=)\left[m\omega^2 J\right]. \quad (3.12)$$

By analogy, we could conclude that (corrected) rotational impedance would be,

$$Z_R = \alpha(v,\ f,\ \omega,\ \tau) \times \frac{\omega}{\tau} (\approx) p^2 \times \frac{\omega}{\tau} = \frac{\omega}{(\tau / p^2)} (\approx) \frac{m\omega^2 L}{\tau}\ (=) \left\{ Z'_R (=) \left[\frac{Kg}{s}\right]\right\}, \quad (3.13)$$

and, that most probably we should change (redefine) the meaning of Torque (= τ = **dL/dt**), and Moment of Inertia (**J** = **L** / ω), in order to create desired symmetry between rotation and linear motion, but obviously this artificial and dimensional fitting (given by (3.10) until (3.13)) already looks as a very unrealistic, or too complicated and not natural option.

The second, more realistic possibility is to treat impedance of rotation as fully analog to magnetic impedance, $Z_R = \alpha'(v, f, \omega, \tau) \cdot \frac{\tau}{\omega} \Leftrightarrow \frac{i}{u} = Z_{mag.}$:

$$\left\{Z_{el.} \times Z_{mag} = \frac{u}{i} \times \frac{i}{u} = 1\right\} \Rightarrow \left\{Z_m \times Z_R = \frac{v}{f} \times \alpha'(v, f, \omega, \tau) \times \frac{\tau}{\omega} = 1\right\},$$

$$\Rightarrow Z_m \cdot Z_R = \frac{v}{f} \cdot S \cdot \frac{\tau}{\omega} = 1 \Leftrightarrow Z_R = S \cdot \frac{\tau}{\omega}$$
$$\alpha'(v, f, \omega, \tau) = \text{Surface} = S(=)\left[m^2\right] \quad (3.9.1)$$

and to introduce redefinition of Torque as, $\tau_{new} = S \cdot \tau_{old}$ (or new torque should be dimensionally equal to the product between old torque and a surface area captured by rotation),

$$\tau_{new} = S \times \tau_{old} = S \times \frac{dL}{dt} = S \times \frac{d(J\omega)}{dt} (=) \left[\frac{kg \times m^4}{s^2}\right]. \quad (3.9.2)$$

Of course, we could also introduce new Angular momentum, or new Moment of Inertia, or new Angular Velocity (whatever shows the most practical and logical, but only one of them at the same time), as for instance:

$$L_{new} = S \times L_{old} = S \times J\omega (=) \left[\frac{kg \times m^4}{s} \right], \text{ or}$$
$$J_{new} = S \times J_{old} (=) \left[kg \times m^4 \right], \text{ or} \qquad . \qquad (3.9.3)$$
$$\omega_{new} = \frac{\omega_{old}}{S} (=) \left[\frac{1}{m^2 \times s} \right]$$

Since only one of expressions (3.9.1)-(3.9.3) obviously should be the best and correct choice regarding completing the concept of analogies introduced in this paper, it is also clear that again (after selecting new, more relevant definitions of parameters in question) we should make backward corrections regarding all analogies between linear and rotational motion.

It is still premature to conclude that impedance of rotating motion is really equal to (3.9.1), but at least **this was an interesting and challenging, brainstorming exercise in order to show that linear motion and rotation should be better united (if we would like to establish a stronger analogy platform between Maxwell Electromagnetic Theory and Mechanics, than currently known)**. Going backwards from (3.9.1) we should again correct all before established analogies related to rotation (just to mention the consequences of (3.9.1)).

Based on extended analogies from T.3.1, T.3.2 and T.3.3 (regardless of used terminology and symbols that could be further improved), many ideas and analyses can open new and/or reinforce already presented hypotheses (see **[3]**), leading us to a more general theory of gravitation, complemented with the field effects of rotation (which is the task to be fully realized in some other paper). It is also clear that here we are only touching the bottom levels of possible unification concepts for constructing the higher level of Universal Field Theory, searching for indicative and provoking facts and analogies that will support establishment of a New Field Theory *(The more pragmatic and more general field unification platform will be introduced in the chapter 4. and 5. of this paper. See development of equations (4.22)-(4.29), (5.15)-(5.16)).*

The fact is that analogies could serve as a powerful unifying and predicting research tool, only if properly formulated, and if all of bottom-line, elementary elements of formulated analogy charts and tables are fully mutually coherent and analogically replaceable. As we can see, until present, such level of bottom-line coherence and symmetry (regarding establisher analogies) is remarkably present but still not fully satisfied.

Another approach to analogies would be to start from Energy (in any of its packaging formats) considering it as the primary building entity of everything what we deal with in our universe, and then express all other physics-related values (such as: Charges, Reactance, Voltages, Impedances, Resistances, Displacements and Power, found in T.3.1, T.3.2 and T.3.3) in terms of Energy. This way we could try to formulate more powerful analogies and symmetries leading to generally acceptable and Unified Field Theory.

[♣ <u>COMMENTS & FREE-THINKING CORNER</u>:

3.4.1. The Hyperspace Communications and Light

If we hypothetically imagine that light is the manifestation of some oscillating phenomena connected to multidimensional world (which has more than four dimensions, or which has different set of dimensions, comparing to our perceptible world), we could try to communicate with this (multidimensional) world just manipulating and modulating the dynamic and transient aspects of light that are under our control (in our 4-dimensional universe). For instance, let us imagine that we create a convenient closed mirror-wall container, then introduce certain light source inside, and let light to reach its equilibrium state (after almost countless number of reflections, scattering and interference effects inside of such mirror-wall container). Practically, we can say that certain amount of (electromagnetic) energy is continuously being introduced and present inside of a closed space of mirror-wall container. If our container is in the state of rest (no mechanical movements), the internal "light energy fluid" in its equilibrium will create certain (stable) center of gravity, or center of inertia, somewhere in internal space of the container, and input light energy would be partially and continuously transformed only into output heat dissipation on the container walls. This light container is an object that we see as a fully closed body in our four-dimensional universe. Contrary, the "light-energy fluid" inside of such container is only virtually bounded by the closed mirror walls of the container, but effectively not bounded at all, if our hypothetical starting point that light belongs to a multidimensional world (which has more than four dimensions) is correct. The only action we could make to create a meaningful information going out of our four-dimensional world is to modulate (dynamically and non-stationary) the carrier "light energy fluid" inside of the container (by external mechanical and electrical oscillations and complex motion, or rotation applied on the mirror-walls of the light container, or producing similar effects by some other electrodynamics means like applying frequency and amplitude modulation...), this way permanently moving and/or rotating effective center of inertia of the light mass captured in the container. Such locally unbalanced modulating energy (or information) cannot be transformed only into heat energy, and part of it should penetrate into multidimensional world, since energy and momentum conservation should also be valid in a multidimensional world (of course if such hypothetical world exists). A similar communication system (apparatus) combined with convenient sensors could be used as a receiver of signals coming from multidimensional world into our 4-dimensional world (of course all of that is still highly hypothetical and oversimplified). By the opinion of the author of this paper some kind of above proposed experimental arrangement should be a probable connecting channel between our 4-dimensional world and higher dimensions of some more complex universe, if the initial hypothesis about light nature is correct (of course given example should be considered only as a big simplification of the apparatus for future communications between multidimensional worlds, and still highly speculative). If a light is not the convenient carrier (or messenger) for making contacts with multidimensional world, we could try to find some other phenomenology (which exist in our 4-dimensional world and penetrates into higher dimensions, if such dimensions exist), and basically reproduce the above described process of locally unbalanced, dynamic (and non-stationary) modulation (where effective center of gravity would have an unstable position, this way carrying the information towards higher dimensions). ♣]

4. DE BROGLIE MATTER WAVES, QUANTUM THEORY AND GRAVITATION

4.0 WAVE FUNCTIONS, WAVE VELOCITIES AND UNCERTAINTY RELATIONS

The concept of modeling wave packets or wave groups, which in certain important aspects are serving to simulate moving particles, has extremely significant place in understanding the particle-wave duality and Uncertainty Relations (even much more important than presently seen in Physics literature). What we can find in literature regarding wave velocities (group and phase velocity) in relation with an equivalent particle velocity is often oversimplified, artificially constructed, subjects-and-objects-jumpy and narrow-condition valid. The other disadvantage (found in contemporary physics literature) in presenting and analyzing the same problematic is that different authors are using different mathematical strategies. Consequently, often everything (regarding waveforms velocities) looks either not enough convincing, or mostly made to produce needed results, without giving non-doubtful impression about general applicability of such results and conclusions to all possible wave related phenomena known in the Nature. This is probably the case because most of authors dealing with such items are targeting only specific areas covered by Quantum Theory, forging their ways of presentation to be quantum mainstream correct, without taking any chance to challenge certain step stones of contemporary Quantum Theory.

Here we will attempt to significantly rectify and generalize concepts dealing with wave motion velocities in general and in relation to a “wave packet equivalent particle” velocity. The first and very important step would be to establish (or accept) the most applicable, most common, and sufficiently rich and universal mathematical model that would be able to cower all kind of (physics-related) wave and oscillatory motions. Something as that could presently look as the very ambitious task, but it will be shown as realistic and feasible if we would use Analytic Signals modeling, based on Hilbert Transform.

Let us start from an arbitrary and energy finite waveform $\Psi(t)$, which is transformed using the Analytic Signal model, in a form of the product of certain amplitude and phase function, see (4.0.1). This is the closest, ready-made model where we have amplitude and phase function separated (like in cases of signals’ modulating techniques), and where we will prove (later on) that the amplitude function would propagate with a group velocity, and the phase function would have phase velocity. It is also important to underline that all kinds of wave functions (here marked as $\Psi(t)$) describing real energy finite motions, known in Physics, are presentable using the Analytic Signal modeling (first time introduced by D. Gabor), and this is one reason more to establish this analysis using the Analytic Signal concepts. Of course, later we will show more profoundly and with more arguments why Analytic Signal modeling is taken as the optimal and natural waveforms modeling.

Most of contemporary (and old) analyses elaborating this problematic (and dealing with wave functions analyses in any other aspect) are still not significantly using the Analytic Signal concepts for wave functions modeling. Since different interpretation languages and frameworks are in use regarding analyses of wave motions, this will be shown as a big disadvantage in the house of modern mathematical physics, making that important multidisciplinary unifications and generalizations are presenting difficult tasks. The formal differences when using the Analytic Signal or Fourier Transform for presenting different wave functions are not so much evident and immediately clear. This is most probably the reason why the scientific community in number of cases continues (by intellectual inertia) to use the old and well established, but not far-reaching, methods based mostly on Fourier analysis. They are almost repeating and in some cases “creatively” copying in their publications what somebody else made long time ago. It is necessary to make an effort and enter into a new universe of dynamic and instantaneous Analytic Signals and Hilbert transform-based waveforms modeling and analysis in order to catch all finesse and advantages of such world, and to distinguish it from the ordinary Fourier transform based averaging methods. We could say that the applicability differences between Fourier analysis and Analytic Signal concepts are comparable to differences between the operations with Real Numbers and Complex Numbers (where Real Numbers are just a small area of the Complex Numbers Analysis).

Briefly summarizing, Analytic Signal modeling gives a chance to extract the instantaneous (time-dependent and time-evolving) amplitude and phase signal functions, signal frequency and power, all of them in time-space and frequency domains (including all mutually analogical amplitude and phase spectral functions), from any arbitrary finite waveform, which could be of certain relevance in Physics. It is also worth mentioning that starting from a wave function presented as a Complex Analytic Signal, development of Schrödinger, Klein-Gordon and many other well-known differential wave equations (known in Physics) is a relatively easy task.

Going directly to the most useful analytic signal forms for this analysis, we will consider that $\Psi(\mathbf{t})$ is our original, time-domain wave function, wave packet (or wave group), and $\hat{\Psi}(\mathbf{t})$ is its Hilbert transform, both of them real value functions,

$$\begin{aligned}
&\Psi(\mathbf{t}) = \frac{1}{\pi}\int_0^\infty [\mathbf{U}_c(\omega)\cos\omega\mathbf{t} + \mathbf{U}_s(\omega)\sin\omega\mathbf{t}]\mathbf{d}\omega = \frac{1}{\pi}\int_0^\infty [\mathbf{A}(\omega)\cos(\omega\mathbf{t} + \Phi(\omega))]\mathbf{d}\omega = \\
&= \mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t}) = -\mathbf{H}\left[\hat{\Psi}(\mathbf{t})\right], \\
&\hat{\Psi}(\mathbf{t}) = \frac{1}{\pi}\int_0^\infty [\mathbf{U}_c(\omega)\sin\omega\mathbf{t} + \mathbf{U}_s(\omega)\cos\omega\mathbf{t}]\mathbf{d}\omega = \frac{1}{\pi}\int_0^\infty [\mathbf{A}(\omega)\sin(\omega\mathbf{t} + \Phi(\omega))]\mathbf{d}\omega = \\
&= \mathbf{a}(\mathbf{t})\sin\varphi(\mathbf{t}) = \mathbf{H}[\Psi(\mathbf{t})],\ \mathbf{H}\,(=)\ \textbf{Hilbert transform}\ ,
\end{aligned} \tag{4.0.1}$$

Where $\mathbf{a(t)}$ is the instantaneous signal amplitude or signal envelope, $\cos\varphi(\mathbf{t})$ is the signal carrier function, $\varphi(\mathbf{t})$ is the signal phase function, and $\omega(\mathbf{t}) = \mathbf{d}\varphi(\mathbf{t})/\mathbf{dt} = 2\pi\,\mathbf{f}\,(\mathbf{t})$ is the instantaneous signal frequency. Analogical functions in the signal frequency domain are $\mathbf{A}(\omega)$ as the signal amplitude, and $\Phi(\omega)$ as the signal phase function. The Hilbert transform $\mathbf{H}$ is a kind of filter, which shifts phases of all elementary (simple harmonic) components of its input by $-\pi/2$.

The same, time-domain function $\Psi(\mathbf{t})$, transformed into Complex, time-domain Analytic Signal function $\overline{\Psi}(\mathbf{t})$ has the following form,

$$\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1 + jH)\ \Psi\ (t) = \frac{1}{\pi}\int_0^\infty U(\omega)e^{j\omega t}\ d\omega = \frac{1}{\pi}\int_0^\infty A(\omega)e^{j(\omega t + \Phi(\omega))}\ d\omega$$

$$= a(t)e^{j\varphi(t)},\ \Psi(t) = \frac{1}{2}\left[\overline{\Psi}(t) + \overline{\Psi}^*(t)\right],\ \overline{\Psi}^*(t) = \Psi(t) - j\hat{\Psi}(t)\ ,$$

$$a(t) = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)}\ ,\ \dot{a}(t) = \frac{\Psi(t)\dot{\Psi}(t)\text{-}\hat{\Psi}(t)\dot{\hat{\Psi}}(t)}{a(t)} = a(t)\mathrm{Re}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right], \tag{4.0.2}$$

$$\varphi(t) = \mathrm{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)},\ \omega(t) = \frac{\partial\varphi(t)}{\partial t} = \dot{\varphi}(t) = \frac{\Psi(t)\dot{\hat{\Psi}}(t)\text{-}\dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)} = \mathrm{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right].$$

The frequency-domain, wave functions of an Analytic Signal are,

$$\begin{aligned}&\mathbf{A}(\omega) = \mathbf{U}(\omega)\mathbf{e}^{-j\Phi(\omega)}\ ,\Phi(\omega) = -\mathbf{arctan}\left[\mathbf{U}_s(\omega)/\mathbf{U}_c(\omega)\right], \mathbf{j} = \sqrt{-1}\\ &\overline{\mathbf{U}}(\omega) = \mathbf{U}_c(\omega) - \mathbf{j}\mathbf{U}_s(\omega) = \int_{-\infty}^{+\infty}\Psi(\mathbf{t})\mathbf{e}^{-j\omega t}\mathbf{dt} = \mathbf{A}(\omega)\mathbf{e}^{j\Phi(\omega)}\ .\end{aligned} \tag{4.0.3}$$

In the analysis of waveform velocities in this paper, which follows, we will use all (here mentioned) real and complex time-domain and frequency-domain Analytic Signal wave functions, (4.0.1)-(4.0.3).

As we can see, Analytic Signal is a kind of functions-shape modeling where we are "casting" an arbitrary-shaped and finite function into a "mold" of sinus and cosines functions (both in real and complex, time and frequency domains), getting maximum of advantages for wave functions analysis and characterizations already known for simple harmonic wave functions. The power of such modeling is being reinforced by the fact that elementary waveforms known in physics and nature are composed of sinusoidal functions.

For more complete understanding of Analytic Signal functions and regarding Analytic Signal advantages in comparison to all other waveform presentations, it would be useful to go to already published Analytic Signal and Hilbert Transform related literature (in order to focus analysis in this article only to wave functions, wave velocities and closely related topics).

4.0.1. Signal Energy Content

Let us first find the <u>energy $\tilde{\mathbf{E}}$</u> carried by the analytic signal waveform $\Psi(\mathbf{t}) = \mathbf{a(t)cos}\varphi(\mathbf{t})$. Parseval's theorem is connecting time and frequency domains of signal's wave functions, and such expression can have the meaning of signal's (or wave) energy in cases <u>when $\Psi^2(\mathbf{t}) = \mathbf{P(t)} = \mathbf{d}\tilde{\mathbf{E}}/\mathbf{dt}$ is modeled to present instantaneous signal power $\mathbf{P(t)}$</u>, as follows,

$$\tilde{E}=\int_{-\infty}^{+\infty}\Psi^2(t)dt=\int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt=\frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt=\frac{1}{2}\int_{-\infty}^{+\infty}a^2(t)dt=\int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt=$$
$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\overline{U}(\omega)\right|^2 d\omega=\int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega=\frac{1}{\pi}\int_{0}^{\infty}\left[A(\omega)\right]^2 d\omega=\int_{0}^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega=$$
$$=\int_{-\infty}^{+\infty}P(t)dt\ (=)\ [J],$$
$$\rho(\tilde{E})_t=\frac{d\tilde{E}}{dt}=\Psi^2(t)=P(t)\ (\Leftrightarrow)\left[\frac{a(t)}{\sqrt{2}}\right]^2 (=)\ [W]\ ,t\in(-\infty,+\infty)\ , \tag{4.0.4}$$
$$\rho(\tilde{E})_\omega=\frac{d\tilde{E}}{d\omega}=\frac{\Psi^2(t)}{d\omega/dt}(\Leftrightarrow)\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 (=)\left[Js=Ws^2\right],\ \omega\in(0,+\infty),$$
$$\frac{\omega(t)}{\tilde{E}}=\frac{2\left[\Psi(t)\dot{\hat{\Psi}}(t)-\dot{\Psi}(t)\hat{\Psi}(t)\right]}{a^2(t)\int_{-\infty}^{+\infty}a^2(t)dt}=\frac{2\left[\Psi(t)\dot{\hat{\Psi}}(t)-\dot{\Psi}(t)\hat{\Psi}(t)\right]}{\left[\Psi^2(t)+\hat{\Psi}^2(t)\right]\int_{-\infty}^{+\infty}\left[\Psi^2(t)+\hat{\Psi}^2(t)\right]dt}.$$

From the expressions for signal energy (4.0.4), it is obvious that a total signal energy content is captured and propagating only by the signal amplitude (or envelope) function $\mathbf{a(t)}$ or $\mathbf{A(\omega)}$, both in time and frequency domain,

$$\tilde{\mathbf{E}}=\int_{-\infty}^{+\infty}\left[\frac{\mathbf{a(t)}}{\sqrt{\mathbf{2}}}\right]^2\mathbf{dt}=\int_{\mathbf{0}}^{\infty}\left[\frac{\mathbf{A(\omega)}}{\sqrt{\pi}}\right]^2\mathbf{d\omega}, \tag{4.0.5}$$

and it is also obvious that signal phase functions, $\varphi\mathbf{(t)}$ and $\Phi(\omega)$ do not directly participate in a total signal energy content (do not carry signal energy at all). This is very important fact to notice in order to understand the meaning of signal velocities. We can also say that the wave *velocity of the signal amplitude (or signal envelope) is the velocity of the total signal energy propagation*, and this velocity is usually named as a group velocity (in particle-wave duality concepts, group velocity corresponds to particle velocity and a wave packet is the wave equivalent of the particle; -this problematic will be analyzed in details later). The speed of the signal-carrier or phase function/s is usually named as a phase velocity, and it should present the signal carrier velocity, or velocity of elementary signal elements (which are creating a wave packet). Here, the signal carrier functions in a time-domain are $\mathbf{\cos\varphi(t)}$, or $\mathbf{e}^{\mathbf{j\varphi(t)}}$, and in a frequency-domain $\mathbf{\cos\Phi(\omega)}$ or $\mathbf{e}^{\mathbf{j\Phi(\omega)}}$, and we can use them equally and analogically, depending on our preferences for operating with trigonometric or complex functions.

The common and unifying property of Analytic Signals (between their corresponding time and frequency domains) is that all phase functions ($\mathbf{\cos\varphi(t)}$*,* $\mathbf{e}^{\mathbf{j\varphi(t)}}$ *,* $\mathbf{\cos\Phi(\omega)}$*,* $\mathbf{e}^{\mathbf{j\Phi(\omega)}}$*), regardless in which domain formulated, have the same phase velocity **u**. Also all amplitude functions (*$\mathbf{a(t)}$ *,* $\mathbf{A(\omega)}$*), regardless in which domain formulated, have the same group velocity* $\mathbf{v}$*. In addition, there is well-defined equation connecting group and phase velocity of such wave functions (to be shown later). Also, there is experimentally proven and theoretically well-supported knowledge that for Physics-related elementary particles, signals or wave packets, like photons, electrons and/or other energy quanta, both signal velocities (**u** and **v**), M. Planck's wave energy* $\tilde{\mathbf{E}}=\mathbf{hf}$*, Einstein's particle* *energy expression* $\mathbf{E} = \mathbf{mc}^2$*, and de Broglie matter wave wavelength* $\lambda = \mathbf{h}/\mathbf{p}$ *are mutually very closely related. In fact, all of them are fully mathematically compatible and complementary, and analytically united by the equation that is connecting group and phase velocity* $\mathbf{v} = \mathbf{u} - \lambda \mathbf{du}/\mathbf{d}\lambda = -\lambda^2 \mathbf{df}/\mathbf{d}\lambda$ *(this will also be analyzed later in details). The best mathematical environment to exploit all of here mentioned signal properties is the Analytic Signal concept.*

Regarding particle-wave duality concept based on presenting a moving particle, which has certain mass, with an equivalent wave packet, the problem appears in understanding that group and phase velocity of the wave packet can be mutually different. For non-relativistic particle, its phase velocity is close to one half of its group velocity, and in the same time the particle that should be well-represented on that way is known as a compact, stable and time-space well localized (without energy residuals). For the time being, the best answer to such conceptual dilemma is that <u>only the wave packet amplitude function carries the whole packet or particle (motional) energy and propagates with a group or particle velocity</u>, what is mathematically correct and universally valid, based on (4.0.4) and (4.0.5). The other part of the wave packet that is somehow space-time retarded behind the particle like its waving tail (which has a phase velocity), is anyway not carrying the energy, and should not be a big problem for formulating manageable particle-wave duality modeling. Development of expressions for group and phase velocity of a wave packet and relations between them will be presented a bit later, but since this is essential for understanding the particle-wave duality, it is necessary to mention in advance the significance of the problematic we are dealing with (see the development of wave velocities starting from (4.0.6) until (4.0.46)). In the end of this paper, we will again take much closer look to physics-related wave functions structure and properties.

Obviously, the Analytic Signal model is naturally representing arbitrary wave forms (on a generally valid platform) and giving the unique opportunity to extract immediate signal amplitude and phase functions, immediate signal frequency and all other characteristic signal functions, both in time and frequency domain. The striking idea appearing here is to make an attempt to treat all wave motions, oscillations, wave packets and signals (relevant in Physics) using the Analytic Signal modeling. Such modeling is so rich, natural, selective and informative, that also Quantum Theory wave functions framework should be primarily and ontologically based on Analytic Signals modeling, and starting from there, all other wave equations should be logically developed, using clear step-by-step and almost elementary mathematical methods (without introducing add-in, patch-in, fit-in and “fallen from the sky” helping rules). Something like that is shown in the chapter 4.3 (where almost all wave equations and their operators, known in present Quantum Theory, are developed starting from an Analytic Signal wave function).

Since there is a lot of available literature regarding Hilbert transform and Analytic Signal modeling, let us only summarize the most important properties and expressions valid for presenting arbitrary waveforms as Analytic Signals, T.4.0.1.

T.4.0.1

Parallelism between Time and Frequency Domains	Analytic Signal	
	Time Domain	**Frequency Domain**
Complex Signal	$\overline{\Psi}(t) = a(t)e^{j\varphi(t)}$ $= \Psi(t) + j\hat{\Psi}(t)$ $= \frac{1}{\pi}\int_0^{\infty} U(\omega)e^{j\omega t}\, d\omega$ $= \frac{1}{\pi}\int_0^{\infty} \overline{U}(\omega)e^{-j\omega t}\, d\omega$ $= \frac{1}{\pi}\int_0^{\infty} A(\omega)e^{j(\omega t+\Phi(\omega))}\, d\omega$	$\overline{U}(\omega) = A(\omega)e^{j\Phi(\omega)}$ $= U_c(\omega) - jU_s(\omega)$ $= \int_{-\infty}^{+\infty} \Psi(t)e^{-j\omega t}dt$ $= \int_{-\infty}^{+\infty} \overline{\Psi}(t)e^{j\omega t}dt$ $= \int_{-\infty}^{+\infty} a(t)e^{-j(\omega t+\varphi(t))}dt$
Real and imaginary signal components	$\Psi(t) = a(t)\cos\varphi(t) =$ $= -H\left[\hat{\Psi}(t)\right],$ $\hat{\Psi}(t) = a(t)\sin\varphi(t) =$ $= H\left[\Psi(t)\right]$	$U_c(\omega) = A(\omega)\cos\Phi(\omega) =$ $= -H\left[U_s(\omega)\right],$ $U_s(\omega) = A(\omega)\sin\Phi(\omega) =$ $= H\left[U_c(\omega)\right],$
Signal Amplitude	$a(t) = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)}$	$A(\omega) = \sqrt{U_c{}^2(\omega) + U_s{}^2(\omega)}$
Instant Phase	$\varphi(t) = \mathrm{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}$	$\Phi(\omega) = \arctan\frac{U_s(\omega)}{U_c(\omega)}$
Instant Frequency	$\omega(t) = \frac{\partial\varphi(t)}{\partial t}$	$\tau(\omega) = \frac{\partial\Phi(\omega)}{\partial\omega}$
Signal Energy	$\tilde{E} = \int_{-\infty}^{+\infty}\lvert\overline{\Psi}(t)\rvert^2 dt =$ $= \int_{-\infty}^{+\infty}\Psi^2(t)dt =$ $= \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt =$ $= \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt$ $\int_{-\infty}^{+\infty}\Psi(t)\cdot\hat{\Psi}(t)dt = 0$	$\tilde{E} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\lvert\overline{U}(\omega)\rvert^2 d\omega =$ $= \frac{1}{2\pi}\int_{-\infty}^{+\infty}U_c^2(\omega)d\omega =$ $= \frac{1}{2\pi}\int_{-\infty}^{+\infty}U_s^2(\omega)d\omega =$ $= \int_0^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega$ $\int_{-\infty}^{+\infty}U_c(\omega)\cdot U_s(\omega)d\omega = 0$
Central Frequency	$\omega_c = \frac{\int_{[t]}\omega(t)\cdot a^2(t)dt}{\int_{[t]}a^2(t)dt} = 2\pi f_c$	$\omega_c = \frac{\int_{[\omega]}\omega\cdot[A(\omega)]^2 d\omega}{\int_{[\omega]}[A(\omega)]^2 d\omega} = 2\pi f_c$
“Central Time Point”	$t_c = \frac{\int_{[t]}t\cdot a^2(t)dt}{\int_{[t]}a^2(t)dt}$	$t_c = \frac{\int_{[\omega]}\tau(\omega)\cdot[A(\omega)]^2 d\omega}{\int_{[\omega]}[A(\omega)]^2 d\omega}$
Standard Deviation	$\sigma_\omega^2 = \frac{1}{\Delta t}\int_{[t]}\lvert\omega(t)-\omega_c\rvert^2 dt$	$\sigma_t^2 = \frac{1}{\Delta\omega}\int_{[\omega]}\lvert\tau(\omega)-t_c\rvert^2 d\omega$

<table>
<tr><td rowspan="2">Uncertainty Relations</td><td>$$\pi \le \sigma_{\omega} \cdot \sigma_{t} < \left|\omega(t) \cdot \tau(\omega)\right| \cong \omega_{c} \cdot t_{c} \cong \Omega \cdot T \le \frac{\pi}{2 \cdot \delta t \cdot \delta f} = \frac{\pi^{2}}{\delta t \cdot \delta\omega} \ (!?)$$ $$0 < \delta t \cdot \delta\omega = 2\pi \cdot \delta t \cdot \delta f < \pi < \Omega \cdot T \le \frac{\pi}{2 \cdot \delta t \cdot \delta f} = \frac{\pi^{2}}{\delta t \cdot \delta\omega}$$</td></tr>
<tr><td>$$\delta t \cdot \delta f \le 2 \cdot \delta t \cdot \delta f \cdot T \cdot F \le 1/2,$$ $$\delta t \le \frac{1}{2F},\ \delta f \le \frac{1}{2T},\ F \cdot T > \frac{1}{2},$$ $$\delta t \cdot \delta f < \frac{1}{2},\ \delta t \cdot \delta\omega < \pi,\ \Omega \cdot T > \pi$$ $$0 < \delta t \cdot \delta f < \frac{1}{2} < F \cdot T \le \frac{1}{4 \cdot \delta t \cdot \delta f}.$$ Shannon-Kotelnikov optimal signal-sampling relations in time and frequency domains: δt = maximal time sampling interval, δf = maximal frequency sampling interval. $\Omega = 2\pi F$ and T are the total, or absolute signal-lengths in its frequency and time domains; ($\omega = 2\pi f, \delta\omega = 2\pi\,\delta f$).</td></tr>
<tr><td colspan="2">Other forms of Uncertainty Relations (see later in this chapter for more complete development of such relations):
$$0 < \delta t \cdot \delta\omega = 2\pi \cdot \delta t \cdot \delta f < \pi \le \sigma_{\omega} \cdot \sigma_{t} < \Omega \cdot T \le \frac{\pi}{2 \cdot \delta t \cdot \delta f} = \frac{\pi^{2}}{\delta t \cdot \delta\omega},\ \left|\omega(t) \cdot \tau(\omega)\right| \cong \omega_{c} \cdot t_{c} \cong \pi$$ $$0 < \delta t \cdot \delta f < \frac{1}{2} \le \sigma_{f} \cdot \sigma_{t} < F \cdot T \le \frac{1}{4 \cdot \delta t \cdot \delta f},\ \left|f(t) \cdot \tau(\omega)\right| \cong f_{c} \cdot t_{c} \cong \frac{1}{2}$$ $$0 < \delta t \cdot \delta f = \delta x \cdot \delta f_{x} < \frac{1}{2} \le \sigma_{t} \cdot \sigma_{f} = \sigma_{x} \cdot \sigma_{f\text{-}x} < F \cdot T = F_{x} \cdot L \le \frac{1}{4 \cdot \delta t \cdot \delta f} = \frac{1}{4 \cdot \delta x \cdot \delta f_{x}},$$ $$0 < \delta t \cdot \delta\tilde{E} = \delta x \cdot \delta p < \frac{h}{2} \le 2\pi \sigma_{t} \cdot \sigma_{\tilde{E}} = \sigma_{x} \cdot \sigma_{p} < \tilde{E} \cdot T = P \cdot L \le \frac{h}{4 \cdot \delta t \cdot \delta f} = \frac{h}{4 \cdot \delta x \cdot \delta f_{x}}.$$</td></tr>
</table>

4.0.2. Resume of Different Analitic Signal Representations

In number of practical cases related to physics (non dispersive, band-limited, finite signals), Analytic Signal modeling is giving a chance to present certain signal looking as simple-harmonic form $a(t)\cos\varphi(t)$, which has its dominant frequency and its phase function, as for instance,

$$\Psi(t) = a(t)\cos\varphi(t) = a(t)\cos\left[\omega_{0}t + \varphi(t)_{residual}\right],\ \varphi(t) = \omega_{0}t + \varphi(t)_{residual} \ , \ \ \omega_{0} = const. \ .$$

We can also imagine and analyze more complex situations regarding signal-phase functions of non-linear and dispersive signals such as,

$$\Psi(t) = a(t)\cos\varphi(t) = a(t)\cos\left[\omega_0 t + \frac{\omega_1}{T_1}t^2 + \frac{\omega_2}{T_2^2}t^3 + ...\varphi(t)_{residual}\right]$$

$$\varphi(t) = \omega_0 t + \frac{\omega_1}{T_1}t^2 + \frac{\omega_2}{T_2^2}t^3 + ...\varphi(t)_{residual} \quad , \quad \omega_{0,1,2...} = \text{const.} \quad T_{1,2...} = \text{Const.} \quad .$$

Analytic Signals can be presented in number of ways, giving the chance to reveal the internal structure of waveforms from different points of view. Here are summarized several of such possibilities (mainly as superposition or multiplication of elementary signals; -see below).

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)} = a_0(t)[\cos\varphi_0(t) + I\sin\varphi_0(t)] =$$
$$= a_0(t)e^{\sum_{(k)} i_k\varphi_k(t)} = \sum_{(k)} a_k(t)e^{i_k\varphi_k(t)} = \sum_{(k)} \overline{\Psi}_k(t),$$

$$\bar{H}[\Psi(t)] = \overline{\Psi}(t) = \Psi(t) + I\cdot\hat{\Psi}(t) = a(t)\cdot e^{I\cdot\varphi(t)}$$
$$\bar{H} = 1 + I\cdot H, \; I^2 = -1$$

AS SUPERPOSITION	**AS MULTIPLICATION**
$\overline{\Psi}(t) = \sum_{(k)} \overline{\Psi}_k(t) = \sum_{(k)} \Psi_k(t) + I\sum_{(k)} \hat{\Psi}_k(t)$	$\overline{\Psi}(t) = \Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t) + I\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t)$

4.0.2.1. Relations Between Additive and Multiplicative Elements

$$\Psi(t) = \sum_{(k)} \Psi_k(t) = \Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t) = a_0(t)\cos\varphi_0(t) = a(t)\cos\varphi(t) = -H\left[\hat{\Psi}(t)\right]$$
$$\hat{\Psi}(t) = \sum_{(k)} \hat{\Psi}_k(t) = \Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t) = a_0(t)\sin\varphi_0(t) = a(t)\sin\varphi(t) = H[\Psi(t)]$$
$$\overline{\Psi}_k(t) = \Psi_k(t) + i_k\hat{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)}, \Psi_k(t) = -H\left[\hat{\Psi}_k(t)\right], \; \hat{\Psi}_k(t) = H[\Psi_k(t)]$$
$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = \frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}) + \frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\right\}$$

4.0.2.2. Signal Amplitude or Envelope

$$\mathbf{a(t) = a_0(t) = \left|\overline{\Psi}(t)\right| = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)} = a_n(t)\prod_{(i=1)}^{n} \cos\varphi_i(t) = \Psi_{n+1}(t)\prod_{(i=1)}^{n}\cos\varphi_i(t)}$$

$$\mathbf{a_k(t) = \Psi_{k+1}(t) = \left|\overline{\Psi}_k\right| = \sqrt{\Psi_k^2(t) + \hat{\Psi}_k^2(t)} = a_{k+1}(t)\cos\varphi_{k+1}(t) = a_n(t)\prod_{(i=k+1)}^{n}\cos\varphi_i(t),\ k < n}$$

$$\mathbf{a_{n-1}(t) = \Psi_n(t) = \left|\overline{\Psi}_{n-1}(t)\right| = \sqrt{\Psi_{n-1}^2(t) + \hat{\Psi}_{n-1}^2(t)} = a_n(t)\cos\varphi_n(t) = \frac{\Psi_0(t)}{\prod\limits_{(i=0)}^{n-1}\cos\varphi_i(t)}}$$

4.0.2.3. Signal Phase

$$\mathbf{\varphi_0(t) = \varphi(t) = arctg\frac{\hat{\Psi}(t)}{\Psi(t)} = \sqrt{\sum_{(k)}\varphi_k^2(t)},\ \varphi_k(t) = arctg\frac{\hat{\Psi}_k(t)}{\Psi_k(t)}}$$

$$\mathbf{I^2 = i_1^2 = i_2^2 = ... = i_n^2 = -1\ ,\ i_j i_k = 0, \forall\ j \neq k}\ \ \textbf{(hypercomplex imaginary units)}$$

$$\mathbf{I\varphi_0(t) = I\varphi(t) = i_1\varphi_1(t) + i_2\varphi_2(t) + ... + i_n\varphi_n(t) = \sum_{k=1}^{n} i_k\varphi_k(t)}$$

$$\mathbf{e^{i_k\varphi_k(t)} = \cos\varphi_k(t) + i_k\sin\varphi_k(t),\ \varphi_0^2(t) = \sum_{k=1}^{n}\varphi_k^2(t)}$$

4.0.2.4. Signal instant frequency

$$\mathbf{\omega_i(t) = 2\pi f_i(t) = \frac{\partial\varphi_i(t)}{\partial t},\ i = 0,1,2,...k,...n}$$

4.0.2.5. More of interesting relations

$$\overline{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)} = \Psi_k(t) + i_k\hat{\Psi}_k(t)\ ,$$

$$\cos\varphi_k = \frac{1}{2}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}),\ \sin\varphi_k = \frac{1}{2i}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)}),$$

$$\varphi_k(t) = \mathrm{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)},\ \varphi_0{}^2(t) = \sum_{(k)}\varphi_k{}^2(t),\ \ \omega_k(t) = \frac{\partial\varphi_k(t)}{\partial t} = 2\pi f_k(t),$$

$$a_k{}^2(t) = a_{k-1}{}^2(t) + \hat{a}_{k-1}{}^2(t) = \Psi_{k+1}{}^2(t) = \Psi_k{}^2(t) + \hat{\Psi}_k{}^2(t)\ ,$$

$$a_0{}^2(t) = \left|\overline{\Psi}(t)\right|^2 = \Psi^2(t) + \hat{\Psi}^2(t) = \sum_{(k)} a_k{}^2(t) + 2\sum_{(i\neq j)}\Psi_i(t)\Psi_j(t),\ \forall i,j,k \in [1,n]\ .$$

$$\Psi(t) = \frac{a_n(t)}{2^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}),\ \hat{\Psi}(t) = \frac{a_n(t)}{(2i)^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\ ,$$

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = \frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}) + \frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\right\}.$$

4.0.2.6. Hyper-complex Analytic Signal

$$\overline{\Psi}(r,t) = \Psi(r,t) + I \cdot H\left[\Psi(r,t)\right] = \Psi(r,t) + I \cdot \hat{\Psi}(r,t) =$$

$$= \overline{\Psi}_i + \overline{\Psi}_j + \overline{\Psi}_k = \left|\overline{\Psi}(r,t)\right| \cdot e^{I\cdot\varphi(r,t)},\ \overline{\Psi}_{i,j,k} = \Psi_{i,j,k} + \begin{bmatrix} i \\ j \\ k \end{bmatrix} \cdot \hat{\Psi}_{i,j,k} = \left|\overline{\Psi}_{i,j,k}\right| \cdot e^{\begin{bmatrix} i \\ j \\ k \end{bmatrix}\cdot\varphi_{i,j,k}} ,$$

$$\left|\overline{\Psi}(r,t)\right|^2 = \left[\Psi(r,t)\right]^2 + \left[\hat{\Psi}(r,t)\right]^2 = \Psi^2 + \hat{\Psi}^2 = \left|\overline{\Psi}\right|^2, \varphi(r,t) = \operatorname{arctg}\frac{\hat{\Psi}(r,t)}{\Psi(r,t)} = \varphi,$$

$$\left|\overline{\Psi}_{i,j,k}\right|^2 = \left[\Psi_{i,j,k}\right]^2 + \left[\hat{\Psi}_{i,j,k}\right]^2, \varphi_{i,j,k} = \operatorname{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi_{i,j,k}}, \omega_{i,j,k} = \frac{\partial\varphi_{i,j,k}}{\partial t}$$

$$\left|\overline{\Psi}\right|^2 = \left|\overline{\Psi}_i\right|^2 + \left|\overline{\Psi}_j\right|^2 + \left|\overline{\Psi}_k\right|^2 = \Psi^2 + \hat{\Psi}^2 ,$$

$$\Psi = \left|\overline{\Psi}\right| \cdot \cos\varphi,\ \hat{\Psi} = \left|\overline{\Psi}\right| \cdot \sin\varphi = H\left[\Psi\right],$$

$$\Psi_{i,j,k} = \left|\overline{\Psi}_{i,j,k}\right| \cdot \cos\varphi_{i,j,k} ,\ \hat{\Psi}_{i,j,k} = H\left[\Psi_{i,j,k}\right] = \left|\overline{\Psi}_{i,j,k}\right| \cdot \sin\varphi_{i,j,k} ,$$

$$I \cdot \hat{\Psi}(r,t) = i \cdot \hat{\Psi}_i + j \cdot \hat{\Psi}_j + k \cdot \hat{\Psi}_\kappa = e^{I(\frac{\pi}{2}+2m\pi)} \cdot \hat{\Psi}(r,t),$$

$$I \cdot \varphi(r,t) = i \cdot \varphi_i + j \cdot \varphi_j + k \cdot \varphi_k = e^{I(\frac{\pi}{2}+2m\pi)} \cdot \varphi(r,t) ,\ I^2 = i^2 = j^2 = k^2 = -1 ,$$

$$I = i \cdot \frac{\varphi_i}{\varphi} + j \cdot \frac{\varphi_j}{\varphi} + k \cdot \frac{\varphi_\kappa}{\varphi} = i \cdot \frac{\hat{\Psi}_i}{\hat{\Psi}} + j \cdot \frac{\hat{\Psi}_j}{\hat{\Psi}} + k \cdot \frac{\hat{\Psi}_\kappa}{\hat{\Psi}} = e^{I(\frac{\pi}{2}+2m\pi)} ,$$

$$\frac{\varphi_i}{\varphi} = \frac{\hat{\Psi}_i}{\hat{\Psi}} ,\ \frac{\varphi_j}{\varphi} = \frac{\hat{\Psi}_j}{\hat{\Psi}} ,\ \frac{\varphi_k}{\varphi} = \frac{\hat{\Psi}_k}{\hat{\Psi}} ,$$

$$\frac{\varphi_{i,j,k}}{\varphi} = \frac{\hat{\Psi}_{i,j,k}}{\hat{\Psi}} = \frac{H\left[\Psi_{i,j,k}\right]}{H\left[\Psi\right]} = \frac{\operatorname{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi}}{\operatorname{arctg}\frac{\hat{\Psi}}{\Psi}} , \qquad (6.10)$$

$$\Psi = \Psi_i + \Psi_j + \Psi_k ,\ \Psi^2 = \Psi_i^2 + \Psi_j^2 + \Psi_k^2 ,\ \Psi_i\Psi_j + \Psi_j\Psi_k + \Psi_i\Psi_k = 0 ,$$

$$\hat{\Psi} = \hat{\Psi}_i + \hat{\Psi}_j + \hat{\Psi}_k ,\ \hat{\Psi}^2 = \hat{\Psi}_i^2 + \hat{\Psi}_j^2 + \hat{\Psi}_k^2 ,\ \hat{\Psi}_i\hat{\Psi}_j + \hat{\Psi}_j\hat{\Psi}_k + \hat{\Psi}_i\hat{\Psi}_k = 0 ,$$

$$\varphi = \varphi_i + \varphi_j + \varphi_k ,\ \varphi^2 = \varphi_i^2 + \varphi_j^2 + \varphi_k^2 ,\ \varphi_i\varphi_j + \varphi_j\varphi_k + \varphi_i\varphi_k = 0 .$$

4.0.3. Generalized Fourier Transform & Analytic Signal

The Analytic Signal modeling of the wave function can easily be installed in the framework of the Fourier Integral Transform, which exist on the basis of simple-harmonic functions **cos ωt**, as follows,

$$\Psi(t)=a(t)\cos\varphi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})e^{j2\pi ft}df = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\left\{\overline{H}\left[\cos 2\pi ft\right]\right\}df = F^{-1}\left[U(\frac{\omega}{2\pi})\right],$$

$$U(\frac{\omega}{2\pi})=A(\frac{\omega}{2\pi})e^{j\Phi(\frac{\omega}{2\pi})} = \int_{-\infty}^{\infty}\Psi(t)e^{-j2\pi ft}dt = \int_{-\infty}^{\infty}\Psi(t)\left\{\overline{H}^{*}\left[\cos 2\pi ft\right]\right\}dt = F\left[\Psi(t)\right],\quad \omega = 2\pi f,$$

where the meaning of symbols is:

F (=) Direct Fourier transform,

$\mathbf{F^{-1}}$ (=) Inverse Fourier transform,

$\overline{\mathbf{H}}=\mathbf{1}+\mathbf{jH}$ (=) Complex Hilbert transform, $\mathbf{j^2 = -1}$,

$\overline{\mathbf{H}}^{*}=\mathbf{1}-\mathbf{jH}$ (=) Conjugate complex Hilbert transform.

$$\begin{aligned}
&\overline{H}\left[\cos\omega t\right] = e^{j\omega t},\quad H\left[\cos\omega t\right] = \sin\omega t,\\
&\overline{H}^{*}\left[\cos\omega t\right] = e^{-j\omega t},\quad H\left[\sin\omega t\right] = -\cos\omega t,\\
&e^{\pm j\omega t} = (1 \pm jH)\left[\cos\omega t\right],\\
&\overline{H}\left[\Psi(t)\right] = \overline{\Psi}(t) = \Psi(t) + jH\left[\Psi(t)\right] = \Psi(t) + j\hat{\Psi}(t),\\
&\overline{H}^{*}\left[\Psi(t)\right] = \overline{\Psi}^{*}(t) = \Psi(t) - jH\left[\Psi(t)\right] = \Psi(t) - j\hat{\Psi}(t).
\end{aligned}$$

The further generalization of the Fourier integral transformation can be realized by convenient replacement of its simple-harmonic functions-basis **cos(ωt)** by some other, convenient signal basis **α(ω,t)**. Now, the general wave function (in the generalized framework of Fourier transform) can be represented as,

$$\Psi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\left\{\overline{H}\left[\alpha(\omega,t)\right]\right\}df = F^{-1}\left[U(\frac{\omega}{2\pi})\right],$$

$$U(\frac{\omega}{2\pi}) = \int_{-\infty}^{\infty}\Psi(t)\left\{\overline{H}^{*}\left[\alpha(\omega,t)\right]\right\}dt = F\left[\Psi(t)\right].$$

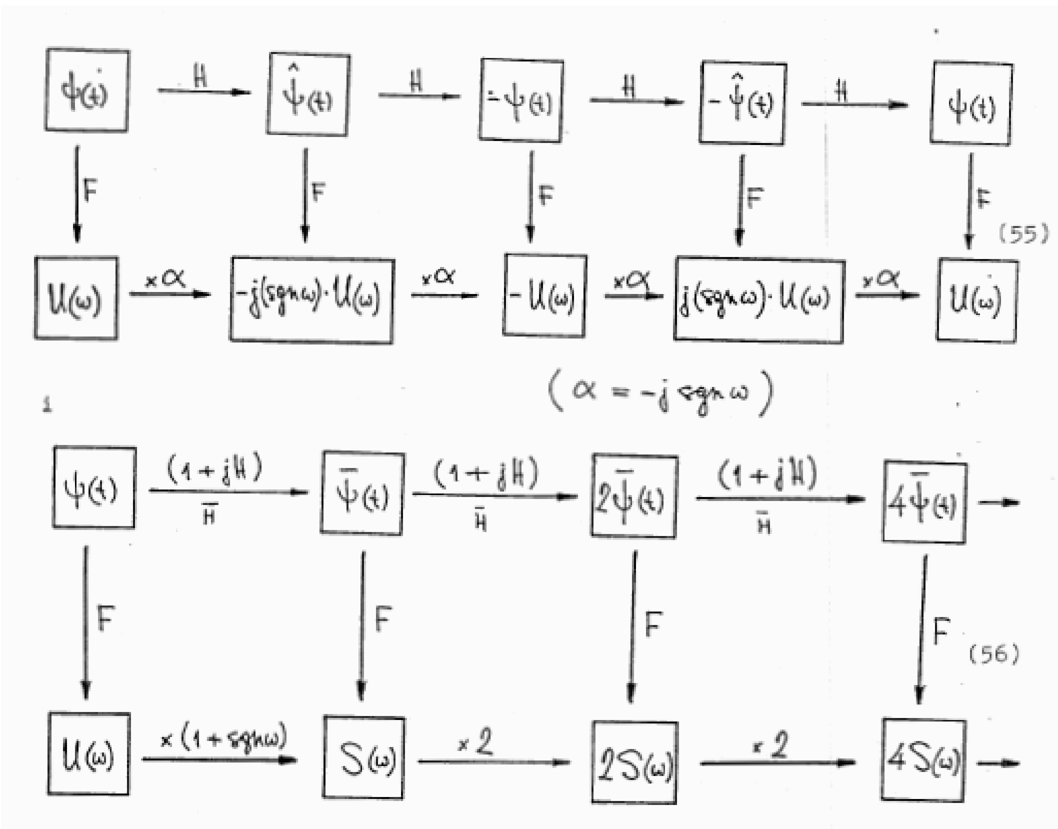
For instance, we could exercise with $\alpha(\omega,t) = \frac{\sin\,\Omega t}{\Omega t}\cos\,(\omega t)$ or $\alpha(\omega,t) = e^{-\beta t}\cdot\frac{\sin\,\Omega t}{\Omega t}\cos\,(\omega t)$, or some other wavelets family (see later (4.0.35) in relation with Kotelnikov-Shannon Theorem).

Here are more of interesting relations connecting Hilbert and Fourier transformation:

$$F\left[\Psi(t)\right]\cdot(-j\cdot\operatorname{sgn}\omega) = F\left[\hat{\Psi}(t)\right] = -j\cdot\operatorname{sgn}\omega\cdot U(\omega),$$
$$F\left[\hat{\Psi}(t)\right]\cdot(-j\cdot\operatorname{sgn}\omega) = -F\left[\Psi(t)\right] = -U(\omega),$$
$$F\left[\Psi(t)\right]\cdot(1+j\cdot\operatorname{sgn}\omega) = F\left[\overline{\Psi}(t)\right] = S(\omega),$$

$$H\left[\Psi(t)\right] = -H\left[\hat{\Psi}(t)\right],\ (1+jH)\cdot\overline{\Psi}(t) = \overline{\Psi}(t),\ H^2 = -j(Y - H),$$
$$H\left[H(\Psi(t))\right] = H^2\left[\Psi(t)\right] = -\Psi(t),\ H^4\left[\Psi(t)\right] = \Psi(t),$$
$$\overline{H}\left[\overline{\Psi}(t)\right] = 2\overline{\Psi}(t) = 2\overline{H}\left[\Psi(t)\right],\ H\left[\overline{\Psi}(t)\right] = -j\overline{\Psi}(t),$$
$$H\cdot\overline{H} = \overline{H}\cdot H = Y,\ H\cdot(\overline{H}\cdot H) = (H\cdot\overline{H})\cdot H,$$
$$Y\left[\overline{\Psi}(t)\right] = 2\overline{\Psi}(t) = H\left[\overline{\Psi}(t)\right],\ Y\left[\Psi(t)\right] = -j\overline{\Psi}(t),$$

$$H\left[t\cdot\Psi(t)\right] = t\cdot\hat{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)\,dt\ ,\ \overline{H}\left[t\cdot\Psi(t)\right] = t\cdot\overline{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)\,dt\ ,$$
$$H\left[(t+\tau)\cdot\Psi(t)\right] = (t+\tau)\cdot\hat{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)\,dt\ ,\ \overline{H}\left[(t+\tau)\cdot\Psi(t)\right] = (t+\tau)\cdot\overline{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)\,dt\ ,$$
$$H\left[\frac{d^n\Psi(t)}{dt}\right] = \frac{d^n\hat{\Psi}(t)}{dt},\ \overline{H}\left[\frac{d^n\Psi(t)}{dt}\right] = \frac{d^n\overline{\Psi}(t)}{dt}\ .$$



4.0.4. Meaning of Complex Analytic Signal Functions in Physics

The practice of using complex functions in Physics and Electronics has long time been considered mostly as a convenience to simplify number of mathematical expressions and equations processing, but in reality has much wider and deeper meaning when properly applied (what has not been quite uniformly and correctly established practice in different disciplines of Physics). Briefly, the only proper extension and generalization of real variable wave function to corresponding complex variable function (with important meaning in Physics) should be formulated as an Analytic Signal function (what is generally still not the case in Physics). Even in electronics or basic electro-technique, where operating with complex functions and current and voltage ***phasors*** *is the common practice, this is not realized on the grounds of an Analytic Signal modeling, but luckily it is formally compatible and not contradictory to such possibility (will be analyzed in the end of this chapter).*
The fact is that significant wave function elements (such as instantaneous amplitude, phase, frequency…) cannot be found if we do not take into account both original wave function Ψ and its Hilbert couple $\hat{\Psi}$, meaning that in reality both of such wave functions should coincidently exist, explaining the real meaning of complex Analytic Signal expressions. In other words, *the nature always makes or creates mutually coupled, phase shifted states and events (which are mutually orthogonal functions), where Hilbert transform and Complex Analytic Signal model are the right and best tools to find such coupled entities* (but often we do not notice or realize immanent and intrinsic existence of such coupled states). Here is a part of the explanation related to Quantum Theory mathematical structure, which is modeling similar reality of mutually coupled states on a formally different (but isomorphic) way. *For instance, well-known and logically controversial particle-wave duality theory, which is mathematically, verbally, experimentally, stochastically and philosophically (but not ontologically and deterministically) explaining how in some cases objects like photons, electrons and other elementary particles could behave like waves and in other cases like particles, should be close to duality of Ψ and its Hilbert couple $\hat{\Psi}$.* We can conditionally say that if Ψ represents certain particle model then $\hat{\Psi}$ represents its wave model (of course, this should be elaborated better). Both Ψ and $\hat{\Psi}$ have the same energy content, the same group and phase velocity (to be shown later), but they are mutually orthogonal (phase shifted for $\pi/2$) and on certain way energetically mutually exclusive since $\int_{-\infty}^{+\infty} \mathbf{\Psi(t)} \cdot \mathbf{\hat{\Psi}(t)dt} = \mathbf{0}$, and this should be the hidden nature of particle-wave duality (which still needs to be transformed into a new, more operational modeling applicable in physics). The celebrated physicist Paul Dirac was even more imaginative, intellectually vocal and convincing than others in conceptualizing an "ocean" of densely packed negative energy "phantom-like" electrons in a pure vacuum state of mater. This way he "invented" anti-matter particle named positron, which without such creative imagination could be found either as certain Ψ-function, or its Hilbert couple $\hat{\Psi}$, or anyway inside of the framework of Analytic Signal functions. Dirac also gave in the same scientific package his imaginative meaning of complex or imaginary functions in relation to relativistic particle energy, what his faithful followers found as exceptional contribution to physics. This way, he "predicted" existence of other anti-matter particles, and some of them were experimentally (and most probably independently from any Dirac's influence and prediction) discovered few years later. This prediction has been presented as a brilliant success of the science of 20th century, also showing that operational Ptolemy-type theories can still exist and that we are in some cases, and during certain period, not able to recognize them as such. Ptolemy is mentioned here in relation to its geocentric teaching that is conceptually and essentially wrong, but practically and mathematically well operational and giving sufficiently correct results. Richard Feynman later upgraded and optimized mentioned Dirac's concepts, making them very practical and operational in the form of Feynman's diagrams. Later on, similar imaginative and seducing concepts evolved towards the Zero Point, Electromagnetic Quantum Vacuum Fluctuations (ZPF). Here, we could simply say that *every real motional state expressed as certain Ψ-function, should have its Hilbert*

couple $\hat{\Psi}$*, which is another and equally real, motional state.* The interpretation how such coupling and imaging (or phase shifting) manifests in Physics is another question, but it is very probably related to de Broglie matter waves, Dirac's, Feynman's and ZPF concepts.

Of course, there is no progress and advance without creative imagination, and eventually wrong concepts, which are on a certain level operational, will be replaced by better concepts. In that name we can introduce another of such concepts, as follows: *"Since atoms as elementary bricks of matter are composed of electrons, protons and neutrons, and all of them manifest particle and wave properties, being presentable as kind of wave packets or wave functions, we can associate to each of them certain wave function* Ψ_i*. From the properties of Analytic Signals (elaborated here), we know that all of them should also have intrinsically coupled and phase shifted Hilbert associates* $\hat{\Psi}_i$*". How such immanent nature of coupled signals structure behaves inside atoms is another question to answer. Is this kind of thinking leading to reinventing positrons, anti-protons, positive, negative, and neutrally charged particles? Another imaginative aspect of Hilbert-type duality should be duality of linear and rotational motion, meaning that every particle (or wave) in linear motion should have its motional Hilbert couple, which could be some kind of associated rotational motion (and vice-versa).* Is de Broglie mater-wave duality describing something familiar to that?

What is interesting regarding physics-related, real energy finite signals (found in our universe) is that such signals are not only time dependent functions, but in most of cases well-integrated and compact, space-time dependent. However, from our point of view (related to measurements and observations in certain space-location) we often see signals as only time or space dependent functions. In fact, most of the above presented mathematical relations (starting from (4.0.1) to T.4.0.1) could equally and reasonably exist (having physics-related meaning) if we simply replace a time variable $\mathbf{t}$ with corresponding space variable $\mathbf{x}$, as for instance: $\mathbf{t} \rightarrow \mathbf{x}$, $\omega = \omega_t = 2\pi\ \mathbf{f} \rightarrow \omega_x = 2\pi \mathbf{f}_x$, $\Psi(\mathbf{t}) \rightarrow \Psi(\mathbf{x})$. *The reason for such time-space symmetry, and integration is that whichever wave function, found as a good model for representing certain sufficiently stable particle, energy state or other, relatively stable (non-transient and non-dispersive) motion, should intrinsically have a structure that takes care about its space and time parameters` matching and integrity. This is creating mutual harmony and unity between them (usually expressed by simple mathematical relations between relevant time and space related parameters).* If this has not been the case in our universe, we would not have time-space stable objects and recognizable patterns of different motions, which are respecting basic conservation laws of Physics (see later (4.0.46)). It will be shown later that mentioned time-space integrity, stability and synchronization between relevant time, space and their frequency intervals is also closely related to relevant group and phase velocity, and to optimal time and space mutually dependent signal sampling intervals known in Signal Analysis in relation to Shannon-Kotelnikov signal sampling theorem. Analysis of that kind also leads to an extension, revision and generalization of Uncertainty Relations, originating in Physics from W. Heisenberg.

The unfortunate event (regarding quantum physics) is that Dennis Gabor, the inventor of the Analytic Signal concept, came too late with his invention, after the step stones of Quantum Theory have been "strongly established" and celebrated founders of Quantum Theory (maybe) were already too tired and not ready to make new house redesign. In addition, it could probably be embarrassing to admit that few of the Nobel prizes were already associated to something that maybe was not the best possible and brilliant creation…and in order to keep established harmony the most appropriate was to continue as nothing serious happened. A little bit of hocus-pocus, magic and mystery (partially formulated by present Quantum Theory) has anyway been compatible to the western ideology and state of its subconscious mind. In addition, most of the followers of the Orthodox Quantum Theory teachings are proudly asking themselves and suggesting to the others: Where the problem is, taking into account that Quantum Theory is mathematically working very well (similar to Ptolemy geocentric system which resisted as the unique and most accurate teaching for very long time).

4.0.5. Wave Packets and mathematical strategies in formulating Wave Velocities

Let us first introduce generic wave properties regarding most elementary, simple harmonic waves as usually modeled in physics.

Oscillations at a particular point (only time dependent) can be presented as,

$$\psi(t) = a \cdot \cos 2\pi ft = a \cdot \cos \omega t \tag{4.0.6}$$

Traveling waves (in one-dimensional motion) are characterized by,

$$\begin{aligned}
&\psi(t, x) = a \cdot \cos 2\pi\ f(t - \frac{x}{u}) = a \cdot \cos 2\pi(ft - \frac{x}{\lambda}) = a \cdot \cos(\omega t - kx) \\
&\omega = 2\pi f \ (=) \text{ angular frequency}, \ k = \frac{2\pi}{\lambda} \ (=) \text{ wave number}, \\
&u \ (=) \text{ phase velocity},
\end{aligned} \tag{4.0.7}$$

and traveling waves as 3-dimensional motion,

$$\psi(t, x) = a \cdot \cos(\omega t - k\vec{r}), \ \vec{r} = \vec{r}(x, y, z) \ . \tag{4.0.8}$$

All other, more complex waveforms or wave packets are presentable as integral or discrete superposition of elementary, simple harmonic waves, given by (4.0.1).

If wave velocity of certain waveform (or wave packet) is independent of wavelength, each elementary wave (and thus the wave packet) travels at the same speed.

If wave velocity depends upon wavelength, each elementary wave travels at a different speed, compared to the wave packet speed (or group speed).

The general condition (regarding an arbitrary waveform) for extracting the waveform phase velocity, $\mathbf{u}$, is that signal phase function would become constant, meaning that we would be able to travel parallel with signal phase, seeing always the same phase point (being linked to the same phase value: or we would not see that signal carrier function $\cos\varphi(t)$ is propagating or being time dependent. Of course, the first step would be to present the waveform as an Analytic Signal function). Satisfying such condition will mean that we are also traveling parallel to a wave in question, with wave's phase velocity. Mathematically, this could be summarized as,

$$\left\{\begin{aligned} &\varphi(t) \to \varphi(t, x) = \text{const.} \\ &\omega t \to \omega t - kx \\ &\varphi^{'}(t, x) = \varphi^{''}(t, x) = \text{const.} \end{aligned}\right\} \Rightarrow \left\{\frac{dx}{dt}, \frac{\Delta x}{\Delta t}\right\} (= u = \text{phase velocity}). \tag{4.0.9}$$

Group velocity, $\mathbf{v}$, is the velocity of the signal amplitude or its envelope function $\mathbf{a(t)}$. Now the same situation could be visualized if we imagine that we are traveling parallel to signal amplitude and we would always see only one point of the amplitude function (for instance signal envelope peak value).

$$\left\{\begin{array}{l} \mathbf{a(t) \to a(t,x) = const.} \\ \omega \mathbf{t} \to \omega \mathbf{t - kx} \\ \mathbf{a^{'}(t,x) = a^{''}(t,x) = const.} \end{array}\right\} \Rightarrow \left\{\frac{\mathbf{dx}}{\mathbf{dt}}, \frac{\partial \mathbf{x}}{\partial \mathbf{t}}\right\} \mathbf{(= v = group\ velocity)}, \tag{4.0.10}$$

We could also create new analytic signal form, which is equal only to the signal amplitude function, and apply similar method as in case of phase velocity (here we are treating the signal amplitude function as a new wave function, which would have its own, newly calculated phase and amplitude functions).

$$\left\{\begin{array}{l} \mathbf{a(t) \to a(t,x) = (\sqrt{a^2(t,x) + [H(a(t,x))]^2})\cos\left[\arctan\frac{H(a(t,x))}{a(t,x)}\right] =} \\ \mathbf{= a_1(t,x)\cos\varphi_1(t,x);\ \ \varphi_1(t,x) = const.,}\quad \omega \mathbf{t} \to \omega \mathbf{t - kx} \\ \mathbf{a_1(t,x) = \sqrt{a^2(t,x) + [H(a(t,x))]^2},\ \varphi_1(t,x) = \arctan\frac{H(a(t,x))}{a(t,x)}} \end{array}\right\} \Rightarrow \tag{4.0.11}$$

$$\Rightarrow \left\{\frac{\mathbf{dx}}{\mathbf{dt}}, \frac{\partial \mathbf{x}}{\partial \mathbf{t}}, \frac{\Delta \mathbf{x}}{\Delta \mathbf{t}}\right\}_1 \mathbf{(\Rightarrow v = group\ velocity)}\ .$$

Let us now compare the "**external and internal**" signal structure in a signal time-domain, (4.0.1). *Internally*, (inside of the integral) we have the infinitesimal superposition of elementary waveforms,

$$\mathbf{U_c(\omega)\cos\omega t + U_s(\omega)\sin\omega t = A(\omega)\cos(\omega t + \Phi(\omega))}, \tag{4.0.12}$$

Externally (after integration) we have a similar analytic signal form, $\mathbf{\Psi(t) = a(t)\cos\varphi(t)}$. In the following several steps (1°, 2°, 3° …), it will be shown how group and phase velocities can be found, and what the consequences regarding modeling elementary waveforms that are building elements of other more complex waveforms, are. Terms "**internal and external**" signal structure are conditionally introduced in this paper, before we find terminology that is more appropriate.

1°

In both cases (time-wise and frequency-wise: (4.0.1) – (4.0.3)) there is an amplitude function ($\mathbf{A(\omega)}$ or $\mathbf{a(t)}$) and a phase function as an argument of the cosine function ($\cos(\omega t + \Phi(\omega)) = \cos\Phi(\omega,t)$ or $\mathbf{\cos\varphi(t)}$), and we could say that both, "**internal and external**" signal waveforms are presenting Analytic Signal forms (basically being created on the same way and looking mutually similar).

For the signal integrity and its existence as the lasting traveling wave form it would be necessary that group and phase velocities of "**internal and external**" wave functions are mutually equal (in other words, here we will analyze only non-dispersive traveling waves).

Since the integration of $\mathbf{A(\omega)\cos(\omega t + \Phi(\omega))}$ is made taking into account only frequency ω (not time) it is almost obvious that *the wave function* $\mathbf{A(\omega)\cos(\omega t + \Phi(\omega))}$ *and wave function* $\mathbf{a(t)\cos\varphi(t)}$ *should have the same velocities*. Of course, all wave forms in question should be conveniently presented in time and space coordinates (briefly: $\omega \mathbf{t} \to \omega \mathbf{t - kx}$), what is applicable in all cases of non-dispersive traveling waves.

2°

We can also compare the "**external and internal**" signal structure, (4.0.1) – (4.0.3), in a signal frequency domain in order to address differently the group and phase wave velocity. This time we will take the complex signal forms in order to have more condensed mathematical expressions and easier comparison.

$$\begin{aligned}&\overline{\mathbf{U}}(\omega) = \mathbf{A}(\omega)\mathbf{e}^{-j\Phi(\omega)} = \mathbf{U}_c(\omega) - j\mathbf{U}_s(\omega) = \int_{-\infty}^{+\infty}\overline{\Psi}(\mathbf{t})\mathbf{e}^{j\omega t}\,\mathbf{dt} = \int_{-\infty}^{+\infty}\mathbf{a}(\mathbf{t})\mathbf{e}^{j(\omega t+\varphi(t))}\,\mathbf{dt}\\ &\overline{\Psi}(\mathbf{t}) = \mathbf{a}(\mathbf{t})\mathbf{e}^{-j\varphi(t)} = \Psi(\mathbf{t}) + j\hat{\Psi}(\mathbf{t}) = \frac{1}{\pi}\int_0^{\infty}\mathbf{U}(\omega)\mathbf{e}^{j\omega t}\,\mathbf{d}\omega = \frac{1}{\pi}\int_0^{\infty}\mathbf{A}(\omega)\mathbf{e}^{j(\omega t+\Phi(\omega))}\,\mathbf{d}\omega\end{aligned} \tag{4.0.13}$$

The formal analogy is obvious: both complex signal forms, $\overline{\mathbf{U}}(\omega)$ and $\overline{\Psi}(\mathbf{t})$ (one in the frequency domain, and the other in the time domain, representing the same signal or the same wave) should have the same wave velocities; - the phase functions, $\Phi(\omega)$ and $\varphi(\mathbf{t})$, should generate a phase velocity **u**, and the amplitude functions, $\mathbf{A}(\omega)$ and $\mathbf{a}(\mathbf{t})$ should generate a group velocity **v**. Also, we see again that all "**internal and external**" signal functions are clearly presenting the structure of Analytic Signal forms (of course, differently formulated). Before we start searching for mathematical expressions of wave velocities, all wave forms in question should be extended to have time and space coordinates, or to represent traveling and non-dispersive waveforms (briefly: $\omega t \to \omega t - \mathbf{kx},\ \omega = \omega(\mathbf{k})$).

Obviously, the forms of group and phase wave velocities should equally (and analogically) be presentable in terms of ordinary time-space (**t, x**) variables, and in terms of spectral variables (ω, **k**), $\omega = \omega(\mathbf{k}) = \omega_t = 2\pi \mathbf{f}_t = 2\pi\mathbf{f}$, $\mathbf{k} = \omega_x = 2\pi\mathbf{f}_x = \frac{2\pi}{\lambda}$,

$$\left\{\begin{array}{l}\varphi(\mathbf{t},\mathbf{x}) = \mathbf{const.}\\ \omega t \to \omega t - \mathbf{kx},\ \omega = \omega(\mathbf{k})\\ \Phi(\omega,\mathbf{k}) = \mathbf{Const.}\end{array}\right\} \Rightarrow \left\{\frac{\mathbf{dx}}{\mathbf{dt}}, \frac{\Delta\mathbf{x}}{\Delta\mathbf{t}}, \frac{\omega}{\mathbf{k}}\right\}(=)\mathbf{u} = \mathbf{phase\ velocity,} \tag{4.0.14}$$

$$\left\{\begin{array}{l}\mathbf{a}(\mathbf{t},\mathbf{x}) = \mathbf{const.}\\ \omega t \to \omega t - \mathbf{kx},\ \omega = \omega(\mathbf{k})\\ \mathbf{A}(\omega,\mathbf{k}) = \mathbf{Const.}\end{array}\right\} \Rightarrow \left\{\frac{\mathbf{dx}}{\mathbf{dt}}, \frac{\partial\mathbf{x}}{\partial\mathbf{t}}, \frac{\mathbf{d}\omega}{\mathbf{dk}}\right\}(=)\mathbf{v} = \mathbf{group\ velocity}. \tag{4.0.15}$$

3°

Let us now unite the wave-velocities search strategies for "**external and internal**" signal structures, both in time and frequency signal domains. We will now create the simplest possible wave group that is composed only of two elementary waveforms that are mutually infinitesimally close (where closeness is measured by small differences between their space, time and frequency variables), and find its phase and group velocity. By the nature of mathematical formulation of all Analytic Signals, we can say that amplitude or envelope functions $\mathbf{a}(\mathbf{t},\mathbf{x})$ and $\mathbf{A}(\omega,\mathbf{k})$ are placed in the lower frequency spectrum area (being slowly evolving), compared to carrier or phase functions $\mathbf{e}^{j(\omega t - kx + \varphi(t,x))}$ and $\mathbf{e}^{j(\omega t - kx + \Phi(\omega,k))}$ (and by the way it is also good time to say, this is also well-known property of analytic signals, not analyzed here). Consequently, we have a chance to simplify determination of group and phase velocity, since when making superposition of two infinitesimally close wave elements we will be able to consider that their amplitude functions remain constant (and that only signal carrier phase functions are significant variables). Practically, instead of integrating in total integral limits, we will take only two of sub-integral elementary wave forms (both in time and frequency domains), that are mutually infinitesimally close, and find what their superposition will create,

$$\left\{\begin{array}{l} \overline{y}(t,x) = a(t,x)e^{j(\omega t-kx+\varphi(t,x))} = a(t,x)e^{j\Theta(t,x)} \\ \overline{Y}(\omega,k) = A(\omega,k)e^{j(\omega t-kx+\Phi(\omega,k))} = A(\omega,k)e^{j\Theta(\omega,k)} \\ \Theta(t,x) = \omega t - kx + \varphi(t,x) \\ \Theta(\omega,k) = \omega t - kx + \Phi(\omega,k) \\ \overline{U}(\omega,k) = A(\omega,k)e^{-j\Phi(\omega)} = \int_{-\infty}^{+\infty} a(t,x)e^{j(\omega t-kx+\varphi(t))}\,dt = \int_{-\infty}^{+\infty} \overline{y}(t,x)\cdot dt \\ \overline{\Psi}(t,x) = a(t,x)e^{-j\varphi(t)} = \frac{1}{\pi}\int_{0}^{\infty} A(\omega,k)e^{j(\omega t-kx+\Phi(\omega))}\,d\omega = \frac{1}{\pi}\int_{0}^{\infty}\overline{Y}(\omega,k)\cdot d\omega \end{array}\right\} \Rightarrow \quad (4.0.16)$$

$$\begin{aligned} \overline{\Psi}_{1+2} &= \frac{1}{2}\left[\vec{y}_1(t,x) + \overline{y}_2(t,x)\right] = \frac{1}{2}a(t,x)\cdot\left\{e^{j[\Theta(t,x)-d\Theta(t,x)]} + e^{j[\Theta(t,x)+d\Theta(t,x)]}\right\} \\ \overline{U}_{1+2} &= \frac{1}{2}\left[\overline{Y}_1(\omega,k) + \overline{Y}_2(\omega,k)\right] = \frac{1}{2}A(\omega,k)\cdot\left\{e^{j[\Theta(\omega,k)-d\Theta(\omega,k)]} + e^{j[\Theta(\omega,k)+d\Theta(\omega,k)]}\right\} \end{aligned} \quad (4.0.17)$$

$$\begin{aligned} \overline{\Psi}_{1+2} &= \frac{1}{2}a(t,x)e^{j\Theta(t,x)}\cdot\left[e^{-jd\Theta(t,x)} + e^{+jd\Theta(t,x)}\right] = a(t,x)e^{j\Theta(t,x)}\cdot\cos\left[d\Theta(t,x)\right] = \\ &= a(t,x)e^{j\varphi(t,x)}\cdot\cos\left[d\Theta(t,x)\right]\cdot e^{j(\omega t-kx)} = \overline{\Psi}(t,x)\cdot\cos\left[d\Theta(t,x)\right]\cdot e^{j(\omega t-kx)} = \\ &= \overline{\Psi}(t,x)\cdot\cos\left[d(\omega t-kx)\right]\cdot\cos\left[d\varphi(t,x)\right]\cdot e^{j(\omega t-kx)} \\ &-------------- \\ \overline{U}_{1+2} &= \frac{1}{2}A(\omega,k)e^{j\Theta(\omega,k)}\cdot\left[e^{-jd\Theta(\omega,k)} + e^{+jd\Theta(\omega,k)}\right] = A(\omega,k)e^{j\Theta(\omega,k)}\cdot\cos\left[d\Theta(\omega,k)\right] = \\ &= A(\omega,k)e^{j\Phi(\omega,k)}\cdot\cos\left[d\Theta(\omega,k)\right]\cdot e^{j(\omega t-kx)} = \overline{U}(\omega,k)\cdot\cos\left[d\Theta(\omega,k)\right]\cdot e^{j(\omega t-kx)} = \\ &= \overline{U}(\omega,k)\cdot\cos\left[d(\omega t-kx)\right]\cdot\cos\left[d\Phi(\omega,k)\right]\cdot e^{j(\omega t-kx)} \end{aligned} \quad (4.0.18)$$

$$\begin{aligned} \cos\left[d\Theta(t,x)\right] &= \cos\left\{d\left[\omega t - kx + \varphi(t,x)\right]\right\} = \\ &= \cos\left[d(\omega t-kx)\right]\cdot\cos\left[d\varphi(t,x)\right] - \sin\left[d(\omega t-kx)\right]\cdot\sin\left[d\varphi(t,x)\right] = \\ &= \cos\left[d(\omega t-kx)\right]\cdot\cos\left[d\varphi(t,x)\right] \end{aligned}$$

$$\begin{aligned} \cos\left[d\Theta(\omega,k)\right] &= \cos\left\{d\left[\omega t - kx + \Phi(\omega,k)\right]\right\} = \\ &= \cos\left[d(\omega t-kx)\right]\cdot\cos\left[d\Phi(\omega,k)\right] - \sin\left[d(\omega t-kx)\right]\cdot\sin\left[d\Phi(\omega,k)\right] = \\ &= \cos\left[d(\omega t-kx)\right]\cdot\cos\left[d\Phi(\omega,k)\right] \end{aligned}$$

General conditions to be satisfied (regarding (4.0.18)) in order to find phase velocity are: $\varphi(\mathbf{t},\mathbf{x}) = \mathbf{const.}$ and $\Phi(\omega,\mathbf{k}) = \mathbf{Const.}$, making that $\mathbf{d}\varphi(\mathbf{t},\mathbf{x}) = \mathbf{0}$ and $\mathbf{d}\Phi(\omega,\mathbf{k}) = \mathbf{0}$, what simplifies the expressions of elementary waveforms in time and frequency domains,

$$\left\{\begin{array}{l} \overline{\Psi}_{1+2} = \overline{\Psi}(\mathbf{t},\mathbf{x}) \cdot \cos[\mathbf{d}(\omega\mathbf{t} - \mathbf{kx})] \cdot \mathbf{e}^{\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})} \\ ---------------- \\ \overline{\mathbf{U}}_{1+2} = \overline{\mathbf{U}}(\omega,\mathbf{k}) \cdot \cos[\mathbf{d}(\omega\mathbf{t} - \mathbf{kx})] \cdot \mathbf{e}^{\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})} \end{array}\right\} \Leftrightarrow \left\{ \frac{\overline{\Psi}_{1+2}}{\overline{\mathbf{U}}_{1+2}} = \frac{\overline{\Psi}(\mathbf{t},\mathbf{x})}{\overline{\mathbf{U}}(\omega,\mathbf{k})} \right\} \qquad (4.0.19)$$

Obviously, the only common carrier frequency, phase function member in both elementary wave functions ($\overline{\Psi}_{1+2}$ and $\overline{\mathbf{U}}_{1+2}$) is $\mathbf{e}^{\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}$, and their velocities should be the signal phase velocity, found as usually (when the phase is constant),

$$\begin{array}{l} \omega\mathbf{t} - \mathbf{kx} = \mathbf{const} \Leftrightarrow \omega\mathbf{t}_1 - \mathbf{kx}_1 = \omega\mathbf{t}_2 - \mathbf{kx}_2 \Leftrightarrow \omega(\mathbf{t}_2 - \mathbf{t}_1) - \mathbf{k}(\mathbf{x}_2 - \mathbf{x}_1) = \mathbf{0} \Leftrightarrow \\ \Leftrightarrow \omega\Delta\mathbf{t} - \mathbf{k}\Delta\mathbf{x} = \mathbf{0} \Leftrightarrow \dfrac{\Delta\mathbf{x}}{\Delta\mathbf{t}} = \dfrac{\omega}{\mathbf{k}} \end{array} \qquad (4.0.20)$$

Since in this analysis we started creating the superposition of two infinitesimally close elementary waves, (4.0.16), it is obvious that their phase velocity should be found as,

$$\mathbf{u} = (\lim \frac{\Delta\mathbf{x}}{\Delta\mathbf{t}})_{\Delta\mathbf{t}\to\mathbf{0}} = \frac{\mathbf{dx}}{\mathbf{dt}} = \frac{\omega}{\mathbf{k}} = \mathbf{phase\ velocity}\,. \qquad (4.0.21)$$

From the other common member, $\cos[\mathbf{d}(\omega\mathbf{t} - \mathbf{kx})]$, of two elementary wave forms ($\overline{\Psi}_{1+2}$ and $\overline{\mathbf{U}}_{1+2}$), we would be able to find the group velocity, again using the same constant phase argument as in the case of phase velocity,

$$\begin{array}{l} \omega\mathbf{t} - \mathbf{kx} = \mathbf{const} \Leftrightarrow \mathbf{d}(\omega\mathbf{t} - \mathbf{kx}) = \mathbf{0} \Leftrightarrow \mathbf{k}(\dfrac{\omega}{\mathbf{k}} - \dfrac{\mathbf{dx}}{\mathbf{dt}})\mathbf{dt} - (\mathbf{x} - \mathbf{t}\dfrac{\mathbf{d}\omega}{\mathbf{dk}})\mathbf{dk} = \mathbf{0} \Leftrightarrow \\ \Leftrightarrow \left[\dfrac{\mathbf{dx}}{\mathbf{dt}} = \dfrac{\omega}{\mathbf{k}} = \mathbf{phase\ velocity}\right] \mathbf{and} \left[\mathbf{x} - \mathbf{t}\dfrac{\mathbf{d}\omega}{\mathbf{dk}} = \mathbf{0}\right] \Rightarrow \mathbf{x} = \mathbf{t}\dfrac{\mathbf{d}\omega}{\mathbf{dk}} \Rightarrow \\ \Rightarrow \mathbf{v} = \dfrac{\partial\mathbf{x}}{\partial\mathbf{t}} = \dfrac{\mathbf{d}\omega}{\mathbf{dk}} = \mathbf{group\ velocity} \end{array} \qquad (4.0.22)$$

It is also possible to find the functional connection between phase and group velocity in the following form,

$$\mathbf{v} = \mathbf{u} + \mathbf{k}\frac{\mathbf{du}}{\mathbf{dk}} = \frac{\mathbf{d}\omega}{\mathbf{dk}}\,. \qquad (4.0.23)$$

Of course, we could immediately find the elementary sub-integral, waveforms as,

$$\left\{\begin{array}{l}\overline{\Psi}_{1+2}=\overline{\Psi}(\mathbf{t},\mathbf{x})\cdot\cos\left[\mathbf{d}(\omega\mathbf{t}-\mathbf{kx})\right]\cdot\mathbf{e}^{\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}=\overline{\Psi}(\mathbf{t},\mathbf{x})\cdot\mathbf{e}^{\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}=\mathbf{a}(\mathbf{t},\mathbf{x})\cdot\mathbf{e}^{\mathbf{j}\varphi(\mathbf{t},\mathbf{x})}\cdot\mathbf{e}^{\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}\\ ---------------\\ \overline{\mathbf{U}}_{1+2}=\overline{\mathbf{U}}(\omega,\mathbf{k})\cdot\cos\left[\mathbf{d}(\omega\mathbf{t}-\mathbf{kx})\right]\cdot\mathbf{e}^{\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}=\overline{\mathbf{U}}(\omega,\mathbf{k})\cdot\mathbf{e}^{\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}=\mathbf{A}(\omega,\mathbf{k})\cdot\mathbf{e}^{\mathbf{j}\Phi(\omega,\mathbf{k})}\cdot\mathbf{e}^{\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}\end{array}\right\}\Leftrightarrow$$

$$\Leftrightarrow\left\{\begin{array}{l}\overline{\Psi}(\mathbf{t},\mathbf{x})=\overline{\Psi}_{1+2}\cdot\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}=\mathbf{a}(\mathbf{t},\mathbf{x})\cdot\mathbf{e}^{\mathbf{j}\varphi(\mathbf{t},\mathbf{x})}\\ \overline{\mathbf{U}}(\omega,\mathbf{k})=\overline{\mathbf{U}}_{1+2}\cdot\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}=\mathbf{A}(\omega,\mathbf{k})\cdot\mathbf{e}^{\mathbf{j}\Phi(\omega,\mathbf{k})}\end{array}\right\}\Leftrightarrow\left\{\frac{\overline{\Psi}_{1+2}}{\overline{\mathbf{U}}_{1+2}}=\frac{\overline{\Psi}(\mathbf{t},\mathbf{x})}{\overline{\mathbf{U}}(\omega,\mathbf{k})}\right\},$$

$$\begin{array}{l}\overline{U}(\omega,k)=A(\omega,k)e^{-j\Phi(\omega,k)}=\int_{-\infty}^{+\infty}\overline{y}(t,x)\,dt=\int_{-\infty}^{+\infty}\overline{\Psi}_{1+2}\,dt=\int_{-\infty}^{+\infty}\overline{\Psi}(t,x)\cdot e^{j(\omega t-kx)}\,dt\\ \overline{\Psi}(t,x)=a(t,x)e^{-j\varphi(t,x)}=\frac{1}{\pi}\int_0^{\infty}\overline{Y}(\omega,k)\cdot d\omega=\frac{1}{\pi}\int_0^{\infty}\overline{U}_{1+2}\cdot d\omega=\frac{1}{\pi}\int_0^{\infty}\overline{U}(\omega,k)\cdot e^{j(\omega t-kx)}\cdot d\omega.\end{array}\tag{4.0.24}$$

but going too fast towards the final result, we would not be able to find the expression for the group velocity. The group velocity here sounds a little bit strange, since the amplitude modulating function is $\cos\left[\mathbf{d}(\omega\mathbf{t}-\mathbf{kx})\right]=1$, making a signal amplitude flat. We should not forget that here we started with the superposition of two infinitesimally close elementary waveforms, what was the important condition to treat both of their amplitude functions as virtually constant (and if this was not the case some other amplitude modulating function such as $\cos\left[\Delta(\omega\mathbf{t}-\mathbf{kx})\right]$ would materialize, and have much bigger influence on the signal amplitude). We have here also the proof that the superposition of only two elementary waveforms (resulting in $\overline{\Psi}_{1+2}$ and $\overline{\mathbf{U}}_{1+2}$) is sufficiently good for representing the original waveform when searching for group and phase velocity of the integral waveform. This is possible since the original signal phase and amplitude are not lost in this process (both in time and frequency terms). This way we are also confirming that "**external and internal**" signal structure, in time and frequency domain, have the same wave velocities (where the original waveform is equal to the superposition result of an infinite number of such elementary waveforms).

4°

Since we already know that sub-integral elementary waveform would generate the same wave velocities as the original waveform (here we already applied the names: internal and external waveforms), we can construct another way of finding expressions for wave velocities. Let us consider that the wave function $\Psi(\mathbf{t})$ is the narrow frequency band wave-packet, or wave-group, which should be a wave-model of certain moving-particle. In other words, the wave-packet $\Psi(\mathbf{t})$ presents the (discrete or infinitesimal) superposition of numbers of mutually similar and narrow-zone concentrated elementary waves $\mathbf{y(t)}=\mathbf{A}(\omega)\cos(\omega\mathbf{t}+\Phi(\omega))$. By introducing the assumption that the frequency-band of the wave-packet $\omega\in\Omega$ is very narrow, we would be in a position to treat the signal amplitude function $\mathbf{A}(\omega)$ as approximately constant, $\mathbf{A}(\omega)\cong\mathbf{A}(\omega_0)\cong\mathbf{const.}$ in the area around central frequency point $\omega_0\in\Omega$. Under given conditions we will be able to make the following approximations,

$$\left\{\begin{array}{l}\Psi(t) = \frac{1}{\pi}\int_0^{\infty}\left[U_c(\omega)\cos\omega t + U_s(\omega)\sin\omega t\right]d\omega = \\ = \frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\cos(\omega t + \Phi(\omega))\right]d\omega = a(t)\cos\varphi(t)\end{array}\right\} \Rightarrow$$

$$\begin{array}{l}\Psi(t) = a(t)\cos\varphi(t) = \frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\cos(\omega t + \Phi(\omega))\right]d\omega \cong \\ \cong \frac{A(\omega_0)}{\pi}\int_{[\Omega]}\left[\cos(\omega t + \Phi(\omega))\right]d\omega \cong \\ \cong \frac{\Omega}{\pi}A(\omega_0)\sum_{[\omega_i\in\Omega]}\cos(\omega_i t + \Phi(\omega_i)) = \frac{\Omega}{\pi}A(\omega_0)\sum_{[\omega_i\in\Omega]}\cos\Theta(\omega_i) \\ \Theta(\omega) = \omega t + \Phi(\omega)\end{array} \tag{4.0.25}$$

Now, since we already know that the superposition of only two elementary waveforms would carry the information about characteristic signal velocities, we can accelerate this process assuming that the total wave-packet has only two elementary waveforms,

$$\begin{array}{l}\Psi(t) = a(t)\cos\varphi(t) \cong \frac{A(\omega_0)}{\pi}\int_{[\Omega]}\left[\cos(\omega t + \Phi(\omega))\right]d\omega \cong \frac{\Omega}{\pi}A(\omega_0)\sum_{[\omega_i\in\Omega]}\cos\Theta(\omega_i) \Rightarrow \\ \sum_{[\omega_i\in\Omega]}\cos\Theta(\omega_i) \Leftrightarrow \cos\left[\Theta(\omega_0) - \delta\Theta\right] + \cos\left[\Theta(\omega_0) + \delta\Theta\right] = \cos\left[\Theta_0 - \delta\Theta\right] + \cos\left[\Theta_0 + \delta\Theta\right] = \\ = 2\cos\delta\Theta\cdot\cos\Theta_0\ ,\ \frac{\Omega}{\pi}A(\omega_0) = const.\end{array} \tag{4.0.26}$$

Let us now extend the signal to have all other space and frequency related variables,

$$\begin{array}{l}\omega t \to \omega t - kx;\ \omega = \omega(k);\ \Theta(\omega) = \omega t + \Phi(\omega) \to \Theta(\omega,k) = \omega t - kx + \Phi(\omega,k) \Rightarrow \\ 2\cos\delta\Theta\cdot\cos\Theta_0 = 2\cos\left\{\delta\left[\omega t - kx + \Phi(\omega,k)\right]\right\}\cdot\cos\left[\omega_0 t - k_0 x + \Phi(\omega_0, k_0)\right] = \\ = 2\cos\left[k\left(\frac{\omega}{k} - \frac{\delta x}{\delta t}\right)\cdot\delta t + \left(x - t\frac{\delta\omega}{\delta k}\right)\cdot\delta k + \left(\frac{\delta\omega}{\delta k} - \frac{\delta x}{\delta t}\right)\delta k\cdot\delta t\right]\cdot\cos\left[\omega_0 t - k_0 x + \Phi(\omega_0, k_0)\right],\end{array}$$

$$\left\{\begin{array}{l}\delta\left[\omega t - kx + \Phi(\omega,k)\right] = (\omega\cdot\delta t + t\cdot\delta\omega) + \delta\omega\cdot\delta k - (k\cdot\delta x + x\cdot\delta k) - \delta k\cdot\delta x + \delta\Phi(\omega,k) = \\ = k\left(\frac{\omega}{k} - \frac{\delta x}{\delta t}\right)\cdot\delta t + \left(x - t\frac{\delta\omega}{\delta k}\right)\cdot\delta k + \left(\frac{\delta\omega}{\delta k} - \frac{\delta x}{\delta t}\right)\delta k\cdot\delta t;\ \Phi(\omega,k) = const. \Leftrightarrow \delta\Phi(\omega,k) = 0\end{array}\right\} \tag{4.0.27}$$

The wave function $2\cos\delta\Theta\cdot\cos\Theta_0$ has the form of an amplitude-modulated elementary wave, where $\cos\Theta_0$ presents the carrier-function and $2\cos\delta\Theta$ presents its amplitude function. Both, amplitude and carrier function have different phases, and consequently they should have different, but mutually related velocities: where velocity of the amplitude function will be the group-velocity = **v** and velocity of the carrier function will be the phase-velocity = **u**.

Now we can find the phase velocity, as usual, (when phase of the carrier-function is constant),

$$\omega_0 t - k_0 x + \Phi(\omega_0, k_0) = \text{const.} \Leftrightarrow u = \frac{dx}{dt} = \frac{\omega_0}{k_0} = \frac{\omega}{k} = \text{phase velocity}, \ (\omega, \omega_0) \in \Omega, \quad (4.0.28)$$

In addition, from the constant-phase of the amplitude signal-member we will be able to find the group velocity as,

$$\begin{aligned} & k\left(\frac{\omega}{k} - \frac{\delta x}{\delta t}\right) \cdot \delta t + \left(x - t\frac{\delta \omega}{\delta k}\right) \cdot \delta k + \left(\frac{\delta \omega}{\delta k} - \frac{\delta x}{\delta t}\right)\delta k \cdot \delta t = \text{Const.} \Leftrightarrow \\ & \frac{\omega}{k} = \frac{\delta x}{\delta t} = \frac{dx}{dt} = u, \ \left(x - t\frac{\delta \omega}{\delta k}\right) \cdot \delta k + \left(\frac{\delta \omega}{\delta k} - \frac{\delta x}{\delta t}\right)\delta k \cdot \delta t = \text{Const.} \Rightarrow \\ & \Rightarrow \frac{\delta \omega}{\delta k} = \frac{\delta x}{\delta t} = \frac{dx}{dt} = v = \text{group velocity} \end{aligned} \quad (4.0.29)$$

5°

What we can conclude and summarize from already presented steps (from 1° to 4°) regarding waveform velocities is:

a) That the most elementary (and non-dispersive) waves, which are building elements of all other waveforms we know in physics, should have forms of certain simple harmonic, sinusoidal functions both in time and space coordinates.

b) That there is a big level of structural symmetry and integrity regarding constructing well-operating mathematical models of wave functions in time, space or time-space domains ($\Psi(t), \Psi(x), \Psi(t,x)$), and that similar symmetry is also extending to time-domain-frequency and space-domain-frequency spectral signal-functions ($A(\omega), A(k), A(\omega,k)$ and $\Phi(\omega), \Phi(k), \Phi(\omega,k)$). Mentioned symmetry is particularly well-exposed if Analytic Signals modeling is applied, as for instance: Every waveform element, or most elementary signal form of certain more complex waveform, "**Internally and Externally**", is carrying all important information regarding waveform velocities, both in time and frequency domains, where "**internally and externally**" has the same meaning already explained in earlier steps. In other words saying the same, it is clear that all time-space and frequency parameters of relevance for conceptualizing different wave motions in the world of Physics are mutually dependent and well-united, as for instance: $x = x(t), \ \omega = \omega(k), \ v = v(x, \omega)...$

c) That the total signal energy is carried only by signal amplitude-function either in time of frequency domain, which is propagating by group velocity.

d) That the only sufficiently narrow, time-space-frequency limited and finite energy, elementary (band-limited in all domains) signal forms are of the biggest relevance in wave motions analyses. Such building blocks are synthesizing all other more complex waveforms. It seems that the highest preferences of Nature are in communicating between all relatively stable energy-carrying states using such band-limited elementary waves.

Let us now respect here summarized facts (a, b, c and d) and construct the following, elementary and band-limited wave-packet,

$$\Psi(t,x) = a(t,x)\cos\varphi(t,x) = \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega}\left[A(\omega)\cos(\omega t - kx + \Phi(\omega))\right]d\omega =$$
$$= \frac{1}{\pi}\int_{k_0-\Delta k}^{k_0+\Delta k}\left[A(k)\cos(\omega t - kx + \Phi(k))\right]dk \; . \tag{4.0.30}$$

Since we are analyzing a band-limited and finite, very narrow elementary wave-packet, we would be able to express the dispersion function $\omega = \omega(k)$ in the close vicinity of $\omega_0, k_0; \omega_0 = \omega(k_0)$ as,

$$\omega(k) = \omega(k_0) + (k - k_0)(\frac{d\omega}{dk})_{(k=k_0)} + \frac{1}{2}(k-k_0)^2(\frac{d^2\omega}{dk^2})_{(k=k_0)} + ... \cong$$
$$\cong \omega(k_0) + (k - k_0)(\frac{d\omega}{dk})_{(k=k_0)} \; . \tag{4.0.31}$$

Now, the elementary and band-limited wave-packet can be found as,

$$\Psi(t,x) = \frac{A(k_0)}{\pi}\int_{k_0-\Delta k}^{k_0+\Delta k}\cos\left[\omega t + (k-k_0)(\frac{d\omega}{dk})_0 \cdot t - kx + \Phi(k_0)\right]dk =$$
$$= \frac{2A(k_0)}{\pi}\Delta k \frac{\sin \Delta k \cdot \left[(\frac{d\omega}{dk})_0 \cdot t - x + \Phi(k_0)\right]}{\Delta k \cdot \left[(\frac{d\omega}{dk})_0 \cdot t - x + \Phi(k_0)\right]}\cos(\omega_0 t - k_0 x) = a(t,x)\cos\varphi(t,x) \, , \tag{4.0.32}$$
$$\frac{2A(k_0)}{\pi}\Delta k = \text{constant}, \; \Phi(k_0) = \text{CONST.}, \cos(\omega_0 t - k_0 x) = \cos\varphi(t,x).$$

The constant phase of the carrier function will generate the phase velocity, and the constant phase of the amplitude function will generate the group velocity (as in all of the cases presented earlier),

$$\omega_0 t - k_0 x = \text{const.} \Rightarrow u = \frac{dx}{dt} = \frac{\omega_0}{k_0} = \text{phase velocity}$$
$$(\frac{d\omega}{dk})_0 \cdot t - x + \Phi(k_0) = \text{Const.} \Rightarrow v = \frac{dx}{dt} = (\frac{d\omega}{dk})_0 = \text{group velocity}. \tag{4.0.33}$$

Almost the same example in different and mutually similar variants is very often present in most of the Quantum Theory books in the chapters explaining group and phase velocity. Here, it is a little bit more convincing, more general and clearer why, and when, conclusions based on such bands-limited elementary signals are correct. We can also show that any other, more complex or arbitrary (energy-finite) waveform can be presented as the superposition of the above found elementary signals. In Mathematics, modern Telecommunication Theory and Digital Signal Processing practice, we can find a number of methods and formulas for discrete signal representations or signals sampling. This means that time-continuous signals, or wave functions, could be fully represented (errorless, without residuals) if we just implement sufficiently short time-increments of signal sampling, and create discrete series of such signal-samples. For instance, if a continuous wave-function, $\Psi(t)$ is a frequency band-limited (and in the same time it should also be an energy-finite function), by applying Kotelnikov-Shannon sampling theorem, we can express it in terms of its samples-values $\Psi(n \cdot \delta t)$ as for instance (see **[8]**),

$$\begin{aligned}
&\Psi(t) = a(t)\cos\varphi(t) = \sum_{n=-\infty}^{+\infty} \Psi(n \cdot \delta t) \frac{\sin \Omega(t - n \cdot \delta t)}{\Omega(t - n \cdot \delta t)} = \\
&= \sum_{n=-\infty}^{+\infty} a(n \cdot \delta t) \frac{\sin\Omega\,(t - n \cdot \delta t)}{\Omega(t - n \cdot \delta t)} \cos\varphi(n \cdot \delta t),\ \Psi(n \cdot \delta t) = a(n \cdot \delta t)\cos\varphi(n \cdot \delta t),
\end{aligned} \tag{4.0.34}$$

Where Ω is the highest frequency in the spectrum of $\Psi(t)$, and we could also consider that Ω is the total frequency duration of the signal $\Psi(t)$.

Since sampling frequency-domain of signal-amplitude, Ω_L, is always in a lower frequency range than the frequency range of its phase function $\Omega = \Omega_H$, and since the total signal energy is captured only by the signal amplitude-function, we should also be able to present the same signal as:

$$\begin{aligned}
&\Psi(t) = a(t)\cos\varphi(t) = \sum_{n=-\infty}^{+\infty} a(n \cdot \delta t) \frac{\sin\Omega\,(t - n \cdot \delta t)}{\Omega(t - n \cdot \delta t)} \cos\varphi(n \cdot \delta t) = \\
&= \left[\sum_{n=-\infty}^{+\infty} a(n \cdot \delta t_L) \frac{\sin \Omega_L (t - n \cdot \delta t_L)}{\Omega_L (t - n \cdot \delta t_L)} \right] \cdot \left[\sum_{n=-\infty}^{+\infty} \cos\varphi(n \cdot \delta t) \frac{\sin \Omega_H (t - n \cdot \delta t)}{\Omega_H (t - n \cdot \delta t)} \right], \\
&\overline{\Psi}(t) = \sum_{n=-\infty}^{+\infty} a(n \cdot \delta t) \frac{\sin\Omega\,(t - n \cdot \delta t)}{\Omega(t - n \cdot \delta t)} e^{j\varphi(n \cdot \delta t)} = a(t)e^{j\varphi(t)},
\end{aligned} \tag{4.0.35}$$

$$\begin{aligned}
&a(t) = \sum_{n=-\infty}^{+\infty} a(n \cdot \delta t_L) \frac{\sin \Omega_L (t - n \cdot \delta t_L)}{\Omega_L (t - n \cdot \delta t_L)},\ \cos\varphi(t) = \sum_{n=-\infty}^{+\infty} \cos\varphi(n \cdot \delta t) \frac{\sin \Omega_H (t - n \cdot \delta t)}{\Omega_H (t - n \cdot \delta t)} \\
&\delta t \le \frac{\pi}{\Omega} = \frac{1}{2F} < \delta t_L\ ,\ \Omega = \Omega_H = 2\pi F = 2\pi F_H\, ,\ \delta t_L \le \frac{\pi}{\Omega_L} = \frac{1}{2F_L}\ ,\ \Omega_L = 2\pi F_L < \Omega.
\end{aligned} \tag{4.0.36}$$

As we can see by simple comparison, the sampled waveforms under the summation signs, given by (4.0.35) and (4.0.36), have the same form as an elementary wave-packet (4.0.32), what is giving much more weight and importance to Kotelnikov-Shannon sampling theorem (regarding modeling natural waveforms). *In fact, (4.0.35) and (4.0.36) are also defining kind of signals atomizing, packaging and formatting, what would become important later when we start making closer equivalency relations between wave-packets and moving-particles (see also (4.0.38)-(4.0.44)).*

Now, by applying analogy with already analyzed examples (regarding waveform velocities), we can express the same signal, (4.0.35), as the function of extended time-space variables as,

$$\left.\begin{cases} \mathbf{x}=\mathbf{x(t)},\ \omega=\omega(\mathbf{k}), \\ \omega\mathbf{t}\to\omega\mathbf{t}-\mathbf{kx},\ \omega\cdot\delta\mathbf{t}\to\omega\cdot\delta\mathbf{t}-\mathbf{k}\cdot\delta\mathbf{x}, \\ \Omega\mathbf{t}\to\Omega\mathbf{t}-\mathbf{Kx},\ \Omega\cdot\delta\mathbf{t}\to\Omega\cdot\delta\mathbf{t}-\mathbf{K}\cdot\delta\mathbf{x} \\ \Omega(\mathbf{t}-\mathbf{n}\cdot\delta\mathbf{t})\to\Omega(\mathbf{t}-\mathbf{n}\cdot\delta\mathbf{t})-\mathbf{Kx}=\mathbf{K}\cdot\left[(\frac{\delta\omega}{\delta\mathbf{k}})\cdot(\mathbf{t}-\mathbf{n}\cdot\delta\mathbf{t})-\mathbf{x}\right] \\ \mathbf{n}\cdot\delta\mathbf{t}\to\mathbf{n}\cdot\delta\mathbf{t}-\frac{\mathbf{K}}{\Omega}\mathbf{n}\cdot\delta\mathbf{x},\ \frac{\delta\omega}{\delta\mathbf{k}}=\frac{\Omega}{\mathbf{K}},\ \Omega=2\pi\mathbf{F},\ \mathbf{K}=2\pi\mathbf{F}_x \\ \delta\mathbf{t}\le\frac{\pi}{\Omega}=\frac{1}{2\mathbf{F}},\quad \delta\mathbf{f}\le\frac{1}{2\mathbf{T}},\ \delta\mathbf{x}\le\frac{1}{2\mathbf{K}} \end{cases}\right\}\Rightarrow \tag{4.0.37}$$

$$\begin{aligned}\Psi(\mathbf{t})&=\mathbf{a(t)}\cdot\cos\varphi(\mathbf{t})=\left[\sum_{n=-\infty}^{+\infty}\mathbf{a}(\mathbf{n}\cdot\delta\mathbf{t}_L)\frac{\sin\Omega_L(\mathbf{t}-\mathbf{n}\cdot\delta\mathbf{t}_L)}{\Omega_L(\mathbf{t}-\mathbf{n}\cdot\delta\mathbf{t}_L)}\right]\cdot\cos\varphi(\mathbf{t})= \\ &=\left[\sum_{n=-\infty}^{+\infty}\mathbf{a}(\mathbf{n}\cdot\delta\mathbf{t}_L)\frac{\sin\Omega_L(\mathbf{t}-\mathbf{n}\cdot\delta\mathbf{t}_L)}{\Omega_L(\mathbf{t}-\mathbf{n}\cdot\delta\mathbf{t}_L)}\right]\cdot\cos\varphi(\mathbf{t})= \\ &=\left[\sum_{n=-\infty}^{+\infty}\mathbf{a}(\mathbf{n}\cdot\delta\mathbf{t}_L)\frac{\sin\ \mathbf{K}_L\cdot\left[(\frac{\delta\omega}{\delta\mathbf{k}})\cdot(\mathbf{t}-\mathbf{n}\cdot\delta\mathbf{t}_L)-\mathbf{x}\right]}{\mathbf{K}_L\cdot\left[(\frac{\delta\omega}{\delta\mathbf{k}})\cdot(\mathbf{t}-\mathbf{n}\cdot\delta\mathbf{t}_L)-\mathbf{x}\right]}\right]\cdot\cos\varphi(\mathbf{t}),\end{aligned} \tag{4.0.38}$$

$$\mathbf{A(f)}=\mathbf{A}(\frac{\omega}{2\pi})=\sum_{n=-\infty}^{+\infty}\mathbf{A}(\mathbf{n}\cdot\delta\mathbf{f}_L)\frac{\sin 2\pi\mathbf{T}_L(\mathbf{f}-\mathbf{n}\cdot\delta\mathbf{f}_L)}{2\pi\mathbf{T}_L(\mathbf{f}-\mathbf{n}\cdot\delta\mathbf{f}_L)},\ \delta\mathbf{f}_L\le\frac{1}{2\mathbf{T}_L}. \tag{4.0.39}$$

Let us consider that there is sufficiently high number of samples, N and M, that would fully represent or reconstruct the same amplitude wave-functions, (4.0.38) and (4.0.39), both in time and frequency domain (in order to capture the same and total signal energy amount). Kotelnikov-Shannon sampling theorem combined with Parseval's theorem is giving the option to express the signal-energy as,

$$\begin{aligned}&\tilde{\mathbf{E}}=\int_{-\infty}^{+\infty}|\Psi(\mathbf{t})|^2\mathbf{dt}=\frac{1}{\pi}\int_0^{\infty}\mathbf{A}^2(\omega)\,\mathbf{d}\omega=\frac{1}{2}\int_{-\infty}^{+\infty}\mathbf{a}^2(\mathbf{t})\,\mathbf{dt}=\delta\mathbf{t}\cdot\sum_{(n)}|\Psi(\mathbf{n}\cdot\delta\mathbf{t})|^2 \\ &=\frac{1}{2}\cdot\delta\mathbf{t}_L\cdot\sum_{(n)}\mathbf{a}^2(\mathbf{n}\cdot\delta\mathbf{t}_L)=2\cdot\delta\mathbf{f}_L\cdot\sum_{(m)}\mathbf{A}^2(\mathbf{m}\cdot\delta\mathbf{f}_L)=\frac{\delta\omega_L}{\pi}\cdot\sum_{(m)}\mathbf{A}^2(\mathbf{m}\cdot\delta\mathbf{f}_L), \\ &\mathbf{F}_L<\mathbf{F}_H,\ \delta\mathbf{t}_L>\delta\mathbf{t}_H=\delta\mathbf{t},\ \mathbf{n}\in[1,2,3...\mathbf{N}],\ \mathbf{m}\in[1,2,3...\mathbf{M}], \\ &\mathbf{T}_L=\mathbf{N}\cdot\delta\mathbf{t}_L=\frac{1}{2\mathbf{F}_L},\ \mathbf{F}_L=\frac{\Omega_L}{2\pi}=\mathbf{M}\cdot\delta\mathbf{f}_L=\frac{1}{2\mathbf{T}_L},\ \Omega_L\mathbf{T}_L>\pi, \\ &\frac{\delta\mathbf{f}_L}{\delta\mathbf{t}_L}=\frac{\delta\omega_L}{2\pi\cdot\delta\mathbf{t}_L}=\frac{\mathbf{N}}{\mathbf{M}}\cdot\frac{\mathbf{F}_L}{\mathbf{T}_L}=\frac{1}{2\pi}\cdot\frac{\mathbf{N}}{\mathbf{M}}\cdot\frac{\Omega_L}{\mathbf{T}_L}=\frac{\sum_{(n)}\mathbf{a}^2(\mathbf{n}\cdot\delta\mathbf{t}_L)}{8\pi\cdot\sum_{(m)}\mathbf{A}^2(\mathbf{m}\cdot\delta\mathbf{f}_L)}, \\ &\frac{1}{2}\cdot\frac{\mathbf{N}}{\mathbf{M}}\cdot\frac{\pi}{\mathbf{T}_L^2}<\frac{\delta\mathbf{f}_L}{\delta\mathbf{t}_L}=\frac{\delta\omega_L}{2\pi\cdot\delta\mathbf{t}_L}=\frac{\mathbf{N}}{\mathbf{M}}\cdot\frac{\mathbf{F}_L}{\mathbf{T}_L}=\frac{1}{2\pi}\cdot\frac{\mathbf{N}}{\mathbf{M}}\cdot\frac{\Omega_L}{\mathbf{T}_L}<\frac{1}{2\pi}\cdot\frac{\mathbf{N}}{\mathbf{M}}\cdot\frac{\Omega_L^2}{\pi}.\end{aligned} \tag{4.0.40}$$

Since the amplitude-function frequency interval $\mathbf{F_L}$ could be for orders of magnitude lower than carrier-function-frequency $\mathbf{F_H} = \mathbf{F}$, it is clear that also number of samples necessary to reconstruct the signal amplitude $\mathbf{a(t)}$ could be for an order of magnitude lower than the number of samples which is reconstructing the total wave function $\Psi\mathbf{(t)}$. Here it is also the beginning of the explanation how particles with non-zero rest masses could be created by certain superposition or packaging of elementary wave-packets.

If we are interested in exploring the signal-energy options, we can continue developing only the signal amplitude function (as an Analytic Signal function), making the next similar step, as for instance, and

$$\left\{\begin{array}{l} \mathbf{a_0(t)} = \mathbf{a(t)} = \sum_{\mathbf{n}=-\infty}^{+\infty} \mathbf{a(n\cdot\delta t_L)} \frac{\sin \Omega_L (\mathbf{t}-\mathbf{n}\cdot\delta \mathbf{t}_L)}{\Omega_L (\mathbf{t}-\mathbf{n}\cdot\delta \mathbf{t}_L)} \Leftrightarrow \\ \Leftrightarrow \tilde{\mathbf{E}}_0 = \tilde{\mathbf{E}} = \frac{1}{2}\cdot\delta \mathbf{t}_{L0}\cdot\sum_{(\mathbf{n})} \mathbf{a}^2(\mathbf{n}\cdot\delta \mathbf{t}_{L0}) = 2\cdot\delta \mathbf{f}_{L0}\cdot\sum_{(\mathbf{m})} \mathbf{A}^2(\mathbf{m}\cdot\delta \mathbf{f}_{L0}) \\ \delta \mathbf{t}_{L0} = \delta \mathbf{t}_L \ , \ \delta \mathbf{f}_{L0} = \delta \mathbf{f}_L \end{array}\right\} \Rightarrow \qquad (4.0.41)$$

$$\Rightarrow \left\{\begin{array}{l} \mathbf{a_1(t)} = \sum_{\mathbf{n}=-\infty}^{+\infty} \mathbf{a_1(n\cdot\delta t_{L1})} \frac{\sin \Omega_{L1} (\mathbf{t}-\mathbf{n}\cdot\delta \mathbf{t}_{L1})}{\Omega_{L1} (\mathbf{t}-\mathbf{n}\cdot\delta \mathbf{t}_{L1})}, \\ \mathbf{a_1(t)} = \sqrt{\mathbf{a}_0^2(\mathbf{t}) + \{\mathbf{H}[\mathbf{a_0(t)}]\}^2} \Leftrightarrow \\ \Leftrightarrow \tilde{\mathbf{E}}_1 = \frac{\mathbf{1}}{\mathbf{2}}\cdot\delta \mathbf{t}_{L1}\cdot\sum_{(\mathbf{n})} \mathbf{a}_1^2(\mathbf{n}\cdot\delta \mathbf{t}_{L1}) = \mathbf{2}\cdot\delta \mathbf{f}_{L1}\cdot\sum_{(\mathbf{m})} \mathbf{A}_1^2(\mathbf{m}\cdot\delta \mathbf{f}_{L1}) \\ \Leftrightarrow \tilde{\mathbf{E}}_0 = \frac{\mathbf{1}}{\mathbf{2}^2}\cdot\delta \mathbf{t}_{L0}\cdot\delta \mathbf{t}_{L1}\cdot\sum_{(\mathbf{n})} \mathbf{a}_1^2(\mathbf{n}\cdot\delta \mathbf{t}_{L1}) = \mathbf{2}^2\cdot\delta \mathbf{f}_{L0}\cdot\delta \mathbf{f}_{L1}\cdot\sum_{(\mathbf{m})} \mathbf{A}^2(\mathbf{m}\cdot\delta \mathbf{f}_{L1}) \end{array}\right\} \Rightarrow \ (4.0.42)$$

$$\Rightarrow \left\{\begin{array}{l} \mathbf{a_2(t)} = \sum_{\mathbf{n}=-\infty}^{+\infty} \mathbf{a_2(n\cdot\delta t_{L2})} \frac{\sin \Omega_{L2} (\mathbf{t}-\mathbf{n}\cdot\delta \mathbf{t}_{L2})}{\Omega_{L2} (\mathbf{t}-\mathbf{n}\cdot\delta \mathbf{t}_{L2})}, \\ \mathbf{a_2(t)} = \sqrt{\mathbf{a}_1^2(\mathbf{t}) + \{\mathbf{H}[\mathbf{a_1(t)}]\}^2} \Leftrightarrow \\ \Leftrightarrow \tilde{\mathbf{E}}_2 = \frac{\mathbf{1}}{\mathbf{2}}\cdot\delta \mathbf{t}_{L2}\cdot\sum_{(\mathbf{n})} \mathbf{a}_2^2(\mathbf{n}\cdot\delta \mathbf{t}_{L2}) = \mathbf{2}\cdot\delta \mathbf{f}_{L2}\cdot\sum_{(\mathbf{m})} \mathbf{A}_2^2(\mathbf{m}\cdot\delta \mathbf{f}_{L2}) \Rightarrow \\ \Leftrightarrow \tilde{\mathbf{E}}_0 = \frac{\mathbf{1}}{\mathbf{2}^3}\cdot\delta \mathbf{t}_{L0}\cdot\delta \mathbf{t}_{L1}\cdot\delta \mathbf{t}_{L2}\cdot\sum_{(\mathbf{n})} \mathbf{a}_2^2(\mathbf{n}\cdot\delta \mathbf{t}_{L2}) = \mathbf{2}^3\cdot\delta \mathbf{f}_{L0}\cdot\delta \mathbf{f}_{L1}\cdot\delta \mathbf{f}_{L2}\cdot\sum_{(\mathbf{m})} \mathbf{A}^2(\mathbf{m}\cdot\delta \mathbf{f}_{L2}) \end{array}\right\} \Rightarrow \ \mathbf{(4.0.43)}$$

…
…

$$\Rightarrow \begin{Bmatrix} \mathbf{a_k(t)} = \sum_{n=-\infty}^{+\infty} \mathbf{a_k(n \cdot \delta t_{Lk}) \frac{\sin \Omega_{Lk}(t - n \cdot \delta t_{Lk})}{\Omega_{Lk}(t - n \cdot \delta t_{Lk})}}, \\ \mathbf{a_k(t)} = \sqrt{\mathbf{a^2_{k-1}(t)} + \{\mathbf{H[a_{k-1}(t)]}\}^2} \Leftrightarrow \\ \Leftrightarrow \mathbf{\tilde{E}_k} = \frac{1}{2} \cdot \delta \mathbf{t_{Lk}} \cdot \sum_{(n)} \mathbf{a^2_k(n \cdot \delta t_{Lk})} = 2 \cdot \delta \mathbf{f_{Lk}} \cdot \sum_{(n)} \mathbf{A^2_2(n \cdot \delta f_{Lk})} \Rightarrow \\ \Leftrightarrow \mathbf{\tilde{E}_0} = \frac{1}{2^{k+1}} \cdot \prod_{n=0}^{k} \delta \mathbf{t_{Ln}} \cdot \sum_{(n)} \mathbf{a^2_k(n \cdot \delta t_{Lk})} = 2^{k+1} \cdot \prod_{n=0}^{k} \delta \mathbf{f_{Ln}} \cdot \sum_{(n)} \mathbf{A^2(n \cdot \delta f_{Lk})} \end{Bmatrix}, \qquad (4.0.44)$$

Until the level "k" when we would arrive to the simplest amplitude function (since this mathematical process should converge to certain finite result, because initially we assumed that we are dealing only with energy-finite functions). This process, (4.0.40)-(4.0.44), looks very motivating for brainstorming. It could be that *here we are just unveiling new signal-atomizing and formatting techniques and probably coming closer to the explanation why and when Planck's and de Broglie relations regarding wave-packet energy and particle wave-properties are working well.*

4.0.6. Traveling Wave-Packets and Waves-Dispersion

Velocities of the latest waveforms (4.0.41)-(4.0.44) are again equal to already known group and phase velocity (found as before). In addition, we are also able to extract another property of the group velocity, which is basically showing that the same proportionality is conserved between signal's time and space domain frequency-durations (of course, valid only in all cases of traveling and non-dispersive waves, where signal propagation velocity is frequency independent),

$$\mathbf{v} = \frac{\delta\omega}{\delta \mathbf{k}} = \frac{\Omega_{Lm}}{\mathbf{K}_{Lm}} = \frac{\Omega_{Hm}}{\mathbf{K}_{Hm}} = \frac{\delta \mathbf{x}}{\delta \mathbf{t}} = \frac{\mathbf{dx}}{\mathbf{dt}} = \frac{\mathbf{d}\omega}{\mathbf{dk}}, \ \mathbf{m} = 0, 1, 2, 3, ... \qquad (4.0.45)$$

For the group and phase velocities (as found in all previously shown examples) it seems essential that a waveform time-domain function, $\Psi(\mathbf{t})$ should be fully compliant and extendable to an equivalent waveform in a time-space-domain $\Psi(\mathbf{t}, \mathbf{x})$. Moreover, the new *time-space waveform,* $\Psi(\mathbf{t}, \mathbf{x})$*, should represent a non-dispersive and traveling wave (meaning that signal propagation velocities are not frequency dependent).* As we have seen, there are very simple rules of such waveform transformation/s (based on a much more general symmetry of space and time variables, which is in the background) that can be described as:

a) *First, an arbitrary waveform should be presented as a superposition of elementary waves where simple harmonic functions are involved, such as* $\cos\omega t$ *or* $\sin\omega t$*, or* $\frac{\sin\Omega t}{\Omega t}\cos(\omega t)$ *…and then,*

b) *Time and frequency variables which are arguments of simple-harmonic functions should be extended as for instance,*

$$\begin{aligned}
&\mathbf{x} = \mathbf{x(t)},\ \omega = \omega(\mathbf{k}),\\
&\omega\mathbf{t} \to \omega\mathbf{t} - \mathbf{kx},\ \omega\cdot\delta\mathbf{t} \to \omega\cdot\delta\mathbf{t} - \mathbf{k}\cdot\delta\mathbf{x},\\
&\Omega\mathbf{t} \to \Omega\mathbf{t} - \mathbf{Kx},\ \Omega\cdot\delta\mathbf{t} \to \Omega\cdot\delta\mathbf{t} - \mathbf{K}\cdot\delta\mathbf{x},
\end{aligned}$$

Or *(4.0.46)*

$$\begin{aligned}
&\mathbf{x} = \mathbf{x(t)},\ \omega = \omega(\mathbf{k}),\\
&\mathbf{kx} \to \mathbf{kx} - \omega\mathbf{t},\ \mathbf{k}\cdot\delta\mathbf{x} \to \mathbf{k}\cdot\delta\mathbf{x} - \omega\cdot\delta\mathbf{t},\\
&\mathbf{Kx} \to \mathbf{Kx} - \Omega\mathbf{t},\ \mathbf{K}\cdot\delta\mathbf{x} \to \mathbf{K}\cdot\delta\mathbf{x} - \Omega\cdot\delta\mathbf{t}\ .
\end{aligned}$$

In (1.46) all minus "-" signs could be replaced by plus "+" signs, changing only a direction of propagation of a waveform in question. Furthermore, in some cases, we could combine inwards and outwards signals propagation, without influencing already established conclusions regarding signal integrity and signal velocities.

Such simple rules can be well explained and supported by analyzing a number of elementary wave phenomena known in Physics, where only simple harmonic waveforms and oscillations are involved. Obviously, here we are just generalizing or applying mentioned rules to any other arbitrary waveform, which presents non-dispersive traveling wave. Effectively, matter-waves and oscillations related to the world of micro-physics, electrons, atoms and photons… are in most of the cases presentable by energy-finite functions, composed of simple harmonic and elementary waveforms or wavelets (applicable also in cases of macro-particles' motions, which are effectively composed of similar simple harmonic waveforms).

In fact, instead of generalizing $\Psi(\mathbf{t}) \to \Psi(\mathbf{t},\mathbf{x})$ *we could start from* $\Psi(\mathbf{t},\mathbf{x}) \to \Psi(\mathbf{t},\mathbf{0})$ *and go to* $\Psi(\mathbf{t}), \mathbf{x} = \mathbf{const.}$ *(and then deductively extract above-mentioned rules), but for the purpose of the analysis presented here it was mathematically simpler to start from* $\Psi(\mathbf{t})$*). There are many different analyses (in available literature) regarding waves dispersion, taking into account number of possible situations. Here, we are limiting our framework to the fields directly applicable to basic particle-wave phenomenology among elementary particles, micro-particles, photons and de Broglie matter waves (such as different interactions, diffractions and interferences related to elementary-particles, waves, photons, to Compton and Photoelectric Effect, to particles annihilation, or creation etc.). In all such examples, it looks that Nature is showing or respecting very simple unification and interaction rules between wave-packets and particles (where group velocity has its very important place).*

4.0.7. Wave-Packets already known in Physics

From a number of different experimental observations (Davisson-Germer, G.P. Thomson, Photo Effect and Compton Effect…), we are empirically faced with the reality that particles regardless of which kind and origin, detected in experiments, exhibit wave-like diffraction and interference effects with a de Broglie wavelength $\lambda = \mathbf{h}/\tilde{\mathbf{p}}$ *(*$\tilde{p} = \sqrt{2mE} = \gamma\ mv,\ h = 6.626176 \cdot 10^{-34} J \cdot s$*). Here, in addition, the same value for Planck's constant is also found valid for black body radiation formula, for photons or any other kind of particles. This is the reason to introduce the concept that certain equivalent wave group or wave packet could replace moving particle.*

As we will see, the wave-packet or wave-group is modeled to produce group wave-velocity that should be equal to a certain real equivalent-particle velocity. In addition, it will be shown that the wave-packet energy should be equal to the equivalent-particle kinetic energy. For instance, an electromagnetic quant of energy, or photon, behaving as a wave-packet, is experimentally showing waves or particles properties. In Physics is widely demonstrated applicability and compatibility of Planck's wave-packet energy-expression as $\tilde{\mathbf{E}} = \mathbf{hf}$ (originally found and firstly applied only for photons, and later extended to other particles), with Energy and Momentum conservation laws, as well as with de Broglie matter-waves wavelength, $\lambda = \mathbf{h}/\tilde{\mathbf{p}}$ (where $\tilde{\mathbf{m}} = \mathbf{hf}/\mathbf{c}^2,\ \tilde{\mathbf{p}} = \mathbf{hf}/\mathbf{c},\ \mathbf{u} = \lambda\mathbf{f}$), without precisely and completely showing what really makes those relations correct. Mentioned mathematics was working well in explaining a number of experimental enigmas, and this has been one of the justifications of Planck's energy expression, and de Broglie's wavelength). The answer to the most simple question how and why only one characteristic frequency (multiplied by Planck's constant) can represent the motional wave-energy of a wave-group (or what means that frequency) should be found. The first intuitive and logical starting point could be to imagine that this is just the mean frequency, $\bar{\mathbf{f}}$, of the corresponding narrow-band, elementary matter-wave-group calculated in relation to its energy (where a wave group, or wave packet, or de Broglie matter-wave is composed of infinite number of elementary waves, covering certain, not too wide, frequency interval: $\mathbf{0} \leq \mathbf{f}_{min.} \leq \bar{\mathbf{f}} \leq \mathbf{f}_{\mathbf{max.}} < \infty$). The energy of such wave-group (connecting Parseval's and Planck's energy forms) is,

$$\tilde{E} = \int_{-\infty}^{+\infty} \Psi^2(t)\ dt = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t) \cdot dt = \frac{1}{\pi}\int_0^{\infty} A^2(\omega\ )\ d\omega = h\bar{f} = h\frac{u}{\lambda} = pu\quad . \tag{4.0.47}$$

Now (by definition) we can find the mean frequency of such wave-packet as,

$$\bar{f} = \frac{\frac{1}{\pi}\int_0^{\infty} f \cdot [A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty} [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty} f \cdot [A(\omega)]^2 d\omega}{\tilde{E}}, \tag{4.0.48}$$

Moreover, replace it into the wave energy expression, (4.0.47),

$$\tilde{E} = \frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\right]^2 d\omega = h\overline{f} = h\frac{\frac{1}{\pi}\int_0^{\infty} f\cdot\left[A(\omega)\right]^2 d\omega}{\tilde{E}} \Rightarrow \tilde{E}^2 = \frac{h}{\pi}\int_0^{\infty} f\cdot\left[A(\omega)\right]^2 d\omega . \qquad (4.0.49)$$

Using one of the most general formulas valid for all definite integrals (and applying it to (4.0.49)), we can prove that the wave energy (of a wave-group) should be equal to the product between Planck's constant and mean frequency of the wave group in question, as follows:

$$\left\{\begin{array}{l} \int_a^b f(x)\cdot g(x)dx = f(c)\int_a^b g(x)dx \, , a < c < b \, , g(x) \geq 0 \, , \\ f(x) \text{ and } g(x) - \text{continuous in } \left[a \leq x \leq b\right], \\ f(x) = f \, , g(x) = \left[A(\omega)\right]^2 > 0 \, , x = \omega \in (0, \infty) \end{array}\right\} \Rightarrow$$

$$\begin{aligned} &\Rightarrow \tilde{\mathbf{E}}^2 = \frac{\mathbf{h}}{\pi}\int_0^{\infty} \mathbf{f}\cdot\left[\mathbf{A}(\omega)\right]^2 \mathbf{d}\omega = \frac{\mathbf{h}}{\pi}\cdot\mathbf{f}\cdot\int_0^{\infty}\left[\mathbf{A}(\omega)\right]^2 \mathbf{d}\omega = \mathbf{hf}\cdot\tilde{\mathbf{E}} = \mathbf{h}\overline{\mathbf{f}}\cdot\tilde{\mathbf{E}} \Rightarrow \\ &\Rightarrow \tilde{\mathbf{E}} = \mathbf{hf} \, , \, \Delta\tilde{\mathbf{E}} = \mathbf{h}\cdot\Delta\mathbf{f} \, . \end{aligned} \qquad (4.0.50)$$

If Planck's energy of a photon or wave-group deals with a mean frequency of that wave-group, the same should be valid for de Broglie wavelength, as well as for its phase and group velocities. Consequently, all of them could be treated as mean values describing the motion of an effective center of inertia, or center of gravity of that narrow banded wave-group). Consequently, we do not need to specifically mark them as mean-values, as it was the case with mean-frequency (since we know that all of them should anyway, by their nature of formation and existence) be mean values $\overline{\mathbf{f}} = \mathbf{f}, \ \overline{\lambda} = \lambda, \ \overline{\mathbf{u}} = \mathbf{u}, \ \overline{\mathbf{v}} = \mathbf{v}$).

For instance, for sufficiently narrow banded signals, mean-values of a group and phase velocity should also be presentable as:

$$\overline{v}_g = \overline{v} = \frac{\frac{1}{\pi}\int_0^{\infty} v\cdot\left[A(\omega)\right]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\right]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty}\frac{d\omega}{dk}\cdot\left[A(\omega)\right]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\right]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty}\frac{d\omega}{dk}\cdot\left[A(\omega)\right]^2 d\omega}{\tilde{E}} = \frac{\delta\omega}{\delta k}, \qquad (4.0.51)$$

$$\overline{v}_f = \overline{u} = \frac{\frac{1}{\pi}\int_0^{\infty} u\cdot\left[A(\omega)\right]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\right]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty}\frac{\omega}{k}\cdot\left[A(\omega)\right]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\right]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty}\frac{\omega}{k}\cdot\left[A(\omega)\right]^2 d\omega}{\tilde{E}} = \frac{\omega_0}{k_0}. \qquad (4.0.52)$$

From the expressions for mean-group and mean-phase velocity (in cases of narrow band waveforms, when we can approximate the signal frequency-domain amplitude as a constant) we should also be able to find Planck's expression for energy of an elementary wave-group, $\mathbf{E} = \mathbf{hf} = \frac{\mathbf{h}}{\mathbf{2\pi}}\omega$, as follows,

$$\left\{ \begin{array}{l} \overline{\mathbf{v}}_{\mathbf{g}} = \overline{\mathbf{v}} = \dfrac{\frac{\mathbf{1}}{\pi}\int_{\mathbf{0}}^{\infty}\frac{\mathbf{d\omega}}{\mathbf{dk}}\cdot[\mathbf{A}(\omega)]^2\mathbf{d\omega}}{\tilde{\mathbf{E}}} = \dfrac{\delta\omega}{\delta\mathbf{k}} \Rightarrow \tilde{\mathbf{E}} = \frac{\mathbf{1}}{\pi}\int_{\mathbf{0}}^{\infty}\frac{\delta\mathbf{k}}{\delta\omega}\cdot\frac{\mathbf{d\omega}}{\mathbf{dk}}\cdot[\mathbf{A}(\omega)]^2\mathbf{d\omega} \Rightarrow \\ \Rightarrow \mathbf{d\tilde{E}} \cong \dfrac{[\mathbf{A}(\omega)]^2}{\pi}\mathbf{d\omega} = \mathbf{2}[\mathbf{A}(\omega)]^2\mathbf{df} \end{array} \right\} \vee$$

$$\vee \left\{ \begin{array}{l} \overline{\mathrm{v}}_{\mathrm{f}} = \overline{\mathrm{u}} = \dfrac{\frac{1}{\pi}\int_{0}^{\infty}\frac{\omega}{\mathrm{k}}\cdot[\mathrm{A}(\omega)]^2 \mathrm{d}\omega}{\tilde{\mathrm{E}}} = \dfrac{\omega_0}{\mathrm{k}_0} \Rightarrow \tilde{\mathrm{E}} = \frac{1}{\pi}\int_{0}^{\infty}\frac{\mathrm{k}_0}{\omega_0}\frac{\omega}{\mathrm{k}}\cdot[\mathrm{A}(\omega)]^2 \mathrm{d}\omega \\ \Rightarrow \mathrm{d}\tilde{\mathrm{E}} \cong \dfrac{[\mathrm{A}(\omega)]^2}{\pi}\mathrm{d}\omega = 2[\mathrm{A}(\omega)]^2 \mathrm{df} \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \delta\tilde{\mathbf{E}} = 2[\mathbf{A}(\omega_0)]^2 \cdot \delta\mathbf{f} \\ \tilde{\mathbf{E}} = 2[\mathbf{A}(\omega_0)]^2 \cdot \mathbf{f} = \mathbf{hf} \\ \mathbf{A}(\omega) \cong \mathbf{A}(\omega_0) \cong \mathbf{Const.} \\ 2[\mathbf{A}(\omega_0)]^2 = \mathbf{h} = \mathbf{Planck\ constant} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} \Delta\tilde{\mathbf{E}} = \mathbf{h}\cdot\Delta\mathbf{f}\ ,\ \delta\tilde{\mathbf{E}} = \mathbf{h}\cdot\delta\mathbf{f} \\ \dfrac{\Delta\tilde{\mathbf{E}}}{\Delta\mathbf{f}} = \dfrac{\tilde{\mathbf{E}}}{\mathbf{f}} = \mathbf{h}, \dfrac{\Delta\tilde{\mathbf{E}}}{\tilde{\mathbf{E}}} = \dfrac{\Delta\mathbf{f}}{\mathbf{f}} \end{array} \right\}. \tag{4.0.53}$$

Of course, since here parts of conclusions are based on "narrow-band" approximations, we can only get a good feeling regarding the background conditions that are supporting Planck's expression for an elementary quant of wave-energy. Briefly summarizing, we can say that *Planck's energy-expression should really be well applicable only to narrow band elementary wave-packet forms* (but what that means in all of its aspects, and how it is fully connected to all other moving macro particles with non-zero rest masses is still to be found). Luckily we already know that Planck's energy-quant (or energy-packet concept), united with de Broglie wavelength and relativistic particle-energy expressions is perfectly applicable in explaining number of experiments (such as Compton and Photoelectric effect etc.). *Presently we see clearly that Nature is using its own methods of waveforms sampling, analysis and synthesis, applying them equally to waves and particles, basically moving certain energy content from one packaging-format to another (conveniently linking infinitesimal and discrete signal processing), and this is already a very good starting point for further analyses.*

*[♣ **COMMENTS & FREE-THINKING CORNER:***

We could additionally test the Planck's radiation law, regarding photon energy $\mathbf{E} = \mathbf{hf} = \frac{\mathbf{h}}{\mathbf{2\pi}}\omega$. *It is well proven that a photon has the wave energy equal to the product between Planck's constant **h** and photon's frequency **f**. Photon is also a wave phenomena and it should be presentable using a certain time-domain wave function* $\psi(\mathbf{t}) = \mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})$, *expressed in the form of an Analytic Signal. Since the*

Analytic Signal presentation gives the chance to extract immediate signal amplitude $\mathbf{a(t)}$, *phase* $\varphi\mathbf{(t)}$, *and frequency* $\omega\mathbf{(t)} = \frac{\partial\varphi\mathbf{(t)}}{\partial\mathbf{t}} = 2\pi\mathbf{f(t)}$, *let us extend and test the meaning of Planck's energy when: instead of constant photon frequency **f** (valid for a single photon) we take its wave-group, mean wave-frequency,* $2\pi\bar{\mathbf{f}} = \bar{\boldsymbol{\omega}}$, *of the time-domain photon wave-function* $\psi\mathbf{(t)}$. *Since we already know that the photon is a "relatively concentrated wave-group", its time-variable frequency-function would also be very narrow band limited (and could easily be replaced by photon's mean frequency value). However, just for mathematically exercising such opportunity, we will proceed with this idea, and maybe find some additional conditions applicable to all physics related wave-groups, as follows,*

$$\tilde{E} = hf = \frac{h}{2\pi}\omega(t) \quad (\Leftrightarrow) \quad \tilde{E} = h\bar{f} = \frac{h}{2\pi}\bar{\omega}$$

$$\tilde{E} = \int_{[T]} \Psi^2(t)dt = \int_{[T]} [a(t)\cos\varphi(t)]^2\, dt = \int_{[T]} a^2(t)dt$$

$$\Psi(t) = a(t)\cos\varphi(t) = -H\left[\hat{\Psi}(t)\right] \ , \ \hat{\Psi}(t) = a(t)\sin\varphi(t) = H\left[\Psi(t)\right]$$

$$a(t) = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)} \quad ; \quad \varphi(t) = \arctan\frac{\hat{\Psi}(t)}{\Psi(t)} \quad ,$$

$$P(t) = \frac{d\tilde{E}}{dt} = \Psi^2(t) \ (\Leftrightarrow) \left[\frac{a(t)}{\sqrt{2}}\right]^2 \ (=) \ [W] \ , t \in (-\infty, +\infty) \ ,$$

$$P(\omega) = \frac{d\tilde{E}}{d\omega} = \frac{\Psi^2(t)}{d\omega/dt} = \left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 \ (=) \ \left[Js = Ws^2\right] , \ \omega \in (0, +\infty) \ .$$

$$\omega(t) = \frac{\partial\varphi(t)}{\partial t} = 2\pi f(t) \quad (\Leftrightarrow) \quad \bar{\omega}(t) = \left\langle\frac{\partial\varphi(t)}{\partial t}\right\rangle = 2\pi\langle f(t)\rangle = 2\pi\bar{f} = \bar{\omega}$$

$$\bar{\omega} = \frac{1}{T}\int_{[T]} \omega(t)dt \quad (\Leftrightarrow) \quad \frac{\frac{1}{T}\int_{[T]} \omega(t)\cdot a^2(t)\cdot dt}{\int_{[T]} a^2(t)\cdot dt} \quad (=) \quad \tilde{E}\frac{2\pi}{h} \Rightarrow$$

$$\Rightarrow \frac{\tilde{E}}{\bar{\omega}} = \frac{\left[\int_{[T]} a^2(t)\cdot dt\right]^2}{\frac{1}{T}\int_{[T]} \omega(t)\cdot a^2(t)\cdot dt} = \frac{\int_{[T]} a^2(t)\cdot dt}{\frac{1}{T}\int_{[T]} \omega(t)dt} = \tag{4.0.54}$$

$$= \frac{a^2(t)\int_{-\infty}^{+\infty} a^2(t)dt}{2\left[\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)\right]} = \frac{h}{2\pi} = \text{ Const.}$$

*More about photon wave function can be found in the Annex at the end of this paper: **WAVE FUNCTION OF THE BLACK BODY RADIATION AND A PHOTON**)*

*Depending on how we calculate the mean frequency, we should be able to prove at least one of the above given relations (see the last line). In addition, we should be able to find the family of wave functions that are describing photon or any other elementary wave-packet in a time domain. In any case, we should be able to see how universal Planck's energy law could be regarding the energy of arbitrary wave functions. See more about similar problem in the following article from Dr. Juluis S. Bendat: **THE HILBERT TRANSFORM AND APPLICATIONS TO CORRELATION MEASUREMENTS.***

What is very interesting in the analysis of Dr. Bendat and others is that a group or phase velocity is directly proportional to the square root of signal frequency $\mathbf{v} = \mathbf{v_g} \approx \sqrt{\mathbf{f}}$ *. This is in an agreement with Planck's elementary-quanta radiation-energy expression* $\tilde{\mathbf{E}} = \mathbf{hf}$ *, since we already know that any motional or kinetic energy is proportional to the square of the group velocity and it should be directly proportional to the dominant wave-packet frequency* $\mathbf{E_k} = \tilde{\mathbf{E}} \approx \mathbf{v}^2 = \mathbf{v_g^2} \approx \mathbf{f}$ *. It should be very natural and deterministic, mathematical explanation regarding Planck's elementary quantum energy, and here we are approaching well to it. ♣]*

4.0.8 Uncertainty Relations and Waveform Velocities

In physics literature, the Uncertainty Principle is usually linked to Heisenberg's Uncertainty Relations. For real, correct and full understanding of a number of relations that are linked to the Uncertainty Principles, it would be, for the time being, better to forget that Heisenberg made any invention regarding Uncertainty. In other words, we should know (or learn) that Uncertainty is not married almost exclusively with Quantum Theory. It is also the current case that Uncertainty, as presented in contemporary Physics (mostly in Quantum Mechanics), is often applied as a very useful supporting background for a number of semi-mystifications, and justifications of a number of logical, conceptual and methodological Uncertainties in Physics (author's comment).

A more general approach to Uncertainty relations (in connection with the quantum manifestations of energy formats) should start from a finite wave function Ψ(t). Here we are treating the square of the wave function as a power, (4.0.4): $\Psi^2(\mathbf{t}) = \mathbf{Power} = \mathbf{dE}/\mathbf{dt}$, but we could also treat Ψ(t) as a dimensionless function without influencing the results of the analysis that follows.

It is an advantage of Analytical Signals that they cover only natural domains of real time and frequency: $-\infty < \mathbf{t} < \infty$, $0 \leq \mathbf{f} < \infty$, opposite to the traditional Signal Analysis (Fourier Analysis), where frequency can also take negative values. In many other aspects, Analytic Signals are giving equivalent results, as in the case of Fourier Signal Analysis including producing some additional, time-frequency dependent, dynamic and spectral signal properties what Fourier analysis is not able to perform.

The mutual relations between time and frequency signal-domain-segments and/or their isolated points of certain waveform $\Psi(\mathbf{t})$, are not compatible for 1:1 (one-to-one) mapping or imaging. Moreover, they can be mutually related and precisely localized on both sides only within certain approximating relations given by the following Uncertainty relations (or domains-imaging restrictions):

$$\begin{aligned} & 0 < \delta\mathbf{t} \cdot \delta\mathbf{f} < \frac{1}{2} \leq \mathbf{F} \cdot \mathbf{T} \leq \frac{1}{4 \cdot \delta\mathbf{t} \cdot \delta\mathbf{f}}, (\delta\mathbf{t} \leq \frac{1}{2\mathbf{F}}) << \mathbf{T}, (\delta\mathbf{f} \leq \frac{1}{2\mathbf{T}}) << \mathbf{F}, \\ & 0 < \delta\mathbf{t} \cdot \delta\omega = 2\pi \cdot \delta\mathbf{t} \cdot \delta\mathbf{f} < \pi \leq \Omega \cdot \mathbf{T} \leq \frac{\pi}{2 \cdot \delta\mathbf{t} \cdot \delta\mathbf{f}} = \frac{\pi^2}{\delta\mathbf{t} \cdot \delta\omega}, \end{aligned} \tag{4.0.55}$$

where $\mathbf{T}$ is the total time-duration of the signal, $\mathbf{F} = \Omega/2\pi$ is its total frequency-duration, $\delta\mathbf{t}$ is the maximal time-sampling interval of the signal, and $\delta\mathbf{f} = \delta\omega/2\pi$ is the maximal frequency-sampling interval (all of them compliant to Nyquist and Shannon-Kotelnikov signal sampling theorems; $t \in [\mathbf{T}]$, $0 \leq \mathbf{T} < \infty$, $-\infty < \mathbf{t} < \infty$, and $\mathbf{f} \in [\mathbf{F}]$, $0 \leq \mathbf{F} < \infty$, $0 \leq \mathbf{f} < \infty$, $2\pi\mathbf{F} = \Omega, 2\pi\mathbf{f} = \omega$). Of course, only mathematically judging, here we know that an absolute time and frequency signal durations (total signal lengths) cannot be precisely found in both time and frequency domains. Only in the frames of capturing the dominant part of the signal energy in both domains (for instance to have 99 % of total signal energy in both domains) we can speak about definite and total signal durations. The other fact known from Physics is that Nature (or our universe) is always presenting a kind of filtering and modifying media for all signals and waves propagation. Thus, even mathematically ideal, unlimited or infinite spectral durations would be transformed into finite signal-lengths (simply low energy and very high frequency spectral components of the signal would be absorbed, dissipated, or filtered by the media where signal propagates). In relation to such kind of background, we are able to use the terms of finite signal durations in all its domains. The next important mathematical foundation compatible to operations with limited and finite spectral or domain lengths (in time, space and all frequency-related domains) can be found in signal analysis and synthesis (or in sampling rules) defined by Kotelnikov and Shannon's theorem. It will be again addressed later.

We know that Nature (or everything what is in motion and what belongs to Physics) also respects mentioned domains imaging restrictions. In Physics, such mathematical relations are "domesticated" as Uncertainty Relations and widely applied. In fact, Physics literature made in certain cases "more bad than good" by popularizing, over-simplifying, mystifying and incompletely explaining what Uncertainty Relations should mean (instead, firstly clarifying all mathematical aspects of domains imaging restrictions, and then starting with a bottom-line, and simple physics related, conceptual explanations). Since we already know that Nature really respects certain Uncertainty Relations (maybe not exactly as we presently describe such Uncertainty in Physics), we can safely say that Nature also respects our time and frequency signal modeling techniques (Fourier and Hilbert transform, Nyquist sampling rules, Analytic Signal concept, Kotelnikov-Shannon theorem etc.). Consequently, we can say that whatever we see as a motional form in our Universe is composed of elementary simple harmonic signal-forms and can be decomposed on such elementary waveforms.

It will be shown later that even stable and virtually non-moving forms (particles with non-zero rest masses) were in their past (during creation) assembled by a convenient superposition of certain elementary waveforms. In such particles related situations, it seems that Nature has conveniently transformed all of its domains' Uncertainties, (4.0.55), of freely traveling waveforms (here recognizable by the symbol $\leq$) into stabilized waveform Certainties (symbol =). This is like in cases of formation of stable elementary particles and atoms (which are also stable in states of rest, presenting objects where standing-waves structures are involved as building elements). In all other dynamic cases, when something is moving, Uncertainty relations (4.0.55) are applicable in their original mathematical forms. If we compare the basic Uncertainty relations (4.0.55) from this paper, with similar Uncertainty relations found in Quantum Theory (only concerning introductory, mathematics related uncertainty relations), we will notice certain differences between them. For instance, in the contemporary Quantum Theory significant, dominant or total signal durations, are captured differently than here (taking shorter intervals). This produces different quantitative relations such as: instead of $\mathbf{F}\cdot\mathbf{T}\geq\frac{1}{2}$ in Quantum Theory, we often find $\mathbf{F}\cdot\mathbf{T}\geq 1$, but in reality, both of such uncertainty relations would become mutually identical if the same signal-intervals were taken into consideration. The same problem gets much more complex in contemporary Quantum Theory, when other assumptions and statistics and probability related concepts are applied, what presently looks like jumping from the strong and mathematically clear platform into multilevel uncertainties with a lot of arbitrary and non-deterministic environment-parameters.

Since energy finite signals or waves in real physics related cases propagate in certain space, during certain time, similar uncertainty relations should exist taking into account space-related parameters (length along axis of propagation $\mathbf{x}$, and spatial-periodicity $\mathbf{k}=\mathbf{2}\pi\cdot\mathbf{f}_{\mathbf{x}}$). Just for making analogies, the ordinary meaning of frequency in a time-domain could also be considered as a time-related-frequency $\omega=\mathbf{2}\pi\cdot\mathbf{f}_{\mathbf{t}}=\mathbf{2}\pi\cdot\mathbf{f}$, and $\mathbf{k}=\mathbf{2}\pi\cdot\mathbf{f}_{\mathbf{x}}$ would be a space-related frequency. By analogy, with time-frequency-domain Uncertainty Relations (4.0.55), to speed up this process, we could create similar (and extended) uncertainty relations where time is analogically replaced by corresponding signal propagation length or signal duration, $\mathbf{t}\leftrightarrow\mathbf{x},\ \mathbf{T}\leftrightarrow\mathbf{L}$. Moreover, time related frequency is replaced by space related frequency, $\omega\leftrightarrow\mathbf{k},\ \Omega\leftrightarrow\mathbf{K}$, where **L** is the total signal length or total signal spatial duration, and $\mathbf{K}=\mathbf{2}\pi\,\mathbf{F}_{\mathbf{x}}$ is the total signal spatial frequency duration. Since here we take into account the same waveform $\Psi(\mathbf{t},\mathbf{x})$ in different domains, it is obvious that relevant Uncertainty Relations in time and space domains should be mutually united or coupled. Thus, they keep stable proportionality between all relevant signal domain intervals (in order that such waveform would be compact, progressive and non-dispersive), and in this way they make an extended Uncertainty Relations chain,

$$\begin{aligned}&\mathbf{0}<\delta\mathbf{t}\cdot\delta\omega=\mathbf{2}\pi\cdot\delta\mathbf{t}\cdot\delta\mathbf{f}=\delta\mathbf{x}\cdot\delta\mathbf{k}=\mathbf{2}\pi\cdot\delta\mathbf{x}\cdot\delta\mathbf{f}_{\mathbf{x}}<\pi\leq\Omega\cdot\mathbf{T}=\mathbf{K}\cdot\mathbf{L}\,,\\&\mathbf{0}<\delta\mathbf{t}\cdot\delta\mathbf{f}=\delta\mathbf{x}\cdot\frac{\delta\mathbf{k}}{\mathbf{2}\pi}=\delta\mathbf{x}\cdot\delta\mathbf{f}_{\mathbf{x}}<\frac{\mathbf{1}}{2}\Rightarrow\ \bar{\mathbf{v}}=\frac{\delta\mathbf{x}}{\delta\mathbf{t}}=\frac{\delta\omega}{\delta\mathbf{k}}\Leftrightarrow\left\{\frac{\mathbf{d}\,\mathbf{x}}{\mathbf{d}\,\mathbf{t}}=\frac{\mathbf{d}\omega}{\mathbf{dk}}\right\}.\end{aligned}\tag{4.0.56}$$

We can also see that the average velocity $\bar{\mathbf{v}}$ associated to extended Uncertainty relations (4.0.56), at least dimensionally corresponds to the signal's $\Psi(\mathbf{t},\mathbf{x})$, group velocity (and that such group velocity is a measure of proportionality between corresponding original and spectral domains durations). Later we will see that the velocity found here really has strong grounds in Physics (meaning that *as long the signal or an equivalent particle (represented by such signal) are in waving or motion, they will have the same group and phase velocity, and relevant wave energy would correspond to the particle kinetic energy*).

Here, in order to speed up the conclusion making process, we simply applied analogies to create extended uncertainty relations (4.0.56), but in Physics, such relations are an experimentally known and mathematically well-supported fact. By creating δ-differences, it is obvious that we only take into account elementary sampling intervals that are very small steps defined by Nyquist-Kotelnikov-Shannon signal-sampling rules. By comparing such signal-intervals with total signal-durations in time and frequency domains, it becomes clear that δ-differences present very small signal atomizing intervals. By multiplying the last form of Uncertainty Relations (4.0.56) with Planck's constant, taking into account Planck's energy of a wave packet, and de Broglie wavelength, we can easily formulate the following Uncertainty Relations,

$$\left\{\begin{array}{l} 0 < h \cdot \delta t \cdot \delta f = h \cdot \delta x \cdot \frac{\delta k}{2\pi} = h \cdot \delta x \cdot \delta f_x < \frac{h}{2} < h \cdot F \cdot T = h \cdot F_x \cdot L = h \cdot \frac{K}{2\pi} \cdot L \quad , \\ k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p \quad , \quad \lambda = \frac{h}{p} \quad , \quad h = \text{const.} \quad , \quad \tilde{E} = hf \end{array}\right\} \Rightarrow$$

$$0 < \delta t \cdot \delta \tilde{E} = \delta x \cdot \delta p < \frac{h}{2} < \tilde{E} \cdot T = P \cdot L \tag{4.0.57}$$

In Quantum Theory absolute or total signal durations in all domains (effectively real signal dimensions), $\mathbf{F}, \mathbf{T}, \mathbf{F_x}, \mathbf{L}, \mathbf{K}$ *have been designated as* Δ*-intervals,* $\mathbf{F} = \Delta \mathbf{f}$, $\tilde{\mathbf{E}} = \mathbf{h} \cdot \mathbf{F} = \mathbf{h} \cdot \Delta \mathbf{f}$, $\mathbf{T} = \Delta \mathbf{t}$, $\mathbf{L} = \Delta \mathbf{x}$, $\frac{\mathbf{h}}{2\pi} \cdot \mathbf{K} = \Delta \mathbf{p}$*, and such early Quantum-Theory Uncertainty Relations originally have been published as,* $\Delta \tilde{\mathbf{E}} \cdot \Delta \mathbf{t} = \Delta \mathbf{p} \cdot \Delta \mathbf{x} > \frac{\mathbf{h}}{2}$ *, what is fully identical to (4.0.57),* $\tilde{\mathbf{E}} \cdot \mathbf{T} = \mathbf{P} \cdot \mathbf{L} > \frac{\mathbf{h}}{2}$. *Later, in Quantum Theory literature, such absolute and total signal durations are often transformed (philosophically, stochastically and by means of experimental results fitting) into signal standard deviations. The other side of the inequality has also been changed from* $\frac{\mathbf{h}}{2}$ *to* $\mathbf{h}$ *(what is possible when considering differently important signal duration intervals), and renewed Uncertainty Relations got their latest quantum-mechanical form (probably losing a bit of their common sense and logical aspects, and becoming less generally valid) as,*

$$\Delta \tilde{\mathbf{E}} \cdot \Delta \mathbf{t} = \Delta \mathbf{p} \cdot \Delta \mathbf{x} \geq \mathbf{h} \quad . \tag{4.0.58}$$

In the majority of contemporary Quantum Theory presentations regarding Uncertainty Relations (and everywhere else the tendency of using in as many cases as possible the constant $\hbar = \frac{\mathbf{h}}{2\pi}$ *instead of* $\frac{\mathbf{h}}{2}$ *or* $\mathbf{h}$ *) is fashionable and overwhelming (in some cases maybe not fully mathematically defendable, but somehow "stochastically thinking" acceptable, and anyway in the same order of magnitude as* $\frac{\mathbf{h}}{2}$ *or* $\mathbf{h}$*). Here we will try to avoid such approach staying less quantum-fashionable.*

In this paper, we will still keep as very relevant the original and mathematically strongest and fully defendable form of Uncertainty Relations, where absolute or total signal durations are explicitly involved,

$$\mathbf{0} < \delta \mathbf{t} \cdot \delta \tilde{\mathbf{E}} = \delta \mathbf{x} \cdot \delta \mathbf{p} < \frac{\mathbf{h}}{2} < \mathbf{h} \cdot \mathbf{F} \cdot \mathbf{T} = \frac{\mathbf{h}}{2\pi} \cdot \mathbf{K} \cdot \mathbf{L} = \tilde{\mathbf{E}} \cdot \mathbf{T} = \mathbf{P} \cdot \mathbf{L} \ , \tag{4.0.59}$$

Where $\tilde{\mathbf{E}} = \mathbf{hF}$ *should be the total wave energy of the signal, and* $\mathbf{P} = \frac{\mathbf{h}}{2\pi}\mathbf{K}$ *is its total linear moment.*

What we can see is that the complete (Physics related) formulation of Uncertainty Relations is essentially related to the Planck's wave-packet energy expression. As we know from earlier wave-packet velocity analyses, such energy relation is well applicable only to sufficiently "narrow-banded" waveforms. We also know that such wave-packet concept can be applied as the motional-particle model in two aspects: *the average group velocity of the wave-packet corresponds to the particle (center of mass) velocity, and energy of the wave-packet is equal to the kinetic energy of the equivalent-particle,* because in both cases we should consider only motional energy. Also, de Broglie wavelength, easily understandable for wave-packets and photons (which do not have rest masses), just fits well and very much analogically into the moving equivalent particle or particle matter wave concept (where non-zero rest mass exists). Here are the essential grounds of particle-wave duality in Physics: Motional energy of any origin, belonging to any kind of energy carrying entity, has wave properties defined by de Broglie-Planck-Einstein matter-wave formulas.

It is interesting to find what would be regarding the same Uncertainty Relations (4.0.56), if we would like to analyze a sudden signal duration change for an arbitrary, Δ-signal-interval. To answer such a question let us start again from total signal durations in its time and frequency domains,

$$\mathrm{T}\cdot\mathrm{F} = \mathrm{L}\cdot\mathrm{F_x} > \frac{1}{2},\ \mathrm{T}\cdot\Omega = \mathrm{L}\cdot\mathrm{K} > \pi. \tag{4.0.60}$$

If a certain transformation (4.0.61) happens to a wave function Ψ**(t),** changing its time and frequency lengths, **T** and **F**, for the (mutually dependent or mutually coupled) amounts Δ**t** and Δ**f**, such signal transformation will automatically influence all other space and energy related parameters of Ψ**(x, t)** to change, producing similar Uncertainty relations, already known. Obviously, effective physical signal spatial length **L** and total signal wave-energy $\tilde{\mathbf{E}}$ would also change, as for instance:

$$\left\{\begin{array}{l} \mathrm{T} \to \mathrm{T} \pm \Delta t > 0,\quad \mathrm{F} \to \mathrm{F} \pm \Delta f > 0,\quad \mathrm{L} \to \mathrm{L} \pm \Delta x > 0\ ,\quad \mathrm{K} \to \mathrm{K} \pm \Delta k > 0 \\ \Delta\tilde{\mathrm{E}} = h\,\Delta f = \bar{v}\Delta p = \tilde{\mathrm{F}}\Delta x\ ,\quad \tilde{\mathrm{F}} = \frac{\Delta p}{\Delta t} = \text{force}\ , \\ 0 < \delta t \cdot \delta f = \delta x \cdot \frac{\delta k}{2\pi} = \delta x \cdot \delta f_x < \frac{1}{2} < \mathrm{F}\cdot\mathrm{T} = \frac{1}{2\pi}\cdot\ \mathrm{K}\cdot\mathrm{L} = \mathrm{F_x}\cdot\mathrm{L} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \tilde{\mathbf{E}} \to \tilde{\mathbf{E}} \pm \Delta\tilde{\mathbf{E}}, \\ \bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta \mathbf{t}} = \frac{\Delta\tilde{\mathbf{E}}}{\Delta \mathbf{p}} = \frac{\Delta\omega}{\Delta \mathbf{k}} = \frac{\delta \mathbf{x}}{\delta \mathbf{t}} = \frac{\delta\omega}{\delta \mathbf{k}} = \frac{\delta\tilde{\mathbf{E}}}{\delta \mathbf{p}} \end{array}\right\}. \tag{4.0.61}$$

In cases when time and frequency changes in (4.0.61) would be either positive or negative, we will have:

$$\mathbf{T}\cdot\mathbf{F}>\frac{1}{2}\Leftrightarrow\begin{Bmatrix}(\mathbf{T}+\Delta\mathbf{t})\cdot(\mathbf{F}+\Delta\mathbf{f})>\frac{1}{2}\\(\mathbf{T}-\Delta\mathbf{t})\cdot(\mathbf{F}-\Delta\mathbf{f})>\frac{1}{2}\end{Bmatrix}\Rightarrow\begin{Bmatrix}\left[\mathbf{T}^2-(\Delta\mathbf{t})^2\right]\cdot\left[\mathbf{F}^2-(\Delta\mathbf{f})^2\right]>\frac{1}{4}\\\mathbf{T}\cdot\mathbf{F}+\Delta\mathbf{t}\cdot\Delta\mathbf{f}>\frac{1}{2}\\\mathbf{T}\cdot\Delta\mathbf{f}+\Delta\mathbf{t}\cdot\mathbf{F}>\mathbf{0}\end{Bmatrix}\Leftrightarrow$$

$$\Leftrightarrow\begin{Bmatrix}1-(\frac{\Delta t}{T})^2-(\frac{\Delta f}{F})^2+(\frac{\Delta t}{T})^2\cdot(\frac{\Delta f}{F})^2>\frac{1}{4}\\1+(\frac{\Delta t}{T})\cdot(\frac{\Delta f}{F})>\frac{1}{2T\cdot F},T\cdot F>\frac{1}{2}\\(\frac{\Delta t}{T})+(\frac{\Delta f}{F})>0\end{Bmatrix}. \tag{4.0.62}$$

By analogy (just to shorten the process and avoid lengthy introductions), let us replace the signal total time duration **T** with the signal total spatial length **L**, and signal total frequency duration **F** = **Ft** = **Ω/2π** by signal total spatial frequency duration **Fx** = **K/2π**, and unite results for time and spatial lengths cases,

$$\mathbf{L}\cdot\mathbf{F}_x>\frac{1}{2}\Leftrightarrow\begin{Bmatrix}(\mathbf{L}+\Delta\mathbf{x})\cdot(\mathbf{F}_x+\Delta\mathbf{f}_x)>\frac{1}{2}\\(\mathbf{L}-\Delta\mathbf{x})\cdot(\mathbf{F}_x-\Delta\mathbf{f}_x)>\frac{1}{2}\end{Bmatrix}\Rightarrow\begin{Bmatrix}\left[\mathbf{L}^2-(\Delta\mathbf{x})^2\right]\cdot\left[\mathbf{F}_x^2-(\Delta\mathbf{f}_x)^2\right]>\frac{1}{4}\\\mathbf{L}\cdot\mathbf{F}_x+\Delta\mathbf{x}\cdot\Delta\mathbf{f}_x>\frac{1}{2}\\\mathbf{L}\cdot\Delta\mathbf{f}_x+\Delta\mathbf{x}\cdot\mathbf{F}_x>\mathbf{0}\end{Bmatrix}\Leftrightarrow$$

$$\Leftrightarrow\begin{Bmatrix}1-(\frac{\Delta x}{L})^2-(\frac{\Delta f_x}{F_x})^2+(\frac{\Delta x}{L})^2\cdot(\frac{\Delta f_x}{F_x})^2>\frac{1}{4}\\1+(\frac{\Delta x}{L})\cdot(\frac{\Delta f_x}{F_x})>\frac{1}{2L\cdot F_x},L\cdot F_x>\frac{1}{2}\\(\frac{\Delta x}{L})+(\frac{\Delta f_x}{F_x})>0\end{Bmatrix}\Rightarrow$$

$$\begin{Bmatrix}1-(\frac{\Delta t}{T})^2-(\frac{\Delta f}{F})^2+(\frac{\Delta t}{T})^2\cdot(\frac{\Delta f}{F})^2=1-(\frac{\Delta x}{L})^2-(\frac{\Delta f_x}{F_x})^2+(\frac{\Delta x}{L})^2\cdot(\frac{\Delta f_x}{F_x})^2>\frac{1}{4}\\1+(\frac{\Delta t}{T})\cdot(\frac{\Delta f}{F})=1+(\frac{\Delta x}{L})\cdot(\frac{\Delta f_x}{F_x})>\frac{1}{2T\cdot F}=\frac{1}{2L\cdot F_x},\ T\cdot F=L\cdot F_x>\frac{1}{2}\\(\frac{\Delta t}{T})+(\frac{\Delta f}{F})=(\frac{\Delta x}{L})+(\frac{\Delta f_x}{F_x})>0\end{Bmatrix} \tag{4.0.63}$$

In fact, the real waveforms related domains uncertainties are given by the last set of inequalities in (4.0.63).

Now we can extend uncertainty relations chain for a couple of more members, which implicitly include all possible and arbitrary signals Δ-variations found in (4.0.63),

$$\begin{aligned}
&\mathbf{0} < \delta\mathbf{t}\cdot\delta\mathbf{f} = \delta\mathbf{x}\cdot\delta\mathbf{f}_x < \frac{\mathbf{1}}{\mathbf{2}} < \mathbf{F}\cdot\mathbf{T} = \mathbf{F}_x\cdot\mathbf{L} \leq \frac{\mathbf{1}}{\mathbf{4}\cdot\delta\mathbf{t}\cdot\delta\mathbf{f}} = \frac{\mathbf{1}}{\mathbf{4}\cdot\delta\mathbf{x}\cdot\delta\mathbf{f}_x}, \\
&\mathbf{0} < \delta\mathbf{t}\cdot\delta\tilde{\mathbf{E}} = \delta\mathbf{x}\cdot\delta\mathbf{p} < \frac{\mathbf{h}}{\mathbf{2}} < \tilde{\mathbf{E}}\cdot\mathbf{T} = \mathbf{P}\cdot\mathbf{L} \leq \frac{\mathbf{h}}{\mathbf{4}\cdot\delta\mathbf{t}\cdot\delta\mathbf{f}} = \frac{\mathbf{h}}{\mathbf{4}\cdot\delta\mathbf{x}\cdot\delta\mathbf{f}_x}.
\end{aligned} \tag{4.0.64}$$

Since the signal durations (or lengths) can be changed only for integer number of (Nyquist) sampling intervals $\delta\mathbf{t}, \delta\mathbf{x}$, $\delta\mathbf{f}$ and $\delta\mathbf{f}_x$, in order to keep the non-dispersive signal integrity and stable mutual-proportionality of selected total signal-durations in all domains, the average group-velocity found for small sampling intervals should also be equal to the average group-velocity when much larger space, time and frequency intervals are involved,

$$\begin{aligned}
&\overline{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta\mathbf{t}} = \frac{\delta\mathbf{x}}{\delta\mathbf{t}} = \frac{\mathbf{n}\cdot\delta\mathbf{x}}{\mathbf{n}\cdot\delta\mathbf{t}} \left(= \frac{\delta\mathbf{f}}{\delta\mathbf{f}_x} = \frac{\mathbf{n}\cdot\delta\mathbf{f}}{\mathbf{n}\cdot\delta\mathbf{f}_x} = \frac{\delta\omega}{\delta\mathbf{k}} = \frac{\Delta\omega}{\Delta\mathbf{k}} = \frac{\mathbf{dx}}{\mathbf{dt}} = \frac{\mathbf{d}\omega}{\mathbf{dk}}\right), \\
&\Delta\mathbf{x} = \mathbf{n}\cdot\delta\mathbf{x},\ \Delta\mathbf{t} = \mathbf{n}\cdot\delta\mathbf{t},\ \mathbf{n} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots
\end{aligned} \tag{4.0.65}$$

The most interesting here is that an average group velocity, as domains proportionality measure, is not found as any other velocity and is presently related only to absolute signal lengths. Here, could be a part of the answer why light or electromagnetic waves velocity in open space, $c = 1/\sqrt{\varepsilon_0\mu_0}$ *is always the same (regardless of the observer's or source motion). This is probably the case, because such waveforms (or photons), while propagating in open-space, are manifesting as virtually endless motional waveforms. Their space and time domains are permanently shifted for certain constant amount, expressed by Uncertainty relations, producing constant group wave-speed. Whatever we try to do in order to change the speed of light, the same proportionality would re-appear and again stabilize the photons' group velocity. Once after we succeed to destroy the group-velocity balance between different signal domains, the signal will simply disappear (being dispersed, absorbed or dissipated). For instance, continuing the same brainstorming thinking, we could say that since the light speed is the highest signals speed presently known in Physics, the following limitation should be valid,*

$$\overline{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta\mathbf{t}} = \frac{\delta\mathbf{x}}{\delta\mathbf{t}} = \frac{\mathbf{n}\cdot\delta\mathbf{x}}{\mathbf{n}\cdot\delta\mathbf{t}} = \frac{\delta\mathbf{f}}{\delta\mathbf{f}_x} = \frac{\mathbf{n}\cdot\delta\mathbf{f}}{\mathbf{n}\cdot\delta\mathbf{f}_x} = \frac{\delta\omega}{\delta\mathbf{k}} = \frac{\Delta\omega}{\Delta\mathbf{k}} = \frac{\Omega_L}{\mathbf{K}_L} = \frac{\Omega_H}{\mathbf{K}_H} \leq \mathbf{c}\ , \tag{4.0.66}$$

what could be very useful relation to address additionally the mutual coupling and proportionality limits of absolute signal durations in their space, time and frequency domains. Even if some velocity dependent changes of characteristic signal lengths and intervals would happen (like in case of Lorenz transformations), the average signal group velocity (or domain lengths proportionality) should be conserved.

4.0.9. Uncertainty Relations in Quantum Theory

Understanding Uncertainty relations in physics, (presently still on mathematical level) is also related to our choice of signal duration intervals. Until here we have used (or talked about) real, absolute or total signal interval lengths. Now we will once more extend already established Uncertainty Relations of absolute signal duration intervals, taking into consideration corresponding signal standard deviations intervals.

Since the Orthodox Quantum Mechanics mostly deals with statistical distributions and probabilities, interval lengths are represented by signal variance intervals, which are statistical or standard deviations of certain variables around their mean-values. Consequently, mathematical expressions of basic Uncertainty Relations, when using variance intervals or statistical deviations, present another aspect of Uncertainty Relations (not mentioned before, but very much present in today's Quantum Mechanics literature). This should be properly integrated into a chain of all other, already known Uncertainty Relations. The statistical concept of variance is used to measure the signal's energy spreading in time and frequency domains. For instance, for a finite wave function (4.0.1), we can define the following variances (see **[7]**, pages: 29-37, and **[8]**, pages: 273-277):

$$\begin{aligned}&(\sigma_t)^2 = \Delta^2 t = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty}(t-\langle t\rangle)^2\left|\overline{\Psi}(t)\right|^2 dt = \int_{-\infty}^{+\infty} t^2\frac{\left|\overline{\Psi}(t)\right|^2}{\tilde{E}}dt - \langle t\rangle^2 < T^2,\\ &(\sigma_\omega)^2 = \Delta^2\omega = \frac{1}{\pi\tilde{E}}\int_{0}^{+\infty}(\omega-\langle\omega\rangle)^2\left|A(\omega)\right|^2 d\omega = \frac{1}{\pi}\int_{0}^{+\infty}\omega^2\frac{\left|A(\omega)\right|^2}{\tilde{E}}d\omega - \langle\omega\rangle^2 < (2\pi F)^2,\\ &\omega = 2\pi f,\ \sigma_\omega = 2\pi\sigma_f,\ \tilde{E} = \left\|\overline{\Psi}(t)\right\|^2 = \int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\pi}\int_{0}^{+\infty}\left|A(\omega)\right|^2 d\omega,\end{aligned} \tag{4.0.67}$$

Where mean-time and mean-frequency should be found as:

$$\langle t\rangle = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} t\left|\overline{\Psi}(t)\right|^2 dt,\ \langle\omega\rangle = \frac{1}{\pi\tilde{E}}\int_{0}^{+\infty}\omega\left|A(\omega)\right|^2 d\omega = 2\pi\langle f\rangle = 2\pi\overline{f}\ . \tag{4..68}$$

If two functions, $\Psi(\mathbf{t})$ and $A(\omega)$, form a Fourier-integral pair, then they cannot both be of short duration. This is supported by the scaling theorem,

$$\Psi(\mathbf{at}) \leftrightarrow \frac{\mathbf{1}}{|\mathbf{a}|}\mathbf{A}(\frac{\omega}{\mathbf{a}}), \tag{4.0.69}$$

*where **"a"** is a real constant. The above assertion, (4.0.69), also known as the **Uncertainty Principle**, can be given various interpretations, depending on the meaning of the term "duration".*

Using the time and frequency variances, (4.0.67), as the significant signal duration intervals, found for a finite wave function $\Psi(\mathbf{t})$, it is possible to prove validity of the following Uncertainty Principle (see **[7]** and **[8]**):

$$\begin{aligned}&\text{If } \sqrt{t}\Psi(t)\to 0 \text{ for } |t|\to\infty),\\ &\text{then } 2\pi TF > TF > \sigma_t\sigma_\omega = 2\pi\sigma_t\sigma_f = \sqrt{(\Delta^2 t)(\Delta^2\omega)} = 2\pi\sqrt{(\Delta^2 t)(\Delta^2 f)} \geq \frac{1}{2}.\end{aligned} \tag{4.0.70}$$

In the variance relations (4.0.70) we consider as obvious that absolute (or total) time and frequency durations, **T** and **F,** can never be shorter than time and frequency variances, σ_t and σ_f (and usually they should be much larger than σ_t and σ_f). It is also clear that statistical, (4.0.70), and Quantum Mechanic's aspect of Uncertainty should be fully integrated with absolute interval values Uncertainty Relations, (4.0.64), as for instance,

$$\mathbf{0} < \delta\mathbf{t}\cdot\delta\mathbf{f} = \delta\mathbf{x}\cdot\delta\mathbf{f}_x < \frac{\mathbf{1}}{\mathbf{2}} \leq \sigma_t\cdot\sigma_f = \sigma_x\cdot\sigma_{f\text{-}x} < \mathbf{F}\cdot\mathbf{T} = \mathbf{F}_x\cdot\mathbf{L} \leq \frac{\mathbf{1}}{\mathbf{4}\cdot\delta\mathbf{t}\cdot\delta\mathbf{f}} = \frac{\mathbf{1}}{\mathbf{4}\cdot\delta\mathbf{x}\cdot\delta\mathbf{f}_x},$$
$$\mathbf{0} < \delta\mathbf{t}\cdot\delta\tilde{\mathbf{E}} = \delta\mathbf{x}\cdot\delta\mathbf{p} < \frac{\mathbf{h}}{\mathbf{2}} \leq 2\pi\sigma_t\cdot\sigma_{\tilde{E}} = \sigma_x\cdot\sigma_p < \tilde{\mathbf{E}}\cdot\mathbf{T} = \mathbf{P}\cdot\mathbf{L} \leq \frac{\mathbf{h}}{\mathbf{4}\cdot\delta\mathbf{t}\cdot\delta\mathbf{f}} = \frac{\mathbf{h}}{\mathbf{4}\cdot\delta\mathbf{x}\cdot\delta\mathbf{f}_x},$$

or in cases when normal, Gauss distributions are applicable it should also be valid,

$$\frac{\mathbf{1}}{\mathbf{2}} \leq \sigma_t\cdot\sigma_f = \sigma_x\cdot\sigma_{f\text{-}x} < 36\cdot\sigma_t\cdot\sigma_f = 36\cdot\sigma_x\cdot\sigma_{f\text{-}x} < \mathbf{F}\cdot\mathbf{T} = \mathbf{F}_x\cdot\mathbf{L},$$
$$\frac{\mathbf{h}}{\mathbf{2}} \leq \mathbf{2}\pi\sigma_t\cdot\sigma_{\tilde{E}} = \sigma_x\cdot\sigma_p < 36\cdot\mathbf{2}\pi\sigma_t\cdot\sigma_{\tilde{E}} = \mathbf{36}\cdot\sigma_x\cdot\sigma_p < \tilde{\mathbf{E}}\cdot\mathbf{T} = \mathbf{P}\cdot\mathbf{L}. \tag{4.0.71}$$

In order to maintain the same signal domains proportionality (regarding non-dispersive wave packets and stable particles), the average group velocity (as concluded before for absolute interval lengths, (4.0.65)-(4.0.66)) should also depend (in the same way) on all relevant signal lengths expressed as standard deviations,

$$\bar{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta\mathbf{t}} = \frac{\delta\mathbf{x}}{\delta\mathbf{t}} = \frac{\delta\omega}{\delta\mathbf{k}} = \frac{\Delta\,\omega}{\Delta\,\mathbf{k}} = \frac{\Omega_L}{\mathbf{K}_L} = \frac{\Omega_H}{\mathbf{K}_H} = \frac{\mathbf{dx}}{\mathbf{dt}} = \frac{\mathbf{d}\omega}{\mathbf{dk}} \cong \frac{\sigma_x}{\sigma_t} = \frac{\sigma_\omega}{\sigma_k} = \frac{\sigma_f}{\sigma_{f-x}}. \tag{4.0.72}$$

If the overall signal domains proportionality has not been maintained stable, we would have a mess or disorder in particle waves equivalency situations, as well as stable particles would not be a part of reality of our universe.

Here, as the supporting fact to (4.0.72), is a very convenient place to mention again the well-known and extraordinary expression for electromagnetic waves or photons speed, which is given only in terms of conditionally static materials and space properties (or as a function of electric and magnetic permeability). Such relation is showing that dynamic time domain parameters of certain wave-packet (here photon or light waves) are perfectly united and synchronized with its space and spectral domain parameters,

$$\mathbf{c} = \frac{\mathbf{1}}{\sqrt{\mu\varepsilon}}. \tag{4.0.72-1}$$

Generalizing the same concept about space-time unity and compactness of moving (and finite) entities, objects and particles, we should become familiar with the idea that even stable and solid particles present special stationary states of (at least) 4-dimensional signals (3 space and time dimension). This way, maximal velocity c becomes a somewhat naturally guaranteed boundary factor, which secures the mentioned compactness, integrity and stability (whenever relevant and applicable).

4.0.10. Wave-Group Velocities, Moving Particles and Uncertainty Relations

Let us additionally conceptualize the same idea (about sudden signal duration changes) using the moving particle Energy Momentum 4-vector from the Minkowski-space of Relativity Theory, by (mathematically) introducing mutually coupled changes of a total system-energy and belonging total momentum, applying discrete, central-differentiations method,

$$\overline{P}_4 = \overline{P}\left[\vec{p} = \gamma m\vec{v}, \frac{E}{c} = \gamma mc\right], \ \overline{P}^2 = \vec{p}^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2}, \ E_0 = mc^2, E = \gamma E_0 \Rightarrow \tag{4.0.73}$$

$$\vec{p}^2c^2 + E_0^2 = E^2, \ (p \to p \pm \Delta p) \Leftrightarrow (E \to E \pm \Delta E) \Rightarrow$$

$$\left\{\begin{array}{l} (\mathbf{p}+\Delta\mathbf{p})^2\mathbf{c}^2 + \mathbf{E}_0^2 = (\mathbf{E}+\Delta\mathbf{E})^2 \\ (\mathbf{p}-\Delta\mathbf{p})^2\mathbf{c}^2 + \mathbf{E}_0^2 = (\mathbf{E}-\Delta\mathbf{E})^2 \\ \overline{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta\mathbf{t}} = \frac{\delta\mathbf{x}}{\delta\mathbf{t}} = \frac{\delta\omega}{\delta\mathbf{k}} = \frac{\Delta\omega}{\Delta\mathbf{k}} \end{array}\right\} \Leftrightarrow \left\{\mathbf{c}^2 \cdot \mathbf{p}\,\Delta\mathbf{p} = \mathbf{E}\,\Delta\mathbf{E} \Leftrightarrow \frac{\Delta\mathbf{E}}{\Delta\mathbf{p}} = \mathbf{c}^2\frac{\mathbf{p}}{\mathbf{E}} = \overline{\mathbf{v}}\right\} \Rightarrow \tag{4.0.74}$$

$$\Rightarrow \frac{\Delta\mathbf{x}}{\Delta\mathbf{t}} = \frac{\delta\mathbf{x}}{\delta\mathbf{t}} = \frac{\Delta\mathbf{E}}{\Delta\mathbf{p}} = \mathbf{c}^2\frac{\mathbf{p}}{\mathbf{E}} = \overline{\mathbf{v}} = \mathbf{h}\frac{\Delta\mathbf{f}}{\Delta\mathbf{p}} = \frac{\Delta\tilde{\mathbf{E}}}{\Delta\mathbf{p}} = \frac{\Delta\omega}{\Delta\mathbf{k}} = \frac{\delta\omega}{\delta\mathbf{k}} \leq \mathbf{c}.$$

What we can see from (4.0.74) is that sudden energy-momentum changes in a certain system (moving particle, here) are directly related to its average center-of-mass velocity, which is at the same time equal to the system average-group-velocity, as shown in (4.0.72). *This now gives new and more tangible meaning of Uncertainty Relations, which should be well analyzed, before we make other conclusions. Before, we found that signal domains proportionality was only dimensionally and quantitatively indicating that this could be the signal group velocity (or particle velocity), and now we can safely confirm that this is really the case.*

The idea here is to show that so-called *Uncertainty Relations are directly related to a velocity of matter-waves propagation. The meaning of that is that at the same time when certain motional object or signal would experience sudden change of its energy related parameters, matter waves are automatically created. This produces results captured by Uncertainty Relations (indirectly saying that there is no real Uncertainty in its old and traditional meaning).* For instance, we can express the average group velocity $\overline{\mathbf{v}}$ associated to the transformations (4.0.74) as:

$$\left\{\mathbf{u} = \frac{\omega}{\mathbf{k}}, \ \omega = \mathbf{ku}\right\} \Rightarrow \Delta\omega = (\mathbf{k} + \frac{1}{2}\Delta\mathbf{k})(\mathbf{u} + \frac{1}{2}\Delta\mathbf{u}) - (\mathbf{k} - \frac{1}{2}\Delta\mathbf{k})(\mathbf{u} - \frac{1}{2}\Delta\mathbf{u}) =$$

$$= \mathbf{k}\Delta\mathbf{u} + \mathbf{u}\Delta\mathbf{k} \Leftrightarrow \overline{\mathbf{v}} = \frac{\Delta\omega}{\Delta\mathbf{k}} = \mathbf{u} + \mathbf{k}\frac{\Delta\mathbf{u}}{\Delta\mathbf{k}} \Leftrightarrow \tag{4.0.75}$$

$$\Leftrightarrow \left\{\mathbf{v} = \mathbf{u} + \mathbf{k}\frac{\mathbf{du}}{\mathbf{dk}} = \frac{\mathbf{d}\omega}{\mathbf{dk}} = \frac{\mathbf{d}\tilde{\mathbf{E}}}{\mathbf{dp}} = \frac{\mathbf{dx}}{\mathbf{dt}} = \textbf{immediate group velocity}\right\}.$$

Such group velocity (expressed in terms of finite differences) is fully analog to its differential form where infinitesimal signal changes are involved. By merging average group velocity with Uncertainty Relations, we again see that they are mutually compatible,

$$\left\{\begin{array}{l} \bar{\mathbf{v}} = \mathbf{u} + \mathbf{k}\dfrac{\Delta \mathbf{u}}{\Delta \mathbf{k}} = \dfrac{\Delta \omega}{\Delta \mathbf{k}} = \dfrac{\Delta \tilde{\mathbf{E}}}{\Delta \mathbf{p}} = \dfrac{\Delta \mathbf{x}}{\Delta \mathbf{t}} = \textbf{average group velocity} \\ \textit{and} \\ \left|\Delta \mathbf{x}\, \Delta \mathbf{p}\right| = \left|\Delta \mathbf{t}\, \Delta \tilde{\mathbf{E}}\right| = \mathbf{h}\left|\Delta \mathbf{t}\, \Delta \mathbf{f}\right| > \mathbf{h/2},\ \Delta \tilde{\mathbf{E}} = \mathbf{h}\Delta \mathbf{f}\ , \\ \mathbf{0} < \delta \mathbf{t} \cdot \delta \mathbf{f} = \delta \mathbf{x} \cdot \delta \mathbf{f}_{\mathbf{x}} < \dfrac{1}{2} \le \mathbf{F} \cdot \mathbf{T} = \mathbf{F}_{\mathbf{x}} \cdot \mathbf{L} \le \dfrac{1}{4 \cdot \delta \mathbf{t} \cdot \delta \mathbf{f}} = \dfrac{1}{4 \cdot \delta \mathbf{x} \cdot \delta \mathbf{f}_{\mathbf{x}}} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta \mathbf{t}} = \frac{\Delta \mathbf{E}}{\Delta \mathbf{p}} = \mathbf{h}\frac{\Delta \mathbf{f}}{\Delta \mathbf{p}} = \frac{\Delta \tilde{\mathbf{E}}}{\Delta \mathbf{p}} = \frac{\Delta \omega}{\Delta \mathbf{k}} = \mathbf{u} + \mathbf{k}\frac{\Delta \mathbf{u}}{\Delta \mathbf{k}} = \frac{\delta \mathbf{x}}{\delta \mathbf{t}} = \frac{\delta \omega}{\delta \mathbf{k}} = \frac{\mathbf{d}\,\mathbf{x}}{\mathbf{d}\,\mathbf{t}} = \frac{\mathbf{d}\,\omega}{\mathbf{d}\,\mathbf{k}}. \qquad (4.0.76)$$

*A very interesting fact regarding average group velocity "$\bar{\mathbf{v}}$" in (4.0.75) and (4.0.76), which could pass unnoticed, is that the full analogy between expression for average group velocity "$\bar{\mathbf{v}}$" and expression for immediate group velocity "**v**" are not made as an approximation, automatically by formal and simple replacing of infinitesimal difference "d" with discrete, delta-difference "Δ". The development of the average group velocity (given here) is applying <u>symmetrical central differences</u> on basic definitions of group and phase velocity. Such methodology is fully correct (judging by results), showing that there is a deterministic connection between the physics of continuum and physics of discrete or finite steps (to support better this statement it would be necessary to devote certain time to learn about properties of central, symmetrical differences).*

[♣ COMMENTS & FREE-THINKING CORNER: *Here is the place to address another important aspect of united symmetries and analogies between different (mutually coupled) conservation laws of physics, originating from the concept of Minkowski-space 4-vectors used in Relativity Theory. For instance (4.0.73) presents some kind of space-time unification of energy and momentum conservation laws, making that the following expression is always invariant regarding different reference frames which are mutually in a uniform relative motion,*

$$\vec{\mathbf{p}}_1^2 - \frac{\mathbf{E}_1^2}{\mathbf{c}^2} = \vec{\mathbf{p}}_2^2 - \frac{\mathbf{E}_2^2}{\mathbf{c}^2} = \ldots = \vec{\mathbf{p}}_n^2 - \frac{\mathbf{E}_n^2}{\mathbf{c}^2} = \textbf{invariant}$$

In addition, the relativistic space-time interval is invariant,

$$(\Delta \mathbf{r})_1^2 - \mathbf{c}^2(\Delta \mathbf{t})_1^2 = (\Delta \mathbf{r})_2^2 - \mathbf{c}^2(\Delta \mathbf{t})_2^2 = \ldots = (\Delta \mathbf{r})_n^2 - \mathbf{c}^2(\Delta \mathbf{t})_n^2 = \textbf{invariant}$$
$$-\left[1 - \frac{1}{\mathbf{c}^2}\frac{(\Delta \mathbf{r})_1^2}{(\Delta \mathbf{t})_1^2}\right]\mathbf{c}^2(\Delta \mathbf{t})_1^2 = -\left[1 - \frac{\mathbf{v}_1^2}{\mathbf{c}^2}\right]\mathbf{c}^2(\Delta \mathbf{t})_1^2 = \ldots = -\left[1 - \frac{\mathbf{v}_n^2}{\mathbf{c}^2}\right]\mathbf{c}^2(\Delta \mathbf{t})_n^2 = \textbf{invariant}\,.$$

What we can see from here presented invariant expressions, is that only physical value couples which are relevant for (average) group velocity formulation, are involved in such invariant expressions, and that by analogy we could formulate new invariant expressions such as,

$$(\Delta\, \omega)_1^2 - \mathbf{c}^2(\Delta\, \mathbf{k}\,)_1^2 = (\Delta\, \omega)_2^2 - \mathbf{c}^2(\Delta\, \mathbf{k}\,)_2^2 = \ldots = (\Delta\, \omega)_n^2 - \mathbf{c}^2(\Delta\, \mathbf{k}\,)_n^2 = \textbf{invariant}\,,$$
$$-\left[1 - \frac{\mathbf{v}_1^2}{\mathbf{c}^2}\right]\mathbf{c}^2(\Delta \mathbf{k})_1^2 = \ldots = -\left[1 - \frac{\mathbf{v}_n^2}{\mathbf{c}^2}\right]\mathbf{c}^2(\Delta \mathbf{k})_n^2 = \textbf{invariant}\,,$$

$$(\Delta\theta)_1^2 - \omega_c^2(\Delta t)_1^2 = (\Delta\theta)_2^2 - \omega_c^2(\Delta t)_2^2 = ... = (\Delta\theta)_n^2 - \omega_c^2(\Delta t)_n^2 = \textbf{invariant}\ ,\ \omega = d\theta/dt$$

$$L_1^2 - \frac{E_1^2}{\omega_c^2} = L_2^2 - \frac{E_2^2}{\omega_c^2} = ... = L_n^2 - \frac{E_n^2}{\omega_c^2} = \textbf{invariant}\ ,\ \omega_c = \textbf{Const.},$$

where only absolute signal durations are involved.

In the same way, analogically, it should be possible to formulate a number of new invariant relations (of course still hypothetically valid until being proven from a more general platform). ♣*]*

Now is also possible to prove validity of the following expressions for group and phase velocity,

$$\left\{\begin{array}{l} v = \frac{dE}{dp} = \frac{dE_k}{dp} = \frac{d}{dp}\left[\gamma mc^2\right] = \frac{d}{dp}\left[(\gamma-1)mc^2\right] = \frac{d}{dp}\left[\frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}}\right]\ , \\ u = \frac{E_k}{p} = \lambda\cdot f\ ,\quad v = u + p\frac{du}{dp}\ ,\quad p = \gamma mv\ ,\quad \lambda = \frac{h}{p}\ ,\quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \end{array}\right\} \Rightarrow \tag{4.0.77}$$

$$\Rightarrow\left\{\tilde{E} \Leftrightarrow E_k\right\} \Rightarrow \left\{\begin{array}{l} u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \lambda\cdot f = \frac{\tilde{E}}{p} \le c\ , \\ v = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}\ , \\ k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}\gamma\ mv\ ,\quad \omega = 2\pi\ f = \frac{E_k}{h} \\ E_k = (\gamma-1)mc^2 \end{array}\right\}, \tag{4.0.78}$$

What is directly shown (contrary to similar position of contemporary Quantum Theory) that *wave packet energy corresponds only to motional or kinetic particle energy and that group velocity corresponds to the particle velocity (or that the motional energy is propagating by group velocity). Quantum Theory considers that a wave-packet, which represents certain particle, takes into account total particle energy, including its rest mass, what is not correct.* The phase velocity found here is only relevant for elementary, simple harmonic wave components propagation and it has the same limits as group velocity, $\mathbf{0 \le 2u \le \sqrt{uv} \le v \le c}$.

The other useful relations between motional energy and group and phase velocity are,

$$\left\{\begin{array}{l}\left\{E_k \Leftrightarrow \tilde{E} = pu\,,\ pv = E_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right] = \frac{\gamma^2-1}{\gamma}mc^2\right\}, \\ \left\{(\frac{u}{v}) = \frac{\tilde{E}}{pv} = \frac{pu}{E_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right]} = \frac{1}{1+\sqrt{1-(\frac{v}{c})^2}} = \frac{\gamma}{\gamma+1}\right\}, \\ \left\{\lambda = \frac{h}{p}, k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p, \omega = 2\pi f\,,\ \frac{d\lambda}{\lambda} = -\frac{dp}{p} = -\frac{dk}{k} = -\frac{df}{f}, u = \frac{\omega}{k}, v = \frac{d\omega}{dk}\right\}\end{array}\right\} \Rightarrow \tag{4.0.79}$$

$$\Rightarrow\left\{\begin{array}{l} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1+\frac{uv}{c^2}}\,, \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{f}{f_s} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p} \quad \Rightarrow \\ \Rightarrow \quad 0 \le 2u \le \sqrt{uv} \le v \le c\ , \\ d\tilde{E} = hdf = mc^2 d\gamma, \quad \frac{df}{f} = (\frac{dv}{v})\cdot\frac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} \Rightarrow \frac{\Delta f}{\bar{f}} = (\frac{\Delta v}{\bar{v}})\cdot\frac{1+\sqrt{1-\frac{\bar{v}^2}{c^2}}}{1-\frac{\bar{v}^2}{c^2}}\end{array}\right\} \tag{4.0.80}$$

$$\frac{df}{f} = \frac{d\tilde{E}}{\tilde{E}} = -(\frac{v}{u})\cdot\frac{d\lambda}{\lambda} = (\frac{dv}{v})(1+1/\gamma)\gamma^2 = -(1+\frac{1}{\gamma})\cdot\frac{d\lambda}{\lambda} \approx \left\{\begin{array}{l} 2\frac{dv}{v} = -2\frac{d\lambda}{\lambda}\ , \quad \text{for} \quad v \ll c \\ -\frac{d\lambda}{\lambda}\ , \quad \text{for} \quad v \approx c\end{array}\right\}$$

$$\Rightarrow\left\{\begin{array}{l} \boldsymbol{\frac{\tilde{E}}{mc^2} = \frac{hf}{mc^2} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} - 1 = \gamma - 1 = \frac{\tilde{E}}{E_0},\ E_0 = mc^2 = const.,} \\ \boldsymbol{\frac{\tilde{E}}{\gamma mc^2} = \frac{hf}{\gamma mc^2} = 1 - \sqrt{1-(\frac{v}{c})^2} = 1 - \frac{1}{\gamma} = \frac{\tilde{E}}{E_{total}},\ E_{total} = \gamma mc^2 = \gamma E_0\,,} \\ \boldsymbol{\frac{\tilde{E}}{E_k} = \frac{\tilde{E}}{(\gamma-1)mc^2} = \frac{hf}{(\gamma-1)mc^2} = 1,\ E_{total} = E_0 + E_k = E_t\ ,} \\ \boldsymbol{p^2c^2 + E_0^2 = E_t^2\,, \quad p^2v - pE_t + p_0E_0 = 0,} \\ \boldsymbol{\tilde{E} = pu = -E_0 \pm \sqrt{E_0^2 + p^2c^2} = E_k \left\{= -E_0 + \sqrt{E_0^2 + p^2c^2} = E_0\left[\sqrt{1+(\frac{pc}{E_0})^2} - 1\right]\right\},} \\ \left\{\begin{array}{l}\boldsymbol{\vec{p} + \tilde{p} = \vec{P} = \overrightarrow{const} \Rightarrow d\vec{p} = -d\tilde{p}} \\ \boldsymbol{d\tilde{E} = hdf = d(pu) = dE_k = vdp = c^2 d(\gamma m) = -c^2 d\tilde{m} = -d(\tilde{p}u) =} \\ \boldsymbol{= -vd\tilde{p}\cdot\cos(\vec{p},\tilde{p}) = \{vd\tilde{p}\ or\ -vd\tilde{p}\} = hvdf_s}\end{array}\right\} \\ \boldsymbol{\Rightarrow \Delta E_k = -\Delta\tilde{E},\ \Delta p = -\Delta\tilde{p},\ \Delta L = -\Delta\tilde{L},\ \Delta q = -\Delta\tilde{q}, \Delta\dot{p} = -\Delta\dot{\tilde{p}}, \Delta\dot{L} = -\Delta\dot{\tilde{L}}, ...}\end{array}\right\} \tag{4.0.81}$$

The *Wave-packet and Particle Equivalency or Analogy* gets clearer if we place the corresponding expressions in the same table T.4.0.2, as for instance,

T.4.0.2	**Wave-Packet**	**Particle**
Motional Energy	$\tilde{E} = \int_{-\infty}^{+\infty} \lvert\Psi(t)\rvert^2 dt = \tilde{m}c^2 = hf =$ $= \hbar\omega = \tilde{m}vu = pu = \tilde{p}u =$ $= \frac{1}{\pi}\int_0^{\infty} A^2(\omega)\, d\omega = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)\, dt$	$E_k = (\gamma - 1)mc^2$ $(E_{tot.} = \gamma mc^2 = E_0 + E_k,$ $E_0 = mc^2 = const.)$
Mass	<u>**Photons**</u> $\tilde{m} = \frac{\tilde{E}}{c^2} = \frac{1}{c^2}\int_{-\infty}^{+\infty} \lvert\Psi(t)\rvert^2 dt =$ $= \frac{1}{\pi c^2}\int_0^{\infty} A^2(\omega)\, d\omega =$ $= \frac{1}{2c^2}\int_{-\infty}^{+\infty} a^2(t)\, dt$ <u>**Any Wave-Packet**</u> $\tilde{m} = \frac{\tilde{E}}{vu} =$ $= \frac{1}{\gamma c^2}\int_{-\infty}^{+\infty} \lvert\Psi(t)\rvert^2 dt$ $= \frac{1}{\gamma\pi c^2}\int_0^{\infty} A^2(\omega)\, d\omega$	$m_{mot.} = \frac{E_k}{c^2} = (\gamma - 1)m$ $(m_{tot.} = \frac{E_{tot.}}{c^2} = \gamma m =$ $= m + m_{mot.},$ $m = const.)$
Momentum	$\tilde{p} = \tilde{m}v = \tilde{E}/u =$ $= \frac{1}{\pi c^2}\int_0^{\infty} \frac{d\omega}{dk} \cdot [A(\omega)]^2 \cdot d\omega =$ $= \frac{1}{2c^2}\int_{-\infty}^{+\infty} v(t) \cdot a^2(t) \cdot dt$	$p = \gamma mv = \frac{h}{\lambda}$
Group Velocity	$\underline{v = \frac{d\tilde{E}}{d\tilde{p}} = \frac{d\tilde{E}}{dp} = \frac{d\omega}{dk}},$ $\bar{v}_g = \bar{v} = \frac{\int_0^{\infty} \frac{d\omega}{dk} \cdot [A(\omega)]^2 d\omega}{\int_0^{\infty} [A(\omega)]^2 d\omega} =$ $= \frac{\int_{-\infty}^{+\infty} v(t) \cdot a^2(t)\, dt}{\int_{-\infty}^{+\infty} a^2(t)\, dt} = \frac{\delta\omega}{\delta k}$	$v = \frac{dE_k}{dp}$ $(= \frac{dE_{tot.}}{dp})$
Phase Velocity	$\underline{u = \frac{\tilde{E}}{\tilde{p}} = \frac{\tilde{E}}{p} = \frac{\omega}{k} = \lambda f},$ $\bar{v}_f = \bar{u} = \frac{\int_0^{\infty} \frac{\omega}{k} \cdot [A(\omega)]^2 d\omega}{\int_0^{\infty} [A(\omega)]^2 d\omega} =$ $= \frac{\int_{-\infty}^{+\infty} u(t) \cdot a^2(t)\, dt}{\int_{-\infty}^{+\infty} a^2(t)\, dt} = \frac{\omega_0}{k_0}$	$u = \frac{E_k}{p} = \lambda f$ $(\lambda = \frac{h}{p})$

4.0.11. Generalized Wave Functions and Unified Field Theory

Obviously, particle-waves duality concept favored in this paper creates sufficiently clear frontier between stable particle (which has constant rest mass in its center of mass reference system), and wave or particle-wave phenomena that belongs only to different states of motions. Contrary to such situation, we also know that internal structure of a stable particle is composed of wave and particle-waves constituents, which are properly "geared and fitted" producing (looking only externally) a stable particle (**internally assembled as standing waves and resonant field-structures**). We could also say that *particle presents a stable packaging format for certain group of matter waves (see (4.0.35)-(4.0.44)).* What is missing in such conceptual picture of particle-waves duality is to explain conditions when and how certain dynamic combination (superposition) of waves transforms into a stable particle; -for instance, when motional wave mass given in T.4.0.1 will create a stable particle. Here favored concept is that relatively free, open-space propagating wave-packets, under certain circumstances self-close their waveforms, and become spatially localized objects, being internally structured as certain standing wave, self-stabilized and vortex-like field in resonance. The continuity and energy coupling manifestations between internal particle-wave structures and externally detectable waves and fields (of any origin) de Broglie matter waves, appear every time when particles are in motion (and respecting de Broglie – Planck – Einstein expressions for wavelength and wave-packet energy).

Intuitively, we see that a stable particle (having stable rest-mass) should have certain stationary waving structure (internally properly balanced), and we also know that in many interactions stable particles are manifesting particle-wave duality properties, or could be disintegrated into pure wave energy constituents. It is also known that convenient superposition of pure wave elements could produce a stable particle (electron-positron creation, for instance). Consequently, the general case of a stable particle should be that its internal constituents are composed of wave-mass elements. The proper internal and dynamic "gearing and fitting" of all particle constituents should produce a stable particle, which has stable rest mass (found by applying the rules of Relativity Theory: -connection between total energy, rest energy and particle momentum). Of course, there are intermediary and mixed particle-wave states, which sometimes behave more as particles or as waves.

The way to establish the Unified Field Theory (suggested in this paper) will start from the condition that the square of the wave function should present an active and measurable power-function, $\Psi^2(\mathbf{t},\mathbf{r}) = \mathbf{P}(\mathbf{t},\mathbf{r}) = \mathbf{d}\tilde{\mathbf{E}} / \mathbf{dt} = -\mathbf{dE_k} / \mathbf{dt}$. In other words, if there is certain power or energy content in certain domain, this is good enough to have particles and/or waves there. Since *particles are only particular states of waves packaging formats*, we would need to have sufficiently good and generally applicable matter-waves, wave functions in order to conceptualize and model different matter-states and interactions between them. We already used to express power-functions as products between corresponding current and voltage (in electro-technique), or force and velocity (in mechanics) etc. Here we consider the power-functions as the most relevant wave functions. This will directly enable us to develop new forms of wave equations (valid for Gravitation, Electromagnetism and other fields), formally similar or equivalent to Schrödinger and classical wave equations, but now dealing more with velocities, forces, currents, voltages... Let us first show what possible forms regarding creating such wave functions are, by showing expressions for power (in electric circuits, mechanics, electromagnetic fields etc.; -see (4.23)). This way we will operate with empirically and dimensionally clear and measurable wave functions, being able to explore a number of analogies between them, and to refer to well-developed and proven methods and theories (of Classical Physics), where most of the facts are non-disputable, and where we really know what we are talking about. In some later steps, such dimensional wave functions could easily be normalized (losing dimensionality). Additional modeling can result in getting isomorphic forms of Statistics and Probability familiar functions, like in Quantum Theory, but such aspect of upgraded wave functions would not be analyzed here (at least not before we get a clear picture of what we are dealing with, and how further generalizations could be made). In Quantum Theory, the wave function $\Psi^2(\mathbf{t})$ is conveniently modeled as a probability function, but effectively it behaves in many aspects like normalized and dimensionless Power function. Here it will be closely related to Active Power (or power delivered to a load expressed in Watts as its units). Here, an attempt will be made to connect analogically an arbitrary Power Function $\Psi^2(\mathbf{t})$ (which is a product between corresponding current and voltage, or product between force and velocity, or product between any other relevant, mutually conjugated functions creating power), to a wave-function, as known in Quantum Mechanics. This way we will not go out of deterministic and dimensional frames of such wave functions. Analyzed from the point of view of energy, any wave propagation in time and frequency domain can be mutually (time-frequency) correlated using Parseval's identity (4.0.4). Consequently, the instantaneous (time-domain) Power-signal can be presented as the square of the wave function $\Psi^2(\mathbf{t})$. The analysis of the optimal power transfer (carried by $\Psi^2(\mathbf{t})$) can be extended to any wave-like propagation field (and to arbitrary shaped signals), and profit enormously (in booth, directions) after unifying traditional concepts of Active, Reactive and Apparent power with the $\Psi^2(\mathbf{t})$ wave-function mathematics, based on Analytic Signal methodology.

$$\Psi^2(t,r) = P(t,r) = \frac{d\tilde{E}}{dt} (=) \text{Active Power } (=)$$
$$(=)\left\{\begin{array}{lll} i(t)\cdot u(t) & (=) & [\text{Current}\cdot\text{Voltage}], \text{ or} \\ f(t)\cdot v(t) & (=) & [\text{Force}\cdot\text{Velocity}], \text{ or} \\ \tau(t)\cdot\omega(t) & (=) & [\text{Orb.}-\text{moment}\cdot\text{Angular velocity}], \text{or} \\ (\vec{E}\times\vec{H})\cdot\vec{S} & (=) & \left[\overrightarrow{\text{Pointyng Vector}}\right]\cdot\overrightarrow{\text{Surface}} \\ ----- & (=) & --------- \\ s_1(t)\cdot s_2(t) & (=) & [(\text{signal}-1)\cdot(\text{signal}-2)] \end{array}\right\} \qquad (4.0.82)$$

Since the Electric Circuits Theory is already extremely well developed (being very clear and deterministic theory), we can profit from its modeling and methodology, related to handling optimal active-power transfer, to real, imaginary, and complex power, to real and complex impedances, to pharos complex notation of electric values etc. Eventually we will start applying by analogy, models, structures and conclusions developed in Electronics, to Classical Mechanics, Gravitation, Quantum Mechanics... We are also very familiar with all kinds of measurements regarding electric currents, voltages, electric and magnetic fields, and we know how to produce and use them in many different ways. We also know how to transform electric energy into mechanical motion, oscillations, electromagnetic fields, light, sound etc. Very rich mathematical modeling is well developed and perfectly matched regarding handling all kinds of electric, magnetic and electromagnetic phenomena.

As we can see from (4.0.82) electric power is equal to a product between current and voltage measured in relation to the same load (or same component), and this is the analogical case in mechanics or in any other domain, where power is again equal to a product between certain couple of mutually coupled values. Since here we accepted (at least in this paper) that the square of the wave function corresponds to a relevant power-function, the next idea or proposal is to try to model Quantum Theory matter-wave function as a product between two mutually coupled relevant (conjugated) signals. This way, later we would be able to come closer to developing analogical interpretation between electrical power and square of a mater-wave function. In this way, the complete methodology, modeling and conceptualization known in electromagnetism would naturally be transformed to quantum world wave functions. We will do it in the best possible way, systematically and taking into account all relevant facts (not just literally transforming one set of expressions to the other).

Since we already know that the Analytic Signal modeling gives a very rich mathematical framework for arbitrary signals and wave functions analyses, it should be directly applicable to any current and voltage signals. We will make one step forward, first by transforming all power functions from (4.0.82) into Analytic Signal forms. The advantage here is that none of the functions involved should be only a simple harmonic function (or that voltage and current, that are creating the power function could be arbitrary shaped and frequency-wise wideband; -of course, simple harmonic waveforms are also included here as bottom line, simplest cases). The complexity and mathematical richness of such approach, related to electric currents and voltages, will show a lot of advantages and new opportunities regarding analogically applying similar methodology to any kind of waveform analyses (compared to traditional methods). Consequently, we could ask ourselves ***why not apply it to quantum-mechanical matter-waves*** (of course, reasonably and taking into account all specifics and constraints of matter-waves phenomenology). Doing this, it will be shown that very convenient fields and waves' unification platform can be established, because all such phenomenology would be modeled inside the same and superior mathematical framework.

In the next part of this paper, it will be shown what kind of innovative mathematical modeling will be under our control for analyzing and characterizing wideband and arbitrary shaped signals.

[♣ **COMMENTS & FREE-THINKING CORNER**:

4.0.11.0. Wave Function, Energy, Power and Impedance

We can say that, in this work, wave function is directly related to the power function of some kind of movement, i.e. that it directly represents certain energy transfer. In order to determine the power, it is most commonly implied in physics that we know the two functions: $s_1(t)$ and $s_2(t)$, the product of which defines the power, as mentioned in (4.0.82).

The kind of the energy transfer from its source to the load depends on the coherence relation between the functions $s_1(t)$ and $s_2(t)$, *which determine the* power*. That is how we can define coherence coefficient between $s_1(t)$* and *$s_2(t)$:*

$$K_t = \frac{\int_{-\infty}^{+\infty} s_1(t)\cdot s_2(t)\cdot dt}{\int_{-\infty}^{+\infty} |s_1(t)|\cdot|s_2(t)|\cdot dt} = \frac{\int_{-\infty}^{+\infty} \Psi^2(t)\cdot dt}{\int_{-\infty}^{+\infty} |\Psi^2(t)|\cdot dt}, \tag{4.0.83}$$

And determine their phase functions:

$$\varphi_1(t) = \text{arc tang}\left[\hat{s}_1(t)/s_1(t)\right],\ (s_1(t) = a_1(t)\cos\varphi_1(t) = \frac{1}{\pi}\int_0^{\infty}\left[A_1(\omega)\cos(\omega t + \Phi_1(\omega))\right]d\omega,$$

$$\hat{s}_1(t) = H[s_1(t)] = a_1(t)\sin\varphi_1(t),$$

$$\overline{s}_1(t) = s_1(t) + j\hat{s}_1(t) = a_1(t)\, e^{j\varphi_1(t)}),$$

$$\varphi_2(t) = \text{arc tang}\left[\hat{s}_2(t)/s_2(t)\right],\ (s_2(t) = a_2(t)\cos\varphi_2(t) = \frac{1}{\pi}\int_0^{\infty}\left[A_2(\omega)\cos(\omega t + \Phi_2(\omega))\right]d\omega,$$

$$\hat{s}_2(t) = H[s_2(t)] = a_2(t)\sin\varphi_2(t),$$

$$\overline{s}_2(t) = s_2(t) + j\hat{s}_2(t) = a_2(t)\, e^{j\varphi_2(t)}), \tag{4.0.84}$$

therefore, it is very simple to find an additional criterion about the mutual position (coherence) of $s_1(t)$ and $s_2(t)$ by simply forming their phase difference,

$\Delta\varphi(t) = \varphi_2(t) - \varphi_1(t)$. (4.0.85)

It is also possible to find the amplitude spectral functions for the wave function factors $s_1(t)$ and $s_2(t)$ $A_1(\omega)$ i $A_2(\omega)$, *as well as to find the functions of the appropriate phase functions $\Phi_1(\omega)$ and $\Phi_2(\omega)$ using* definitions from *(*4.0.3*)* and T.4.0.1*.*

Now we can make an attempt to systematize the coherence criteria for the wave function factors $s_1(t)$ and $s_2(t)$ and to connect them with the optimal energy transport analysis:

-If $K_t = 1$, $s_1(t)$ and $s_2(t)$ are totally coherent, or mutually *in phase*, *$\Delta\varphi(t) = 0$, we have an optimal energy transf*er. That is the case of active *or resistive* load *impedances.*

-I*f f $K_t = 0$*, s_1*(t) and $s_2(t)$ are mutually orthogonal or phase* shifted *for $\pi/2$, $|\Delta\varphi(t)| = \pi/2$, no energy circulation from the source to the* load *is possible (i.e. the* load *does not receive any energy).*

-If $K_t = -1$, s1(t) and $s_2(t)$ are in the counter-phase, then there is a nonoptimal energy transfer, i.e. the energy is completely reflected off the load *and returned to its source (the energy has a "-" sign). That is the case of the reactive (or complex)* load *impedances.*

Power *factor coherence criterion* between *$s_1(t)$ and $s_2(t)$ can be widened by comparing their phase spectral functions, which are defined by (*4.0.3*), i.e. by forming the difference:*

$$\Delta\Phi(\omega) = \Phi_2(\omega) - \Phi_1(\omega). \tag{4.0.86}$$

*It is equally justified to introduce the coherence factor into the spectral domain, using amplitude spectral functions (*4.0.3*), analogous to the definition (*4.0.83*),*

$$K_\omega = \frac{\int_0^{+\infty} A_1(\omega)\cdot A_2(\omega)\cdot d\omega}{\int_0^{+\infty} |A_1(\omega)|\cdot|A_2(\omega)|\cdot d\omega} \quad . \tag{4.0.87}$$

While analyzing optimal energy transfer (and introducing the terms like: active, reactive and complex power) great attention should be paid to the fact that, in a general case, Hilbert transformation of wave function elements is not linear, i.e. the following equations are valid:

$$H[\Psi^2(t)] = H[s_1(t)\, s_2(t)] = \mathbf{s_1 s_2} \frac{\mathbf{s_1\hat{s}_2 + \hat{s}_1 s_2}}{\mathbf{s_1 s_2 - \hat{s}_1\hat{s}_2}}, \tag{4.0.88}$$

$$H[s_2(t) / s_1(t)] = \left(\frac{\mathbf{s_2}}{\mathbf{s_1}}\right)\frac{\mathbf{s_1\hat{s}_2 - \hat{s}_1 s_2}}{\mathbf{s_1 s_2 + \hat{s}_1\hat{s}_2}}. \tag{4.0.89}$$

4.0.11.1. Generalized Impedances

In order to analyze the problems related to energy transports, it is necessary that an energy source and its consumer or load exist. Loads are characterized by the term "load impedances". In physics, electrotechnics, electromagnetism, acoustics, and mechanics we can define a universal term "dynamic (time-dependant) load impedance" in the following manner:

$$Z(t) = s_2(t) / s_1(t) = \tilde{S}(t) / s_1^2(t) = s_2^2(t) / \tilde{S}(t) =$$
$$= [\Psi(t) / s_1(t)]^2 = [s_2(t) / \Psi(t)]^2 = \frac{a_2(t)}{a_1(t)}\frac{\cos\varphi_2(t)}{\cos\varphi_1(t)}. \tag{4.0.90}$$

This time it is advisable to think about the way to treat a complex, time dependant impedance (taking (4.0.89) into account), defined as a quotient of appropriate analytical (complex) functions:

$$\overline{Z}(t) = \overline{s}_2(t) / \overline{s}_1(t) = \frac{a_2(t)}{a_1(t)} e^{j[\varphi_2(t)-\varphi_1(t)]} = \frac{a_2(t)}{a_1(t)} e^{j\Delta\varphi(t)} , \qquad (4.0.91)$$

because $\overline{Z}(t) = \overline{s}_2(t) / \overline{s}_1(t) \neq$ *$s_2(t)$ / $s_1(t)$* $+ j H[s_2(t)/s_1(t)]$ (except for simple harmonic wave forms).

In a similar manner we can define frequency dependant, complex impedances, as a quotient of spectral (c*omplex) functions of the appropriate wave function factors:*

$$Z(\omega) = \overline{Z}(\omega) = \frac{s_2(\omega)}{s_1(\omega)} = \frac{A_2(\omega)}{A_1(\omega)} e^{j[\Phi_2(\omega)-\Phi_1(\omega)]} = \frac{A_2(\omega)}{A_1(\omega)} e^{j\Delta\Phi(\omega)} . \qquad (4.0.92)$$

In the example of passive electric impedances (or networks made of a loads *with the resistance R, capaci*tance *C and induc*tance *L), it is possible to present various ways for a more precise defining of the term impedance.* Later on, the *same procedure can be extended to other physics disci*plines as well using "current-force" and "voltage-velocity" analogies (see (1.82)), as for instance:

$$s_1(t) = i(t) = I(t)\cos\varphi_i(t) \; (=) \text{ current}, \; \overline{s}_1(t) = \overline{i}(t) = I(t)e^{j\varphi_i(t)}, \; i(\omega) = I(\omega)e^{i\Phi_i(\omega)}$$
$$s_2(t) = u(t) = U(t)\cos\varphi_u(t) \; (=) \text{ voltage}, \; \overline{s}_2(t) = \overline{u}(t) = U(t)e^{j\varphi_u(t)}, \; u(\omega) = U(\omega)e^{i\Phi_u(\omega)}$$
$$\tilde{S}(t) = d\tilde{E}/dt = \Psi^2(t) = u(t)\, i(t) \; (=) \text{ force}. \qquad (4.0.93)$$

4.0.11.2. Resistive Impedances

Let us assume that the current $i_R(t) = i_R$ runs through the electrical resistance R. In that case, voltage $u_R(t) = u_R$ is formed on the resistance R. It is obvious that the following relations are valid:

$$u_R(t) = Z_R\, i_R = R\, i_R, \; \tilde{S}_R(t) = u_R(t)\, i_R(t) = R\, i_R^2 = \Psi_R^2(t),$$
$$K_t = 1, K_\omega = 1, \; \Delta\varphi(t) = \varphi_u - \varphi_i = 0, \; \Delta\Phi(\omega) = \Phi_u - \Phi_i = 0, \qquad (4.0.94)$$

And from this, the resistive dynamic impedance is:

$$Z_R(t) = u_R(t) / i_R(t) = \overline{Z}_R(t) = Z_R(\omega) = u_R(\omega) / i_R(\omega) = R = [\Psi_R(t) / i_R(t)]^2. \qquad (4.0.95)$$

4.0.11.3. Inductive impedances

*The previous procedure can be extended analogously. If the current $i_L(t) = i_L$ runs through the induc*tance *L, voltage $u_L(t) = u_L$* will be formed on the inductance *L. The validity of the following relations is obvious (or can be proved):*

$$u_L(t) = Z_L\, i_L = -L(di_L/dt), \; \tilde{S}_L(t) = u_L(t)\, i_L(t) = -L\, i_L(di_L/dt) = \Psi_L^2(t),$$

a) $U(\omega) = -\omega L I(\omega)$,
$\Delta\Phi(\omega) = \Phi_u - \Phi_i = +\pi/2$,

b) $U(\omega) = +\ \omega L I(\omega)$,
$\Delta\Phi(\omega) = \Phi_u - \Phi_i = \{+3\pi/2, -\pi/2\}$,

$$\Delta\varphi(t) = \varphi_u - \varphi_i = arctg\left[\frac{\frac{d\varphi_i}{dt}}{\frac{1}{I(t)}\frac{dI(t)}{dt}}\right] +/- \pi \quad , \qquad (4.0.96)$$

from this, the inductive impedance is:

$$Z_L(t) = u_L(t) / i_L(t) = \frac{U_L(t)}{I_L(t)}\frac{\cos\varphi_u(t)}{\cos\varphi_i(t)} = (-L / i_L)\ (di_L/dt)\ = [\Psi_L(t) / i_L(t)]^2$$

$$= (\frac{d\varphi_i(t)}{dt})L\ \mathrm{tg}\varphi_i(t)\ - \frac{1}{I(t)}\frac{dI(t)}{d(t)}L.$$

$$\overline{Z}_L(t) = \frac{\overline{u}_L(t)}{\overline{i}_L(t)} = \frac{U_L(t)}{I_L(t)}e^{j\Delta\varphi(t)} = -j(\frac{d\varphi_i(t)}{dt})L - \frac{1}{I(t)}\frac{dI(t)}{d(t)}L \quad ,$$

$$Z_L(\omega) = u_L(\omega) / i_L(\omega) = \frac{U(\omega)}{I(\omega)}e^{j\Delta\Phi(\omega)} = \{[+j\omega L = +j\frac{U(\omega)}{I(\omega)}\ ;\ U(\omega) = -\ \omega L I(\omega)],$$

$$ili\quad [-j\omega L = -j\frac{U(\omega)}{I(\omega)}\ ;\ U(\omega) = +\ \omega L I(\omega)]\}. \qquad (4.0.97)$$

4.0.11.4. Capacitive Impedances

Finally, if current $i_C(t) = i_C$ *runs through capaci*tance *C, then voltage* $u_C(t) = u_C$ *will be formed on the capaci*tance *C. The validity of the following relations is obvious (or can be proved):*

$$u_c(t) = Z_c i_c = \frac{1}{C}\int_0^t i_c(t)dt,\ \tilde{S}_C(t) = u_C(t)\ i_C(t) = \frac{i_C(t)}{C}\int_0^t i_c(t)dt = \Psi_c^2(t), \qquad (4.0.98)$$

$$U(\omega) = \frac{1}{\omega C}\ I(\omega) \quad ,$$

$$\Delta\Phi(\omega) = \Phi_u - \Phi_i = -\pi/2\ ,$$

$$\Delta\varphi(t) = \varphi_u - \varphi_i = -\pi/2$$

from this, capacitance impedances are:

$$Z_c(t) = u_c(t) / i_c(t) = \frac{U_C(t)}{I_C(t)}\frac{\cos\varphi_u(t)}{\cos\varphi_i(t)} = \frac{1}{C i_C(t)}\int_0^t i_c(t)dt = [\Psi_c(t) / i_c(t)]^2 = \ ,$$

$$\overline{Z}_C(t) = \frac{\overline{u}_c(t)}{\overline{i}_c(t)} = \frac{U_C(t)}{I_C(t)}e^{j\Delta\varphi(t)} \quad ,$$

$$Z_c(\omega) = u_c(\omega) / i_c(\omega) = \frac{U(\omega)}{I(\omega)}e^{j\Delta\Phi(\omega)} = -j\frac{1}{\omega C} = -j\frac{U(\omega)}{I(\omega)}\ . \qquad (4.0.99)$$

4.0.11.5. R-L-C Impedances for Simple Harmonic Currents

If we consider only the cases when through an impedance a simple harmonic stationary current $i(t) = I_0 \cos\omega t$, ($I_0 = const.$) runs, then the previous generalized impedances are simplified to the following:

$$Z_R(t) = \overline{Z}_R(t) = R,\ \ Z_R(\omega) = R,$$

$$Z_L(t) = \omega L(\ tg\ \omega t\ ) = X_L(tg\ \omega t),\ \overline{Z}_L(t) = -j\omega L = -j\,X_L\ ,\ \ Z_L(\omega) = \ j\omega L = jX_L,$$

$$Z_C(t) \ = (-1/\omega C)\ (\ tg\ \omega t\ ) = -X_C(tg\ \omega t),\ \overline{Z}_C(t) = -j\frac{1}{\omega C} = -jX_C,\ \ Z_C(\omega) = -j\frac{1}{\omega C} = -jX_C. \tag{4.0.100}$$

In the electrotechnics of stationary or simple harmonic currents we can notice the advantage of introducing complex electric impedances, which simplify time dependant impedances (4.0.100). If we introduce the transformation of *a real current function into a complex current function*, *like this:*

$$i(t) = I_0 \cos\omega t \ \leftrightarrow I_0\, e^{-j\omega t} = \ \overline{i}(t) = \ \overline{I}\,, \tag{4.0.101}$$

and if we use the complex current form (4.0.101) to determine the impedances (4.0.100), we will get so-called complex stationary-current impedance forms, where we (formally) no longer see the difference among so-called time dependant, complex or frequency dependant impedances (which are evident in (4.0.100)), i.e. for all impedance forms, the following is valid:

$$Z_R = R,$$

$$Z_L = j\omega L = j\,X_L, \tag{4.0.102}$$

$$Z_C(t) \ = 1\ /\ j\omega C = -\ j\ X_C\,.$$

The impedance forms given in (4.0.102), although being practical (useful) and simple, hide within themselves the real identity of the term “impedances” and most of related facts about it.

The formal way of imaging (4.0.101), which is in accordance with the definition of impedance, is nothing else but a replacement of one real function by an appropriate complex, analytical function; therefore, if the current function i(t) is arbitrary, and not simple harmonic, we can make the replacement (with an appropriate analytical signal function (4.0.1) and (4.0.2), analogous to (4.0.101)):

$$i(t) \ \leftrightarrow \ i(t) + j\ H[i(t)] = i(t) + j\,\hat{i}(t) \ = I_0(t)\, e^{j\varphi(t)} \ = \ \overline{i}(t)\,. \tag{4.0.103}$$

Mathematical formalism (4.0.103), related to simple harmonic (stationary) currents and voltages, is very well developed in electrotechnics, whereas the same issue concerning nonperiodical and non-harmonic (arbitrary functions of current and voltage) is rather badly dealt with in terms of mathematics. It would be good to use the elements of the previously described formalism to analyze optimal transmission of electrical signals (of the energy, power, currents, etc.) in the cases of their arbitrary, nonperiodical waveforms (for arbitrary load impedances). If we do not wish to make a complete redefinition of the term “complex impedances”, and we want that definition to keep its general meaning (within the frame previously established) and to preserve the continuity with the usual treatment of that term, we have to limit the meaning of “complex impedance” only to its appropriate frequency dependant impedance forms given in (4.0.100), because it is obvious that time dependant *impedances will be changeable according to the modeling factor tanωt (which is not at all practical for any selection or quantification of impedances).*

For now (in the examples given above), we will rely on methodology established in electrotechnics, because that issue is very comprehensive for us, and, besides, it is extremely well developed, in comparison to the other fields in physics. There is only one small step to the analogous broadening and usage of the same way of thinking to other fields, such as acoustics, fluid mechanics, electromagnetic waves, vibrations, quantum wave mechanics, etc.

4.0.11.6. Power Function

The concept of the complex, active and reactive power, which operates well in the electrotechnics of stationary (simple harmonic) currents and voltages, is, in the general case of arbitrary forms of currents and voltages, inapplicable (at least not in any way known so far). Certainly, it is always possible to talk about the active power transfer, when a system operates as an active (resistive) load and it satisfies the criteria from (4.0.94). All other cases can be treated as cases of the circulation of reactive and/or complex power. In fact, the key criterions for the determination of optimal energy transfer are *coherence relations among the* power *factors (see (*4.0.83*) to (*4.0.99*)). It is good to complement the previous claim with the comparison of the following real and complex wave function forms (by asking the question: which of these relations or parts of the relations, can stand for the complex, and which one/s for the reactive force, and to what extent such a terminology and classification makes sense and practical significance):*

$$\Psi^2(t) = s_1 s_2 \ ,$$

$$\overline{s}_1 \overline{s}_2 = (s_1 s_2 - \hat{s}_1 \hat{s}_2) + j(s_1 \hat{s}_2 + \hat{s}_1 s_2) \ ,$$

$$\overline{\Psi}^2(t) = s_1 s_2 - \left\{H\left[\sqrt{s_1 s_2}\right]\right\}^2 + 2j\sqrt{s_1 s_2}\, H\left[\sqrt{s_1 s_2}\right], \qquad (4.0.104)$$

$$\left|\overline{\Psi}(t)\right|^2 = s_1 s_2 + \left\{H\left[\sqrt{s_1 s_2}\right]\right\}^2 ,$$

$$s_1 s_2 + jH\left[s_1 s_2\right] = \overline{s}_{12} \ .$$

It is obvious that none of the relations from (4.0.104) makes a direct way of introducing and total understanding of the terms: complex, active and reactive power. In order to have an appropriate way of looking at the issues related to the term ‘power’, we have to hold on to a standpoint, which gives a definition that, on the resistive load, the current and voltage functions will be completely coherent, i.e. in the phase, and that this will be valid for both time and frequency domain of those functions, when the power is completely active. All other cases indicate the presence of complex or reactive power (or the absence of the optimal energy transfer). What is also very important is that, if there is any other, more general mathematical analysis of the problems mentioned above, it must take into account all cases of the classical treatment of active, reactive and complex power discussed so far. This is a great advantage and facility in the study of various wave natural phenomena, because nature in a way strives towards creating harmonic oscillations, which can often be very well presented by simple harmonic functions (in most of the cases of interest).

The methodology introduced beginning with (4.0.82) to (4.0.104) opens new (generalized) possibilities for the analysis and measurement of the energy transport from the source to its load, and gives the possibility of generalized load-type classification (for arbitrary wave functions). It is of a special significance that, by the application of this methodology (with certain mathematical extra work on the previously opened questions), we can overcome confusions and delusions that have existed until now, or the partially introduced and partially valid (but, very often, mutually not matching) mathematical formalisms and theories, to analyze various forms of energy transfer regarding oscillations and matter-wave phenomena. Naturally, it is clear that, the introduction of the complex analysis into the wave movement analysis, beside other things, plays a great role in the simplification of the mathematical apparatus used, in the way that, instead of solving integral-differential equations, the whole analysis is actually based on rather elementary algebra operations. ♣**]**

Extension of the Electrical Power Definition to Arbitrary (Non-Sinusoidal) Voltage and Current Signals and Consequences Regarding Novel Understanding of Quantum-Mechanical Wave Function

List of Symbols:

$\mathbf{p(t)} =$ **Instantaneous Power**

$\hat{\mathbf{p}}(\mathbf{t}) =$ **Hilbert Transform of Instantaneous Power**

$\overline{\mathbf{p}}(\mathbf{t}) =$ **Instantaneous Power in the form of Complex Analytic Signal**

$|\overline{\mathbf{p}}(\mathbf{t})| =$ **Absolute Value of Instantaneous Power**

$\mathbf{P(t)} =$ **Amplitude function of Instantaneous (real or active) Power**

$\mathbf{S(t)} =$ **Amplitude function of Instantaneous Apparent Power**

$\mathbf{Q(t)} =$ **Amplitude function of Instantaneous Reactive Power**

$\overline{\mathbf{u}}(\mathbf{t}) =$ **Instantaneous Voltage in the form of complex Analytic Signal**

$\mathbf{u(t)} =$ **Instantaneous Voltage**

$\hat{\mathbf{u}}(\mathbf{t}) =$ **Hilbert Transform of Instantaneous Voltage**

$\overline{\mathbf{i}}(\mathbf{t}) =$ **Instantaneous Current in the form of Complex Analytic Signal**

$\mathbf{i(t)} =$ **Instantaneous Current**

$\hat{\mathbf{i}}(\mathbf{t}) =$ **Hilbert Transform of Instantaneous Current**

$\mathbf{U(t)} =$ **Amplitude function of Instantaneous Voltage**

$\mathbf{I(t)} =$ **Amplitude function of Instantaneous Current**

$\mathbf{U_{rms}} =$ **Effective, RMS Voltage**

$\mathbf{I_{rms}} =$ **Effective, RMS Current**

$\overline{\mathbf{Z}} = \mathbf{R} - \mathbf{jX} =$ **Complex Impedance,** $|\overline{\mathbf{Z}}| = \mathbf{Z} = \sqrt{\mathbf{R}^2 + \mathbf{X}^2}$

$\mathbf{R} =$ **Resistance,** $\mathbf{X} =$ **Reactance**

$\mathbf{T} =$ **Time interval**

$\varphi_\mathbf{p}(\mathbf{t}) =$ **Phase function of the Instantaneous Power**

$\omega_\mathbf{p} =$ **Frequency of the Instantaneous Power**

$\varphi_\mathbf{u} =$ **Phase function of the Voltage signal**

$\omega_\mathbf{u} =$ **Frequency of the Voltage signal**

$\varphi_\mathbf{i} =$ **Phase function of the Current signal**

$\omega_\mathbf{i} =$ **Frequency of the Current signal**

$\mathbf{j} = \sqrt{-1} =$ **Imaginary unit**

$\mathbf{H}[\ ]\ (=)$ **Hilbert Transform**

$\mathbf{PF} = \cos\theta =$ **Power Factor**

$\Psi(\mathbf{t}) =$ **Wave function**

$\mathbf{a(t)} =$ **Amplitude of a Wave function**

$\varphi(\mathbf{t}) =$ **Phase of a Wave function**

Analytic Signal and Electrical Power Characterization (comparative tables)

<table>
<tr><th>Instantaneous Power
(Apparent Power & Analytic Signal)</th><th>Averaged Complex Power
(Averaged Apparent Power & Phasor notation)</th><th>Generalized Complex Power
(Generalized Phasor Notation)</th></tr>
<tr><td>

$\overline{p}(t) = p(t) + j\cdot\hat{p}(t) = |\overline{p}(t)|\cdot e^{j\varphi_p(t)}$,

$\varphi_p(t) = arctg\frac{\hat{p}(t)}{p(t)} = \varphi_u(t)+\varphi_i(t) =$

$= \theta(t) + 2\varphi_u(t) = 2\varphi_i(t) - \theta(t)$,

$\theta(t) = \varphi_i(t)-\varphi_u(t),\ \omega_p = \frac{\partial\varphi_p(t)}{\partial t}$, $(j^2 = -1)$

</td><td>

$\overline{S} = \frac{1}{2}\overline{U}\cdot\overline{I}^* = P - jQ = Se^{-j\theta} = U_{rms}I_{rms}e^{-j\theta}$

$\theta(t) = \angle[u(t), i(t)] = \varphi_i(t) - \varphi_u(t) = \varphi_i - \varphi_u =$

$= \theta_i - \theta_u = arctg\frac{Q}{P} = \theta,\ \omega = \frac{\partial\varphi_u(t)}{\partial t} = \frac{\partial\varphi_i(t)}{\partial t}$

$(\varphi_u(t) = \omega t + \theta_u, \varphi_i(t) = \omega t + \theta_i)$

</td><td>

$\overline{S}(t) = \frac{1}{2}\overline{u}(t)\cdot\overline{i}^*(t) = P(t) - jQ(t) =$

$= S(t)e^{-j\theta(t)} = \frac{1}{2}U(t)I(t)e^{-j\theta(t)}$

$\theta(t) = \varphi_i(t) - \varphi_u(t) = arctg\frac{Q(t)}{P(t)}$

</td></tr>
<tr><td>

$p(t) = u(t)\cdot i(t) = |\overline{p}(t)|\cdot\cos\varphi_p(t) = |\overline{p}(t)|\cdot\cos(\varphi_u(t)+\varphi_i(t))$

$= U(t)\cdot I(t)\cdot\cos\varphi_u(t)\cdot\cos\varphi_i(t)$

$|\overline{p}(t)| = \sqrt{p(t)^2 + \hat{p}(t)^2} = U(t)I(t)\frac{\cos\varphi_u(t)\cos\varphi_i(t)}{\cos\varphi_p(t)} =$

$= \frac{p(t)}{\cos\varphi_p(t)} = 2S(t)\cdot\frac{\cos\varphi_u(t)\cos\varphi_i(t)}{\cos\varphi_p(t)}$

$\hat{p}(t) = H[p(t)] = |\overline{p}(t)|\cdot\sin\varphi_p(t) = p(t)\cdot\tan(\varphi_u(t)+\varphi_i(t))$,

$H[\ ]$ (=) **Hilbert transformation,**

$u(t) = U(t)\cos\varphi_u(t),\quad \overline{u}(t) = u(t) + j\cdot\hat{u}(t) = U(t)\cdot e^{j\varphi_u(t)}$,

$U(t) = \sqrt{u(t)^2 + \hat{u}(t)^2}$, $\varphi_u(t) = arctg\frac{\hat{u}(t)}{u(t)}, \omega_u = \frac{\partial\varphi_u}{\partial t}$,

$i(t) = I(t)\cos\varphi_i(t),\quad \overline{i}(t) = i(t) + j\cdot\hat{i}(t) = I(t)\cdot e^{j\varphi_i(t)}$,

$I(t) = \sqrt{i(t)^2 + \hat{i}(t)^2}$, $\varphi_i(t) = arctg\frac{\hat{i}(t)}{i(t)}, \omega_i = \frac{\partial\varphi_i}{\partial t}$.

$ui + \hat{u}\hat{i} = U(t)I(t)\cos\theta(t),\ u\hat{i} - \hat{u}i = U(t)I(t)\sin\theta(t)$

$ui = U(t)I(t)\cos\varphi_u(t)\cos\varphi_i(t)$,

$\hat{u}\hat{i} = U(t)I(t)\sin\varphi_u(t)\sin\varphi_i(t)$

</td><td>

$\overline{U} = \sqrt{2}U_{rms}e^{j\varphi_u}, \overline{I} = \sqrt{2}I_{rms}e^{j\varphi_i} = \sqrt{2}I_{rms}e^{j(\varphi_u+\theta)}$,

$\overline{I}^* = \sqrt{2}I_{rms}e^{-j\varphi_i} = \sqrt{2}I_{rms}e^{-j(\varphi_u+\theta)}$,

$P = U_{rms}I_{rms}\cos\theta$ = Active Power (=) [W]

$Q = U_{rms}I_{rms}\sin\theta$ = Reactive Power (=) [VAR]

$S = \sqrt{P^2 + Q^2} = U_{rms}I_{rms}$ (=) [VA]

$PF = \frac{P}{S} = \cos\theta$ = Power Factor,

$U_{rms} = \sqrt{\frac{1}{T}\int_{[T]} u^2(t)dt}$, $I_{rms} = \sqrt{\frac{1}{T}\int_{[T]} i^2(t)dt}$.

$\overline{Z} = |Z|e^{-j\theta} = \frac{U_{rms}}{I_{rms}}e^{-j\theta} = R - jX =$

$= \sqrt{R^2 + X^2}\cdot e^{-j\theta}$

</td><td>

$P(t) = S(t)\cos\theta(t) = \frac{1}{2}(ui + \hat{u}\hat{i})$

$Q(t) = S(t)\sin\theta(t) = \frac{1}{2}(u\hat{i} - \hat{u}i)$

$S(t) = \sqrt{P(t)^2 + Q(t)^2} = \frac{1}{2}U(t)I(t) =$

$= |\overline{p}(t)|\frac{\cos\varphi_p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)} =$

$= \frac{p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)} = \frac{P(t)}{\cos\theta(t)}$,

$PF(t) = \frac{P(t)}{S(t)} = \cos\theta(t) = \frac{ui + \hat{u}\hat{i}}{U(t)I(t)}$

$U(t) = \sqrt{u(t)^2 + \hat{u}(t)^2}$,

$I(t) = \sqrt{i(t)^2 + \hat{i}(t)^2}$

$\overline{u}(t) = U(t)e^{j\varphi_u(t)}$,

$\overline{i}(t) = I(t)e^{j\varphi_i(t)} = I(t)e^{j[\varphi_u(t)+\theta(t)]}$

$\overline{Z}(t) = \frac{\overline{u}(t)}{\overline{i}(t)} = \frac{U(t)}{I(t)}e^{-j\theta(t)} =$

$R(t) - jX(t) = \sqrt{R^2(t) + X^2(t)}\cdot e^{-j\theta(t)}$

</td></tr>
</table>

Exstension of the Electrical Power Definition

4.0.12. Evolution of the RMS concept

Based on the traditional Phasor (complex) presentation of sinusoidal currents and voltages we have:

$$\left\langle p(t)\right\rangle = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt \text{ (=) Average Instantaneous Power}$$

$$\left\langle p(t)\right\rangle = \frac{U_{rms}{}^2}{R} = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt = \frac{1}{T}\int_{[T]} u(t)\frac{u(t)}{R}dt =$$

$$= \frac{1}{RT}\int_{[T]} u(t)^2 dt = RI_{rms}{}^2 = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt =$$

$$= \frac{1}{T}\int_{[T]} Ri(t)i(t)dt = \frac{R}{T}\int_{[T]} i(t)^2 dt \text{ (=) Average Active Power} \Rightarrow$$

$$\Rightarrow U_{rms} = \sqrt{\frac{1}{T}\int_{[T]} u^2(t)dt}\ ,\ I_{rms} = \sqrt{\frac{1}{T}\int_{[T]} i^2(t)dt}$$

$$P = U_{rms}I_{rms}\cos\theta = \text{Active Average Power}\,(=)\left[W\right]$$

$$Q = U_{rms}I_{rms}\sin\theta = \text{Reactive Average Power}\,(=)\left[VAR\right]$$

$$S = \sqrt{P^2 + Q^2} = U_{rms}I_{rms} = \text{Average Apparent Power (=)}\left[VA\right]$$

$$PF = \frac{P}{S} = \cos\theta = \text{Power Factor},$$

$$\overline{U} = \sqrt{2}U_{rms}e^{j\theta_u}, \overline{I} = \sqrt{2}I_{rms}e^{j\theta_i}\ , \overline{I}^* = \sqrt{2}I_{rms}e^{-j\theta_i}\ , \overline{S} = \frac{1}{2}\overline{U}\cdot\overline{I}^*\ .$$

It is important to underline that only the Active Power P is the power delivered to the load, and that the Reactive Power Q is the power reflected from the load and sent back to its energy source. We already know all of that from the basic electro-technique regarding alternative currents and voltages (usually only related to sinusoidal and constant operating frequency signals in electric energy distribution systems). Here we shall extend and generalize the same concept of Active, Reactive and Apparent Power, to the propagation of any arbitrary shaped and multi-frequency, or large frequency-band signals. The generalization platform for new Active, Reactive and Apparent power definition will be related to Analytic, complex signal (and Hilbert Transform) that gives the possibility to present an arbitrary shaped time domain signal into corresponding Complex and Sinusoidal-like signal.

After replacing the Instantaneous Power with its Analytic Signal form, we will find that the traditional concept of RMS currents and voltages remains basically unchanged:

$$\langle \overline{p}(t) \rangle = \frac{1}{T} \int_{[T]} \overline{p}(t)dt = \frac{1}{T} \int_{[T]} |\overline{p}(t)| e^{j\varphi_p(t)} dt =$$

$$= \langle p(t) \rangle + j \cdot \langle \hat{p}(t) \rangle \text{ (=) Average Instantaneous Complex Power}$$

$$\langle \overline{p}(t) \rangle = \frac{U_{A-rms}^{\ 2}}{R} + j\frac{U_{R-rms}^{\ 2}}{R} = \frac{1}{T} \int_{[T]} \overline{p}(t)dt = \frac{1}{T} \int_{[T]} p(t)dt + j\frac{1}{T} \int_{[T]} \hat{p}(t)dt =$$

$$= \frac{1}{RT} \int_{[T]} u(t)^2 dt + j\frac{1}{T} \int_{[T]} \hat{p}(t)dt = \frac{U_{rms}^{\ 2}}{R}(1+j) =$$

$$= R \cdot I_{A-rms}^{\ 2} + jR \cdot I_{R-rms}^{\ 2} = \frac{R}{T} \int_{[T]} i(t)^2 dt + j\frac{1}{T} \int_{[T]} \hat{p}(t)dt = RI_{rms}^{\ 2}(1+j) \Rightarrow$$

$$\Rightarrow U_{A-rms} = \sqrt{\frac{1}{T} \int_{[T]} u^2(t)dt} = \sqrt{\frac{R}{T} \int_{[T]} p(t)dt} = U_{R-rms} = \sqrt{\frac{R}{T} \int_{[T]} \hat{p}(t)dt} = U_{rms} \ ,$$

$$I_{A-rms} = \sqrt{\frac{1}{T} \int_{[T]} i^2(t)dt} = \sqrt{\frac{1}{RT} \int_{[T]} p(t)dt} = I_{R-rms} = \sqrt{\frac{1}{RT} \int_{[T]} \hat{p}(t)dt} = I_{rms},$$

$$\langle \overline{p}(t) \rangle = \langle p(t) \rangle + j \cdot \langle \hat{p}(t) \rangle = \langle p(t) \rangle (1+j) = \langle \hat{p}(t) \rangle (1+j).$$

Based on complex Analytic Signal forms of voltage and current functions we can now generalize the Phasor notation concept defining Instantaneous, Complex, Apparent, Active and Reactive Power (to be applicable to arbitrary signal shapes), as follows:

$$\overline{S}(t) = \overline{u}(t) \cdot \overline{i}^*(t) = P(t) - jQ(t) = S(t)e^{-j\theta(t)} = U(t)I(t)e^{-j\theta(t)} \ ,$$

$$\theta(t) = \varphi_i(t) - \varphi_u(t) = \text{arctg}\frac{Q(t)}{P(t)} = \text{arctg}\frac{\hat{i}(t)}{i(t)} - \text{arctg}\frac{\hat{u}(t)}{u(t)} \ ,$$

$$P(t) = S(t)\cos\theta(t) = \frac{1}{2}(ui + \hat{u}\hat{i}) = \text{Power delivered to a load} \ ,$$

$$Q(t) = S(t)\sin\theta(t) = \frac{1}{2}(u\hat{i} - \hat{u}i) = \text{Power reflected from a load} \ ,$$

$$S(t) = \sqrt{P(t)^2 + Q(t)^2} = \frac{1}{2}U(t)I(t) = |\overline{p}(t)| \frac{\cos\varphi_p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)} =$$

$$= \frac{p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)} = \frac{P(t)}{\cos\theta(t)} = \frac{Q(t)}{\sin\theta(t)},$$

$$PF(t) = \frac{P(t)}{S(t)} = \cos\theta(t) = \frac{ui + \hat{u}\hat{i}}{U(t)I(t)} \ , \left\{ \cos\theta(t) = 1 \Leftrightarrow (u\hat{i} = \hat{u}i) \right\}$$

$$U(t) = \sqrt{u(t)^2 + \hat{u}(t)^2} \left\{ = u(t)\sqrt{1 + (\frac{\hat{i}}{i})^2} = u(t)\sqrt{1 + (\frac{\hat{u}}{u})^2}, \text{for } \cos\theta(t) = 1 \right\} ,$$

$$I(t) = \sqrt{i(t)^2 + \hat{i}(t)^2} \left\{ = i(t)\sqrt{1 + (\frac{\hat{i}}{i})^2} = i(t)\sqrt{1 + (\frac{\hat{u}}{u})^2}, \text{for } \cos\theta(t) = 1 \right\} ,$$

$$\overline{u}(t) = U(t)e^{j\varphi_u(t)}, \ \overline{i}(t) = I(t)e^{j\varphi_i(t)} = I(t)e^{j[\varphi_u(t)+\theta(t)]} \ ,$$

$$\overline{i}^*(t) = I(t)e^{-j\varphi_i(t)} = I(t)e^{-j[\varphi_u(t)+\theta(t)]}$$

$$\overline{Z}(t) = \frac{\overline{u}(t)}{\overline{i}(t)} = \frac{U(t)}{I(t)}e^{-j\theta(t)} = R(t) - jX(t) = \sqrt{R^2(t) + X^2(t)} \cdot e^{-j\theta(t)}$$

$$\left\{ = \frac{u(t)}{i(t)}e^{-j\theta(t)}, \text{ for } \cos\theta(t) = 1 \right\}.$$

In majority of energy transfer systems we do not care too much about immediate, transient, time-domain signals, since mathematically it could be complicated to use such functions (especially for arbitrary-shaped signals). What counts much more in practice of power and energy conversion and distribution systems are effective, rms, mean, averaged and other numerically expressed values (applicable to sufficiently representative frequency and/or time signal intervals).

In order to avoid time dependence of the above expressions, we shall determine all corresponding average values, as follows:

$$\langle p(t)\rangle = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt = \frac{1}{T}\int_{[T]} \left|\overline{p}(t)\right|\cos\varphi_p(t)dt =$$

$$= \frac{1}{T}\int_{[T]} U(t)I(t)\cos\varphi_u(t)\cos\varphi_i(t)dt =$$

$$= \frac{{U_{rms}}^2}{R} = \frac{1}{RT}\int_{[T]} U(t)^2\cos\varphi^2{}_u(t)dt \;\; = R{I_{rms}}^2 =$$

$$= \frac{R}{T}\int_{[T]} I(t)^2\cos\varphi^2{}_i(t)dt\,(=)\text{ Average Active Power} \Rightarrow$$

$$U_{rms} = \sqrt{\frac{1}{T}\int_{[T]} U(t)^2\cos\varphi^2{}_u(t)dt} = \sqrt{\frac{1}{T}\int_{[T]} u(t)^2 dt}\;,$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_{[T]} I(t)^2(t)\cos\varphi^2{}_i(t)dt} = \sqrt{\frac{1}{T}\int_{[T]} i(t)^2 dt}$$

$$P = U_{rms}I_{rms}\langle\cos\theta(t)\rangle \;\; = \frac{1}{T}\int_{[T]} P(t)dt =$$

$$= \frac{1}{T}\int_{[T]} S(t)\cos\theta(t)dt = \text{Active Average Power}$$

$$Q = U_{rms}I_{rms}\langle\sin\theta(t)\rangle \;\; = \frac{1}{T}\int_{[T]} Q(t)dt =$$

$$= \frac{1}{T}\int_{[T]} S(t)\sin\theta(t)dt = \text{Reactive Average Power}$$

$$\overline{S} = P - jQ = Se^{-j\theta} = \text{ Complex Apparent Power}$$

$$\theta = \operatorname{arctg}\frac{Q}{P} = \operatorname{arctg}\frac{\langle\sin\theta(t)\rangle}{\langle\cos\theta(t)\rangle}$$

$$S = \frac{1}{T}\int_{[T]} \sqrt{P(t)^2 + Q(t)^2}dt = U_{rms}I_{rms} =$$

$$= \sqrt{P^2 + Q^2} = \text{ Average Apparent Power}$$

Now it will be possible to extend the meaning of average electrical impedance for the general case of arbitrary voltage and current signals, as for instance:

$$\overline{S} = P - jQ = Se^{-j\theta} = ZI^2_{rms}e^{-j\theta} = \frac{U^2_{rms}}{Z}e^{-j\theta} = (R - jX)I^2_{rms}$$

$$\overline{Z} = R - jX = Ze^{-j\theta} = \text{Average Complex Impedance}$$

$$Z = \left|\overline{Z}\right| = \frac{U_{rms}}{I_{rms}} = \sqrt{R^2 + X^2} = \sqrt{\frac{\int_{[T]} u(t)^2 dt}{\int_{[T]} i(t)^2 dt}} = \sqrt{\frac{\int_{[T]} U(t)^2 \cos\varphi^2_u(t)dt}{\int_{[T]} I(t)^2 \cos\varphi^2_i(t)dt}},$$

$$\langle\theta\rangle = \theta = \arctan\frac{X}{R} = \arctan\frac{Q}{P} = \arctan\left[\frac{\langle\sin\theta(t)\rangle}{\langle\cos\theta(t)\rangle}\right], \quad \overline{\theta} = \arctan\left[\frac{\overline{\sin\theta(t)}}{\overline{\cos\theta(t)}}\right]$$

$$R = \frac{P}{I^2_{rms}} = Z \cdot \langle\cos\theta(t)\rangle = \text{Average Active (Real) Impedance}$$

$$X = \frac{Q}{I^2_{rms}} = Z \cdot \langle\sin\theta(t)\rangle = \text{Average Reactive (Imaginary) Impedance, or Reactance}$$

$$\frac{X}{R} = \frac{Q}{P} = \frac{\langle\sin\theta(t)\rangle}{\langle\cos\theta(t)\rangle} = \tan\theta = \text{Average Quality Factor}, \quad \tan\overline{\theta} = \frac{\overline{\sin\theta(t)}}{\overline{\cos\theta(t)}}$$

$$\left\{\cos\theta(t), \langle\cos\theta(t)\rangle, \overline{\cos\theta(t)}, \cos\overline{\theta}\right\} = \text{Different Power Factors}$$

Practically, in power management systems (after introducing Analytic Signal methodology) we shall be able to apply innovative concepts based on averaged and rms signal values, easily measurable using existing technology, and valid for arbitrary-shaped signals (without need to have explicit time and frequency expressions). This way, many of traditionally known concepts of power and frequency regulation will be generalized and could be significantly optimized. Also Quantum mechanical energy exchanges, quantum states and wave functions can be explained using "*active and reactive wave functions*" (similar to active and reactive power components). Stable atoms could be described as reactive resonant (multi-dimensional or multi-component) circuit structures, where Quality Factor = $\mathbf{tan}\,\theta$, approaches infinity ($\theta \rightarrow \frac{\pi}{\mathbf{2}}$), or where all internal, stationary atom waves and other movements behave (analogically) like currents and voltages in pure loss-less capacitive-inductive resonant-oscillating circuits, without resistive components.

Here we will attempt to connect arbitrary Power Function (product between current and voltage, or product between force and speed, or product between any other relevant, mutually conjugated functions creating power) to a wave function, as known in Quantum Mechanics. Energy-wise analyzed, any wave propagation in time and frequency domain can be mutually (time-frequency) correlated using Parseval's identity. Consequently, the immediate (time-domain) Power-signal can be presented as the square of the wave-function $\Psi^2(\mathbf{t})$*, and analysis of the optimal power transfer can be extended to any wave propagation field (and to arbitrary shaped signals), and profit enormously (in booth, directions) after unifying traditional concepts of Active, Reactive and Apparent power with the* $\Psi^2(\mathbf{t})$ *wave-function mathematics, based on Analytic Signal methodology.*

In Quantum Mechanics the wave function $\Psi^2(\mathbf{t})$ *is conveniently modeled as a probability function, but in many aspects effectively behaves like normalized and dimensionless Power function, and here it will be closely related to Active Power, or power delivered to a load (expressed in Watts as its units).*

Let us start from the immediate electrical power found as a product between corresponding voltage and current signals, where both of them are arbitrary-shaped functions. We can show that such active-power function (which transfers the power from its source to its load) can be presented as,

$$\Psi^2(\mathbf{t}) = \mathbf{P(t)} = \mathbf{S(t)}\cos\theta\mathbf{(t)} = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{ui} + \hat{\mathbf{u}}\hat{\mathbf{i}}) = \mathbf{Q(t)} \cdot \mathbf{cotan}\theta\mathbf{(t)}\ (=)\left[\mathbf{W}\right] .$$

The power reflected from a load, or Reactive Power, can be given as:

$$\mathbf{Q(t)} = \mathbf{S(t)}\sin\theta\mathbf{(t)} = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{u}\hat{\mathbf{i}} - \hat{\mathbf{u}}\mathbf{i}) = \Psi^2(\mathbf{t}) \cdot \tan\theta(\mathbf{t}) = \mathbf{P}(\mathbf{t}) \cdot \tan\theta(\mathbf{t})\,(=)\left[\mathbf{VAR}\right]$$

Electric Power and Energy transfer analysis (especially for arbitrary voltage and current signal forms) can be related to a wave-function analysis if we establish the wave function (or more precisely, the square of the wave function) in the following way:

$$\mathrm{P(t)} = \Psi^2(\mathrm{t}) = \left[\mathrm{a(t)}\cos\varphi(\mathrm{t})\right]^2 = \text{Wave function},\ \mathrm{t} \in \left[\mathrm{T}\right],$$

$$\Psi(\mathrm{t}) = \mathrm{a(t)}\cos\varphi(\mathrm{t}),\ \hat{\Psi}(\mathrm{t}) = \mathrm{a(t)}\sin\varphi(\mathrm{t}),$$

$$\overline{\Psi}(\mathrm{t}) = \Psi(\mathrm{t}) + \mathrm{j}\hat{\Psi}(\mathrm{t}) = \Psi(\mathrm{t}) + \mathrm{jH}\left[\Psi(\mathrm{t})\right] = \mathrm{a(t)}e^{\mathrm{j}\varphi(\mathrm{t})} = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\mathrm{U}(\omega)e^{-\mathrm{j}\omega\mathrm{t}}\mathrm{d}\omega =$$

$$= \frac{1}{\pi^2}\int_{0}^{+\infty}\mathrm{U}(\omega)e^{-\mathrm{j}\omega\mathrm{t}}\mathrm{d}\omega = \frac{1}{\pi}\int_{(0,+\infty)}\mathrm{A}(\omega)e^{-\mathrm{j}\omega\mathrm{t}}\mathrm{d}\omega,$$

$$\mathrm{U}(\omega) = \mathrm{U_c}(\omega) - \mathrm{j\,U_s}(\omega) = \int_{(-\infty,+\infty)}\overline{\Psi}(\mathrm{t})\ e^{\mathrm{j}\omega\mathrm{t}}\mathrm{dt} = \mathrm{A}(\omega)e^{-\mathrm{j}\Phi(\omega)},$$

$$\mathrm{U_c}(\omega) = \mathrm{A}(\omega)\cos\Phi(\omega),\ \mathrm{U_s}(\omega) = \mathrm{A}(\omega)\sin\Phi(\omega),$$

$$\mathrm{a(t)} = \sqrt{\Psi(\mathrm{t})^2 + \hat{\Psi}(\mathrm{t})^2},\quad \varphi(\mathrm{t}) = \mathrm{arctg}\frac{\hat{\Psi}(\mathrm{t})}{\Psi(\mathrm{t})},$$

$$\mathrm{A}^2(\omega) = \mathrm{U}^2{}_{\mathrm{c}}(\omega) + \mathrm{U}^2{}_{\mathrm{s}}(\omega),\quad \Phi(\omega) = \mathrm{arctg}\frac{\mathrm{U_s}(\omega)}{\mathrm{U_c}(\omega)},$$

$$\mathrm{T}\cdot\left\langle\mathrm{P(t)}\right\rangle = \int_{-\infty}^{+\infty}\mathrm{P(t)dt} = \int_{-\infty}^{+\infty}\Psi^2(\mathrm{t})\mathrm{dt} = \int_{-\infty}^{+\infty}\hat{\Psi}^2(\mathrm{t})\mathrm{dt} = \frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(\mathrm{t})\right|^2\mathrm{dt} =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty}\left|\Psi(\mathrm{t}) + \mathrm{j}\hat{\Psi}(\mathrm{t})\right|^2\mathrm{dt} = \mathrm{T}\cdot\left\langle\hat{\mathrm{P}}(\mathrm{t})\right\rangle = \int_{-\infty}^{+\infty}\hat{\mathrm{P}}(\mathrm{t})\mathrm{dt} =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty}\mathrm{a}^2(\mathrm{t})\mathrm{dt} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\mathrm{U}(\omega)\right|^2\mathrm{d}\omega = \frac{1}{\pi}\int_{0}^{\infty}\left[\mathrm{A}(\omega)\right]^2\mathrm{d}\omega,$$

As we can see, any physics-related and finite wave function Ψ has its Hilbert couple $\hat{\Psi}$. Both of them are presentable as a product of two other functions, since to create an instantaneous power function $\Psi^2(\mathbf{t})$, (see (4.0.4)) it is essential to make the product of two relevant, mutually conjugated signals, like current and voltage, velocity and force, or some other equally important couple of conjugated signals (see 4.0.82)):

$$\Psi^2(\mathbf{t}) = \mathbf{P}(\mathbf{t}) = \mathbf{S}(\mathbf{t})\cos\theta(\mathbf{t}) = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{ui}+\hat{\mathbf{u}}\hat{\mathbf{i}}) = \Psi_1{}^2(\mathbf{t}) + \Psi_2{}^2(\mathbf{t})\ ,\ \Psi_1^2 = \frac{\mathbf{ui}}{\mathbf{2}},\ \Psi_2^2 = \frac{\hat{\mathbf{u}}\hat{\mathbf{i}}}{\mathbf{2}}.$$

It shouldn't be too big success to causally explain quantum-mechanical diffraction, superposition and interference effects, when a "single-wave-object" and/or a single particle (like an electron, or photon) passes the plate with (at least) two small, diffraction holes, because in reality there isn't a single particle or wave object. There are always minimum two of mutually Hilbert-coupled wave elements and their mixed products, in some way force-energy-coupled with their environment, extending the number of interaction participants, and we need all of them in order to find belonging amplitude, phase and frequency functions). What could look as a bit unusual quantum interaction, or interference of a single wave or particle with itself, in fact presents an interaction of at least 2 wave entities with some other, third object, here with a plate with diffraction holes ($\Psi^2(\mathbf{t}) = \Psi_1{}^2(\mathbf{t}) + \Psi_2{}^2(\mathbf{t})$). Somehow Nature always creates complementary and conjugated couples of important elements (signals, particles, energy states…) belonging to all kind of matter motions, or we can also say that every object (or energy state) in our universe has its non-separable and conjugated, phase-shifted image (defined by Analytic Signal concepts). Consequently, the quantum-mechanical wave function and wave energy should represent only a motional energy (or power) composed of minimum two mutually coupled wave functions (see also relevant comments about meaning of complex functions and Analytic signal components given below T.4.0.1).

Here applied mathematics, regarding wave functions $\Psi^2(\mathbf{t}) = \Psi_1{}^2(\mathbf{t}) + \Psi_2{}^2(\mathbf{t})$, after making appropriate normalization/s and generalizations, would start to look as applying Probability Theory laws, like in the contemporary Quantum Theory. Consequently, modern Quantum Theory could also be understood as a generalized mathematical modeling of micro world phenomenology, which conveniently unifies all conservation laws of physics in a joint, normalized and dimensionless, mutually well-correlated theoretical platform, using the framework of Statistics, Probability and modern Signal Analysis. In this way, a new mathematical theory is created, that is only a bit unusual and strange by its appearance, but in reality isomorphic to habitual mathematical modeling of the remaining Physics structure.

Also, a kind of generalized analogy with Norton and Thevenin's theorems (known in Electric Circuit Theory) should also exist (conveniently formulated) in all other fields of Physics and Quantum Theory related to wave motions, since the cause or source of a certain action is producing certain effect, and vice versa, and such events are always mutually coupled.

[♣ COMMENTS & FREE-THINKING CORNER:

4.0.12.1. Standard Deviation in Relation to Load Impedance

*Since the standard deviation is often used to explain Uncertainty relations, let us analyze the meaning of the signal Standard Deviation regarding electric circuits where it is possible to measure voltage, **u**, and current, **i**, on a certain electric load. Electric load can have fully resistive (or active) impedance, **R**, or in general case it can present the complex impedance, **Z** (as the combination of **R**, **L** and **C** elements).*

Since the Standard Deviation presents the power of the average signal deviation from the mean power, and since we are able to measure coincidently (by sampling) the load current and voltage (both of them mutually dependant), we are in the position to formulate the following generalized expression for Standard Deviation, by multiplying voltage and current deviations,

$$\sigma_g^2 = \frac{1}{N}\sqrt{\sum_{(i)}\left|(u_i - \bar{u})(i_i - \bar{i})\right|^2} = \frac{1}{N}\sqrt{\sum_{(i)}(u_i - \bar{u})^2(i_i - \bar{i})^2}$$
$$N(=) \text{ number of samples, } \bar{u} = \frac{1}{N}\sum_{(i)} u_i\,,\ \bar{i} = \frac{1}{N}\sum_{(i)} i_i \qquad (4.0.105)$$
$$\bar{u},\ \bar{i}\ (=) \text{ runnung mean values (time evolving)}$$

Only in case/s when the load is resistive, above-formulated Standard Deviation, (4.0.105) can be transformed into the following expressions (fully equivalent to the contemporary definition of the Standard Deviation),

$$u = Ri,\ \bar{u} = R\bar{i} \Rightarrow \sigma_g^2 \to \sigma^2 \Rightarrow$$
$$\sigma^2 = \frac{1}{N}\frac{1}{R}\sum_{(i)}\left|u_i - \bar{u}\right|^2 = \frac{1}{N}R\sum_{(i)}\left|i_i - \bar{i}\right|^2 = \frac{1}{N}\frac{1}{R}\sum_{(i)}(u_i - \bar{u})^2 = \frac{1}{N}R\sum_{(i)}(i_i - \bar{i})^2$$
$$\sigma^2 = \frac{1}{N}\frac{\bar{i}}{\bar{u}}\sum_{(i)}(u_i - \bar{u})^2 = \frac{1}{N}\frac{\bar{u}}{\bar{i}}\sum_{(i)}(i_i - \bar{i})^2 = \frac{1}{N}\sum_{(i)}\frac{i_i}{u_i}(u_i - \bar{u})^2 = \frac{1}{N}\sum_{(i)}\frac{u_i}{i_i}(i_i - \bar{i})^2$$
$$\sigma^2 = \frac{1}{N}\sqrt{\sum_{(i)}(u_i - \bar{u})^2 \cdot \sum_{(i)}(i_i - \bar{i})^2} \qquad (4.0.106)$$

*If after sampling of a load current and voltage we find that expressions (4.0.105) and (4.0.106) would produce significantly different results, this should mean that the load impedance has non-resistive, complex character (presenting a combination of **R**, **L** and **C** elements). The factor of "Impedance Complexity" can be defined as,*

$$\frac{\sigma^2}{\sigma_g^2} = \sqrt{\frac{\sum_{(i)}(u_i - \bar{u})^2 \cdot \sum_{(i)}(i_i - \bar{i})^2}{\sum_{(i)}(u_i - \bar{u})^2 \cdot (i_i - \bar{i})^2}} \quad \begin{Bmatrix} = 1 \to \text{Resistive Load} \\ \neq 1 \to \text{Complex Load} \end{Bmatrix} \qquad (4.0.107)$$

In other words, when the ratio (4.0.107) is different from 1, contemporary formulation of Standard Deviation (found in all Statistics books) is not the best (and not the most general) representation of the natural signal deviation. If we apply the same situation on arbitrary signals, we could say that the complexity of any signal, obtained as a product of two mutually conjugated signals, can be tested and additionally classified by (4.0.105), (4.0.106) and (4.0.107). Consequently, often used Standard Deviation in Quantum Theory and Physics in number of cases could be very much limited (or in some cases wrong), since not all loads are resistive (electrically, or mechanically, or in some other analogical meaning). Many important laws of Statistics, Thermodynamics, Quantum Theory etc., such as Normal, Gauss distribution, Black body radiation law, Uncertainty Relations etc. are already formulated using the traditional definition of the Standard Deviation. Such standard deviation is intrinsically limited to active or resistive loads (where functions, defining the signal power, are in phase), what is influencing that some of conclusions based on such laws could be wrong (the fact never noticed until present, because of the present incomplete definition of the Standard Deviation). ♣]

4.1. DE BROGLIE MATTER WAVES AND QUANTUM MECHANICS

This chapter presents a natural continuation of all ideas and theoretical step-stones already elaborated in the chapter 4.0 (Wave functions, wave velocities and uncertainty relations). It is strongly recommendable first to read and understand the basics of particle-wave duality found in the chapter 4.0, and later, many of new ideas and concepts from this chapter would be much easier to accept. The next challenging opportunity (presented in this chapter) will be to extend and upgrade the meaning of de Broglie concept of particle-wave duality (briefly saying, towards loosing its duality), thus suggesting the platform for the new Unified Field Theory. Also, significant conceptual upgrade (and modification) of the foundations of Quantum Theory (that is presently the principal platform where particle-wave duality is scientifically addressed) will be initiated in this chapter. The leading concept of innovative understanding of de Broglie mater waves (of this paper) can be summarized by the following statement (that will be more supported later).

Regarding moving particles, in this paper the concept favored is that De Broglie matter waves (apart from external and visible manifestations) are always intrinsically incorporated (rooted) inside a structure of every stable particle in a form of self-sustaining, rotating, stationary and standing-wave fields (as a form of an "energy packaging"). In cases when the particles interact with surrounding environment, such internally parked de Broglie matter waves would "unfold", producing externally (directly and/or indirectly) measurable, wave motion/s and <u>energy and mass exchange</u> manifestations (i.e. external manifestations of de Broglie waves are only a natural extension or expansion of an intrinsic and internal particle/s wave structure). ***The most direct external signs regarding documenting existence of such hidden (internally packed) rotation-related behaviors should be spin and orbital moment attributes of all elementary particles, because if external rotation of some kind has been once in the past involved in the creation of elementary particles, in order to satisfy the total orbital momentum conservation, some permanent, residual and differential, rotating-like characteristic should be permanently linked to the particle properties; therefore, we are presently specifying such properties as intrinsic spin attributes (without clarifying what is really rotating).*** *From the opposite point of view, considering pure waves (energy states without rest mass), such as photons, we can say that such wave states could "solidify or sublimate" by entering an already existing particle structure (being energy momentum absorbed), or under certain conditions (when modulated or captured by presence of torsional external field components) can be directly transformed into particles with non-zero rest mass.* ***In other words, any motional energy presents certain state of mass distribution that is propagating as a kind of waving, and when necessary conditions are met, certain wave packets of such distributed mass can create space-localized and stable particles (when torsional and linear motion components fit into certain mutual agreement and coupling).***

Big part of the phenomenology belonging to particle-wave duality is also closely related to the domain of Thermodynamics and Fluid dynamics (and not exclusively to Quantum Mechanics). That is the fact still not explicitly and sufficiently recognized from such perspective, since Thermodynamics shouldn't be only the discipline dealing with random motions, scattering, and impacts of mutually independent particles, atoms and *molecules, where different interactions, fields, forces and waves between them could be neglected, because every moving particle should be in certain relation with its environment and its matter wave attributes. Again, briefly saying, de Broglie matter waves and all other wave phenomena and oscillations known in Physics naturally belong to the same family of events and should be formulated and analyzed using the same theory (that is presently not the case and that is what this paper intends to address). Also, all fields and wave phenomena (presently known in Physics) should be considered as a natural extension of particles and fluids states towards diversity of other energy or mass states, being a kind of "coupling and gluing medium" in a space between particles (while particles should be considered as condensed energy states of self-sustaining, self-closed and internally-folded forms of rotating mater waves). <u>The same situation regarding Particle-Wave duality would become much simpler and clear later, when we start to structure it mathematically, applying Conservation Laws and very tangible modeling.</u>*

Let us consider several starting points (as the starting background regarding the understanding of particle-wave duality, or better to say- unity), here formulated in the following way:

1. *The initially established concept regarding matter waves was based on de Broglie's explanation of stationary electron orbits of Bohr's planetary atom model. The same wave hypothesis was shown to be essential in explanations of Compton and Photoelectric Effects (combined with Planck's and Einstein's expression for the energy of a wave packet or photon). Then Davisson and Germer demonstrated experimentally the wave properties of massive particles in 1927 in their electron-diffraction experiments. Later, 1930, Estermann and Stern demonstrated diffraction of helium atoms and hydrogen molecules from lithium fluoride crystals (effectively starting the field of atom optics, since they were the first to demonstrate the wave-like properties of atoms). Recent development regarding the understanding of particle-wave duality made possible the production of gaseous Bose-Einstein condensate in which macroscopic numbers of atoms occupy the same quantum state, and where waves associated with each of the atoms are in phase with one another in a way that is directly analogous to the behaviors of photons in laser devices (see chapter 3 in [27]). If we are convinced that a kind of simple harmonic wave (with predictable and quantifiable wavelength and frequency) is associated with particle motion/s, we should ask ourselves what and where the source of such waves is. Logically, based on experimental data and applied wave modeling parameters we use in such situations, it is clear that it should be certain (maybe still strange) kind of "oscillatory circuit" or "wave generator", directly linked to a particle in motion (to its linear momentum and energy), producing de Broglie matter waves. Such "oscillatory source" could be partially related to force and field effects between moving particle and its environment, but could also be an intrinsic part of every particle structure, and here exactly it is the platform we will follow in this paper.*

2. *Briefly, we can safely say that any particle motion is unified (associated, followed, coupled, or at least mathematically presentable) with a corresponding wave motion (based on de Broglie hypothesis) and that all wave motions including de Broglie matter waves, are manifestations of different forms of time variable, motional or kinetic energy (or power). There should be direct correspondence, mutual dependence and equivalence between wave and kinetic energy ($\tilde{\mathbf{E}} \Leftrightarrow \mathbf{E}_k$). Consequently, every quantitative change or (time-space) modulation of motional or total energy (of any origin) should also be a source of (different kind of) matter waves, and should be directly related to action-reaction and inertial forces phenomenology.*

3. *The same mathematical modeling (related to Signal Analysis), used to describe different (mechanical, electrical, electromagnetic, etc.) oscillations, wave motions and similar phenomena in Physics, is (almost) universally applicable to all of them, regardless of the nature of particular wave phenomena. Particle-wave duality theory (or just its mathematical modeling) starts from the intuitive concept that velocity of a real particle,* $\mathbf{v}$*, should be the same as the group velocity,* $\mathbf{v}_g$ *(=* $\mathbf{v}$*), of the wave packet or wave group associated with that particle (based on habitual modeling of a wave group known from Quantum Mechanics). Since every wave group also has its phase velocity,* $\mathbf{u} = \lambda \mathbf{f}$ *, and since there is the well known analytical connection between a group and phase velocity of simple, harmonic, modulated sinusoidal waves (* $\mathbf{v}_g = \mathbf{v} = \mathbf{u} - \lambda \mathbf{du} / \mathbf{d}\lambda$ *), there should also exist one consistent mathematical modeling that will unify all elements of a real particle motion with its associated wave-group replacement. Moreover, we should not forget that Planck's wave energy* $\tilde{\mathbf{E}} = \mathbf{hf}$ *, combined with relativistic expression/s of particle energy (Photoelectric and Compton effects,) produces correct results related to wave-mass momentum calculations and transformations. For instance, a photon that has energy* $\tilde{\mathbf{E}} = \mathbf{hf}$ *, also has an equivalent (particle-like) momentum* $\tilde{\mathbf{p}} = \mathbf{hf} / \mathbf{c}$ *, and equivalent (particle-behaving) mass,* $\tilde{\mathbf{m}} = \mathbf{hf} / \mathbf{c}^2$ *, because the total wave energy of the photon is equal to its total relativistic energy and to its total kinetic energy (since photon has zero rest mass),* $\tilde{\mathbf{E}} = \mathbf{hf} = \tilde{\mathbf{m}}\mathbf{c}^2 = \gamma\, \mathbf{mc}^2 = \mathbf{pv} / (\mathbf{1} + \sqrt{\mathbf{1} - \mathbf{v}^2 / \mathbf{c}^2})$ *,* $\mathbf{v} = \mathbf{c}$*,* $\tilde{\mathbf{p}} = \mathbf{p} = \mathbf{hf} / \mathbf{c}$ *. At the same time, the photon has its intrinsic, elementary angular momentum or spin equal to* $\tilde{\mathrm{L}}_{\mathbf{f}} = \tilde{\mathbf{E}} / \omega = \mathbf{hf} / 2\pi\mathbf{f} = \mathbf{h} / 2\pi$ *(see (2.11.3) and T. 4.0).*

4. *Every particle or wave motion (in fact, a mathematical function describing that motion) that can be characterized by some spectral distribution, characteristic frequency, wavelength, oscillating or waving process, etc., should be an integral part of certain visible or hidden rotation, or torsion-field phenomena (in its original or transformation domain). Since particles, in the frames of de Broglie Particle Wave concept, behave (analogically) as photons regarding linear moments, and since photons also have their intrinsic orbital moments, by applying the same analogy it should be valid that all (at least elementary) particles in linear motion should also have certain wave equivalent to angular momentum (or spin,* $\mathrm{L} = \frac{\mathbf{c}^2}{\mathbf{uv}}(\frac{\mathbf{h}}{2\pi})$, $\Delta \mathrm{L} = \frac{\mathbf{h}}{2\pi}$ *: see the table T.4.0, Photon – Particle Analogies).* <u>*In fact, the big secret of particle-wave duality (or unity) is that somehow all forms of wave and/or kinetic energy (or motional energy in general) of elementary particles and quasi-particles (or wave packets) have the same elementary spin unit, equal to*</u> $\tilde{\mathrm{L}} = \frac{\mathbf{h}}{2\pi} = \frac{\mathbf{E}_{\mathbf{motional}}}{\omega} = \frac{\tilde{\mathbf{E}}}{\omega} = \frac{\mathbf{E}_{\mathbf{k}}}{\omega} = \Delta \mathrm{L}$ *(or, more realistic, as a general case would be to consider integer multiples of* $\frac{\mathrm{h}}{2\pi}$ *). Consequently, it is clear that elements of rotation and linear motion should always be intrinsically coupled in all cases of (charged or neutral) particle and wave motions (* $\frac{\mathrm{L}_0}{\mathrm{p}_0} = \frac{\Delta \mathrm{L}}{\Delta \mathrm{p}} = \frac{\mathrm{L}}{\mathrm{p}} = \lambda \frac{\mathrm{L}}{\mathrm{h}} = \frac{\mathrm{c}^2}{\omega \mathrm{v}} = \frac{\mathrm{c}^2}{\mathrm{v}^2} \cdot \mathrm{r}^* = \frac{\mathrm{c}^2}{\omega^2} \cdot \frac{1}{\mathrm{r}^*}$ *). For instance, electrons always have their intrinsic magnetic and orbital moments mutually coupled, meaning that something equivalent to rotation naturally (or intrinsically) implemented in internal electron structure (connecting rotating mass and rotating electric charge, also known as gyromagnetic ratio) should exist. Also in cases of linear (or circular) macro-motions of electrons, again we have the appearance of rotating (or helix) magnetic field components around their paths. Consequently, similar situation should also be valid for all cases of neutral particles, like atoms, that perform linear motion, producing simultaneously rotation effects of certain field components (belonging to*

their internal, electrically charged constituents), which eventually produce externally measurable or visible consequences, such as different and omnipresent rotation/s in the world of molecules, atoms and elementary particles, as well as rotations of astronomic objects. Here is the reason why Gravitation is still not well integrated into the texture of other important theories of Physics, since its coupled, or conjugate field-component, related to certain kind of rotation (coupled with linear mass motion, like coupling between electric and magnetic fields), is still not adequately taken into account. We could search for such missing rotating component/s (in the world of Gravitation, and non-charged particle motions) by analyzing Coriolis, Centrifugal and Centripetal forces. Here elaborated particle-wave duality concept is strongly related to the necessity of theoretical unification of linear and rotational motions (in the domain of Gravitation and Electromagnetism, see (2.3)-(2.4-3) as the first step in such attempts) on a much more profound and explicit level than presently applied in Physics, since without rotation (or vortex and torsional field components), existence and/or creation of stable elementary particles would not be possible. Most probably, the future upgraded theory of Gravitation will deal with couple/s of mutually conjugated fields related to linear and rotational motions (of any nature or origin). Eventually, we would be able to find that only electrically charged particles and electromagnetic fields (currents and voltages) are in the background of all gravitation-related sources (since electrically neutral masses are composed of electrically charged particles). The situation regarding elementary spin units and/or orbital and linear moments of photons, atoms, electrons and other particles could be much more complex than here presented, but for underlining the idea that Particle-Wave duality should be directly related both to linear and angular moments, as well as their analogies and equivalents (in both directions,) the message given here is already clear enough.

5. *The creation of an electron-positron pair from the energy-momentum content of a sufficiently energetic photon, and the annihilation of an electron-positron pair that produces two photons, are very indicative experimental situations explicitly explaining that internal content of an electron (or positron) mass is just another form of photon-<u>energy packing or unpacking</u> (while satisfying the conservation of total system energy and orbital and linear moments). Something similar (at least by analogy and symmetry) should also be valid for protons (and anti protons), and since neutron presents certain coupled combination of one electron and a proton, we could conclude that quanta of electromagnetic energy (or photons) in different "<u>packing formats</u>" (most probably) create overall mass and atoms diversity in our universe. Here is also the explanation of the nature of particle-wave duality or unity between particles and waves. It could eventually happen that we discover that our universe is composed of a variety of forms, waves, particles and their transients, all of them having profound electromagnetic nature.*

6. *In any case, one of possible well-operating mathematical modeling regarding particle-wave duality situations is already known. This is the mathematical structure of the Orthodox Quantum Mechanics (OQM). The traditional grounds of OQM are very well known and presently (officially) recognized as the "only acceptable", it is the leading and most important picture and framework of contemporary Physics. Only the conceptual picture of the world behind contemporary OQM is unclear or missing, or not sufficiently compatible with intellectual and deterministic visualization. Here, an effort will be made to "dress" the OQM's mathematical modeling into a conceptually much clearer, logical and deterministic shell (using the known pictures and models from other chapters of Physics). In fact, it will be demonstrated that in parallel to probabilistic OQM, there should exist another (isomorphic and more general) level of modeling wave functions and equations, involving conceptually clearer and more tangible reality, which can be used independently to describe the world of particles and waves, which would again, in cases when such wave functions are normalized or "undressed" (losing dimensionality) produce the models already known in OQM.*

4.1.1. Particle-Wave Duality Code

The success of Quantum Mechanics (or its mathematical and predictive power) is mostly related to the proper integration between the essential **P**article-**W**ave **D**uality **C**ode (**PWDC**, the name invented here), and generally valid matter waves equation (or Schrödinger Equation). It should also be underlined that, presently, nobody is using such simplified formulation regarding foundations of Quantum Mechanics. The main part of the **PWDC** was (unintentionally and unknowingly) initially formulated/discovered and applied by L. de Broglie, Max Planck, A. Einstein and Max Bohr (and in this paper it will be explicitly exposed, slightly upgraded and more profoundly explained and applied, than it is the case in today's Quantum Mechanics).

Briefly saying, PWDC is a set of rules, equations and expressions that govern and explain how particles and waves are mutually coupled, how waves can be transformed into particles and vice versa, and how energy can be exchanged between them.

One of the very important wave equation (relevant to micro-world structures and to **PWDC**), first time formulated by E. Schrödinger, will also be upgraded (and generalized) towards much higher level of applicability to any possible wave phenomena (or, at least, it will become much more general than conventional Schrödinger's Equation as known from today's Quantum Mechanics). In fact, it will become evident that conventional Schrödinger Equation is just a simple mathematical consequence of "manipulating" certain generalized integral wave function (formulated in the form of an Analytic Signal, which will be introduced later), and not so strong and unique starting position of Quantum Mechanics that should be postulated as one of the primary sources of wave mechanics (since it can be simply derived from some other, more general mathematical form/s, non-related to Quantum Theory assumptions). It will also be shown that new, generic (mathematical) form/s of universally valid wave equation/s (such as (4.9), (4.10)), that can describe any wave motion in electromagnetism, acoustics, hydrodynamics, etc., including quantum mechanical waves, after proper merging with **PWDC**, would also lead to different forms of generalized Schrödinger's equation, or equation of all matter waves.

An updated Schrödinger-like equation merged with **PWDC** (the one that will be generalized in this paper: see equations starting from (4.9)) will establish higher applicability level of renewed Particle-Wave Duality Theory (the one which is favored in this paper), that is more conceptually causal, clearer and richer in comparison with the traditional Quantum Mechanical picture of the Nature.

To answer the question how matter waves have been created is one of the main objectives of this paper. As an easy introduction (into conceptual understanding of the **PVDC**), let us start analyzing an idealized, a "totally isolated and mutually interacting, two-body" or two-particle system, where the first body is an ordinary-like (known parameters) particle in a rectilinear motion (without elements of rotation), having momentum $\mathbf{p_1}$, and where the second "particle" effectively presents a totality of the surrounding universe, having resulting momentum $\mathbf{p_2}$. We can assume that certain force or interaction field should always exist inside and between mentioned particles. Since we intend to treat this case like an isolated system, the general law of total momentum conservation (First Newton Law) should be satisfied: $\mathbf{P} = \mathbf{p_1} + \mathbf{p_2} = \mathbf{const}$. The particular forces effectively reacting on each entity of the two-body system can be found by applying the Second Newton Law:

$\frac{\mathbf{dP}}{\mathbf{dt}} = \frac{\mathbf{dp_1}}{\mathbf{dt}} + \frac{\mathbf{dp_2}}{\mathbf{dt}} = F_1 + F_2 = \mathbf{0}$, $\frac{\mathbf{dp_1}}{\mathbf{dt}} = F_1 = F_{12}$, $\frac{\mathbf{dp_2}}{\mathbf{dt}} = F_2 = F_{21}$, $F_{12} = -F_{21}$, $F_1 = -F_2$.
The ordinary-like particle (the first body) is characterized by its ordinary (linear motion) particle momentum $\mathbf{p_1}$ and performs the force action F_{12} on its couple (surrounding universe). The second body has its resulting momentum $\mathbf{p_2}$, and performs the force action F_{21} on its couple $\mathbf{p_1}$ (in reality we know that this is only the measure of fields and interaction complexity between such "two bodies"). Later in this paper, an innovative conceptualization of forces and fields between the (known) particle momentum $\mathbf{p_1} = \mathbf{p}$ and resulting, surrounding complex field momentum, $\mathbf{p_2} = \tilde{\mathbf{p}} \Rightarrow \mathbf{P} = \mathbf{p} + \tilde{\mathbf{p}} = \mathbf{const.}$, $\mathbf{dp} = -\mathbf{d}\tilde{\mathbf{p}}$ (which represents a kind of wave motion) would become the starting point for explanation and applications of the **PWDC**. In other cases, if we were dealing with two (known and mutually interacting) moving particles "immersed" in their surrounding environment (what would now effectively become, at least, a three-body system), the previous situation would evolve as:

$\vec{P} = \vec{p}_1 + \vec{p}_2$, $\frac{\mathbf{d}\vec{\mathbf{P}}}{\mathbf{dt}} = \frac{\mathbf{d}\vec{\mathbf{p}}_1}{\mathbf{dt}} + \frac{\mathbf{d}\vec{\mathbf{p}}_2}{\mathbf{dt}} = \vec{F}_1 + \vec{F}_2$, $\frac{\mathbf{d}\vec{\mathbf{p}}_1}{\mathbf{dt}} = \vec{F}_1 + \vec{F}_{12}$, $\frac{\mathbf{d}\vec{\mathbf{p}}_2}{\mathbf{dt}} = \vec{F}_2 + \vec{F}_{21}$, $\vec{F}_{12} = -\vec{F}_{21}$, and significant **PWDC** elements (in this case) would be closely related to "internal" forces between such particles, $F_{12} = -F_{21}$, and to the work produced by such forces (acting on the path $\mathbf{r_{12}}$, which connects two moving particles): $E_{12} = \int_{(1)}^{(2)} \vec{F}_{12} d\vec{r}_{12} = E_{kr}$ (see (4.3)-(4.8); in this case forces $\vec{F}_1, \vec{F}_2$ are external forces). The new understanding of the **PWDC** (regarding forces between mutually approaching particles) should also be complemented with the generalized Newton-Coulomb force law, presented in the chapter 2 (see equations from (2.3) to (2.9)), showing that "internal forces" between two objects could be composed of many different, static and dynamic force components (some of them still hypothetical, but it should also be stated that Newton-Coulomb force laws are not sufficient to capture all possible forces between two objects).

How and why the forces between interacting and approaching objects create matter waves is the principal question to answer. If we would just have two mutually approaching particles, without having any of significant force field interactions between them, the particles would simply pass the zone of interaction (or, more correctly, pass the zone of non-interaction) and disappear in an open space in front of them (or, in cases of plastic or any other collision, we would get results predictable by conservation laws). In the opposite case, if in process of approaching, and in the near interaction zone, the particles are permanently affected by central forces between them, initially rectilinear motions of both of them would start transforming to helix-paths motions, with torsional-fields motional elements. Such rotating motion elements would start creating matter waves, and we would be able to associate the wavelength and frequency to such motions, what De Broglie mathematically mastered or fitted regarding wavelength, and what is the concept being favored in this paper, but elaborated much wider and in much more details than in de Broglie foundations. If interacting particles already had linear and orbital moments (and maybe spins), before starting the interaction, the same situation regarding forces and matter waves would become much more mathematically complex (since the care should be taken to satisfy all conservation laws). **In this paper a step further is also made, introducing the idea (with certain hypothetical elements) that Physics has still not mastered the knowledge of all possible dynamic forces between two moving objects, suggesting that linear and orbital moments could also be active charges for such forces (see the second chapter: equations from (2.3) to (2.9)).**

We could also imagine that all interacting particles and objects (including their environment) behave like being mutually connected with some kind of (invisible) springs, creating a multidimensional and elastic space structure (or matrix) that is able to oscillate, or to produce de Broglie matter waves, if any of its nodes or elements is excited, activated or set in motion, and PWDC describes important elements of such waving behaviors. Furthermore, all nodal elements (particles, interacting objects, atoms, molecules, etc.) of the mentioned elastic space-matrix structure themselves (internally) present stabilized, oscillating and/or resonant structures, where again, PWDC is the framework describing how such coupled oscillators are constructed internally and how they mutually communicate and synchronize (externally).

The contemporary physics deals very much traditionally, one-sided and straightforward when analyzing two-body interactions, mostly from the point of view of classical mechanics of rigid particles, and this is one of the problems we shall meet when we will start introducing and explaining the full meaning of the **PWDC**. In reality, similar to electric circuit theory and electromagnetic theory, where we analyze electric currents, voltages, field components and electric power (present in a certain electric network), and apply well-known concepts about active, reactive and apparent power, we should be able to create (at least by analogy) a very similar concept, or a general case for (mechanical and other) forces, velocities and power associated to moving particles and their interactions, which is still not a generally accepted practice. In this way, we could introduce the meaning of Complex, Active, Reactive and RMS forces and velocities (of any origin and nature), as well as exploit the terms of Active, Reactive and Apparent mechanical power, and show that any motion (in Physics) has very rich and balanced nature between intrinsic, mutually interacting corpuscular and wave properties of its constituents (which will be presented in the following chapters of this paper).

The message of this paper is that only ***"ENERGY IN MOTION"****, or* ***"CURENT OF ENERGY"*** *(regardless of its origin), or simply* **POWER (= dE/dt,** *applying the analogy with* ***alternate,*** *electric currents and/or voltages, or with electromagnetic waves), presents everything we should relate to matter waves, or de Broglie waves (where the total time-space fluctuating and* **alternating POWER** *can have electromagnetic, mechanical, linear motion, rotational and/or other components). Later on it will be shown that the celebrated Schrödinger's equation and Quantum Wave Mechanics were also (implicitly, and effectively) formulated using similar ideas, practically normalizing and averaging various power-related functions (making them dimensionless), and conveniently applying general rules of Probability Theory, Statistics and Signal spectrum analysis on such wave equations and functions, while respecting ("in average") all Conservation Laws known in Physics.*

The initial formulation and explanation of the **PWDC** we shall start briefly (without presenting any particular justification), with principal formulas and results already known (see T 4.0) and used in all contemporary presentations of particle-wave duality concepts, as for example with de Broglie matter-wavelength, $\lambda = \mathbf{h} / \mathbf{p} = 2\pi / \mathbf{k}$, and Planck's expression for wave packet (or photon) energy, $\tilde{\mathbf{E}} = \mathbf{hf}$,

$$\left\{\lambda = \frac{h}{p}\ ,\ u = \lambda f,\ \tilde{E} = hf\right\} \Leftrightarrow \left\{\frac{\lambda}{v} = \frac{h}{pv} \Leftrightarrow \frac{\lambda f}{vf} = \frac{u}{vf} = \frac{h}{pv}\right\} \Leftrightarrow \left\{\frac{u}{v} = \frac{hf}{pv} = \frac{\tilde{E}}{pv}\right\}\ , \qquad (4.1)$$

where $\mathbf{v}$ and $\mathbf{u} = \lambda\mathbf{f}$ are group and phase velocities, $\gamma\,\mathbf{mv} = \mathbf{p} = \mathbf{hk}/2\pi$ is the particle momentum of a moving particle, $\gamma = (1 - v^2/c^2)^{-1/2}$, $f_t = f = \omega/2\pi$ is the time-domain frequency of associated de Broglie wave, $\mathbf{f_s} = \mathbf{k}/2\pi$ is its space-domain frequency, $\mathbf{h}$ is Planck's constant, and $\tilde{E} = pu = hf \Leftrightarrow E_k$ is the (motional and/or wave) energy of de Broglie wave packet, where $E_k = pv/(1 + \sqrt{1 - v^2/c^2})$ is the kinetic energy of the particle in linear motion. We often use relations and expressions given in (4.1) without thinking too much how de Broglie and Planck came to such simple and useful relations. *In reality, the basic particle-wave duality relations (4.1) are formulated or postulated (or mathematically fitted, but not essentially explained) by searching for the best possible solution/s, or missing mathematical (and conceptual) link/s, to explain some phenomena and models in physics (for instance in order to better explain Bohr's hydrogen atom model and thermal radiation of a black body, and also to serve similar purposes in explaining many other micro-world phenomena, such as Photoelectric, Compton effect, etc.).*

For a moment we do not need to state explicitly what are all possible mutual relation/s (or quantitative connections) between the wave energy $\tilde{\mathbf{E}}$, wave momentum $\tilde{\mathbf{p}}$, kinetic (particle) energy $\mathbf{E_k}$, and its (particle) momentum $\mathbf{p}$, except to consider that certain equivalency and coupling between them exists, and that both entities (particle and wave entity) have the same (group) velocity, $\mathbf{v} = \mathbf{v_g}$ (Later on, the exact connections between all of them will be found: see (4.2). Currently we shall simply use the equivalency relation $\tilde{\mathbf{E}} = \mathbf{pu} = \mathbf{hf} \Leftrightarrow \mathbf{E_k}$, since any wave energy is also a kind of motional or kinetic energy).

For a moving particle we know the differential of its kinetic energy in the form $\mathbf{dE_k} = \mathbf{vdp}$, and by analogy, for wave energy (of an associated wave group) the following similar relation: $\mathbf{d\tilde{E}} = \mathbf{hdf} = \mathbf{d(pu)} = -\mathbf{vd\tilde{p}}\cos(\mathbf{p}, \tilde{\mathbf{p}}) \Leftrightarrow \mathbf{dE_k} = \mathbf{vdp}$, should be valid. Differential (or infinitesimal) energy expressions are interesting and important because they have the same mathematical forms in Classical and Relativistic Mechanics, such as:

$$\left\{\begin{array}{l} E_k = \frac{1}{2}mv^2 = \frac{pv}{2} \\ m = m_0 = \text{const.} \\ p = mv \end{array}\right\} \Rightarrow dE_k = vdp\ ,\ \left\{\begin{array}{l} E_k = mc^2(\gamma - 1) \\ m = m_0 = \text{const.} \\ p = \gamma\, mv \\ E_{tot.} = \gamma\, mc^2 \end{array}\right\} \Rightarrow dE_k = vdp = mc^2 d\gamma = dE_{tot.}\ ,$$

and we can use them in certain differential equations of interest (as for instance in the equation that connects group and phase velocity, (4.2)), taking into account all boundary conditions in the final phase of integration process when solving such equations.

Let us now try to connect and unify all particular, above mentioned expressions regarding particle and corresponding wave group. From the last, right-hand part of the equation (4.1), we can come closer to the conclusion that de Broglie wave length is not the most significant qualification of particle-wave duality, and that its fuller qualification should be the relation between phase and group velocity of de Broglie wave packet. Practically, we shall exploit and merge all relations from (4.1) with the following relations: $\tilde{\mathbf{E}} = \mathbf{hf} = \mathbf{pu} \Leftrightarrow \mathbf{E_k}, \mathbf{dE_k} = \mathbf{vdp} \Leftrightarrow \mathbf{d\tilde{E}} = \mathbf{hdf} = \mathbf{d(pu)}$, and $\mathbf{v} = \mathbf{u} - \lambda\mathbf{du}/\mathbf{d}\lambda$, and create results given in (4.2). It is not absolutely necessary, but we could also apply to (4.1) conclusions based on analogies, using the last part of (1.19), in order to get more explicit relations between particle and wave characteristics of de Broglie wave packet, as shown in (4.2). In fact, the complete explanation and justification of **PWDC** relations will be postponed, in order to, firstly, present the summarized skeleton of the most important ideas of this paper, and, later on, it will be shown that introducing a bit hypothetical wave quantities $(\tilde{\mathbf{p}}, \tilde{\mathbf{E}})$ becomes very much logical and defendable in explaining universal laws of inertia. **Whatever we understand under different inertial effects (in mechanics and electromagnetism) in reality should present interaction between two complementary fields or forces appearing in processes of sudden and non-uniform changes of motional energy. Our conceptual problem is that we understand well what that means in electromagnetic environment (different induction laws), but in mechanics, our understanding stops with Newton law of inertia, and usually we do not search there for the field component that should complement Gravitation, like in complementary relations between electric and magnetic fields. It is also not excluded that all inertia-related effects have their profound roots in electromagnetic induction laws.**

If corpuscular and wave momentum $(\mathbf{p}, \tilde{\mathbf{p}})$ are mutually collinear vectors ($\cos(\mathbf{p}, \tilde{\mathbf{p}}) = \mathbf{1} \text{ or } -\mathbf{1}$) we will find that there is only one consistent and unifying result, given by (4.2), which extends relations (4.1), relevant for basic understanding of particle-wave duality (here supported only by a very simplified mathematical procedure, by listing initial statements and going directly to their algebraic implications), which can be improvised and summarized as follows (see also **[5]**):

$$\left\{\begin{array}{l}
\left\{E_k \Leftrightarrow \tilde{E} = pu\,,\ pv = E_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right] = \frac{\gamma^2-1}{\gamma}mc^2\right\}, \\
\left\{(\frac{u}{v}) = \frac{\tilde{E}}{pv} = \frac{pu}{E_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right]} = \frac{1}{1+\sqrt{1-(\frac{v}{c})^2}} = \frac{\gamma}{\gamma+1}\right\}, \\
\left\{\lambda = \frac{h}{p}, k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p, \omega = 2\pi f\,, \frac{d\lambda}{\lambda} = -\frac{dp}{p} = -\frac{dk}{k} = -\frac{df}{f}, u = \frac{\omega}{k}, v = \frac{d\omega}{dk}\right\}
\end{array}\right\} \Rightarrow$$

$$\Rightarrow\begin{cases} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1+\frac{uv}{c^2}}\ , \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{f}{f_s} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p} \quad \Rightarrow \\ \Rightarrow \quad 0 \le 2u \le \sqrt{uv} \le v \le c\ , \\ d\tilde{E} = hdf = mc^2 d\gamma, \quad \frac{df}{f} = (\frac{dv}{v})\cdot\frac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} \Rightarrow \frac{\Delta f}{\overline{f}} = (\frac{\Delta v}{\overline{v}})\cdot\frac{1+\sqrt{1-\frac{\overline{v}^2}{c^2}}}{1-\frac{\overline{v}^2}{c^2}} \end{cases}$$

$$\Rightarrow\begin{cases} \frac{\tilde{\mathbf{E}}}{\mathbf{mc^2}} = \frac{\mathbf{hf}}{\mathbf{mc^2}} = \frac{\mathbf{1}}{\sqrt{\mathbf{1}-(\frac{\mathbf{v}}{\mathbf{c}})^2}} - \mathbf{1} = \gamma - \mathbf{1} = \frac{\tilde{\mathbf{E}}}{\mathbf{E_0}},\ \mathbf{E_0} = \mathbf{mc^2} = \mathbf{const.}, \\ \frac{\tilde{\mathbf{E}}}{\gamma\mathbf{mc^2}} = \frac{\mathbf{hf}}{\gamma\mathbf{mc^2}} = \mathbf{1} - \sqrt{\mathbf{1}-(\frac{\mathbf{v}}{\mathbf{c}})^2} = \mathbf{1} - \frac{\mathbf{1}}{\gamma} = \frac{\tilde{\mathbf{E}}}{\mathbf{E_{total}}},\ \mathbf{E_{total}} = \gamma\mathbf{mc^2} = \gamma\mathbf{E_0}\ , \\ \frac{\tilde{\mathbf{E}}}{\mathbf{E_k}} = \frac{\tilde{\mathbf{E}}}{(\gamma-\mathbf{1})\mathbf{mc^2}} = \frac{\mathbf{hf}}{(\gamma-\mathbf{1})\mathbf{mc^2}} = \mathbf{1},\ \mathbf{E_{total}} = \mathbf{E_0} + \mathbf{E_k} = \mathbf{E_t}\ , \\ \mathbf{p^2c^2} + \mathbf{E_0^2} = \mathbf{E_t^2}, \quad \mathbf{p^2v} - \mathbf{pE_t} + \mathbf{p_0E_0} = \mathbf{0}, \\ \tilde{\mathbf{E}} = \mathbf{pu} = -\mathbf{E_0} \pm \sqrt{\mathbf{E_0^2}+\mathbf{p^2c^2}} = \mathbf{E_k}\ \left\{ = -\mathbf{E_0} + \sqrt{\mathbf{E_0^2}+\mathbf{p^2c^2}} = \mathbf{E_0}\left[\sqrt{\mathbf{1}+(\frac{\mathbf{pc}}{\mathbf{E_0}})^2} - \mathbf{1}\right]\right\}, \\ \begin{Bmatrix} \vec{\mathbf{p}} + \tilde{\mathbf{p}} = \vec{\mathbf{P}} = \overrightarrow{\mathbf{const}} \Rightarrow \mathbf{d}\vec{\mathbf{p}} = -\mathbf{d}\tilde{\mathbf{p}} \\ \mathbf{d}\tilde{\mathbf{E}} = \mathbf{hdf} = \mathbf{d(pu)} = \mathbf{dE_k} = \mathbf{vdp} = \mathbf{c^2d(\gamma m)} = -\mathbf{c^2d}\tilde{\mathbf{m}} = -\mathbf{d(\tilde{p}u)} = \\ \underline{= -\mathbf{vd}\tilde{\mathbf{p}}\cdot\mathbf{cos}(\vec{\mathbf{p}},\tilde{\mathbf{p}}) = \{\mathbf{vd}\tilde{\mathbf{p}}\ \mathbf{or}\ -\mathbf{vd}\tilde{\mathbf{p}}\} = \mathbf{hvdf_s}} \end{Bmatrix} \\ \Rightarrow \Delta\mathbf{E_k} = -\Delta\tilde{\mathbf{E}},\ \Delta\mathbf{p} = -\Delta\tilde{\mathbf{p}},\ \Delta\mathbf{L} = -\Delta\tilde{\mathbf{L}},\ \Delta\mathbf{q} = -\Delta\tilde{\mathbf{q}}, \Delta\dot{\mathbf{p}} = -\Delta\dot{\tilde{\mathbf{p}}}, \Delta\dot{\mathbf{L}} = -\Delta\dot{\tilde{\mathbf{L}}}, ... \end{cases}, \quad (4.2)$$

As we can see (from (4.2) and later in this chapter), here we are also dealing with generalized concept of "action-reaction" and inertial forces (which should be an equivalent to the Third Newton Law: ($\Delta\mathbf{p} = -\Delta\tilde{\mathbf{p}}$, $\Delta\mathbf{L} = -\Delta\tilde{\mathbf{L}}$, $\Delta\mathbf{q} = -\Delta\tilde{\mathbf{q}}$ ...) $\Rightarrow$ ($\Delta\dot{p} = -\Delta\dot{\tilde{p}}$, $\Delta\dot{\mathbf{L}} = -\Delta\dot{\tilde{\mathbf{L}}}$, $\Delta\dot{\mathbf{q}} = -\Delta\dot{\tilde{\mathbf{q}}}$ ...), and which becomes explicitly evident after implementing the time differentiation on characteristic momentum and charge relations, regardless of their field nature (applicable to gravitation, electromagnetic fields, rotation, etc.).

Practically, it will become much clearer (later, in this and other chapters) that equations and relations found in (4.1) and (4.2) describe the essential **P**article-**W**ave **D**uality **C**ode, or **PWDC** (see also (4.3), tables T.4.0 and T 5.3, and Uncertainty relations from Chapter 5, as the important complements to **PWDC**).

Let us now make an attempt to compare (analogically, dimensionally and mathematically) basic characteristics, equations and formulas applicable to a photon (which is a wave packet without rest mass) and to a real moving particle that has a rest mass. The comparison photon-particle will be presented in the T.4.0, based on data from (2.11.3), (4.1) and (4.2). Some values and symbols found in T.4.0 are formally introduced there only for the purpose of making simpler and more indicative (dimensionally correct) analogies and mathematical expressions, in order to initiate new ideas that would appear in this paper later on. The leading (new and partially still hypothetical) idea regarding the comparison between a photon and a real particle is to show that internal (or intrinsic) particle wave properties (such as its characteristic wavelength and frequency) are closely related to similar external wave attributes of a particle (such as de Broglie wavelength and frequency), all of that also in the function of explaining the meaning of the **PWDC**. In fact, oversimplifying the situation, we would try to show that real particles are some kind of field-folded, self-stabilized, standing-wave structures.

Motional or kinetic particle energy could be treated as usually, having any positive value (velocity dependent), if this energy is measured externally, in the space where a particle is in motion. If we make an attempt to solve the relativistic equation that connects all energy aspects of a single particle, we will find that one of the solutions for kinetic energy could be the negative energy amount that corresponds to the particle rest mass energy.

$$\left\{\begin{array}{l} E^2 = E_0^2 + p^2c^2 = (E_0 + E_k)^2 = E_0^2 + 2E_0E_k + E_k^2 \quad , \\ E_0 = mc^2 \quad , \quad E = \gamma mc^2, \quad E_k = E - E_0 = (\gamma - 1)mc^2 \quad , \quad \gamma = (1 - v^2/c^2)^{-1/2} \end{array}\right\} \Rightarrow$$

$$E_k^2 + 2E_0E_k - p^2c^2 = 0 \ \Rightarrow$$

$$E_k = -E_0 \pm \sqrt{E_0^2 + p^2c^2} = -E_0 \pm E \Rightarrow$$

$$E_k = \left\{\begin{array}{l} +E_k \\ -E_0 \end{array}\right\} = \left\{\begin{array}{l} (\gamma - 1)mc^2 \\ -mc^2 \end{array}\right\} \quad (=)$$

$$(=) \quad \left\{\begin{array}{l} \text{motional particle energy "in external space"} \\ \text{motional particle energy internaly captured by its rest mass} \end{array}\right\}$$

Such result could look illogical and could be neglected or considered unrealistic, but if we take into account that internal particle structure (that creates its rest mass) is also composed of motional-field energy components (or matter waves), well-packed, self-stabilized and internally closed, we could consider another conceptual approach, which is that negative motional energy belongs to the ordinary motional energy that is internally “frozen” or captured by the particle rest mass.

T.4.0. Photon – Particle Analogies

	PHOTON	MOVING PARTICLE		
	Expressions related to wave or motional energy	Expressions related to states of rest	Expressions related to motional energy states	Expressions related to a total energy content
Energy	$\tilde{\mathbf{E}} = \mathbf{hf}$	$\mathbf{E_0} = \mathbf{mc^2}$	$\mathbf{E_k} = (\gamma - 1)\mathbf{mc^2}$	$\mathbf{E_t} = \mathbf{E_0} + \mathbf{E_k} =$ $= \gamma \mathbf{mc^2}$
Frequency	$\mathbf{f} = \frac{\tilde{\mathbf{E}}}{\mathbf{h}}$	$\mathbf{f_0} = \frac{\mathbf{E_0}}{\mathbf{h}} = \frac{\mathbf{mc^2}}{\mathbf{h}}$	$f = \frac{E_k}{h} = f_t - f_0 =$ $= (\gamma - 1)f_0 = \frac{\gamma(\Delta f)^*}{\gamma - 1} =$ $= \frac{c^2}{uv}(\Delta f)^*$	$f_t = f_0 + f = \frac{E_t}{h} =$ $= \frac{\gamma mc^2}{h} = \gamma f_0$
Mass	$\tilde{\mathbf{m}} = \frac{\tilde{\mathbf{E}}}{\mathbf{c^2}} = \frac{\mathbf{hf}}{\mathbf{c^2}}$	$\mathbf{m} = \frac{\mathbf{E_0}}{\mathbf{c^2}}$	$\Delta\mathbf{m} = \frac{\mathbf{E_k}}{\mathbf{c^2}} =$ $= (\gamma - 1)\mathbf{m}$	$\mathbf{m_t} = \frac{\mathbf{E_t}}{\mathbf{c^2}} = \gamma\mathbf{m} =$ $= \mathbf{m} + \Delta\mathbf{m}$
	$\frac{\tilde{\mathbf{E}}}{\tilde{\mathbf{m}}} = \mathbf{c^2}$	$\frac{\mathbf{E_0}}{\mathbf{m}} = \frac{\mathbf{E_k}}{\Delta\mathbf{m}} = \frac{\mathbf{E_t}}{\mathbf{m_t}} = \mathbf{c^2}$		
-Linear Momentum & Energy	$\tilde{p} = \frac{\tilde{E}}{c^2}c = \frac{h}{\lambda} =$ $= \frac{hf}{c} = \tilde{m}c$	$p_0 = \frac{E_0}{c^2}v = \frac{h}{\lambda_0} =$ $= \frac{p}{\gamma} = mv$	$\Delta p = \frac{E_k}{c^2}v =$ $= (\gamma - 1)mv =$ $= (\gamma - 1)p_0 =$ $= \frac{h}{\lambda}(\frac{\gamma - 1}{\gamma}) =$ $= \frac{h}{\lambda_0}(\gamma - 1) = \frac{p_0}{\lambda}\Delta\lambda$ $= \frac{p_0\lambda_0}{(\Delta\lambda)^*} = (\Delta m) \cdot v$	$p = \frac{E_t}{c^2}v = \frac{h}{\lambda} =$ $= \frac{\Delta p}{\gamma - 1} =$ $= p_0 + \Delta p =$ $= \gamma p_0 = \gamma mv$
	$\lambda = \frac{h}{\tilde{p}} = \frac{h}{\tilde{m}c} =$ $= \frac{c}{f}$	$\lambda_0 = \frac{h}{p_0} = \frac{h}{mv} =$ $= \gamma\lambda = \frac{u}{f_0}(\frac{\gamma}{\gamma - 1})$	$(\Delta\lambda)^* = \frac{h}{\Delta p} = \frac{h}{(\Delta m) \cdot v}$ $= \frac{\lambda_0}{\gamma - 1} = \frac{\lambda\lambda_0}{\Delta\lambda} = (\frac{c}{f})(\frac{c}{v}) =$ $= \frac{u}{(\Delta f)^*} = \lambda\frac{f}{(\Delta f)^*}$	$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} =$ $= \frac{\lambda_0}{\gamma} = \frac{u}{f}$

<table>
<tr>
<td></td>
<td>$\tilde{E} = hf = \tilde{m}c^2 =$
$= \tilde{p}c$
$(c = \lambda\ f = u = v)$</td>
<td colspan="3">$p^2c^2 + E_0^2 = E_t^2 = (E_0 + E_k)^2$, $\quad p = h/\lambda$, $\lambda = hc/\sqrt{E_t^2 - E_0^2}$
$\gamma = (1 - v^2/c^2)^{-1/2} = (1 - uv/c^2)^{-1}$, $\quad u = v/(1 + 1/\gamma) = \lambda f$
$(\frac{\gamma - 1}{\gamma})\lambda_0 f_0 = (\Delta\ \lambda)^*(\Delta\ f)^* = \lambda f = u$, $\frac{(\Delta\ f)^*}{f} = \frac{uv}{c^2} = \frac{\gamma - 1}{\gamma} = \frac{\lambda}{(\Delta\lambda)^*}$
$p\lambda = p_0\lambda_0 = (\Delta p)(\Delta\lambda)^* = h$, $\frac{p}{p_0} = \frac{\lambda_0}{\lambda} = \gamma$,
$\Delta\lambda = \lambda_0 - \lambda = \lambda(\gamma - 1) = \frac{\lambda\lambda_0\Delta p}{h} = \frac{h\Delta p}{pp_0} = \lambda\frac{\Delta p}{p_0} = \frac{\lambda\lambda_0}{(\Delta\lambda)^*}$</td>
</tr>
<tr>
<td rowspan="2">Angular Momentum

see (2.9.5)</td>
<td>$\tilde{L} = \frac{\tilde{E}}{\omega} = \frac{hf}{\omega} = \frac{h}{2\pi}$</td>
<td>$L_0 = \frac{E_0}{\omega} =$
$= \frac{c^2}{\gamma uv}(\frac{h}{2\pi})$</td>
<td>$\Delta L = \frac{E_k}{\omega} = \frac{h}{2\pi}$
$= \frac{(\gamma - 1)c^2}{\gamma uv}(\frac{h}{2\pi})$</td>
<td>$L = \frac{E_t}{\omega} = L_0 + \Delta L =$
$= \frac{c^2}{uv}(\frac{h}{2\pi})$</td>
</tr>
<tr>
<td>$\frac{\tilde{L}}{\tilde{p}} = \frac{\lambda}{2\pi} = \frac{c}{\omega} = \lambda\frac{\tilde{L}}{h}$</td>
<td colspan="3">$\frac{L_0}{p_0} = \frac{\Delta L}{\Delta p} = \frac{L}{p} = \lambda\frac{L}{h} = \frac{c}{\omega}(\frac{c}{v}) = \frac{c^2}{v^2}\cdot r^* = \frac{c^2}{\omega^2}\cdot\frac{1}{r^*}$</td>
</tr>
</table>

(The meaning of analogies and terms found in this table will be explained and analyzed later: see (4.1)-(4.8))

*[♣ **COMMENTS & FREE-THINKING CORNER**: How to properly address the spin and orbital moments characteristics should also be very important for understanding the **PWDC**. Let us start with a photon, by testing the concept that a photon already presents the unity of two motional energy components: one that belongs to its linear motion and the other that represents the amount of energy related to its spinning (leaving aside all questions regarding the application of proper terminology, in order to have an open way to faster and easier conceptual understanding). Both of mentioned energy components are measured by the amount of total energy of a photon, that is:* $\tilde{E} = hf = \frac{h}{2\pi}\omega = \tilde{m}c^2 = \tilde{p}c$. *We shall now try to isolate or separate the two photon energy components, and to show that both of them are (intrinsically) included in* $\tilde{E} = hf = \frac{h}{2\pi}\omega$. *In order to do that, let us use differential (infinitesimal) expressions for energy. For photon's total energy (which is at the same time its total motional energy) we will have:*

$$\left\{\tilde{E} = hf = \frac{h}{2\pi}\omega = \tilde{m}c^2\right\} \Leftrightarrow \left\{d\tilde{E} = hdf = \frac{h}{2\pi}d\omega\right\}.$$

To get an idea how to separately express linear motion energy component and orbital, rotational or spin-related energy component let us again use analogical thinking. For instance, if a moving macro-particle is in linear motion (without having elements of rotation), the following will be valid:

$$\left\{\begin{array}{l} E_k = mc^2(\gamma - 1) \\ m = m_0 = \text{const.} \\ p = \gamma mv \\ E_{tot.} = \gamma mc^2 \end{array}\right\} \Rightarrow \left\{dE_k = vdp = mc^2 d\gamma = dE_{tot.}\right\}$$

By analogy, if the same particle only rotates or spins (without progressing with any linear motion), its differential motional energy (related to rotation) will be: $\mathbf{dE_k} = \omega\mathbf{dL}$ *, for, if the analogy is applicable, this should be valid:*

$\{dE_k = vdp\}_{linear-motion} \Leftrightarrow \{dE_k = \omega d\mathbf{L}\}_{rotation}$. *It is clear that here we are applying the analogies:* ***linear velocity replaced by angular velocity***, $\mathbf{v} \leftrightarrow \omega$ *and* ***linear moment replaced by orbital moment***, $dp \leftrightarrow d\mathbf{L}$.

Let us now present the total photon energy (again in its differential form) as the superposition of the two motional energy components,

$$d\tilde{E} = hdf = \frac{h}{2\pi} d\omega = \underline{vd\tilde{p} + \omega d\tilde{\mathbf{L}}} .$$

since we are also familiar with calculating photon linear momentum (based on its energy and speed: Compton and Photoelectric effects) as

$$\left\{\tilde{p} = \tilde{m}c = \frac{h}{2\pi\ c}\omega = \frac{hf}{c}\ ,\quad v = u = c\right\} \Rightarrow \left\{d\tilde{p} = \frac{h}{2\pi c} d\omega = \frac{h}{c} df\right\}.$$

The total photon energy (which now includes linear and rotational motion components) can be found as:

$$d\tilde{E} = hdf = \frac{h}{2\pi} d\omega = \underline{vd\tilde{p} + \omega d\tilde{\mathbf{L}}} = c\frac{h}{2\pi c} d\omega + \omega d\tilde{\mathbf{L}} = \frac{h}{2\pi} d\omega + \omega d\tilde{\mathbf{L}} .$$

The last differential energy equation can only be valid if rotation-related motion component presents some kind of constant parameters spinning, where orbital moment or spin should take only constant numbers,

$$\{\omega d\tilde{\mathbf{L}} = 0\} \Rightarrow \{d\tilde{\mathbf{L}} = 0\} \Rightarrow \tilde{\mathbf{L}} = \text{const.} \quad \left\{= n \cdot \frac{h}{2\pi},\quad n = 1,2,3,...\right\}.$$

Here could be an explanation where the rotation related component is and why a photon has a constant spin (which is also valid for other elementary particles). The same kind of analogical thinking could be extended to real macro-particles (ones that have non-zero rest mass), understanding that internal and intrinsic rotation is captured by a particle rest mass (becoming externally "non-visible" and not measurable because a number of elementary domains inside a macro-particle is randomly packed neutralizing the total, resulting orbital moment). Of course, we know that real macro-particles have number of atoms, and that each atom has number of more elementary particles (all of them having constant and closely related magnetic and orbital moments). All of that makes this situation more complicated, but the conceptual understanding would stay simple and clear: The hidden rotation-related motional component is captured by a number of mutually coupled elementary domains that create rest mass (see also (2.5-1) to (2.11), (4.42) to (4.45) and (5.4.1) to (5.10)).

For instance, in order to additionally support the concept that rotational wave motion is somehow captured and "frozen" by particle rest mass, let us first find the wave function of certain moving particle or corresponding wave packet,

$$\begin{bmatrix} E^2 = E_0^2 + c^2p^2 \\ E = E_0 + E_k = \gamma mc^2 \\ E_0 = mc^2 = \text{const.}\ , \\ E_k = (\gamma - 1)mc^2 \\ p = \gamma mv,\ dE_k = vdp \end{bmatrix} \Rightarrow \Psi^2 = \frac{dE_k}{dt}.$$

In cases when linear and rotational motions are combined (as complex motion elements of the same particle or the same wave packet), by analogy we could extend the meaning of the wave function as:

$$\begin{bmatrix} \Psi^2 = \frac{dE_k}{dt} = v\frac{dp}{dt} \\ v\frac{dp}{dt} \leftrightarrow \omega\frac{d\mathbf{L}}{dt} \end{bmatrix} \Rightarrow \Psi^2 = \frac{dE_{k\text{-linear}}}{dt} + \frac{dE_{k\text{-rotation}}}{dt} = v\frac{dp}{dt} + \omega\frac{d\mathbf{L}}{dt} = vF + \omega\tau \Rightarrow$$

$$\Rightarrow E_0^2 = -2\int_{[\mathbf{L}]} E\omega d\mathbf{L} = -2mc^2\int_{[\mathbf{L}]} \gamma\omega d\mathbf{L} = \left(mc^2\right)^2 \Rightarrow$$

$$\Rightarrow \int_{[\mathbf{L}]} \gamma\omega d\mathbf{L} = -\frac{mc^2}{2} \Leftrightarrow \int_{[v]} \gamma dE_{k\text{-rotation}} = \int_{[v]} \frac{dE_{k\text{-rotation}}}{\sqrt{1-(\frac{v}{c})^2}} = -\frac{mc^2}{2} \Rightarrow$$

$$\Rightarrow E_0 = mc^2 = -2\int_{[v]} \gamma dE_{k\text{-rotation}} = \text{const.} \quad ,$$

which clearly shows that rest mass is created from some kind of properly packed rotational energy. What is important here is just to start familiarizing with the idea or concept that every motion has linear and torsional (or rotational) components, but rotation is in cases of particles often hidden by formation of rest masses. Later we will have enough mathematical and modeling margins to be able to pay attention on details and proper equations. ♣**]**

The more natural and conceptually clearer explanation of **PWDC** (compared to above formulated mathematical merging of particle and wave entities: (4.1), (4.2) and T.4.0) can also be formulated as:

a) First, we should know (or accept) that there isn't a case of a single particle, fully isolated and without any interaction with its vicinity. Any single particle or energy state belongs to a much more general case of (at least) two-body relations, where the first body is that single particle, and the second body is its vicinity (or totality of surrounding universe). This way (in any possible interaction, or when describing any possible particle/s or quasiparticles/s state/s), we effectively deal with at least a two-body system (or with interactions between two bodies). When we analyze the general two-body problem traditionally (or only mechanically), we can formulate the most important relations describing such interaction, given by results from (4.5) until (4.8). Effectively, by analyzing how we came to all results and relations (4.5)-(4.8), we can conclude later on that **PWDC** from (4.1) and (4.2) is an equivalent mathematical modeling way to explain the general two-body problem *by creating direct parallelism between particle (or mechanical) and wave entities of two-bodies* in question. Differently formulated, we will see that the general conclusion regarding two-body or two-particle relations presents the real source of **PWDC** (which will become more understandable after we make an equivalent wave presentation of the mechanical two-body situation). When analyzing the two-body problem in the light of **PWDC** we can also formulate the generalized, Newton-like "action-reaction", inertial forces (valid for all possible field situations: in gravitation, in electric and magnetic fields, etc.). Of course, the statements above are still a bit formalistic (and unverified) before we see the development of all mentioned relations, (4.3)-(4.8), which is just temporarily postponed for the purpose of this introductory **PWDC** formulation.

b) The second, also very important element of particle-wave duality connection is just coming from the fact that two bodies, mentioned in a), mutually interact even before any mechanical contact between them is established (again, see the development of (4.3)-(4.8), etc., as a support). We can call this interaction a field-channel, or particle-wave connection, ***and in reality this is just a modus of conserving (and redistributing) energy and momentum (both linear and orbital)***. By introducing the idea that certain field or wave connection should always exist (in any two-body situation), we can mathematically analyze what happens when two (or much bigger number of) elementary waves interact (of course, this is the example of a simple superposition of elementary (sinusoidal) waves in order to create the wave group, or wave packet model, well analyzed in almost any modern Physics and Quantum Mechanics book). What we create this way is called the wave packet. For such waving form we are in the position to find its group and phase velocity, and also find the essential relation between them, given by the equation: $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}$. When we analyze this situation, we can find that de Broglie wavelength , Planck's relation for wave energy $\tilde{\mathbf{E}} = \mathbf{hf}$, and Einstein's mass energy relation, $\lambda = h/p$ $\mathbf{E_{total}}$ = $\boldsymbol{\gamma}\mathbf{mc^2}$ are simply (mathematically) integrated in the structure of the equation that connects group and phase velocity (this way supporting and/or deriving every one of them using the combination of others), and all of them are shown to be mutually compatible and essential to explain (non-mechanically) the two-body situation a), this way automatically explaining the **PWDC** relations from (4.1) and (4.2):

$$\left\{\begin{array}{c} \left[\lambda = \frac{h}{p}\right] \\ \\ \left[\tilde{E} = hf\right] \\ \\ \left[\begin{array}{l} dE_k = vdp = mc^2 d\gamma = dE_{tot} = d\tilde{E} \\ E_{tot} = \gamma mc^2,\ E_k = (\gamma - 1)mc^2 \end{array}\right] \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = \frac{dE_k}{dp} = \frac{d\tilde{E}}{dp} = \frac{d\omega}{dk} \\ u = v/(1 + 1/\gamma) = \lambda f = \frac{E_k}{p} = \frac{\tilde{E}}{p} = \frac{\omega}{k} \\ 0 \le 2u \le \sqrt{uv} \le v \le c \end{array}\right\},$$

$$\frac{df}{f} = \frac{d\tilde{E}}{\tilde{E}} = -(\frac{v}{u}) \cdot \frac{d\lambda}{\lambda} = (\frac{dv}{v})(1 + 1/\gamma)\gamma^2 = -(1 + \frac{1}{\gamma}) \cdot \frac{d\lambda}{\lambda} \approx \left\{\begin{array}{l} 2\frac{dv}{v} = -2\frac{d\lambda}{\lambda} \ , \quad \text{for} \quad v << c \\ -\frac{d\lambda}{\lambda} \ , \quad \text{for} \quad v \approx c \end{array}\right\}.$$

In addition, we also know that in explaining all experimental situations regarding particle-photon interactions (such as Compton, Photoelectric, electron-positron creation from a high energy photon, or their annihilation, and other familiar effects), the equality between wave and particle energy interpretation is fully applicable and valid, $\tilde{E} = hf = \tilde{m}c^2$, $\tilde{p} = \frac{hf}{c}$, $\lambda = \frac{h}{\tilde{p}}$, and basically related only to motional (or kinetic) energies of both interacting participants.

In other words, if we accept this approach, the real sources of particle-wave duality become much more realistic (tangible and deterministic) than a contemporary probability platform of Quantum Theory in explaining the same phenomena (for more of complementary explanation of **PWDC** see also the chapter 5., especially T.5.3).

Since particles behave as photons (or waves) and vice versa, regarding the analogy related to their "linear moments", energy and mass, and since photons also have their intrinsic orbital moments, _by reversing the same analogy (now regarding rotational motion aspects) it should be valid that particles in linear motion should also have certain wave equivalent to angular momentum_ (or spin, $L = \frac{c^2}{uv}(\frac{h}{2\pi})$), ***because such analogies should be applicable in both directions***. <u>It is also indicative that photon spin, and the spin belonging to kinetic elementary particle energy, are both equal to</u> $\frac{h}{2\pi}$ (see (2.11.3), T 4.0 and (4.8)). Consequently, it is clear that rotation and linear motion should always be intrinsically coupled in all cases of (charged or neutral) particle motions. For instance, moving electron/s create rotating magnetic field/s around their paths, and something similar should also be valid for all cases of neutral particles that perform linear motion (producing simultaneously effects of rotation of certain field components, eventually having measurable consequences such as omnipresent rotation/s in the world of atoms and elementary particles, as well as rotations of astronomic objects, possibly involving couple/s of some other fields. For instance, planetary rotation around the sun could be quantifiable as: $L = \frac{c^2}{uv}(\frac{h}{2\pi}) \cong \frac{2c^2}{v^2}(\frac{h}{2\pi}), u \cong \frac{v}{2} << c$). Whenever we have the dynamic coupling of linear motion with rotation, this means that matter waves have been created along certain helix line of two complementary field components, which present a content of particle motional energy, and that de Broglie wavelength and frequency should be in a direct relation with mentioned helix field structure. **The most direct externally measurable signs regarding documenting the existence of intrinsic rotation-related behaviors of all elementary particles should be their spin and orbital moment attributes.**

In cases of stationary inter-atomic, closed or circular movements, a much more general picture (regarding the same situation) is given by Wilson-Sommerfeld rules (see 5.4.1).

c) **<u>Matter Waves & Rotation</u>**: In cases of rotation, particle angular velocity (or mechanical rotating frequency) should not be mixed with angular velocity (or frequency) of belonging matter waves, since mentioned frequencies are mutually connected by certain velocity dependent function, where both group and phase velocity should be taken into account. It is often the situation that in many analyses regarding spin and orbital moments, for instance, people do not make sufficiently clear difference between them (related to relevant group and phase velocity), making a big conceptual confusion regarding understanding particle-wave duality (or unity).

<u>[♣ COMMENTS & FREE-THINKING CORNER:</u> *Example: Let us imagine that certain particle is rotating, having tangential velocity* $v = v_g = \omega_m R = \omega_{gm} R$ *, where the radius of rotation is* ***R,*** *and* $\omega_m = \omega_{gm}$ *is the mechanical, angular particle velocity (number of full rotations per second), and let us find all particle and matter-wave parameters associated to such movement.* ***Practically, the same concept of a wave packet, which has its group and phase velocity, in cases or linear motions should be analogically extended to the rotating wave packet that has group and phase angular velocity, as for instance,***

$$\mathbf{v = v_g = R\omega_{gm} = \frac{d\omega}{dk} (=) \text{ group wave velocity} (=) \text{particle, tangential velocity},}$$

$$\omega_{gm} = \mathbf{\frac{v}{R} = 2\pi f_{gm} (=) \text{group angular velocity or frequency} (=) \text{particle angular velocity},}$$

$$\mathbf{u = v_f = R\omega = \frac{\omega}{k} = \lambda f = v/(1+\sqrt{1-\frac{v^2}{c^2}})\ (=) \text{ phase, wave velocity}, Rk = 1,}$$

$$\omega = \omega_f = \mathbf{\frac{u}{R} = 2\pi f (=) \text{angular wave frequency},}$$

$$\mathbf{k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p (=) \text{ wave vector}, \ \tilde{E} = hf\ (=) \text{wave} - \text{packet energy},}$$

$$\omega = \omega_{gm}\ \mathbf{/(1+\sqrt{1-\frac{v^2}{c^2}}) = \omega_{gm}\ /(1+\sqrt{1-\frac{\omega_{gm}^2}{\omega_c^2}})},$$

$$\frac{\omega_{gm}}{\omega} = \mathbf{1+\sqrt{1-\frac{v^2}{c^2}} = 1+\sqrt{1-\frac{\omega_{gm}^2}{\omega_c^2}} = \frac{k}{\omega}\frac{d\omega}{dk} = \frac{1}{u}\frac{d\omega}{dk} = \frac{f_{gm}}{f} = \frac{v}{u}}$$

$$(0 \le v \le c) \Rightarrow 1 \le \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = \frac{v}{u} \le 2$$

$$(v << c) \Rightarrow v = 2u \Rightarrow \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = 2$$ *(See later (4.3))*

$$(v \cong c) \Rightarrow (v = u) \cong c \Rightarrow \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = 1$$

(Indexing: m (=) mechanical, f (=) phase, g (=) group)

*Matter waves associated to any particle motion are practically defined by **PWDC** relations (equations (4.2)-(4.3)), and not directly equal to particle mechanical rotating parameters, and we should be very careful in making a difference between mechanical rotation (mechanical angular speed) and orbital frequency of associated matter waves. In cases of inter-atomic circular motions, where standing matter waves are an intrinsic structural property, it can be:*

$$\mathbf{If: 2\pi R = n\lambda \Rightarrow Rk = n = 1, 2, 3... \Rightarrow}$$

$$\omega_{gm} = \mathbf{\frac{1}{R}\frac{d\omega}{dk}, \frac{\omega_{gm}}{\omega} = \frac{v}{u} = \frac{k}{\omega}\frac{d\omega}{dk}}$$

Usually, such precise differentiation between mechanical and mater wave parameters has never been made, influencing that many ad hoc and exotic postulates and suspicious, or theory-correcting and supporting statements (regarding elementary particles and atom structure) have legitimacy in Orthodox Quantum Mechanics (in order to explain certain conflicting situations such as: gyromagnetic ratio, spin attributes, correspondence principle, orbital and magnetic moments etc.). In cases when radius related to rotation is not constant, the example above should be appropriately modified (and accorded with Wilson-Sommerfeld rules: see (5.4.1)). It would be very fruitful to elaborate more profoundly the ideas mentioned here. ♣].

d) The next very important part of understanding the PWDC should also be found in the possibility to create an equivalent and isomorphic mathematical structure that represents (or replaces) deterministic concept given under a) and b), using the framework of Probability, Statistics and Signal Analysis (which is officially still not admitted and unrecognized case of today's Orthodox Quantum Mechanics, which found a good way how to make its mathematical structure working well, even perfectly well when formulated by words of the non-conditional believers of Quantum Theory). Here we will try to show that if once (in the past when Quantum Theory got its foundations) we had had a better, more natural and more general mathematical modeling, many of present Quantum Mechanical ad hoc theoretical discoveries, postulates and rules would have been just elementary (mathematical) products of that more general and better mathematical modeling. Later on it will be shown that Schrödinger's and several other wave equations, known as big achievements of Orthodox Quantum Theory, are just simple (mathematical) consequences of using the much more general model, applicable to any wave function, known as the complex Analytic Signal (see equations from 4.9 to 4.28, and basics of Analytic Signals in the chapter 4.0). It will also be shown that all forms of Uncertainty relations (or Heisenberg relations), presently placed in the basket of big achievements of Quantum Theory, are just simple laws of Signal Analysis, equally valid for micro and macro world when characteristic wave functions (describing motions in a micro and macro world) are correctly treated and presented within the same mathematical framework (see Chapter 5), since it is natural that all fundamental laws of (future, more advanced) physics should be equally valid on all size scales (what is still not the case of contemporary physics). In fact we already know that Orthodox (or probabilistic) Quantum Theory mathematically works very much correctly, but we could also try to see it as an equivalent and alternative mathematical modeling which replaces much more tangible particle wave platform of a) and b), and one of the objectives of this paper is to show how such equivalent (or isomorphic) modeling has been successfully realized, and to replace it later with more advanced and more general concepts. The current mainstream tendency of modern physics is to present Probabilistic Quantum Theory as the unique, most general and the most correct view of micro world almost predestinated to be never changed (contrary to the message of this paper). Of course, we should be conscious that Statistics and Probability theory are irreplaceable and essentially important whenever we analyze sets and events containing virtually countless number of elements. *We could also ask ourselves in which tangible domain/s of physics (apart from not so conceptually tangible Quantum Theory), Probability and Statistics play the essential (and irreplaceable) role, and we will find that this is Thermodynamics. For Thermodynamics we cod say that it is probabilistically deterministic theory, and for present Quantum Theory we could say that it is probabilistically non-deterministic. It is maybe too early to say that Quantum Theory and Thermodynamics should be united into one single theory (as many other Physics domains, too), but it is just the time to start being creatively suspicious and curious when answering the question why both theories are essentially and intrinsically merged with Statistics and Probability, except that regarding Thermodynamics, nobody seriously claims that there is no experimentally verifiable causality there.* Whichever the correct case is, we are faced with the situation that the present Quantum Theory, after many years of enormous research investments and involvement of generations of brain and manpower in building it, is now working very well. The reasons are related to the fact that modeling, fitting and matching between the theory and relevant experimental results are made as very operational and mathematically correct (but from the conceptual point of view much more questions are left open than answered). This paper is addressing new conceptual picture of the particle-wave duality and indirectly offers an alternative to Probabilistic Quantum Theory teaching.

e) Since present Quantum Theory (mathematically) generates good results, which are experimentally verifiable, paradoxically, we do not have a better choice than to compare results of any new ***particle-wave*** theory with corresponding results of Quantum Theory, at least in the basic early steps, and to exploit many of fundamental results of present Quantum Theory in process of building its replacement theory. The admirers and defenders of the present Quantum Theory would say that it is forbidden, useless and worthless to search for the new and better theory (because this will bring nothing new, etc.), but the objective of this paper is just to open such a process.

f) We can also find that several of the elements of here formulated **PWDC** are also incorporated in the N. Bohr's hydrogen atom model (although artificially and with problematic assumptions), which sufficiently and surprisingly correctly operates in the frames of its definition and already known applications (in spectroscopy, for instance).

What follows in the next part of this chapter (and later) is more of step-by-step explanation of a), b), c), d), etc. statements, or the explanation of the **PWDC** and its integration with generalized wave equations, <u>effectively transforming the Particle-Wave Duality concept into Particle-Wave Unity</u>.

[♣ <u>COMMENTS & FREE-THINKING CORNER:</u>

4.1.1.1. The Resume regarding PWDC

*A brief and simplified summary of all above given explanation/s regarding **PWDC** can also be presented in the following way:*

1. *Relativity theory shoved or implicated that there is a simple relation of direct proportionality between any mass and total energy that could be produced by fully transforming that mass into radiation,*

$$E_0 = mc^2, \quad E_{tot.} = \gamma mc^2 = E_0 + E_k = E_0 + (\gamma - 1)mc^2 = \sqrt{E_0^2 + p^2c^2} .$$

2. *The most important conceptual understanding of frequency dependant matter wave energy, which is fully equivalent to particle motional, or kinetic energy, is related to the fact that total photon energy can be expressed as the product of the Planck's constant and frequency of the photon wave packet,* $E_f = hf$*, and consequently (since there is a known proportionality between mass and energy), the photon momentum was correctly found as* $p_f = hf / c = m_f c$ *(and proven applicable and correct in analyzes of different interactions between photons as waves, or quasiparticles and real particles).*

3. *Since a photon has certain energy, we should be able to present this energy in two different ways, as for instance:* $E_f = hf = \sqrt{E_{0f}^2 + p_f^2c^2} = p_f c = E_{kf}$*. In reality, since photon rest mass equals zero, there is only a photon's kinetic energy* $E_f = hf = p_f c = E_{kf} = m_f c^2$*, and in a number of applications this concept (and all equivalency relations for photon energy and momentum) showed to be correct.*

4. *Going backwards, we can apply the same conclusion, or analogy, to any real particle (which has a rest mass), accepting that particle kinetic energy is presentable as the product between Planck constant and characteristic particle's matter wave frequency* $E_k = (\gamma - 1)mc^2 = \tilde{E} = hf$*. Doing it that way we are able to find the frequency of de Broglie*

matter waves as $f = E_k/h = (\gamma-1)mc^2/h = \tilde{E}/h$. *Now, we can find the phase velocity of matter waves as* $u = \lambda f = \frac{h}{p} f = \frac{E_k}{p} = \frac{(\gamma-1)mc^2}{\gamma mv} = \frac{v}{1+\sqrt{1-v^2/c^2}}$.

5. *The relation between phase and group velocity of a matter wave packet is also known in the form:* $v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda}$, *and, combining two latest forms of phase and group velocities, we can get:* $v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u(1+\sqrt{1-v^2/c^2})$, *implicating validity of the following differential relations:* $d\tilde{E} = d\left[(\gamma-1)mc^2\right] = mc^2 d\gamma = hdf = vdp = d(pu)$, *and practically confirming mathematical consistency of all above introduced equivalency and analogy based relations (named in this paper as* ***PWDC*** *=* ***P****article-****W****ave* ***D****uality* ***C****ode).*
6. *It is even more interesting to notice that in the same framework (from 1 to 5) the spin of motional energy (photon spin and kinetic energy spin, T 4.0) is always equal to* $\frac{\tilde{E}}{\omega} = \frac{E_k}{\omega} = \frac{h}{2\pi}$ *(or to a multiple of* $\frac{\mathbf{h}}{2\pi}$*, see (4.8)).*
7. *Since the above given particle wave equivalency relations are found valid only if we use the wave packet model (as a replacement for a particle in motion), consequently, we have an argument more to say that matter waves (or corresponding wave functions) should exist as forms of harmonic, modulated sinusoidal signals (naturally satisfying the rules of Fourier signal and spectrum analysis). It is of essential importance to notice that a rest mass (or rest energy) does not belong directly to matter wave energy (opposite to many current presentations in modern physics books, regarding matter wave properties), but certainly, when in motion, rest mass presents the source of mater waves. Analyzing Compton Effect and many other elementary interactions known in Quantum Mechanics, we can easily prove the statement that only kinetic or motional energy presents the active mater wave energy. The next consequence of this concept is that the rest mass of the particle itself should present (internally and intrinsically) a "passive mater wave energy" in the form of a closed, self-sustainable, stationary and standing wave structure, which only externally looks like a stable and closed shell particle. This internally closed mater wave energy would become externally measurable (directly or indirectly) in all cases when a particle interacts with its environment participating in any kind of motion.*
8. *We can also rethink about the meaning of phase velocity* ***u****, based on (4.2), because modern interpretations of this velocity in most Quantum Mechanics and Physics books are different from (4.2), (in some general literature we can only find the relation* $\mathbf{v} = 2\mathbf{u}$*, for* $\mathbf{v} \ll \mathbf{c}$*, which is in agreement with (4.2)). For instance, thermal black body radiation can also be analyzed using group and phase velocity relations from (4.2). We can try to estimate what really happens inside a black body cavity where we have complex, random motion of hot gas particles, and random light emission, absorption and photons scattering. We only know from Planck's results the resulting spectral distribution of outgoing light emission, in case when we make a small hole on the surface of the black body and let photons be radiated and measured in the external free space of the black body. This external light radiation is characterized by free photons where each photon has the same phase and group velocity* $\mathbf{v} = \mathbf{u} = \mathbf{c} = \mathbf{constant}$*, but inside the black body cavity this is not the case, since there are many mechanical and field interactions between photons, gas particles and cavity walls, and we have a large range of spectral dependency between group and phase velocities,* $0 \le 2u \le \sqrt{uv} \le v \le c$. *A big number of wave packets (de Broglie matter wave groups) inside the black cavity permanently interact (among themselves, as well as with the cavity and gas particles) and we cannot consider them free wave groups, or stable and/or standing wave formations. It is logical (as the starting point in an analysis of such case) to imagine that particle and/or group velocity of such wave groups is directly proportional to the black body temperature, and when gas temperature (inside a black body radiator) is relatively low, than we should dominantly have non-relativistic particle*

*velocities ($v << c \Rightarrow v \approx 2u$), and when temperature is sufficiently (or extremely) high we should dominantly have the case of relativistic particle velocities ($v \approx c \Leftrightarrow v \approx u$), according to (4.2). Let us create the following table, comparing relativistic and non-relativistic wave group energies of an internal black body situation (or any similar situation), searching for the practical meaning of the **PWDC** found in (4.1) and (4.2).*

T.4.1. Interacting and coupled wave groups inside the black body cavity

<table>
<tr><th>Non relativistic group velocities:
$\mathbf{v << c \Rightarrow v \approx 2u}$
(Lower temperatures)</th><th>Relativistic group velocities:
$\mathbf{v \approx c \Rightarrow v \approx u}$
(Very high temperatures)</th></tr>
<tr><td>$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} \approx 2u,$$ $$\frac{du}{u} \approx -\frac{d\lambda}{\lambda};\ \frac{df}{f} \approx -2\frac{d\lambda}{\lambda},$$ $$\ln\left|\frac{u}{u_0}\right| \approx -\ln\left|\frac{\lambda}{\lambda_0}\right| \Rightarrow \ln\left|\frac{u\lambda}{u_0\lambda_0}\right| \approx 0$$ $$\Rightarrow u\lambda = f\lambda^2 \approx u_0\lambda_0 = \text{Const.}$$ $$\Rightarrow f \approx \frac{\text{Const.}}{\lambda^2}, v \approx 2u \approx 2\cdot\frac{\text{Const.}}{\lambda},$$ $$\left[\tilde{E}\right]_{\text{Low temp.}} = hf \approx h\cdot\frac{\text{Const.}}{\lambda^2} =$$ $$= \frac{hf^2}{u^2}\cdot\text{Const.} \approx \frac{hf^2}{v^2}\cdot 4\cdot\text{Const.},$$ $$\left[\frac{d\tilde{E}}{d\lambda}\right]_{\text{Low temp.}} \approx -2h\cdot\frac{\text{Const.}}{\lambda^3} =$$ $$= -\frac{2hf^3}{u^3}\cdot\text{Const.} \approx -\frac{2hf^3}{v^3}\cdot 8\cdot\text{Const.}$$</td><td>$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} \approx u,$$ $$\lambda\frac{du}{u} \approx 0;\ \frac{df}{f} \approx -\frac{d\lambda}{\lambda} \Rightarrow$$ $$\ln\left|\frac{f}{f_0}\right| \approx -\ln\left|\frac{\lambda}{\lambda_0}\right| \Rightarrow \ln\left|\frac{f\lambda}{f_0\lambda_0}\right| \approx 0$$ $$\Rightarrow f\lambda \approx f_0\lambda_0 = u_0 = \text{const.}$$ $$\Rightarrow f \approx \frac{\text{const.}}{\lambda}, v \approx u \approx \text{const.}$$ $$\left[\tilde{E}\right]_{\text{High temp.}} = hf \approx h\cdot\frac{\text{const.}}{\lambda} =$$ $$= \frac{hf}{u}\cdot\text{const.} \approx \frac{hf}{v}\cdot\text{const.} \geq \frac{hf}{c}\cdot\text{const.},$$ $$\left[\frac{d\tilde{E}}{d\lambda}\right]_{\text{High temp.}} = -h\cdot\frac{\text{const.}}{\lambda^2} =$$ $$= -\frac{hf^2}{u^2}\cdot\text{const.} \approx -\frac{hf^2}{v^2}\cdot\text{const.} \leq$$ $$\leq -\frac{hf^2}{c^2}\cdot\text{const.}$$</td></tr>
</table>

It is difficult to say what is the real situation inside a black body cavity, but we can imagine that numbers of Secondary Emission, Photoelectric and Compton events are permanently created and combined with thermal motion of gas particles etc. The analyzed situation is not directly comparable with external black body radiation (Planck's result), since (in above given table) we found only internal wave energy states in the black body cavity, but the final results are very much indicative and provoking. For instance, we see that in the case of lower temperatures we have the wave energy function proportional to $\tilde{\mathbf{E}}_{\mathbf{Low}} \cong \mathbf{A}\cdot\frac{\mathbf{hf}^2}{\mathbf{v}^2}, \mathbf{v} << \mathbf{c}, \mathbf{A} = \mathbf{const}.$ *and in the case of very high temperatures the same energy is proportional to* $\tilde{\mathbf{E}}_{\mathbf{High}} \cong \mathbf{B}\cdot\frac{\mathbf{hf}}{\mathbf{v}}, \mathbf{v} \approx \mathbf{c}, \mathbf{B} = \mathbf{const}.$*, meaning that internal wave groups have an enormous tendency to lose energy with temperature increase (which is not contradictory to the conclusions of Planck's radiation law). Obviously, here we see that there is a big difference between free wave groups, like free photons in open space, and mutually interacting (de Broglie) mater waves (inside of a limited space of a cavity). On the contrary, in most analyzes of similar situations in modern Quantum Mechanics, we do not find that such differentiation is explicitly underlined and properly treated (mostly we find that de Broglie matter waves are treated similar to free photons or some other free wave groups, or artificial probability waves). Also, in mathematical development of Planck's radiation law we can only find certain particularly suitable (and artificial) modeling situations where the phase velocities of black body (internal)* *photons are always treated as the velocity of free (externally radiated) photons, or as* $u = \lambda f = v = c = \text{Constant}$*. The results given in the table above are also applicable to any other situation when we analyze relativistic or non-relativistic motions.*

Another instructive exercise is to exploit the symmetry with mutual replacements of corpuscular and wave momentum ($\vec{p} \leftrightarrow \tilde{p}$ *), presented in the following table, T 4.1.1.*

T 4.1.1

Symmetry based on equivalence: $\vec{\mathbf{p}} \leftrightarrow \tilde{\mathbf{p}}$	
$\mathbf{E_k} = \frac{\vec{\mathbf{p}}\vec{\mathbf{v}}}{\mathbf{1}+\sqrt{\mathbf{1}-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}$	$\tilde{\mathbf{E}} = \mathbf{pu}$
$\mathbf{E_k} = \tilde{\mathbf{p}}\mathbf{u}$	$\tilde{\mathbf{E}} = \frac{\tilde{\mathbf{p}}\vec{\mathbf{v}}}{\mathbf{1}+\sqrt{\mathbf{1}-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}$
$\Rightarrow \left\{ (\frac{\tilde{E}}{E_k})^2 = \left[\frac{\tilde{p}}{p}\cos(\vec{p},\tilde{p})\right]^2 = (\frac{p}{\tilde{p}})^2 = (\frac{m}{\tilde{m}})^2 = \left[\frac{u}{v}(1+\sqrt{1-\frac{v^2}{c^2}})\right]^2 = \left[\frac{v\cos(\vec{p},\tilde{p})}{u(1+\sqrt{1-\frac{v^2}{c^2}})}\right]^2 = \cos(\vec{p},\tilde{p}) \right\}$	
$\Rightarrow \left\{ \lambda = \frac{h}{p} = (\frac{h}{\tilde{p}})\frac{v}{u(1+\sqrt{1-\frac{v^2}{c^2}})} = (\frac{h}{\tilde{p}})\frac{u(1+\sqrt{1-\frac{v^2}{c^2}})}{v\cos(\vec{p},\tilde{p})} = (\frac{v}{f})\frac{\sqrt{\cos(\vec{p},\tilde{p})}}{1+\sqrt{1-\frac{v^2}{c^2}}} \right\}$	
$\{\cos(\vec{p},\tilde{p}) = 1\} \Rightarrow \left\{ (\frac{\tilde{E}}{E_k})^2 = (\frac{p}{\tilde{p}})^2 = (\frac{m}{\tilde{m}})^2 = \left[\frac{u}{v}(1+\sqrt{1-\frac{v^2}{c^2}})\right]^2 = \left[\frac{v}{u(1+\sqrt{1-\frac{v^2}{c^2}})}\right]^2 = 1, \quad E_k = \tilde{E},\ p = \tilde{p}, u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}},\ \lambda = \frac{h}{p} = \frac{h}{\tilde{p}} \right\},$ $\{\cos(\vec{p},\tilde{p}) = -1\} \Rightarrow \left\{ (\frac{\tilde{E}}{E_k})^2 = (\frac{p}{\tilde{p}})^2 = (\frac{m}{\tilde{m}})^2 = \left[\frac{u}{v}(1+\sqrt{1-\frac{v^2}{c^2}})\right]^2 = \left[\frac{v}{u(1+\sqrt{1-\frac{v^2}{c^2}})}\right]^2 = -1, \quad E_k = -j\tilde{E},\ p = j\tilde{p}, u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}},\ j^2 = -1 \right\}$	

From the above-given table we can conclude that corpuscular and wave momentum of a particle wave object should act in the same direction, since the only logical and reasonable result is that the angle between them should stay in the following limits: $0 < \cos(\vec{p},\tilde{p}) \le 1 \Rightarrow -\frac{\pi}{2} \le \left[\angle(\vec{p},\tilde{p})\right] \le \frac{\pi}{2}$. ♣]

4.1.2. De Broglie Matter Waves and Hidden Rotation

The position of the author of this paper (formulated on a conceptual and common sense level) is that linear particle motion (especially when we analyze micro and elementary particles) should also have (or create) certain elements of particle rotation and associated torsion field components (where such torsion filed components could be of different origins and manifested in different ways). Some of possibilities that could support such concept will be mentioned here. Since certain (at present, mathematical) analogy between electric and gravitation field is obvious, as well as the analogy between magnetic field and "field of mass rotation" (see T.1.6, T.2.2, **[3]** and **[4]**), this can open the way towards establishing a new, Maxwell-like General Theory of Gravitation (at least by suggesting initial ideas, as follows). Presently known phenomenology of de Broglie's matter waves strongly confirms that any rectilinear motion of some micro-particle with momentum (quantity of motion) can be dualistically presented by certain wave, the wavelength of which is directly dependent on its linear momentum. The nature of waving (or oscillations in general) is often connected to some kind of rotation that is associated with a wave source, but such initial rotation is not always and easily recognizable. Indirectly, we could conclude that any rectilinear motion is likely to be accompanied by "field of mass rotation" or literally by elements of visible mechanical mass rotation (since de Broglie matter waves are a kind of oscillations linked to linear motion), and here we will try to explain the origins of such a hidden rotation. This could mean that some associated form of field and/or mass rotation follows any rectilinear motion (that cannot always be directly and clearly detectable, but it certainly exists on a certain phenomenological, or energy level, currently not well understood, because we know that de Broglie waves indirectly and eventually produce measurable and mathematically predictable experimental consequences). We could also imagine that our universe has more dimensions than we are able to detect, and that mentioned associated waving has some (or all) of its components linked to such multidimensional universe (but this would not be necessary theory-saving concept at the time). **At least, we know that all subatomic micro particles and quasiparticles have their very real intrinsic spin and orbital moment/s characteristics, and behave as de Broglie matter waves (regardless of the fact that Quantum Theory avoids formulations about particle rotation using the terminology such as electron clouds, energy states etc.). We also know that rotation is also very much typical in movements of planets, solar systems, galaxies and other astronomic entities, meaning that rotation cannot be just a hazard or random play of Nature.**

From the point of view of modern Quantum Mechanics, de Broglie waves are effectively treated as probability or "possibility" waves (conceptually not meaning too much). We also know that this is just one of today's most successful mathematical models (or theories) of the micro world. In this paper it will be shown that de Broglie mater waves should also have much more general, tangible and deterministic nature.

In almost all known situations when de Broglie wave phenomenology has been considered a relevant event, we can find the interactions of two bodies, particles/quasiparticles, where one (test) moving particle or its energy mass equivalent, $m_1 = m$*, is significantly smaller than the other particle,* $m_2 = M >> m$. A "bigger particle", M, is usually a scattering target, diffraction plate, atom nucleus, instrument in certain laboratory etc. This way, if we analyze such situation in the center of a mass coordinate system, we can always (dynamically) transform rectilinear motion of the test, small moving particle, **m,** to a kind of equivalent, rotation-similar motion of a reduced test mass, $\mu = m_r = mM/(m+M) \approx m$ around its center of gravity (that has a significantly bigger mass, $m_c = m + M \approx M$). The rotating circle radius of $\mathbf{m_r}$ around $\mathbf{m_c}$ will be equal to the distance between interacting particles, $\mathbf{r_{12}}$ (and in general case would have a space-time evolving value). If we now make an effort to visualize de Broglie's waves as an associated rotational (or orbital) field manifestation present around the particle in rectilinear motion (that has circular, helix, or spiral wave shape around the particle path), de Broglie wavelength could be associated with the perimeter length measured along that spiral or helix line, between two neighboring quasi-circles (see the illustration on Fig.4.1, and the equations under (4.3)).

Since de Broglie matter waves are in most of known cases only indirectly detectable for motions of electrons, protons, neutrons, atoms molecules and other subatomic micro-particles, all of them having certain spin properties (internally or externally measurable), this is an extremely indicative situation which supports the hypothesis that some associated form of field rotation (similar to the helix path on Fig.4.1) should naturally accompany every rectilinear motion (much in the same way as electrical currents and fields are accompanied by magnetic fields).

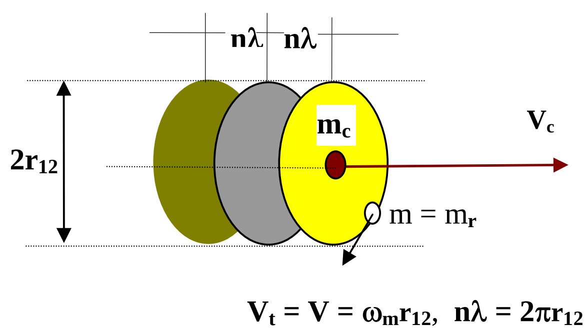


Fig.4.1. Two-particle problem presented in equivalent center of mass system

Let us now analyze in details the above-introduced common sense concept of de Broglie waves (presented with Fig.4.1 and based on equations (4.1) and (4.2)), by analyzing the (dynamically equivalent) quasi-rotation of a small particle ($\mu = m_r = mM/(m+M) \approx m$) around a much bigger particle (**M**), or around its center of mass ($\mathbf{m_c}$ = $m + M \approx M$), understanding that in the Laboratory System of coordinates, practically only mass **m** with velocity **v** moves towards mass **M**, and that mass **M** is standstill in the same system. The results of the same analysis will not change if we consider that a Laboratory System in question is fixed to the big mass **M**. In order to comply with de Broglie hypothesis (or in order to rediscover expression for de Broglie wavelength) we should be able to show that de Broglie matter wavelength (in the Laboratory System) originates from the effective or equivalent particle rotation around its center of mass (in its Center of Mass System), making the final (or essential) result:

$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{2\pi r_{12}}{n}$, $n = 1, 2, 3, ...$ Here we say that the small particle $\mu = \mathbf{m_r} \approx \mathbf{m}$ effectively rotates in its Center of Mass System because from the analysis of the two-body problem we know that the total, internal angular momentum of such system exist, and can be expressed as: $\mathbb{L} = \mathbb{L}_m = \mathbb{J}_m \omega_m = m_r r_{12} \times v_{12} = m_r r_{12} \times v \cong m r_{12} \times v$, making it possible to find the value that should be equivalent to the particle angular velocity $\omega = \omega_m = v_{12} / r_{12} = v / r$ (valid for Center of Mass System). Since the small "rotating particle" at the same time makes:

a) **Linear motion in its Laboratory System,**
b) **As well as a kind of circular path motion in its Center of Mass System** (having in both situations the same linear speed equal $\mathbf{v}$, because of valid approximations and initial conditions: $m_r = mM/(m + M) \approx m$, and because mass $\mathbf{M}$ is considered standstill),

we could express the particle kinetic energy in two usual ways for linear and rotational motions. For instance, in Classical Mechanics, kinetic energy of a linear particle motion is given by $\mathbf{E_k} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{mv}^2$, and if a particle is rotating, we have an analog expression $E_k = \frac{1}{2}\mathbb{J}\omega_m^2$, where ω_m is the angular, "mechanical" frequency (of rotation), and $\mathbb{J}$ particle moment of inertia. Since in this example, the same particle is in linear motion and in a kind of rotation (depending on the point of view), and because of valid approximations, which make particle kinetic energy in a Laboratory and Center of Mass System quantitatively (almost) equal, we will have, $E_k = \frac{1}{2}mv^2 = \frac{1}{2}pv = \frac{1}{2}\mathbb{J}\omega_m^2 = \frac{1}{2}\mathbb{L}\mathbb{J}$, or in differential form it should be valid that $dE_k = vdp = \omega_m d\,\mathbb{L} = hdf = c^2 d(\gamma m)$. ***Also, because of mentioned approximations and considering only cases when r_{12} is significantly larger than any other space dimension of the rotating particle in question, particle's relevant moment of inertia will be $\mathbb{J}$ = mr_{12}^2, and we can get the same result if rotating particle is (mathematically) replaced by certain distributed mass, thin-walls rotating torus of equivalent wave energy formation.***

The same kinetic energy equivalency between linear and rotational nature of particle motion in question can also be expressed using the following relativistic formulations:

$$\left\{\begin{array}{l} E_k = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbb{J}\omega_m^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \left(\frac{\mathbb{J}4\pi^2 f_m^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbb{J}4\pi^2}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{v^2}{(2\pi r_{12})^2} = \right. \\ = \frac{\mathbb{J}v^2}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{1}{(r_{12})^2} = \frac{\gamma m r_{12}^2 v^2}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{1}{(r_{12})^2} = \frac{\gamma m v^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \left.\frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}}\right) = pu \end{array}\right\} \Leftrightarrow$$

$$dE_k = vdp = \omega_m d\mathbb{L} = hdf = c^2 d(\gamma m)$$

$$\Leftrightarrow \left\{ \begin{array}{l} \mathbf{J} = \gamma \, m r_{12}^2 = \mathbf{J}_m, \; v = \omega_m r_{12}, \; \gamma = (1 - v^2 / c^2)^{-0.5}, \\ 2\pi r_{12} = n\lambda = n \dfrac{h}{p} = \dfrac{v}{f_m} = n \dfrac{u}{f}, \; u = \lambda f, \; \hbar = \dfrac{h}{2\pi}, n = 1, 2, 3, ... \\ p = \gamma \, m v = h \dfrac{k}{2\pi} = \hbar k, \; \tilde{p} = \gamma \, \tilde{m} v, k = \dfrac{2\pi}{\lambda}, \\ \tilde{E} = pu = h \dfrac{\omega}{2\pi} = hf = \hbar\omega = \gamma \, m v u = E_k \end{array} \right\}, \tag{4.3}$$

$\mathbf{f_m} = \omega_\mathbf{m} / \mathbf{2}\pi$ (=) **frequency of mechanical rotation**, $\mathbf{f_m} \neq \mathbf{f}$,

$\mathbf{f} = \omega / \mathbf{2}\pi = \mathbf{u} / \lambda = \mathbf{n f_m} / (\mathbf{1} + \sqrt{\mathbf{1} - \mathbf{v}^2 / \mathbf{c}^2})$ (=) **de Broglie − wave frequency**,

$$1 \leq \left[\frac{v}{u} = \frac{pv}{\tilde{E}} = \frac{n \cdot f_m}{f} = 1 + \sqrt{1 - \frac{v^2}{c^2}} \right] \leq 2, \; n = 1, 2, 3, ... \; ,$$

$$\frac{1}{2} \leq \left[\frac{u}{v} = 1 + \frac{\lambda}{v} \frac{du}{d\lambda} = 1 + (\frac{h}{pv}) \cdot (\frac{du}{d\lambda}) = \frac{hf}{pv} = \frac{\hbar\omega}{pv} \right] \leq 1 \; .$$

We could also present dynamically the motion of the big mass **M** as performing a kind of rotation (with much smaller radius than $\mathbf{r_{12}}$) around the same center of mass, in its Center of Mass System, similarly as we did for the small mass **m**, but taking into account already accepted approximations, this would not bring us any new conceptual or mathematical benefit in the case analyzed here.

See later the equations from (5.4.1) until (5.4.10); Uncertainty, Chapter 5, where similar and equivalent concept is elaborated from a little bit different perspective.

As we can see, from (4.3), combined with results from (4.2), we do not even need to have a real, visible (continuous and full-circle) rotation of two particles in the same plane (in the Laboratory System) in order to “generate” de Broglie wavelength or frequency. Any of two (somehow interacting) particles (**m** and **M,** where **m** could also be a photon), in linear motion, can be presented as a sort of rotation of a mass $\mathbf{m_r} = \mathbf{mM} / (\mathbf{m} + \mathbf{M})$ and mass $\mathbf{m_c}$ = **m** + **M** around their common center of mass (or center of inertia), and effectively this kind of quasi-rotation is described by de Broglie matter waves, since it shows the following basic relation is valid: $2\pi r_i = n\lambda_i = n \frac{h}{p_i} = \frac{v_i}{f_m} = 2\pi \frac{v_i}{\omega_m}$ (which secures structural stability and continuity of described motion). Of course, mentioned particle rotation, visible in the plane of the Center of Inertia (perpendicular to the velocity of the center of mass), can also be certain angular swing, without creating full circle/s in a Laboratory System, but a kind of associated field and wave motion around it will present matter waves in question. If **M** presents the big *mass in our Laboratory System*, and if $(\mathbf{M} >>> \mathbf{m}) \Rightarrow (\mathbf{m_r} \cong \mathbf{m})$, consequently, the motion of every small particle **m** in such system can be treated (approximately) as the motion of $\mathbf{m_r} \approx \mathbf{m}$ in the Center of Mass System, satisfying relations given in (4.3). **This way, it becomes clear that the binding and surrounding (mutually interacting) fields in the space between m and M create de Broglie matter waves as a way of energy exchange and force coupling between them**, also satisfying the following differential energy balance: $dE_k = v dp = \omega_m d\,\mathbf{L} = h df = c^2 d(\gamma m)$ (see the second chapter: Tables T.2.4, T.2.5 and T.2.6 and equations from (2.1) to (2.11.5)). Now it also becomes clear when and why de

Broglie relation $\lambda = h/p$ is valid and applicable. We should also be very careful in noticing significant difference between mechanical rotating or revolving frequency of the rotating particle, and matter waves frequency which is a field related parameter (see (4.3)).

Here, we should underline that for the time internally interacting nature of structural elements of a particle is fully neglected regarding its intrinsic orbital and linear moments (and regarding all other electric and magnetic properties, standing waves and rotating states; -see equations (2.11.3), (2.11.4) and (2.11.5) from the second chapter).

De Broglie matter waves should also belong to all other wave phenomena already known in Physics, including some of (hypothetical) fields proposed in the second chapter of this paper by force laws (2.1) and (2.2). The challenging question that appears here is whether all mater waves (of different nature, regarding how we see and measure them) have their profound, hidden or recognizable origins in the world of electromagnetism. **Since Maxwell equations are the ones of hydrodynamics, fluid-flow type of equations, this should lead to the conclusion that theory dealing with electromagnetic phenomena is essentially also a part of classical mechanics concepts.**

We now see and know, or at least have a clear concept regarding **what, where, when, why and how** de Broglie matter waves are produced (see also: Fig.4.1.2, T.4.3 and equations (2.5.1), (4.18), (4.5-1)-(4.5-3), dealing with unity of linear and rotational motions).

[♣ COMMENTS & FREE-THINKING CORNER**: In parallel with here given conceptualization, we could also say that de Broglie matter waves could be presentable as products of certain “equivalent to antenna, or resonant circuit oscillations”, where moving mass, force coupled with its environment, intrinsically creates a kind of mass-spring or inductance-capacitance oscillating circuit (where missing oscillatory circuit elements belong to the particle environment). Here we could also apply a much wider analogy with electric or mechanical oscillatory circuits in order to deduce what should be the unknown oscillatory circuit elements that complement the motion of the known mass, since we already know some of important parameters of de Broglie waves (in other words, we know certain results and we would search what produces such results). Similarly, as known from electrical oscillatory circuits, where we know that total energy circulates between inductive and capacitive elements (following certain sinusoidal function, and periodically being either fully electrical or fully magnetic, but having constant, total amount of energy) we would be able to conclude that total motional particle energy should fluctuate from the kind of linear motion kinetic energy to its complementary rotational motion energy and vice versa, and this could be the reason why de Broglie waves are detectable only as consequences or final acts of certain interactions of particles (behaving as well hidden, or almost artificial mater waves). This hypothetical matter wave concept regarding “equivalent oscillatory, resonant and antenna type circuits” should be much better elaborated later, and also connected with concepts and results from (4.3). **♣]

We can now find the quantitative meaning of de Broglie wavelength in relation with center-of-mass axial (or linear) motion. Let us determine the shortest axial distance, $\mathbf{\Delta S}$, between two successive quasi-circles (on the helix line) from the Fig. 4.1, as $\Delta \mathbf{S} = \mathbf{v_c} \cdot \Delta \mathbf{t} \cong \frac{\mathbf{mv}}{\mathbf{M}} \cdot \frac{\mathbf{1}}{\mathbf{f_m}} \cong \frac{\mathbf{mv}}{\mathbf{M}} \cdot \frac{\mathbf{n}\lambda}{\mathbf{v}} = n\frac{m}{M}\lambda <<< \lambda, \ \ \mathbf{n} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots \mathbf{M} >>> \mathbf{m}$. It is clear that de Broglie mater waves are related only to a (kind of) circular motion, since for the distance $\mathbf{\Delta S}$ in axial direction (when particle $\mathbf{m}$ would make one full circle, during the time interval $\Delta \mathbf{t} = \mathbf{1}/\mathbf{f_m}$) we shall get the length that is much shorter than relevant de Broglie wavelength ($n\frac{m}{M}\lambda <<< \lambda$).

The real laboratory (or "kitchen"), where de Broglie matter waves are created is the situation described by "virtual objects" interaction/s in the Center of Mass System. The real (and initial) interaction participants (in the frames of the above introduced concept, Fig.4.1) are masses $m_1 = m$ and $m_2 = M$, but in the Center of Mass System we replace them with "virtual reaction participants" m_r and m_c, because only in the Center of Mass System we are able to associate rotational motion to such situation (m_r rotating around m_c). After introducing the concept of "virtual rotation", which is in fact mathematically and from the point of view of Conservation Laws fully equivalent to the real situation in the Laboratory System (with real interaction participants), we are able to answer the questions regarding what should be the frequency and wavelength associated to such rotation, and we find that de Broglie matter waves have the same frequency and wavelength. This way, a little bit indirectly, we can conclude that what counts dominantly regarding the creation of matter waves is a complex of relations and fields between mutually approaching objects in their Center of Mass System. Such potentially interacting objects are already creating mutual couplings and new interaction participants (long before being united or experiencing any kind of impact and scattering), what is becoming mathematically explicable when analyzed in the Center of Mass System. The same situation is explained with much more of mathematical details at the end of this chapter, and here is the proper time to briefly consult Fig.4.1.2 and T.4.3.

We could also connect the total angular moment of the two-body configuration in the Laboratory System, with the angular moment of the same configuration found in the Center of Mass System, as follows: $\mathbb{L}_{\mathbf{Lab.}} = \mathbb{L}_{\mathbf{m}} + \mathbb{L}_{\mathbf{c}} = \mathbb{J}_{\mathbf{m}}\omega_{\mathbf{m}} + \mathbb{J}_{\mathbf{c}}\omega_{\mathbf{c}} = \mathbb{L}_{\mathbf{m}} + \mathbf{m}_{\mathbf{c}}\,\mathbf{r}_{\mathbf{c}} \times \mathbf{v}_{\mathbf{c}}$, and we would see that this approach could also lead to recognition of de Broglie matter wave frequency and wavelength, as we have found in the case of equations (4.3), where principal rotation was linked only to an angular movement of reduced mass in its Center of Mass System.

In reality we already know (mostly based on experiments) that all elementary particles and photons have their intrinsic spin characteristics (or attributes), but we are often not able to notice the presence of a visible mechanical rotation. In this paper we have been saying that regardless of not being able to see mechanical or some other kind of rotation (in cases where spin is involved), something that is by externally measurable effects equivalent to rotation (or to spinning) should internally exist (belonging to the internal particle structure). Being more explicit, based on ideas presented in this paper, we are already familiar to the concept that relatively stable particles (and quasiparticles) are nothing else than specific "packaging or parking formats" of certain fields in the form of rotating standing waves. Such internal matter wave rotation is externally measurable as particle spin attribute (meaning that at the same time a kind of sophisticated and unusual rotation really exists, but not externally visible). In other words, without spin attributes (or without mater wave rotation) we would not have stable particles, because there should be a cause that would bend and place certain wave in a closed circular area, and make it stable there, by creating standing waves on such self-closed circular area (see chapter 2, equations (2.11.3), (2.11.4) and (2.11.5). The same situation (regarding rotation and spin characteristics of elementary particles) is very well explained (conceptually and quantitatively) by David L. Bergman and his colleagues, [16]- [22], "Common Sense Science".

We also know that every electrically charged particle moving in magnetic field will follow a similar spiral (or helix) line as presented in Fig.4.1, caused by Lorentz force (in case of electrically charged particles we only have stronger interacting fields between such particles than between electrically neutral particles, and most of above given mathematics and logic is again applicable). Since dominant and stable elementary constituents of all atoms and other known particles in our universe are electrons and protons, or electrically charged particles (also including their combinations in the form of neutrons, as well as positrons and antiprotons), we can safely say that (most probably) dominant interaction field responsible for the creation of de Broglie matter waves should have an electromagnetic origin, and that the most important coordinate frame for modeling such interactions is the Center of Mass System.

When the above described quasi-rotational movement and rotation-like field structure separates and becomes a self-sustaining object (for instance, like rotating ring, or toroidal, closed space vortex form), which has certain rest mass and spin as its permanent characteristics, it should depend on kinetic energies and mutual positions, paths and forces between interaction participants. In order to be more explicit, it is worthwhile to underline that every motion of mutually approaching objects (particles, quasiparticles, particles and waves etc.) naturally creates additional energy carrying, rotating elements with associated orbital moments in the zone of their interaction, this way establishing specific conditions for creating new particles and/or waves, or producing different interference and diffraction effects. If interacting objects already have different spin moment characteristics, long before an interaction happens, the appearance of additional rotating elements in the interaction zone would be an even much more obvious consequence (see T.2.4, T.2.5 and T.2.6) and equations (2.11.3), (2.11.4) and (2.11.5)). After the necessary motional energy threshold (in an interaction) is reached, we expect the creation of electrons, positrons, photons and/or other particles and quasiparticles (which is the known experimental fact in particles' physics). In reality, most probably the presence of radial or central attractive force components (between interacting elements) is somehow, conveniently balanced with certain centrifugal force components (when interacting objects are mutually close enough), this way producing self-sustaining (non-collapsing or non-expanding) closed vortex structures, such as electrons, positrons, protons, etc. In such situations, usually a couple of mutually complementary and rotating fields are intrinsically involved, similar to cases of complementarities and coupling between electric and magnetic fields, and of course, every closed circular field structure, in order to be stable and self-sustaining, should automatically create standing wave structures: $2\pi r_i = n\lambda_i = n\frac{h}{p_i} = \frac{v_i}{f_m} = 2\pi\frac{v_i}{\omega_m}$*), known as de Broglie matter waves. In fact, elementary particle models based on rotating ring, toroidal field structure/s are already well established and perfectly explained (conceptually and theoretically), and mathematically tested, to produce very precise results, previously known (in Quantum Mechanics) only after measurements without being conceptually explained (see works of David L. Bergman and his colleagues [16]- [22], "Common Sense Science"). It should not be just a matter of hazard, and probability, in any sense, that everything we see, find, analyze and measure in our universe rotates: galaxies, stars, solar systems, planets, atoms, elementary particles, photons, etc. Even in cases when it seems that certain objects experience only linear motion (for instance rockets sent from the earth to outer space), linear motion is only an external, visible and dominant manifestation, but internally, inside the moving solid mass structure everything rotates, keeping only the external 3-D object-frame and structural object-matrix stable.*

After understanding where the hidden place of rotation in rectilinear motion is, we can conceptually visualize what de Broglie waves phenomenology means (Fig.4.1, mathematically formulated by (4.2) and (4.3)), and we also leave the door open for introducing "Field/s of Rotation" or Torsion fields in the (new) Theory of Gravitation united with Faraday-Maxwell Electromagnetic Theory. The following table of analogies, T.4.2, is sufficiently illustrative to support such ideas (regarding intrinsic coupling between linear motion and rotation, also extended hypothetically to electric and magnetic field; See chapter 5. for more: T.5.3 and equations (5.1) – (5.4-1).

T.4.2. Wavelength analogies in different frameworks

<table>
<tr><th>Mater Wave Analogies</th><th>Linear Motion</th><th>Rotation</th><th>Electric Field</th><th>Magnetic Field</th></tr>
<tr><td>Characteristic Charge</td><td>Linear Momentum
$\mathbf{p}$</td><td>Orbital Momentum
$L = \mathbf{pR}$</td><td>Electric Charge
$\mathbf{q_e = q}$</td><td>"Magnetic Charge"
$\mathbf{q_m = \Phi}$</td></tr>
<tr><td>Mater Wave
Periodicity</td><td>Linear Path Periodicity
$\lambda = \frac{\mathbf{h}}{\mathbf{p}}$
(Linear Wavelength)</td><td>Angular Motion Periodicity
$\alpha = \frac{\mathbf{h}}{L}$
(Angular Wavelength)</td><td>"Electric Periodicity"
$\lambda_e = \frac{\mathbf{h}}{\mathbf{q_e}} = \mathbf{q_m}$</td><td>"Magnetic Periodicity"
$\lambda_m = \frac{\mathbf{h}}{\mathbf{q_m}} = \mathbf{q_e}$</td></tr>
<tr><td rowspan="2">Standing Waves on a circular self-closed zone</td><td>$\mathbf{n}\lambda = 2\pi\mathbf{R}$
$\mathbf{p} = \mathbf{n}\frac{\mathbf{h}}{2\pi}\cdot\frac{1}{\mathbf{R}}$</td><td>$\mathbf{n}\alpha = 2\pi$
$L = \mathbf{n}\frac{\mathbf{h}}{2\pi}$</td><td colspan="2" rowspan="2">$\lambda_e\lambda_m = \mathbf{q_e q_m = h}$</td></tr>
<tr><td colspan="2">$\alpha L = \lambda p = h \ , \ \alpha = \frac{\lambda}{R} = \frac{2\pi}{n}$</td></tr>
</table>

(Periodicity – here invented, unifying formulation, $\mathbf{q_m = \Phi}$ *is not a free and independent magnetic charge)*

Simplifying the same situation, we can say that any rectilinear motion of a particle should always be accompanied by certain angular (and oscillating) field components, and also in many cases certain level of real mass rotation should appear (for instance: rotation and spinning of planets around the sun, rotation of galaxies, rotation and spinning of electrons and other elementary particles, etc.). Since a single, isolated and totally free particle cannot exist, practically we always have a two-particle system: a test particle, **m**, and the rest of its surrounding universe, $\mathbf{M} >> \mathbf{m}$, (even particles like the planet Earth are very small comparing to our Sun etc.), and practically, we can also say that in our universe, there is no linear and straight-line uniform motion, except in some more or less approximate and limited (laboratory) conditions (see [4]).

We should not immediately conclude that the frequency of mechanical rotation, $\mathbf{f_m}$, of the particle **m** (number of full particle revolutions per second) directly corresponds to its associated de Broglie wave frequency, **f**, since the rotating particle, surrounded by certain field, presents the source of matter waves, making wavelike perturbations (or wave groups) in its vicinity. When we come to wave propagation, we should not forget that every wave group has its group and phase velocity, and that mathematical connection between group and phase velocity is given by non-linear equations,

$v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = 2u/(1 + \frac{uv}{c^2})$, found in (4.2), producing the clear difference between (mechanical) frequency of the particle rotation and frequency of its (associated, de Broglie) rotational field components, such as: $\mathbf{1 \leq n\frac{f_m}{f} \leq 2}$, (see (4.3)).

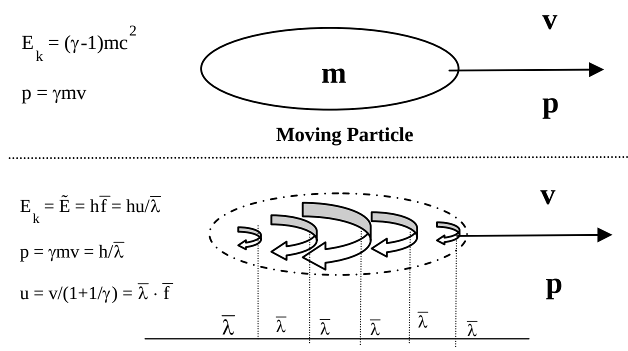
From another point of view we can say that every linear motion of certain particle belongs to a much more general case of quasi-elastic collision between that particle and its vicinity (since in such a case the rest mass of the system does not change, and conservation of kinetic energy is satisfied). In such cases the “elastic collision” system should be considered as an interaction of the (relatively free) moving particle with the rest of a surrounding universe (where rotation of the particle along its path of movement becomes immanent and obvious only for an observer in the Center of Mass System, which could be also fixed to the Laboratory System). In fact, every moving particle, or ordinary mass in rotation, or just particle transient angular swing, should synchronously be coupled with corresponding inertial torsion-field swing, creating de Broglie matter waves around it.

We also know (based on experiments in particles’ physics) that all elementary particles, atoms and molecules behave as dual particle wave objects, and that all of them also have intrinsic wave structure (and that all of them have intrinsic spin attributes). De Broglie waves and torsion fields should only be a natural (external) extension of particles’ intrinsic (internal) wave properties. When a change in particle motion is created, this will influence internal (and elastic) field perturbation of the particle intrinsic field structure, and such wave perturbation will have important, externally manifesting consequences, with properties of rotating de Broglie waves (see also T. 5.3 to understand a more general nature of de Broglie Waves and elementary particles). The most logical conclusion is that de Broglie matter waves already (or always) exist, creating intrinsic, internally rotating form of standing wave particle/s structure, when particles are in a state/s of relative rest, long before we can (externally) detect anything regarding PWD phenomenology, and becoming externally (directly or indirectly) measurable in all situations when particles mutually interact and change their previous state/s of motion and energy *(see [16] - [22])*. In other words, if certain “packaging” or folding of de Broglie stationary or standing waves have been everything what we understand under the (internal) content and intrinsic structure of stable particles, when such particles move, this internal wave content starts unfolding, producing (radiating, or “leaking”) de Broglie waves, externally (creating energy flow, coupling force/s and communicating channels between interacting particles). Classical Mechanic’s analysis of two-body interactions (only partially given here) completely neglects internal mater wave particle structure, and also neglects the associated external mater wave manifestations (what we are trying to address in this paper). Oversimplifying, we could say that stable particles in states of relative rest (with non-zero rest masses) present “parking, packaging or folding domains” of their internal de Broglie matter waves, manifesting as resonating, rotating and standing waves, 3-D forms. Such internally captured de Broglie waves get their “external promotion/s” in all cases of particle interactions (by unfolding internally captured matter waves).

The author of this paper supports and promotes the concept that complementary rotational motion energy member of total motional particle energy is directly related to de Broglie matter waves (or saying the same differently: de Broglie or matter waves present the manifestation of intrinsic nature and couple between linear and rotational motional energy components). In other words, total motional or kinetic energy of a particle is equivalent to its matter wave energy (excluding the amount of energy blocked inside of the particle rest mass). For instance, we know that moving particle, which has linear motion moment p, also has de Broglie wavelength $\lambda = h/p$**, phase velocity** $u = \lambda f = \frac{E_k}{p}$**, group velocity** $v = u - \lambda \frac{du}{d\lambda} = \frac{dE_k}{dp} = -\lambda^2 \frac{df}{d\lambda}$ **and angular velocity** $\omega = 2\pi f = 2\pi \frac{E_k}{h}$ **, and conjugated or complementary rotational particle motional energy components should also be related to the same (mater wave) parameters. We should only be very careful not to immediately identify matter waves frequency** $\omega = 2\pi f = 2\pi \frac{E_k}{h}$ **with a mechanical revolving frequency of a particle** $\omega = L/J_0 = \omega_m$**, since they are mathematically connected with certain velocity dependent function, and generally not equal, because of the specific functional relation between group and phase velocity (see PWDC and equations (4.2) and (4.3) for additional explanations).**

The origins of mechanical rotation of astronomic size macro objects are only an extension of rotation and torsion field properties of its micro-particle constituents, and well-explained and strongly coupled to atomic scale magnetic moment perturbations caused by attractive force of gravitation inside of big astronomic masses (see works of P. Savic and R. Kasanin published by Serbian Academy of Sciences, SANU, Department for Natural and Mathematical Sciences, in the period 1960-1980, and summarized in the book: “Od atoma do nebeskih tela”, author Pavle Savic, publisher “Radnicki univerzitet Radivoj Cirpanov”, Novi Sad, 1978, Yugoslavia - Serbia).

Based on the above-presented intrinsic nature of matter related to strong coupling/s between linear motion and rotation, and coupling between electric and magnetic fields, we are now in the position to propose a significantly simplified conceptualization of the particle-wave duality, illustrated on Fig.4.1.1, applied on a moving particle (also familiar to the situation presented on the Fig.4.1). What really rotates and oscillates in such situation/s, where externally we do not see rotation and oscillations, should be strongly related to the well-known connections between Orbital and Spin Angular Moments, and corresponding Magnetic Dipole Moments of atoms and elementary particles. Effectively, since the internal structure of mater constituents (atoms) really has electromagnetic nature, where internal electric charges in some ways rotate (creating mutually coupled magnetic and orbital moments), an external particle macro-motion should also have direct influence on such internal orbital moments, described by de Broglie matter waves concept (also in connection with other phenomena such as Van der Vaals attraction forces, cohesion and adhesion forces, other electromagnetic forces, hypothetical forces described by (2.1) and (2.2) etc.).



Wave Group or Wave Packet, pictured below, as an equivalent for a moving particle (above)

Fig.4.1.1. Two ways of presenting a moving particle

Unusual, stochastically behaving diffraction and interference effects of micro particles and photons (one and two-hole diffraction) misled the creators of Quantum Mechanics to (exclusively) associate the probability nature to the de Broglie wave function $\Psi(t)$. In fact, the secret why there are virtually unpredictable (or only statistically predictable) diffraction positions (spots on the screen or photographic plate) regarding single micro particle or photon, m, passing the diffraction hole (target, $M >> m$), is in the above explained nature of rectilinear motion that is always accompanied by immanent (and often hidden) rotation, and effectively, in such situations, we deal with a moving particle and its associated field components (see (2.3) – (2.9), chapter 2.). Such associated field components create coupling and guiding channels with all diffraction holes long before a micro-particle reaches any of the holes. The local time-and-distance dependant motional wave phase ($-\pi \leq \varphi(t, x) \leq +\pi$) of such complex field phenomena, at the moment when the particle m "hits" and passes any of the diffraction holes belonging to a target mass M, dominantly contributes to stochastic-like scattering picture on the back screen (eventually creating typical wave diffraction concentric circles or parallel line zones, but after a very long time, when many single particles pass trough a diffraction hole). Since we are almost unable to know the local and real time wave phase of a particle's rotation field components, it seems (in the beginning phase of a particle diffraction experiment) that micro particles randomly choose to hit arbitrary positions on the one-hole (or many-hole) diffraction screen. In all of such experiments "that virtually confirm stochastic backgrounds of Quantum Theory" the existence of rotating field components has been systematically omitted, since it is unknown and not present as the modeling concept, and nobody has been taking such (hypothetical) possibility into account. Consequently, this way Quantum Theory established its probability grounds, mathematically compensating the missing knowledge about all (immediate) motional field phase components.

[♣ <u>COMMENTS & FREE-THINKING CORNER:</u> Effectively, this has been the implicit (never admitted), quiet victory of a hidden mysticism, metaphysics and religious concepts against causality, objectivity and reality in natural sciences. Somehow, the deep and subconscious soul of Western world society (which is the inventor of Quantum Theory, and presently in overwhelming power on the planet Earth) seems to be infected by the intellectual virus that

indirectly (and wrongly) teaches us that random choice miracles could happen. A number of scientists, philosophers, people in power, mystics, and their naïve followers published almost countless number of (in some cases almost worthless) books and papers, sitting partially and unknowingly on mentioned wrong and mystic grounds, ironically expressing themselves in the name of an objective and neutral science (mixing mathematical objectivity with incomplete concepts and illusions about Physics). ♣]

The two-hole diffraction of a single particle is also a clear case of de Broglie wave diffraction, resulting in a dominant part of a particle passing trough one of two diffraction holes, while the interaction field (and unfolding matter waving) between the particle and both diffraction holes will "flash" two holes at the same time (continuously changing the time-space phase function, making the single particle effectively interact with both holes long before it has the chance to pass to the opposite diffracting plate side). At the opposite screen side, the same interaction field will again make channeling or guiding-influence on the space-time phase of the passing particle (as the consequence of satisfying energy and momentum conservation laws). Here, we also have at least two or three body interaction case, where incident particle, before entering the near proximity of diffraction holes acts as a very much independent object, which gradually becomes a part of the bigger and mutually coupled system (incident particle + diffracting plate), when system center mass and reduced mass start to be parts of certain new interacting process that becomes dominant (in the near field of interaction). As we can see, the real cause of stochastic behaving particle distributions in case of diffraction/s should be related to the fact that certain (transitory and waving) interaction field is always created between the incident particle and its target (or its diffraction hole/s), appearing sufficiently long before interaction (diffraction, scattering, interference, etc.) and sufficiently long after interaction happens (and this interacting or coupling field should have axial and rotational, oscillatory and vortex components). The appearance of such transitory and torsion fields (between moving particle and its target) can also be related to "multi-component dipole-like polarization/s". For instance, an electrically neutral particle in the state of rest will have stable center of mass, stable center of its total electrical charge, stable center (or axis) of its total orbital moments, stable center (or axis) of its magnetic moments etc. On the contrary, a moving particle, especially during acceleration, experiences a "multi-component dipole-like polarization", when every (above mentioned) stable center (or axis) of some field property generates its oscillating "dipole-like moment" (not necessarily all of them acting along the same axis). Such dynamically created and transitory "multi-component dipole moments" will start interacting with their environment and with particle's target, creating de Broglie matter waves (basically originating from the concepts of extended Newton-Coulomb force law/s presented in the second chapter, see equations (2.3) – (2.4-3)-(2.9)). <u>In the Chapter 5. it will also be shown that certain space periodicity and atomization (or quantifying) coincidently exist in all linear and angular motions (as well as in associated electromagnetic fields), meaning that a moving micro particle (or a wave packet) in some cases cannot freely and smoothly select any angular position (in front of it).</u> Obviously, Quantum Mechanics effectively mastered (artificially and purely mathematically) model complexity of such (phase unpredictable) multi-component interactions, mostly using the methodology of Statistics and Probability Theory (sacrificing qualitative and immediate time-space identity of an interaction), by randomizing the missing motional phase

values. Since immediate rotating phase of certain particle motion, after sufficiently long time, anyhow covers a full circle ($-\pi \leq \varphi(t, x) \leq +\pi$), precisely calculable probability that particle/s will or can be found in certain areas should become an experimental fact. Strictly speaking, if we know all of the important elements of certain sufficiently isolated wave motion, we should be able to predict when, where and how an interaction of some wave groups, particles and waves would materialize (see also equations (4.42)-(4.45)).

*[♣ **COMMENTS & FREE-THINKING CORNER:** There could be one unusual comparison between photon (or some wave group) and fluid vortex-shedding phenomenology, known from fluid flow measurements, that supports the existence of torsion field components in connection with linear motion. It is experimentally found that fluid velocity is directly proportional to the vortex-shedding frequency $\mathbf{f}$ (when a non-moving obstacle or "bluff-body" is placed in a moving fluid). In such a case the fluid flow velocity can be measured based on Strouhal relation $\mathbf{v}_{\mathbf{fluid}} = \mathbf{s} \cdot \mathbf{f}$, where the proportionality parameter, $\mathbf{s} = \mathbf{constant}$ (**dimensionally equal to certain wavelength**), is essentially constant over wide velocity ranges and independent of fluid density (see [**12**]). Since motional or kinetic energy is directly proportional to the square of relevant velocity $E_k \sim m\mathbf{v}^2$, then vortex-shedding waves should have energy proportional to their squared frequency $s^2\mathbf{f}^2$. There are many other wave phenomena, like flexural waves, where wave group velocity is proportional to the square root of relevant frequency $v \approx \sqrt{f}$, what is in the agreement with Planck wave energy which is proportional to frequency $\tilde{E} = hf \ (\approx v^2)$. If we now imagine (by analogy with above described situation of vortex-shedding) that any object from our 4-dimensional universe is "immersed" or moving in some kind of fluid (that presents for us still non-detectable hyperspace, or multidimensional universe), we could easily create an association with appearance of de Broglie matter waves, by comparing them to the vortex-shedding phenomenology of real objects (from our detectable universe), being in an unknown and still undetectable "fluid". An interesting reference regarding similar subjects (vortices in fluids and Schrödinger Equation) can be found in the article [**15**] written by R. M. Kiehn. It is also obvious that wave energy (or waves velocities) in cases of different waves could be on a different way dependent on relevant waves' frequency, proportional either to f or f^2, or also to f/v or f^2/v^2, like shown in T.4.1. The micro-world of atoms, subatomic particles and states, and photons is dominantly respecting Planck's-Einstein-de-Broglie energy-wavelength formulations, where $\tilde{E} = hf, \lambda = h/p, u = \lambda f = \omega/k, v = d\omega/dk$, and other wave phenomena from a Marco-objects world could have different wave energy to frequency relations. The hidden intention here is to initiate thinking that for macro objects, like planets and other big objects, also exists certain relevant and characteristic wavelength, analog to de Broglie matter-waves wavelengths, but no more proportional to Planck's constant. The problem in defining de Broglie type of wavelength for macro objects is that such wavelength ($\lambda = h/p$) is meaningless and extremely small, and since macro objects in motion should also create associated waves, like any other micro-world object, the macro-wavelength in question should be differently formulated. ♣].*

In brief, in this paper favors the conceptual model that any Particle Wave object could have: a) elements of its linear motion (as a moving particle), b) elements of particle (mechanical) rotation around the line of its linear motion, and c) associated de Broglie matter wave/s in the form of complex and oscillating field perturbation phenomena, which is composed of axial and torsion field components, where torsion field components are coupled with particle rotation (where the particle kinetic or motional energy corresponds to the energy of a matter wave associated to that particle). If we do not directly see, measure or detect de Broglie waves in the space around moving Particle Wave object, this only means that de Broglie matter waves are still inside the internal and intrinsic waving structure of that object (and will become externally measurable in case of some interaction, impact, diffraction, scattering, interference, etc.).

Mater waves and particle duality concept presented in this paper precisely states that kinetic or motional energy (of micro particles and/or matter waves) can be expressed in two different and mutually fully equivalent ways, such as $\mathbf{E_k = (\gamma - 1)mc^2 = hf}$ ***(while using the wave packet model as an equivalent replacement for a moving particle, apart from the particle rest mass). The analogy of the statement above with properties of stationary electron states in an atom is almost total: when an electron's stationary wave changes its state from a higher energy level,*** $\mathbf{E_{s1}}$***, state (1), to a lower energy level,*** $\mathbf{E_{s2}}$***, state (2), the surplus of energy will be radiated as a photon that will have the frequency,*** $\mathbf{f_{1-2}}$***, equal to a frequency difference between corresponding frequencies of electron stationary waves:*** $E_{s1} - E_{s2} = E_{k1} - E_{k2} = hf_1 - hf_2 = h\Delta f = hf_{1-2}$***.***

Practically, a very similar situation happens (regarding de Broglie waves) when any moving particle (previously) in an energy state $\mathbf{E_1}$***, passes to another energy state*** $\mathbf{E_2}$***, where*** $\mathbf{E_1}$ ***and*** $\mathbf{E_2}$ ***could be both total or kinetic particle energy (because differences between total or only kinetic energy states of the same particle are mutually equal):***

$$\Delta E = E_1 - E_2 = E_{total-1} - E_{total-2} = E_{k1} - E_{k2} = hf_1 - hf_2 = h\Delta f = hf_{1-2} = c^2\Delta m$$

$$\lambda_1 = h/p_1, \lambda_2 = h/p_2, \quad \lambda_{1,2} \cdot f_{1,2} = u_{1,2} = v_{1,2}/(1+\sqrt{1-v_{1,2}^2/c^2})$$

$$E_{k(1,2)} = (\gamma_{1,2} - 1)mc^2 = hf_{1,2}\ , E_{total-1,2} = \gamma_{1,2}mc^2\ .$$

Somehow, the particles' macro-universe reacts (regarding de Broglie matter waves) very much in the same way as a micro-universe of inter-atomic stationary states of elementary particles (except that in the world of macro particles we do not need to have only stationary and standing matter waves). Here could be the starting point of new understanding of Quantum Nature of all matter, fields and waves in our universe: "Quantum properties should be the optimal integer number packaging rules related to stable objects, standing waves (resonant) structures, and their energy momentum "communications". There should also be a lot of other non-quantum phenomenology related to forces, matter and waves in our Universe". For instance, an electron inside an atom has different shape and size parameters compared to relatively free-moving electron in an open space, but to both cases we can associate different de Broglie matter wavelengths and frequencies (the only essential difference could be that a stable inter-atomic state of an electron should also have stable and constant mater wave parameters, and freely moving electron could have time-evolving mater wave parameters). Briefly saying, when waves fold, particles could be created, and similarly, wave sources are often related to unfolding the waves captured by "particle shells".

A quite different question in such situations is if we also deal (in the frames of the same problematic situation) with some presently undetectable multidimensional media (say a kind of fluid, or ether, where de Broglie waving is a natural phenomenon). Mathematically more pragmatic has been the platform that this undetectable field or fluid with de Broglie waves is modeled as the "possibility or probability" distribution of certain event realization, and, as we know, Orthodox Quantum Mechanics has been successfully exploiting this platform (as a perfect modeling replacement for something we do not see or measure directly), losing completely any common sense conceptual platform.

If we do not consider rotation and rotational fields as naturally associated phenomena to all particles in linear motion/s, and if something like that should always exist (regardless of our ignorance), certainly we have a big conceptual problem or a missing link in explaining particle interactions (and, also, we do not know that we have such a problem, because we do not clearly see the problem, and we cannot apply our common sense logic). Consequently, we see the results of particle interactions (particle diffraction and interference, for instance) only as random distributions, because we are unable to take into account an important element of particle motion (its intrinsic rotation), or simply we do not know the motional, immediate, real time particle phase function $\{-\pi \leq [\boldsymbol{\varphi(t, x) = \varphi(\omega t - kx)}] \leq +\pi\}$. *In order to compensate this missing link (missing rotation and phase), we can simply assume that after a long time, after a countless number of experimentally realized interactions (or diffractions), we should get smooth and wave-like probability distribution, or interference picture of what really happens, and this was exactly the case (or assumption) of Orthodox Quantum Mechanics. It looks like a miracle happens, because in average (statistically) we are able to predict the exact future shape of a resulting diffraction picture (event distribution), but we are absolutely unable to predict the immediate, real space-time position of a single event, and Quantum Mechanics concluded that nature (of micro-world) is essentially guided by probability (and not real time) distributions in the form of waves, saying (and documenting, in addition), that a countless number of experiments permanently confirms such a position (consequently, the founders of Quantum Theory convinced their non-critical followers that all strange assumptions and mathematics in Quantum Mechanics should be correct, even if we do not have full and common sense explanation for most of them)!? Closely related to the above-presented concept are the Uncertainty relations, which if not well understood and properly applied, can make Particle-Wave Duality picture only more confusing (see chapter 5. of this paper). How to understand multi-plausible Quantum Reality beyond today's Quantum Mechanics is well analyzed in [13], but if a more common sense logic were used in the very beginning, as proposed here, this multi-plausible reality would certainly lose many elements of its magic.*

4.1.2.1. Example 1: Bohr's Hydrogen Atom Model

As an illustration of here introduced platform for treating de Broglie waves, let us briefly analyze a hydrogen atom in the center-of-mass coordinate system (not going too far from the original Bohr's model). Let us apply (4.1), (4.2) and (4.3) to describe the movement of an electron and a proton around their common center of gravity (in the center of mass system), and to describe their associated de Broglie waves, assuming that the electron and atom nuclei are treated as real rotating charged bodies (slightly modified Bohr's model), having the following characteristics: $\mathbf{m_e}$-electron mass, $\mathbf{m_p}$-nucleus or proton mass, $\mathbf{v_e = \omega_{me} r_e}$-electron velocity around the common center of gravity, $\mathbf{v_p = \omega_{mp} r_p}$-proton velocity around the common center of gravity, $\mathbf{r_e}$-radius of the revolving electron, $\mathbf{r_p}$-radius of the revolving proton, $\mathbf{\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m}$ -mechanical, revolving frequency of the electron and the proton, $\lambda_e = h/\gamma_e m_e v_e = h/p_e$-de Broglie wavelength of the electron wave, $\lambda_p = h / \gamma_p m_p v_p = h / p_p$ -de Broglie wavelength of the proton wave, $\mathbf{f_e}$-de Broglie frequency of the electron wave, $\mathbf{f_p}$ -de Broglie frequency of the proton wave, $\mathbf{u_e = \lambda_e f_e}$-phase velocity of de Broglie electron wave and $\mathbf{u_p = \lambda_p f_p}$-phase velocity of de Broglie proton wave. In

order to satisfy structural stability and non-dissipative nature of a hydrogen atom, the internal angular momentum of the electron and proton orbital motion should also be conserved:
$\gamma_e m_e r_e^p \omega_m = \gamma_p m_p r_p^2 \omega_m \Rightarrow \gamma_e m_e v_e r_e = \gamma_p m_p v_p r_p$, and electron and proton stationary wave structure should be stable and satisfy relations, $\mathbf{n\lambda_e = 2\pi r_e}$, $\mathbf{n\lambda_p = 2\pi r_p}$ (see **[4]** regarding the same situation). We can also imagine that the electron and the proton have the forms of rotating, electrically charged rings, with spiral current paths on their toroids, like circularly closed solenoids (because of the similar reasons already described with results (4.3) and Fig. 4.1), what will only proportionally change their moments of inertia, for certain multiplicative constant on both sides of equation ($J_e\omega_{me} = A\cdot\gamma_e m_e r_e^2\omega_m = J_p\omega_{mp} = A\cdot\gamma_p m_p r_p^2\omega_m$, $A = \text{Const.}$), not changing the final results (see (4.4)). In a few steps, implementing the above-mentioned conditions in the framework of the original Bohr's atom model (and using data from (4.1)-(4.3)), we can find:

$$\sqrt{\frac{\gamma_p m_p}{\gamma_e m_e}} = \frac{r_e}{r_p} = \frac{\lambda_e}{\lambda_p} = \frac{p_p}{p_e} = \frac{\gamma_p m_p v_p}{\gamma_e m_e v_e} = \sqrt{\frac{\gamma_p}{\gamma_e}1836.13} = 42.8503386217\sqrt{\frac{\gamma_p}{\gamma_e}},$$

$$f_m = f_{me} = f_{mp} = f_{m(e,p)} = \omega_m / 2\pi, \quad \gamma_{e,p} = (1 - v_{e,p}^2/c^2)^{-0.5}$$

$$\text{or for } (v_e, v_p, u_e, u_p << c) \Rightarrow$$

$$\sqrt{\frac{m_p}{m_e}} \cong \frac{r_e}{r_p} = \frac{\lambda_e}{\lambda_p} = \frac{p_p}{p_e} \cong \frac{m_p v_p}{m_e v_e} \cong \frac{v_e}{v_p} \cong \frac{u_e}{u_p} \cong \sqrt{1836.13} = 42.8503386217$$

$$f_m \cong \frac{m_e e^4}{4n^3h^3\varepsilon_0^2}, \quad \frac{v_p}{v_e}\sqrt{\frac{m_p}{m_e}} \cong \frac{E_{kp}}{E_{ke}} = \frac{\tilde{E}_p}{\tilde{E}_e} = \frac{m_p u_p}{m_e u_e} = 1,$$

$$f_e = n\frac{f_m}{2}(1 + \frac{u_e^2}{c^2}) \cong n\frac{m_e e^4}{8n^3h^3\varepsilon_0^2}(1 + \frac{u_e^2}{c^2}) \cong n\frac{m_e e^4}{8n^3h^3\varepsilon_0^2} \cong f_p,$$

$$f_p = n\frac{f_m}{2}(1 + \frac{u_p^2}{c^2}) \cong n\frac{m_e e^4}{8n^3h^3\varepsilon_0^2}(1 + \frac{u_p^2}{c^2}) \cong n\frac{m_e e^4}{8n^3h^3\varepsilon_0^2} \cong f_e,$$

$$\frac{f_e}{f_p} \cong 1 + 1836.13\frac{u_p^2}{c^2} \cong 1$$

$$1 < \frac{v_e}{u_e} = n\cdot\frac{f_m}{f_e} = \frac{2}{(1 + \frac{u_e^2}{c^2})} = 1 + \sqrt{1 - \frac{v_e^2}{c^2}} \le 2,$$

$$1 < \frac{v_p}{u_p} = n\cdot\frac{f_m}{f_p} = \frac{2}{(1 + \frac{u_p^2}{c^2})} = 1 + \sqrt{1 - \frac{v_p^2}{c^2}} \le 2, \; n = 1, 2, 3, ... \tag{4.4}$$

It is important to underline that the revolving mechanical frequency of the electron and the proton around their common center of gravity, $\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m$ should not be mixed with de Broglie wave frequency of stationary electron and proton waves, $\omega_m = 2\pi f_m \neq (\omega_e = 2\pi f_e,\ \omega_p = 2\pi f_p)$, and that relation between them is given by $f_{e,p} \le n\cdot f_{m\text{-}e,p} = f_{e,p}\cdot(1 + \sqrt{1 - v_{e,p}^2/c^2}) \le 2f_{e,p}$.

From (4.4) we can also conclude that (wave) energy of a stationary electron wave ($hf_e = (\gamma_e - 1)m_e c^2 = \gamma_e m_e v_e u_e = p_e u_e = E_k$) is fully equal to electron's motional or kinetic energy, meaning that the rest electron mass or its rest energy has no direct participation in this part of the energy.

*[♣ **COMMENTS & FREE-THINKING CORNER**: Obviously, relations equivalent to (4.4) should be valid for planets rotating around their suns, except that we would have the dominance of Gravitation related field/s instead of electromagnetic field, and relevant mass ratio will be a different number (meaning that planets in their solar systems should also have their associated de Broglie waves). For instance, we know that our planet Earth rotates around the Sun and in the same time rotating around its own planetary axis, performing similar motion as presented on Fig.4.1 (and that our complete solar system rotates around our Galaxy center, and that our Galaxy also rotates…). ♣]*

Bohr's hydrogen atom model is a very simple one, very much experimentally tested and proven applicable in all frames of its definition (of course, also having a number of known limitations). By combining Bohr's planetary model with here introduced concept of de Broglie waves (Fig.4.1 and equations (4.1), (4.2) and (4.3)), we are indirectly testing and proving the hypothesis (of this paper), claiming that every rectilinear motion should be accompanied with rotation (and such rotation naturally creates de Broglie waves, producing correct results found in (4.4)). In fact, Bohr's hydrogen atom model could also be used to prove all elements of **PWDC** (**P**article-**W**ave **D**uality **C**ode) found in (4.1)-(4.3).

[♣ **COMMENTS & FREE-THINKING CORNER:** *Wave Theory basically originates from the need to explain Bohr's postulates and that was its first meaningful application. Atom stability and experimentally verifiable spectral series of hydrogen atom postulated modifications of Rutherford's dynamic atom model, resulting with Bohr's atom model. Bohr was the first founder of Quantum Theory concepts in the practical need to explain the orbital hydrogen atom structure and the nature of quantized emissions and absorptions of electromagnetic energy, without giving any better explanation regarding its postulates. The unclear and unexplained situation (in Bohr's atom model) has been the one concerning why and how electron does not emit or absorb light while rotating on its stationary orbit/s, where its momentum is quantized, and which is the analytical connection between the frequency of periodical orbital electron motion and frequency of radiated (or absorbed) photon/s (since in classical Electrodynamics, the two mentioned frequencies are mutually equal). In Bohr's hydrogen atom model, those frequencies are different, except in cases of orbits with integer main quantum number, where such frequencies are approximately equal (which is additionally supported by formulating the "correspondence principle").*

Emission or absorption of photons appears possible only when electrons pass between two stationary orbits, and energy of such photon (directly proportional to its frequency multiplied by Planck's constant) is exactly equal to the difference of corresponding orbital electron energies (this way additionally legitimizing the Planck expression for photon energy). The total stationary orbit electron energy is equal to the sum of its kinetic and potential energy. It looks that Bohr only made a simple hybrid merging between Quantum Theory concepts, Classical Mechanics and Electrodynamics in order to explain already known phenomenology, without profoundly elaborating his theory.

*Luis de Broglie electron wavelength has also been perfectly fitting in the concept of stationary electron orbits, explaining atom stability in a very simple and pragmatic way, saying that the perimeter (or total length) of one stationary orbit should be equal to an integer multiple of the electron matter wave wavelength (similar to standing waves on a string). In reality, we could say that Bohr was the first one who applied the **PWDC**, although without having such intention (and without being conscious what he was really doing regarding the **PWDC**). Unfortunately, Bohr's hydrogen atom model (and planetary atom model in general) looks already as a very obsolete and unnatural concept, comparing it to the works of David L. Bergman and his colleagues [16] - [22]; -"Common Sense Science"). ♣]*

4.1.3. Mater Waves and Conservation Laws

It is also interesting to notice that de Broglie relation, $\lambda = \mathbf{h}/\mathbf{p}$, for mater wave wavelength, (4.1), is not fully explained and step-by-step developed starting from energy and momentum conservation laws. In fact, de Broglie found it by fitting the most logical solution which makes orbital electron wave stable, by introducing postulates of Bohr's hydrogen atom model. Later on, the same (de Broglie) relation was so successful in explaining the wave properties of different particles (diffraction and interference experiments), that nobody else asked the question how this relation could be proved generally valid and developed from a more independent platform (than Bohr's hydrogen atom model). The same situation was with Planck's expression for the photon energy $\tilde{\mathbf{E}} = \mathbf{hf}$, (4.1). **The proper expression was effectively found by the best curve fitting in order to explain measured data regarding spectrum of the black body radiation.** Later on, the same relation combined with de Broglie wavelength was (quietly) generalized to become valid for any matter wave and applied successfully in explaining many quantum interactions (Compton effect, Photo electric effect, etc.).

Let us briefly show how de Broglie wavelength and Planck's wave energy are fully compatible with energy and momentum conservation laws, analyzing the situation presented in Fig.4.1.

Let us start from the most general situation of a two-body interaction, when two particles $\mathbf{m_1}$ and $\mathbf{m_2}$, ($m = m^{rel.} = m_o/\sqrt{1-v^2/c^2} = \gamma\, m_0$) move relatively to each other, respectively having velocities $\mathbf{v_1}$ and $\mathbf{v_2}$, momentum $\mathbf{p_1}$ and $\mathbf{p_2}$, and kinetic energies $\mathbf{E_{k1}}$ and $\mathbf{E_{k2}}$, presenting them in the Laboratory and Center of Mass systems, and applying total Energy and Momentum conservation laws (using already established analogies in earlier chapters and the same symbolic, definitions and designations as in (4.1)-(4.4) and on Fig.4.1, where $\mathbf{E_{oi}}$ and - $\mathbf{m_{oi}}$ are particle energies and masses in the state of rest):

$$E_{tot.} = E_{01} + E_{k1} + E_{02} + E_{k2} = E_{0C} + E_{kc} + E_{kr},$$

$$E_{01} + E_{02} = E_{0C},\ E_{k1} + E_{k2} = E_{kc} + E_{kr}, E_{01} = m_{01}c^2, E_{02} = m_{02}c^2,\ E_{0C} = m_{0C}c^2,$$

$$m_c = m_1 + m_2\,,\ \mu = m_r = \frac{m_1 m_2}{m_1 + m_2},\ E_{12} = \int_{(1)}^{(2)} \vec{F}_{12} d\vec{r}_{12} = E_{kr}\,, F_{12} = F_r = \frac{dp_r}{dt}\,,$$

$$\left.\begin{aligned} E_{k1} &= (\Delta m_1)c^2 = \frac{p_1 v_1}{1+\sqrt{1-v_1^2/c^2}} = p_1 u_1, \\ E_{k2} &= (\Delta m_2)c^2 = \frac{p_2 v_2}{1+\sqrt{1-v_1^2/c^2}} = p_2 u_2, \\ E_{kc} &= (\Delta m_c)c^2 = \frac{p_c v_c}{1+\sqrt{1-v_c^2/c^2}} = p_c u_c \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} & E_{kr} = \frac{p_r v_r}{1+\sqrt{1-v_c^2/c^2}} = p_r u_r = \\ & = \frac{\mathbb{L}_r \omega_r}{1+\sqrt{1-v_c^2/c^2}},\ \mathbb{L}_r = \mathbb{J}_r \omega_r, \\ & E_{kc} = \frac{\mathbb{L}_c \omega_c}{1+\sqrt{1-v_c^2/c^2}},\ \mathbb{L}_c = \mathbb{J}_c \omega_c \end{aligned}\right\} \tag{4.5}$$

$$\vec{p}_1 + \vec{p}_2 = m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{p}_c = m_c\vec{v}_c \,,\ \omega_c = \omega_r = \omega_m \ ,$$
$$\vec{v}_c = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{\vec{p}_1 + \vec{p}_2}{m_c},\ \vec{v}_r = \vec{v}_1 - \vec{v}_2\,,\ u_i = \frac{v_i}{1+\sqrt{1-v_i^2/c^2}}\ .$$

Since only motional (or kinetic) energy should be equal to (de Broglie) matter wave energy, $\mathbf{E_k} = \mathbf{\vec{p}\vec{u}} = \mathbf{hf} = \mathbf{\tilde{E}}$, it should also be possible to apply the (wave) energy and momentum conservation laws, and to prove validity (or mutual compatibility) of de Broglie and Planck's relations (or to prove that there is no other possibility than treating de Broglie mater waves only as a motional energy), as follows:

$$\left\{\begin{array}{l}
\left\{\left[\begin{array}{l}
E_{k1} + E_{k2} = E_{kc} + E_{kr}, \vec{p}_1 + \vec{p}_2 = \vec{p}_c \\
E_{ki} = \vec{p}_i\vec{u}_i \Leftrightarrow \tilde{E}_i = hf_i \\
v_{1,2} = u_{1,2} - \lambda_{1,2}\dfrac{du_{1,2}}{d\lambda_{1,2}} = -\lambda_{1,2}^2\dfrac{df_{1,2}}{d\lambda_{1,2}} \\
u_{1,2} = \lambda_{1,2}f_{1,2} = \dfrac{v_{1,2}}{1+\sqrt{1-v_{1,2}^2/c^2}}
\end{array}\right] \Leftrightarrow \left[\tilde{E}_1 + \tilde{E}_2 = \tilde{E}_c + \tilde{E}_r\right]\right\} \Rightarrow \\
\Rightarrow \left[\begin{array}{l}
\left[f_1 + f_2 = f_c + f_r\right],\ \vec{v}_c = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \dfrac{\vec{p}_1 + \vec{p}_2}{m_1 + m_2}, \\
\left|\vec{v}_r\right| = \left|\vec{v}_1 - \vec{v}_2\right|,\ \left|\vec{u}_r\right| = \left|\vec{u}_1 - \vec{u}_2\right| = \lambda_r f_r
\end{array}\right] \\
\Rightarrow \left[\left|\vec{u}_c\right| = \left|\dfrac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m_2}\right| = \lambda_c f_c = \dfrac{v_c}{1+\sqrt{1-v_c^2/c^2}}\right]
\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l}
\lambda_1 = \dfrac{h}{p_1},\ \lambda_2 = \dfrac{h}{p_2},\ \lambda_c = \dfrac{h}{p_c},\ \lambda_r = \dfrac{h}{p_r} = \lambda_1\dfrac{v_1}{v_r} + \lambda_2\dfrac{v_2}{v_r} \\
\dfrac{1}{\lambda_1^2} + \dfrac{1}{\lambda_2^2} - \dfrac{1}{\lambda_c^2} + \dfrac{2\cos(\vec{p}_1,\vec{p}_2)}{\lambda_1\lambda_2} = 0 \\
\dfrac{v_1}{\lambda_1(1+\sqrt{1-v_1^2/c^2})} + \dfrac{v_2}{\lambda_2(1+\sqrt{1-v_2^2/c^2})} + \dfrac{E_r}{h} = \dfrac{v_c}{\lambda_c(1+\sqrt{1-v_c^2/c^2})}
\end{array}\right\}. \qquad (4.6)$$

In reality, because motional energies and group and phase velocities of mutually approaching objects (as well as all values in (4.5) and (4.6)) are mutually coupled, time and position dependent, or continuously evolving, (since objects somehow mutually communicate by the presence of surrounding fields), we should not take (4.5) and (4.6) as the final and fully correct approach to two-body interactions (because maybe some other interaction participants are missing; - See later (4.5-3)).

More informative and useful relations unifying energy and momentum conservation (and avoiding differences between Relativistic and Classical Mechanics mass interpretation) should be given in the following differential form (found in (4.2)),

$$\left\{\begin{array}{l}\mathbf{E_{k1}(t,x)+E_{k2}(t,x)=E_{kc}(t,x)+E_{kr}(t,x)\Leftrightarrow\tilde{E}_1+\tilde{E}_2=\tilde{E}_c+\tilde{E}_r,}\\ \mathbf{dE_{k1}+dE_{k2}=dE_{kc}+dE_{kr}\Leftrightarrow d\tilde{E}_1+d\tilde{E}_2=d\tilde{E}_c+d\tilde{E}_r, dE_{kr}=v_r dp_r}=\omega_r \mathbf{dL_r}\\ \mathbf{v_i=v_i(t,x)=u_i}-\lambda_i\frac{\mathbf{du_i}}{\mathbf{d\lambda_i}}=-\lambda_i^2\frac{\mathbf{df_i}}{\mathbf{d\lambda_i}}=\frac{\mathbf{d\tilde{E}_i}}{\mathbf{d\tilde{p}_i}}, \mathbf{u_i=u_i(t,x)}=\lambda_i\mathbf{f_i}\end{array}\right\}\Rightarrow \tag{4.7}$$

$$\Rightarrow\left\{\begin{array}{l}\mathbf{d\tilde{E}_i=hdf_i=v_i dp_i=d(p_i u_i)=dE_{ki}=c^2d(\gamma m_i)=-c^2d\tilde{m}_i=-d(\tilde{p}_i u_i)=}\\ \mathbf{=-v_i d\tilde{p}_i\cdot\cos(\tilde{p}_i,\tilde{p}_i)=\left[v_i d\tilde{p}_i\ \ or\ -v_i d\tilde{p}_i\right]}\end{array}\right\},$$

since after applying integration (when solving such equations) we can take into consideration boundary conditions and all stationary, motional, and/or state of rest parameters of certain interaction (see also (4.9-0)).

Here we can also address the idea how to treat forces acting between two mutually approaching particles ($\vec{F}_1,\vec{F}_2,\vec{F}_c,\vec{F}_r$), as for instance:

$$\begin{array}{l}\left\{dE_{k1}+dE_{k2}=dE_{kc}+dE_{kr}\right\}/dt\Rightarrow\vec{v}_1\frac{d\vec{p}_1}{dt}+\vec{v}_2\frac{d\vec{p}_2}{dt}=\vec{v}_c\frac{d\vec{p}_c}{dt}+\vec{v}_r\frac{d\vec{p}_r}{dt}\Leftrightarrow\\ \Leftrightarrow\vec{v}_1\vec{F}_1+\vec{v}_2\vec{F}_2=\vec{v}_c\vec{F}_c+\vec{v}_r\vec{F}_r\ ;\ \vec{F}_1+\vec{F}_2=\vec{F}_c\ ,\ E_{12}=\int_{(1)}^{(2)}\vec{F}_{12}d\vec{r}_{12}=\int_{(1)}^{(2)}\vec{F}_r d\vec{r}_{12}=E_{kr}\,.\end{array} \tag{4.7.1}$$

Particularly interesting cases are when the force $\vec{F}_r=\vec{F}_{12}$ between mutually approaching objects becomes balanced with centrifugal force of quasi-rotational movement of the same objects in their Center of Mass System, $F_r=\frac{dp_r}{dt}=\frac{m_r v_r^2}{r_{12}}(=G\frac{m_r m_c}{r_{12}^2}\pm\ldots\ ?!)$, creating conditions for stable and self-sustaining vortex-toroid formation, leading eventually to stable particle/s formation (see also force expressions (2.1) to (2.9) from the second chapter, in order to understand, conceptually, that such forces should have several static, dynamic and mixed, linear and rotational components).

If we now simplify the situation saying that only one particle moves relatively towards the other (which is in the state of rest), and that the moving particle is much smaller than other particle (equations (4.3), (4.5) and (4.6)) we can, again, demonstrate that the rotation of a reduced mass $\mathbf{m_r}$ in the Center of Mass System is directly responsible for creating de Broglie matter wave/s:

$$m_1=m<<m_2=M,\ v_1=v, v_2=0, E_{k1}=\tilde{E}_1=\tilde{E}_r=pu=hf\ ,\ E_{kc}\cong 0,$$

$$m_c=m+M\cong M,\ m_r\cong m_1=m\ ,\ v_c\cong\frac{m}{M}v\cong 0, v_r=v_1=v, u_c\cong\frac{m}{M}u\cong\frac{1}{2}v_c\cong 0,$$

$$(p_1+p_2=p_c)\Leftrightarrow(\frac{p_1}{h}+\frac{p_2}{h}=\frac{p_c}{h})\Rightarrow(\frac{1}{\lambda_1^2}+\frac{1}{\lambda_2^2}+2\frac{\cos(p_1,p_2)}{\lambda_1\lambda_2}=\frac{1}{\lambda_c^2})$$

$$p=\gamma mv=p_1\cong p_r\,, p_2=0, \cos(\vec{p}_1,\vec{p}_2)=\cos\pi=-1,$$

$$u_r=u=u_1=\lambda f=\frac{hf}{p}=\frac{v}{1+\sqrt{1-v^2/c^2}}, u_2=0,$$

$$f = f_1 \cong f_r = \frac{\mathbf{J}_r \omega_{mr}^2}{h(1+\sqrt{1-v_c^2/c^2})} = \frac{nf_m}{1+\sqrt{1-v_c^2/c^2}} \cong \frac{nf_m}{2}, \omega_{mr} = 2\pi f_m ,$$
$$f_m = nh/4\pi^2 \mathbf{J}_r , \mathbf{L}_r = \mathbf{J}_r \omega_m = nh/2\pi = n\hbar, \; n \in N \Leftrightarrow \{1,2,3...\}, \tag{4.8}$$
$$f_2 \cong f_c \cong \frac{m}{M} f \cong 0, \; \lambda_1 = \frac{h}{p_1} = \frac{h}{\gamma mv} = \frac{h}{p} = \lambda \cong \lambda_r \cong \lambda_c, \; \lambda_2 \cong \frac{1}{2}\lambda .$$

The most interesting in (4.8) is that wavelength $\lambda_2 \cong \frac{1}{2}\lambda = \frac{1}{2}\lambda_1$ also exists, even $\mathbf{p_2} = \mathbf{0}$, probably as the consequence of a kind of “wave mirror imaging effect” of the incident particle $\mathbf{m}$ that creates coupling and interactive field with its (big mass) target, meaning that certain oscillating perturbation should also be measurable on/in the mass $\mathbf{M}$.

Since in the Center of Mass System both moving particles $\mathbf{m_r}$ and $\mathbf{m_c}$ can also be presented as rotating around their common center of inertia, it is clear that this “rotation” directly creates associated de Broglie waves (mathematically recognizable on an energetic, or spectral level, and “visible” in the Center of Mass System, but not explicitly and directly recognizable and “visible” in the original time-space domain, in the Laboratory System). The above-analyzed example, summarized by (4.8), can also be applicable in the case of hydrogen Bohr’s atom model (4.4), where an electron rotates around atom nucleus.

Here we have been talking more about transient, motional elements similar to mechanical rotation (in the Center of Mass System), than about real, full-circle rotational movement (of interacting particles). When particles are in linear motion and approaching each other (without previously having their own orbital moments), obviously that certain transitory angular and vortex field components should be created between them (producing de Broglie matter waves), and both particles will feel (or get) equal, mutually opposite (mechanical) orbital moments. Consequently, the resulting orbital moment (of all mutually interacting objects) equals zero, but energy (or spectral) component associated to such orbital moments could be higher than zero. Generalizing this situation, we can always say that every single particle in linear motion should have certain level of associated rotational components (orbital moments, spin, torsion field structure, etc.), since it always creates a two-body system with the rest of the surrounding universe. Implicitly, here we always assume that between mutually interacting particles there should exist certain field and certain wave carrier or some material medium, regardless of the fact that we are often not able to detect or explain what kind of material wave carrier we are dealing with.

This situation can also be modeled as a dynamical and transient “dipole-formation” (between the moving particle and its vicinity, or its target), where such dipoles could have electrical, magnetic, gravitational, inertial, or some other nature. The above mentioned “dipole states” effectively rotate, or just produce transitory torque swings in a Center of Mass System (because the observed particle moves), and consequently produces angular and vortex field components.

The next consequence could be that Einstein Special Relativity Theory (SRT) is much more limited than it is currently considered to be (valid only under certain assumptions and for uniform, non-accelerated and rectilinear motion, which effectively doesn’t exist without elements of rotation), and that something similar should also be valid for

Maxwell electromagnetic field components. It becomes obvious that SRT, Gravitation and linear motions should be upgraded for the missing rotational field components. Typical examples of interactions that are sources of torsion field components should be all cases of elastic collisions (of course, any other collision types should also create torsion field components and de Broglie matter waves).

In reality (if we just take into account initial particle/s attributes, long before the interaction, and final attributes long after interaction happens) we address the totality of possible interactions between mutually approaching objects $\mathbf{m_1}, \mathbf{m_2}$ (or more correctly between their moments $\mathbf{p_1}, \mathbf{p_2}$) accounting certain coupling (or binding) energy $\mathbf{U_{12}}$, or potential energy $\mathbf{U(r_{12})}$ between them (especially in cases of plastic collisions when after collision we get only one object). Until the point when collision starts and during the transitory process of collision (before colliding objects separate again or stay united) we can treat all collisions in the same way. Later on, if objects separate, this could be the case of elastic collisions, but if objects stay fully united we shall have an ideal plastic collision (and we could also have some other cases in-between).

This way, the ideal plastic collision (realized or not realized) becomes like an asymptotic guiding and modeling frame for treating all collision types (as well as for treating all other interactions between two objects). Here the idea favored is that the most important and decisive elements of one collision process are parameters of that process ($\mathbf{m_c}, \mathbf{m_r}, \mathbf{v_c}, \mathbf{v_r}$), related its Center of Inertia or Center of Mass reference system. The mutually closer interacting objects with masses ($\mathbf{m_1}, \mathbf{m_2}$) and moments ($\mathbf{p_1}, \mathbf{p_2}$) are, the more dominant, and more relevant become (new and calculated) equivalent parameters ($\mathbf{m_c}, \mathbf{m_r}, \mathbf{v_c}, \mathbf{v_r}$) = (central mass, reduced mass, center mass speed, reduced mass speed).

Effectively, in case of plastic collision of two particles (if we take care only about input-output energy balance), the equations (4.5)-(4.8) can be modified accordingly, as for instance:

$$\begin{aligned}
&E_{tot.} = E_{01} + E_{k1} + E_{02} + E_{k2} = E_{0c} + E_{kc} + E_{kr}, \\
&E_{01} + E_{02} = E_{0c},\ E_{k1} + E_{k2} = E_{kc} + E_{kr}, \\
&E_{01} = m_1c^2, E_{02} = m_2c^2, E_{0c} = m_cc^2 + U_{12}, \\
&E_{k1} + E_{k2} = E_{kc} + \left[m_cc^2 + U_{12} - (m_1c^2 + m_2c^2)\right] = E_{kc} + E_{kr}
\end{aligned}$$

$$\begin{aligned}
&E_{k1} = \frac{\gamma_1 m_1 v_1^2}{1+\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{(p_1)^2}{\gamma_1 m_1\left[1+\sqrt{1-\frac{v_1^2}{c^2}}\right]} = \frac{p_1p_c}{\gamma_1 m_c\left[1+\sqrt{1-\frac{v_1^2}{c^2}}\right]} + \frac{p_1p_r}{\gamma_1 m_1\left[1+\sqrt{1-\frac{v_1^2}{c^2}}\right]} = E_{k1c} + E_{k1r} = \\
&= (\gamma_1 - 1)m_1c^2 = p_1v_1 / \left[1+\sqrt{1-\frac{v_1^2}{c^2}}\right] = p_1c\sqrt{\frac{\gamma_1 - 1}{\gamma_1 + 1}},\ p_1 = \gamma_1 m_1 c\sqrt{\gamma_1{}^2 - 1} = \gamma_1 m_1 v_1\ ,
\end{aligned}$$

$$E_{k2}=\frac{\gamma_2 m_2 v_2{}^2}{1+\sqrt{1-\frac{v^2{}_2}{c^2}}}=\frac{(p_2)^2}{\gamma_2 m_2\left[1+\sqrt{1-\frac{v^2{}_2}{c^2}}\right]}=\frac{p_2 p_c}{\gamma_2 m_c\left[1+\sqrt{1-\frac{v^2{}_2}{c^2}}\right]}-\frac{p_2 p_r}{\gamma_2 m_2\left[1+\sqrt{1-\frac{v^2{}_2}{c^2}}\right]}=E_{k2c}-E_{k2r}=$$

$$=(\gamma_2-1)m_2c^2=p_2v_2/\left[1+\sqrt{1-\frac{v^2{}_2}{c^2}}\right]=p_2c\sqrt{\frac{\gamma_2-1}{\gamma_2+1}},\ p_2=\gamma_2 m_2 c\sqrt{\gamma_2{}^2-1}=\gamma_2 m_2 v_2\ ,$$

$$E_{kc}=\frac{\gamma_c m_c v_c{}^2}{1+\sqrt{1-\frac{v^2{}_c}{c^2}}}=\frac{(p_c)^2}{\gamma_c m_c\left[1+\sqrt{1-\frac{v^2{}_c}{c^2}}\right]}=\frac{p_1 p_c}{\gamma_c m_c\left[1+\sqrt{1-\frac{v^2{}_c}{c^2}}\right]}+\frac{p_2 p_c}{\gamma_c m_c\left[1+\sqrt{1-\frac{v^2{}_c}{c^2}}\right]}=E_{kc1}+E_{kc2}=$$

$$=\frac{\gamma_1}{\gamma_c}\cdot\frac{m_1}{m_c}E_{k1}+\frac{\gamma_2}{\gamma_c}\cdot\frac{m_2}{m_c}E_{k2}+\frac{p_1p_2}{\gamma_c m_c}=(\gamma_c-1)m_cc^2=p_cv_c/\left[1+\sqrt{1-\frac{v^2{}_c}{c^2}}\right]=p_cc\sqrt{\frac{\gamma_c-1}{\gamma_c+1}},$$

$$p_c=\gamma_c m_c c\sqrt{\gamma_c{}^2-1}=\gamma_c m_c v_c=\left|\vec{p}_1+\vec{p}_2\right|,\ \vec{p}_c+\vec{p}_r=\vec{p}_c=\vec{p}_1+\vec{p}_2\ ,$$

$$E_{kr}=\frac{p_r v_r}{1+\sqrt{1-\frac{v^2{}_c}{c^2}}}=J_r\omega^2{}_r/\left[1+\sqrt{1-\frac{v^2{}_c}{c^2}}\right]=\frac{\gamma_2}{\gamma_c}\cdot\frac{m_2}{m_c}E_{k1}+\frac{\gamma_1}{\gamma_c}\cdot\frac{m_1}{m_c}E_{k2}-\frac{p_1p_2}{\gamma_c m_c}$$

$$p_r=\left\|\vec{p}_r\right\|_{eff.}=m_rv_r\ \text{(= effective value)},$$

$$\vec{p}_r=\iiint d\vec{p}_r=\vec{0}\ \text{(= total, resulting vectorial field, ??!!)},$$

$$E_{k1}+E_{k2}=E_{kc}+E_{kr}\Rightarrow v_1dp_1+v_2dp_2=v_cdp_c+v_rdp_r\ ,$$

$$\Leftrightarrow\ \frac{\gamma_1 m_1 v_1^2}{1+\sqrt{1-\frac{v_1^2}{c^2}}}+\frac{\gamma_2 m_2 v_2^2}{1+\sqrt{1-\frac{v_2^2}{c^2}}}=\frac{p_c v_c}{1+\sqrt{1-\frac{v_c^2}{c^2}}}+\frac{p_r v_r}{1+\sqrt{1-\frac{v_c^2}{c^2}}},$$

$$\Leftrightarrow E_{k-Lab.} = \left[E_{k1} + E_{k2}\right]_{Lab.} = \left[E_{k1}\right]_{Lab.} + \left[E_{k2}\right]_{Lab.} =$$

$$= \left[E_{k\text{-Translat.}}\right] + \left[E_{k\text{-Rotat.}}\right] = \frac{(p_1 + p_2)v_c}{1 + \sqrt{1 - \frac{v_c^2}{c^2}}} + \frac{\mathbf{J}_r \omega_r^2}{1 + \sqrt{1 - \frac{v_c^2}{c^2}}} ,$$

$$\left[E_{k\text{-Translat.}}\right] = \frac{(p_1 + p_2)v_c}{1 + \sqrt{1 - \frac{v_c^2}{c^2}}} = \frac{\mathbf{J}_c \omega_c^2}{1 + \sqrt{1 - \frac{v_c^2}{c^2}}} , \tag{4.5-1}$$

$$\left[E_{k\text{-Rotat.}}\right] = \frac{\mathbf{J}_r {\omega^2}_r}{1 + \sqrt{1 - \frac{v_c^2}{c^2}}} = \frac{p_r v_r}{1 + \sqrt{1 - \frac{v_c^2}{c^2}}} ,$$

$$2\pi r_i = n\lambda_i = n\frac{h}{p_i} = \frac{v_i}{f_m} = n\frac{u_i}{f_i},\ u_i = \lambda_i f_i\ ,\ \omega_r = \omega_c = 2\pi f_m\ .$$

where indexing "**Lab.**"represents energy states in a Laboratory coordinate system, "**Translat.**" energy states of translation (or linear motion) and index "**Rotat.**" represents energy states of rotation. Based on (4.5-1) we could also upgrade (4.6)-(4.8) in a similar way. In reality, in process of particles' mutual approaching and impact, and just after the impact happens, all energies and moments from (4.5-1) should be presented by time-space evolving functions.

What is very characteristic in (4.5-1) is that every particle ($\mathbf{m_1}, \mathbf{m_2}, \mathbf{m_c}$ *) in the Laboratory System can have certain non-zero momentum* $\vec{\mathbf{p}}_1, \vec{\mathbf{p}}_2, \vec{\mathbf{p}}_c$ *(in a vector form), except the particle* $\mathbf{m_r}$*. The resulting linear macro moment of* $\mathbf{m_r}$*, as a vector equals zero* $\vec{\mathbf{p}}_r = \mathbf{0}$ *(in the Laboratory System), but its effective (**eff.**) non-vectorial moment (that makes contribution in the energy* E_{kr}*) is different from zero,* $p_r = \left\|\vec{p}_r\right\|_{eff.} = \gamma m_r v_r \neq \mathbf{0}$*. This implicates that* $\mathbf{m_r}$ *should be distributed around* $\mathbf{m_c}$ *(in some cases maybe like a toroid) performing rotation, which would create* $\vec{\mathbf{p}}_r = \vec{\mathbf{0}}$*. What should really be distributed around* $\mathbf{m_c}$ *is a kind of inertial and waiving field that has torsion components and motional energy* $\mathbf{E_{kr}}$*.* It is also important to notice that energy conservation of two-body interactions, similar to (4.5-1), in the physics of particle interactions, is usually analyzed without highlighting its direct relation to Particle-Wave Duality and Torsion Fields. In fact, certain expressions in (4.5-1) should be still considered only as "temporarily valid" (as a starting brainstorming initiation) and should be reconfirmed and developed from a much more general platform, such as the one given by T.4.3 (see later).

To reveal the secret about what could happen in the close vicinity of approaching objects is not an easy task because the full picture can be created only if we take into account that certain (known or unknown) carrier medium (or coupling field) should exist between them, producing particle wave phenomenology, that should be analyzed by solving characteristic wave equations describing such process (see (4.25)-(4.37)). Before we develop universal wave equations (suitable to treat such situations), we will try to use simplified modeling and to create clear conceptual picture related to collision processes. The principal objective in presenting different expressions for energy conservation in (4.5)-(4.8) and (4.5-1) is to show that all two-body interactions create a very specific near field, a transitory interaction zone, where all interacting members (real and virtual) mutually "communicate" producing inertial and particle-wave duality effects.

The wave-to-particle transformation, or particle creation, should be a process related to two-body, mutually approaching objects interactions, when their relative energy (in their Center of Mass System), $E_{kr} = E_{12} = \int_{(1)}^{(2)} \vec{F}_{12} d\vec{r}_{12} = p_r u_r$***, either becomes a part of internal energy (and rest mass) of a unified object,*** $\mathbf{m_c}$***, in case of an ideally plastic impact, or before such impact happens, the same relative energy*** $\mathbf{E_{kr}}$ ***will reach certain energy level (and satisfy other necessary conditions, related to relevant conservation laws), sufficient for generating new particles (which are initially not present in the same interaction). The typical example of such interactions is when a very high-energy photon passes close to an atom, generating a couple of electron-positron particles (practically transforming the quasi-rotating wave energy content*** $\mathbf{E_{kr}}$***, of a high-energy incident photon, into (new) real particles with non-zero rest masses).***

*[♣ **COMMENTS & FREE-THINKING CORNER:** If we just imagine (staying in the frames of mechanics) that between interacting objects could exist certain coupling or binding energy* $\mathbf{U_{12}}$*, and that there should be certain energy exchange between energy states of translation and rotation (for the amount* $\boldsymbol{\delta m \cdot c^2}$ *=* $\mathbf{U_{12}} / \mathbf{c^2} \Rightarrow E_{Lab.} = E_{CI} = [E_c]_{Translat.} + [E_r]_{Rotat.} \cong \{[E_c]_{Translat.} - \delta m \cdot c^2\} + \{[E_r]_{Rotat.} + \mathbf{U_{12}} / \mathbf{c^2}\}$*), instead of (4.5-1) we can create the following (very much speculative, but interesting to think about) energy conservation form:*

$$E_{tot.} = E_1 + E_2 = E_c + E_r = (E_c - \delta m \cdot c^2) + (E_r + \delta m \cdot c^2), E_c = E_{0c} + E_{kc}, E_r = E_{0r} + E_{kr},$$
$$E_{tot.} = (E_{01} + E_{k1}) + (E_{02} + E_{k2}) = (E_{0c} + E_{kc}) + (E_{0r} + E_{kr}),$$

$$E_{01} + E_{02} = E_{0c} + E_{0r},\ E_{k1} + E_{k2} = E_{kc} + E_{kr},$$
$$E_{01} = m_1 c^2, E_{02} = m_2 c^2, E_{0c} = (m_c - \delta m_c)c^2,\ E_{0r} = (m_r + \delta m_r)c^2 = \delta m_c c^2,$$

$$E_{k1} = (\gamma_1 - 1)m_1 c^2 = E_1 - E_{01}, E_{k2} = (\gamma_2 - 1)m_2 c^2 = E_2 - E_{02},$$
$$E_{kc} = (\gamma_c - 1)(m_c - \delta m_c)c^2, E_{kr} = (\gamma_r - 1)(m_r + \delta m_r)c^2 = (\gamma_r - 1)\delta m_c c^2,$$

$$\frac{\delta m_c}{\delta m_r} = (\frac{v_r}{v_c})^2,\ \delta m_r = m_r / \left[(\frac{v_r}{v_c})^2 - 1\right] = \frac{v_c}{v_r} \cdot \frac{U_{12}}{c^2} = \delta m_c - m_r \cong \frac{v_c}{v_r} \cdot \delta m,$$

$$\delta m_c = m_r (\frac{v_r}{v_c})^2 / \left[(\frac{v_r}{v_c})^2 - 1\right] = \frac{v_r}{v_c} \cdot \frac{U_{12}}{c^2} = m_r + \delta m_r \cong \frac{v_r}{v_c} \cdot \delta m\ , \qquad (4.5\text{-}2)$$

$$m_c = m_1 + m_2 = \frac{m_1 m_2}{\delta m}\cdot\left(\frac{v_r v_c}{v_r^2 - v_c^2}\right), \delta m \cong \sqrt{\delta m_r \delta m_c} = \frac{v_r}{v_c}\delta m_r = U_{12}/c^2,$$

$$m_r = m_1 m_2/(m_1 + m_2) = \frac{U_{12}}{c^2}\cdot\left(\frac{v_r^2 - v_c^2}{v_r v_c}\right) = \delta m\cdot\left(\frac{v_r^2 - v_c^2}{v_r v_c}\right) = \delta m_c - \delta m_r.$$

If we now compare the first relation from (4.4) for non-relativistic velocities, $\frac{m_p}{m_e} = (\frac{v_e}{v_p})^2 = (\frac{r_e}{r_p})^2 = (\frac{\lambda_e}{\lambda_p})^2 = (\frac{u_e}{u_p})^2$, *with similar mass relation from (4.5-2),* $\frac{\delta m_c}{\delta m_r} = (\frac{v_r}{v_c})^2$, *it becomes obvious where are (hidden) elements of rotation associated to linear motions.* ♣]

Let us again summarize two-body interactions and how and where matter waves are being created (see the illustration on the Fig. 4.1.2. and details from T.4.3).

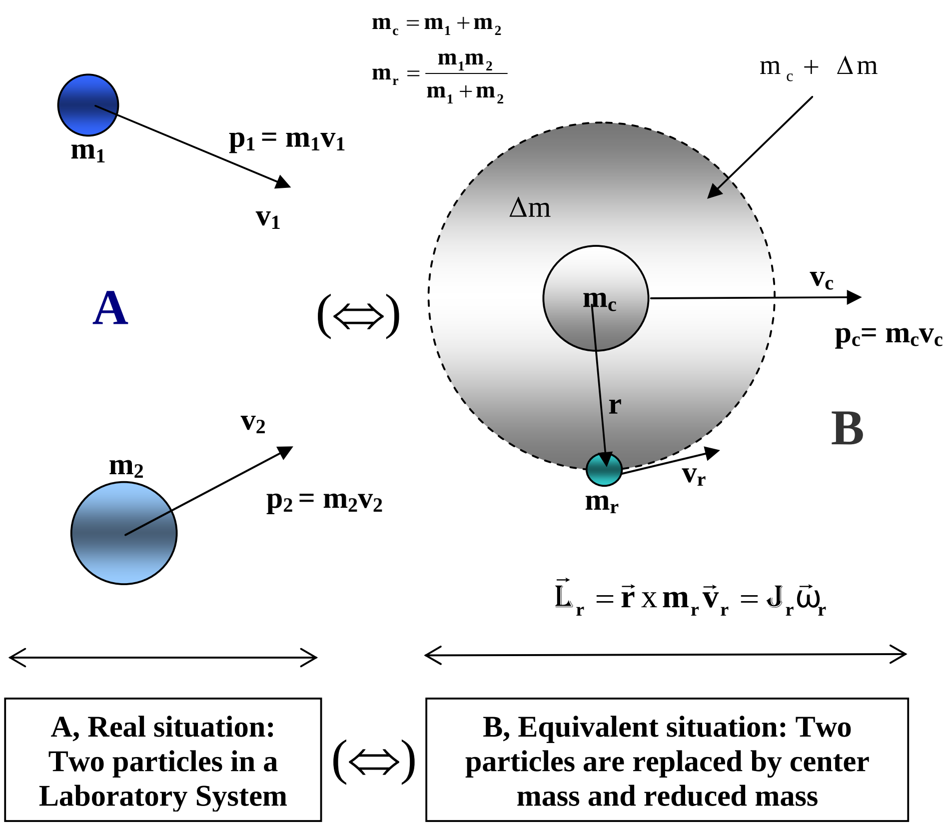


Fig. 4.1.2. Mutually equivalent presentations of a two-body system; A ⇔ B (The plain where m_r performs rotation-like motion around m_c should be considered being perpendicular to the center of mass velocity v_c; -see T.4.3)

The same two-body system from Fig.4.1.2, which is equivalent to the situation from Fig.4.1, can be analyzed (or described) from the point of view of energy and momentum conservation laws, as follows (see the table below; T.4.3). Under two-body interactions here we would understand: Elastic and/or Inelastic Impacts, Particle/s Creation and/or Disintegration/s, Annihilation, Compton and Photoelectric effect etc.

We will just make mutually equivalent and analog mathematical descriptions of the same two-body system (moving masses $\mathbf{m_1}$ and $\mathbf{m_2}$, which are electrically and magnetically neutral, or without other active charges) in two different systems of references; - Laboratory System and Center of Mass System (A and/or B, and A* and/or B*). This will be presented in four different ways, or in four mutually linked (or mutually dependent) systems of references. The principal (original, real) and initial "mass-momentum players" in A are particles $\mathbf{m_1}$ and $\mathbf{m_2}$. The dynamically (or mathematically) equivalent, "mass-momentum players" in B, and B* will be new "virtual particles" $\mathbf{m_r}$ and $\mathbf{m_c}$. In fact, parallel to real interaction participants $\mathbf{m_1}$ and $\mathbf{m_2}$, that are introduced in A (where everything is conceptually obvious and clear), we are introducing additional and a little bit artificial (but dynamically equivalent), "virtual two-body situations", placed in A* and B*, or B, with mutually interacting masses $\mathbf{m_r}$ and $\mathbf{m_c}$, which have known mathematical relations with $\mathbf{m_1}$ and $\mathbf{m_2}$. A* and B* as well as A and B are mutually linked Laboratory and Center of Mass Systems (where A and A* are dealing with $\mathbf{m_1}$ and $\mathbf{m_2}$, and B and B* are dealing with $\mathbf{m_r}$ and $\mathbf{m_c}$). The objective here is to present all possible mathematical relations between moving objects $\mathbf{m_1}$, $\mathbf{m_2}$, $\mathbf{m_r}$ and $\mathbf{m_c}$ in different referential systems A, B, A* and B*, by respecting relevant conservation laws (See table T.4.3). The idea behind all of that is to show that moving objects are entering into certain "mass-momentum communication" initiating (dynamically equivalent) elements of rotation, which are sources of matter-waves phenomenology. From the Laboratory System we see (mutually approaching) objects $\mathbf{m_1}$ and $\mathbf{m_2}$, but how $\mathbf{m_1}$ is noticing $\mathbf{m_2}$ and vice versa is related to "energy-momentum" coupling/s between them, and this part of the analysis should explain the background of matter-waves and particle-wave duality. Let us first make more precise descriptions or definitions of all referential systems, A, B, A* and B*, as:

1° Laboratory System **A**: Is presented with moving and Real Interaction Participants $\mathbf{m_1}$ and $\mathbf{m_2}$.

2° Laboratory System **B**: Is presented with moving and "Virtual Interaction Participants" such as, Center mass $\mathbf{m_c}$, Reduced Mass $\mathbf{m_r}$, Center Mass Velocity $\mathbf{v_c}$, and Reduced Mass Velocity $\mathbf{v} = \mathbf{v_r}$, etc., all of them measured by the observer from the Laboratory System. Here we need to imagine that the same observer, as in the case of the Laboratory system A, would start seeing only motions of $\mathbf{m_c}$ and $\mathbf{m_r}$, instead of seeing $\mathbf{m_1}$ and $\mathbf{m_2}$, and all mathematical relations should be established to make presentations in A and B mutually equivalent.

3° Center of Mass system **A***: The observer is linked to the center of mass, seeing only two, original and initial particles $\mathbf{m_1}$ and $\mathbf{m_2}$, which are also found in the Laboratory System A, now having different velocities and moments. Here, system A* is moving with relative velocity $\mathbf{v_c}$, measured from A.

4° Center of Mass system **B***: The observer is linked to the center of mass, seeing only "Virtual Interaction Participants" such as: Center mass and Reduced mass, $\mathbf{m_c}$ and $\mathbf{m_r}$, analog to the situation in the Laboratory System under B, but this time only $\mathbf{m_r}$ is moving around $\mathbf{m_c}$ and $\mathbf{m_c}$ is in the state of rest (from the point of view of the observer in B*). Here we need to imagine that the same observer, as in the case of the Center of Mass system A*, would start seeing only motions of $\mathbf{m_c}$ and $\mathbf{m_r}$, instead of seeing $\mathbf{m_1}$ and $\mathbf{m_2}$, and all mathematical relations should be established to make presentations in A* and B* mutually equivalent.

The main idea here is to show that particle-wave duality should have its roots and explanation in relation with the Laboratory System B and Center of Mass System B*. It has already been explained (in the beginning of this chapter) that the "kitchen" where matter waves (de Broglie wavelength and frequency) are created is directly related to what happens between $\mathbf{m_r}$ and $\mathbf{m_c}$ in B and B*, since this is the best and maybe the only way to show that $\mathbf{m_r}$ performs rotation around $\mathbf{m_c}$. Of course, here we are talking about something that is mathematically presentable as equivalent to rotation, while paying attention to satisfy all conservation laws and to make mutually equivalent or mutually compatible descriptions between states of motions of real particles ($\mathbf{m_1}$, $\mathbf{m_2}$) and their effective replacements ($\mathbf{m_c}$, $\mathbf{m_r}$). The concept of rotation is directly linked to a concept of frequency, and there is just a small step to imagine creation of certain kind of waves that would have certain wavelength (de Broglie, matter waves wavelength). If we are able to find elements of rotation (frequency and wavelength) related to virtual particles ($\mathbf{m_c}$, $\mathbf{m_r}$), it would be necessary to make just a small step to determine how such elements would appear to an observer from the Laboratory System (and this is what L. de Broglie, A. Einstein, M. Planck and other founders of Wave Quantum Mechanics established, obviously using different methodology; in this paper we are using the abbreviated name **PWDC** = **P**article – **W**ave – **D**uality – **C**ode, to encircle the same domain).

1° With the data presented in T.4.3, **as the first step**, we intend to make this situation mathematically and conceptually much clearer. The table T.4.3 is created by exploiting the complete, formal or mathematical symmetry for all expressions that are related to energies and moments of ($\mathbf{m_c}$, $\mathbf{m_r}$), by making them look like analogous expressions of energies and moments of ($\mathbf{m_1}$, $\mathbf{m_2}$), in all systems of reference (A, A*, B, B*). In fact, we will soon realize by analyzing mutual mathematical consistency and compatibility of data from T.4.3 that certain energy momentum relations in T.4.3 are mathematically non-sustainable and not compatible, especially expressions and relations in connection with reduced mass $\mathbf{m_r}$, and that the biggest mathematical and conceptual challenge in realizing here elaborated strategy would be the question how to address or associate quantity of motion (linear momentum) to the Reduced Mass $\mathbf{m_r}$, and to the Center mass $\mathbf{m_c}$, or saying differently, what would really mean vectorial quantities $\vec{p}_r = m_r\vec{v}_r$ and $\vec{p}_c = \gamma_c m_c \vec{v}_c$, found in T.4.3. The most probable case is that the total initial quantity of motion, or linear motion momentum of both particles ($\mathbf{m_1}$, $\mathbf{m_2}$), would be "given" only to $\mathbf{m_c}$. This way, since $\mathbf{m_r}$ really has certain amount of motional energy, it would be shown that this is only the rotational-motional energy (and that $\mathbf{m_r}$ has nothing related to linear motion; consequently $\mathbf{m_r}$ should only have certain orbital moment or spin).

2° Then, **as the second step**, for the Laboratory System B, we would introduce the assumption that $\mathbf{m_c}$ should be the carrier of the total quantity of rectilinear motion, and that $\mathbf{m_r}$ has only certain amount of rotational motional energy, without having any rectilinear motion momentum, $\vec{\mathbf{p}}_{\mathbf{r}} = \mathbf{0}$ (in other words, $\mathbf{m_r}$ can only make rotation or spinning around $\mathbf{m_c}$). Doing this way we should be able to correct/modify all mutually not-compatible expressions in T.4.3, and exactly explain the origin and meaning of de Broglie matter waves, and the nature of unity between linear and rotational motions (obviously this would be a voluminous mathematical task, well started but still not finalized in this paper).

T.4.3. Laboratory System (A)	Laboratory System (B)	Center of Mass System (A*)	Center of Mass System (B*)
$\mathbf{m_1}$ $\mathbf{m_2}$	$\mathbf{m_c = m_1 + m_2}$ $\mathbf{m_r = \frac{m_1 m_2}{m_1 + m_2}}$	$\mathbf{m_1}$ $\mathbf{m_2}$	$\mathbf{m_c = m_1 + m_2}$ $\mathbf{m_r = \frac{m_1 m_2}{m_1 + m_2}}$
$\vec{\mathbf{v}}_1$ $\vec{\mathbf{v}}_2$	$\vec{\mathbf{v}}_c = \frac{\mathbf{c^2(\vec{p}_1 + \vec{p}_2)}}{\mathbf{E_1 + E_2}}$ $\vec{\mathbf{v}}_r = (\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1) = \vec{\mathbf{v}}$	$\vec{\mathbf{v}}_1^* = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_c$ $\vec{\mathbf{v}}_2^* = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_c$	$\mathbf{v_c^* = 0}$ $\mathbf{v_r^* = v_r = v}$
$\vec{p}_1 = \gamma_1 m_1 \vec{v}_1$ $\vec{p}_2 = \gamma_2 m_2 \vec{v}_2$ $\vec{P}(A) = \vec{p}_1 + \vec{p}_2$	$\vec{p}_c = \gamma_c m_c \vec{v}_c$ $\vec{p}_r = \vec{0},\ \lvert\vec{p}_r\rvert = m_r v_r$ (?!) $\vec{P}(B) = \vec{p}_c + \vec{p}_r$	$\vec{p}_1^* = \gamma_1^* m_1 \vec{v}_1^* = \gamma_1^* m_1 (\vec{v}_1 - \vec{v}_c) =$ $= -m_r(\vec{v}_2 - \vec{v}_1) = -m_r \vec{v}$ $\vec{p}_2^* = \gamma_2^* m_2 \vec{v}_2^* = \gamma_2^* m_2 (\vec{v}_2 - \vec{v}_c) =$ $= -m_r(\vec{v}_2 - \vec{v}_1) = +m_r \vec{v}$ $\vec{P}(A^*) = \vec{p}_1^* + \vec{p}_2^* = 0$	$p_c^* = 0$ $p_r^* = m_r v_r^*$ $P(B^*) = p_r^*$
$E_1 = E_{01} + E_{k1} = \gamma_1 m_1 c^2$ $E_{01} = m_1 c^2$, $E_{k1} = (\gamma_1\text{-}1) m_1 c^2$ $E_2 = E_{02} + E_{k2} = \gamma_2 m_2 c^2$ $E_{02} = m_2 c^2$, $E_{k2} = (\gamma_2\text{-}1) m_2 c^2$ $E(A) = E_1 + E_2 = E(B) =$ $= \gamma_c E(A^*) \geq E(A^*)$ $E_0(A) = E_{01} + E_{02} = E_0(A^*) =$ $= (m_1 + m_2)c^2 = m_c c^2$ $E_k(A) = E_{k1} + E_{k2} =$ $= \gamma_c E_k(A^*) \geq E_k(A^*)$	$E_c = E_{0c} + E_{kc} = \gamma_c m_c c^2$ $E_{0c} = m_c c^2$, $E_{kc} = (\gamma_c\text{-}1) m_c c^2$ $E_r = E_{0r} + E_{kr}$ $E_{0r} = m_r c^2$, $E_{kr} =$ $E(B) = E_c + E_r = E(A) =$ $= \gamma_c E(B^*) \geq E(B^*)$ $E_0(B) = E_{0c} + E_{0r} = E_0(B^*) =$ $= (m_c + m_r)c^2$ $E_k(B) = E_{kc} + E_{kr} =$ $= \gamma_c E_k(B^*) \geq E_k(B^*)$	$E_1^* = E_{01}^* + E_{k1}^* = \gamma_1^* m_1 c^2$ $E_{01}^* = m_1 c^2$, $E_{k1}^* = (\gamma_1^*\text{-}1) m_1 c^2$ $E_2^* = E_{02}^* + E_{k2}^* = \gamma_2^* m_2 c^2$ $E_{02}^* = m_2 c^2$, $E_{k2}^* = (\gamma_2^*\text{-}1) m_2 c^2$ $E(A^*) = E_1^* + E_2^* = E(B^*) =$ $= E(A)/\gamma_c \leq E(A)$ $E_0(A^*) = E_{01}^* + E_{02}^* = E_0(A) =$ $= (m_1 + m_2)c^2 = m_c c^2$ $E_k(A^*) = E_{k1}^* + E_{k2}^* =$ $= E_k(A)/\gamma_c \leq E_k(A)$	$E_c^* = E_{0c}^* + E_{kc}^* = \gamma_c^* m_c c^2 = m_c c^2$ $E_{0c}^* = m_c c^2$, $E_{kc}^* = (\gamma_c^*\text{-}1) m_c c^2 = 0$ $E_r^* = E_{0r}^* + E_{kr}^*$ $E_{0r}^* = m_r c^2$, $E_{kr}^* =$ $E(B^*) = E_c^* + E_r^* = E(A^*) =$ $= E(B)/\gamma_c \leq E(B)$ $E_0(B^*) = E_{0c}^* + E_{0r}^* = E_0(B) =$ $= (m_c + m_r)c^2$ $E_k(B^*) = E_{kc}^* + E_{kr}^* = E_{kr}^* =$ $= E_k(B)/\gamma_c \leq E_k(B)$

$\gamma_1 = (1-\frac{v_1^2}{c^2})^{-0.5}$ $\gamma_2 = (1-\frac{v_2^2}{c^2})^{-0.5}$	$\gamma_c = (1-\frac{v_c^2}{c^2})^{-0.5}$	$\gamma^*_{1,2} = (1-\frac{(v^*_{1,2})^2}{c^2})^{-0.5}$	$\gamma^*_c = 1$
The same situation presented with 4 vectors in the Minkowski Space			
$\overline{P}_1 = \overline{P}_1(\vec{p}_1, \frac{E_1}{c})$ $\overline{P}_2 = \overline{P}_2(\vec{p}_2, \frac{E_2}{c})$ $\overline{P}^2_{1,2} = p^2_{1,2} - \frac{E^2_{1,2}}{c^2} = -m^2_{1,2}c^2$ $\left[\overline{P}(A)\right]^2 = (\overline{P}_1+\overline{P}_2)^2 = \left[\overline{P}(A^*)\right]^2 =$ $= \left[\overline{P}(A)\right]^2 c^2 - \frac{\left[E(A)\right]^2}{c^2} =$ $= -\frac{\left[E_0(A)\right]^2}{c^2} = -(m_1+m_2)^2c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = \dots$ $\overline{P}_1\overline{P}_2 = \overline{P}^*_1\overline{P}^*_2$	$\overline{P}_c = \overline{P}_c(\vec{p}_c, \frac{E_c}{c})$ $\overline{P}_r = \overline{P}_r(\vec{p}_r, \frac{E_r}{c})$ $\overline{P}^2_{c,r} = p^2_{c,r} - \frac{E^2_{c,r}}{c^2} = -m^2_{c,r}c^2$ $\left[\overline{P}(B)\right]^2 = (\overline{P}_c+\overline{P}_r)^2 = \left[\overline{P}(B^*)\right]^2 =$ $= \left[\overline{P}(B)\right]^2 c^2 - \frac{\left[E(B)\right]^2}{c^2} =$ $= -\frac{E_0(B)}{c^2} = -(m_c+m_r)^2c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = \dots$ $\overline{P}_c\overline{P}_r = \overline{P}^*_c\overline{P}^*_r$	$\overline{P}^*_1 = \overline{P}^*_1(\vec{p}^*_1, \frac{E^*_1}{c})$ $\overline{P}^*_2 = \overline{P}^*_2(\vec{p}^*_2, \frac{E^*_2}{c})$ $(\overline{P}^*_{1,2})^2 = (p^*_{1,2})^2 - \frac{(E^*_{1,2})^2}{c^2} = -m^2_{1,2}c^2$ $\left[\overline{P}(A^*)\right]^2 = (\overline{P}^*_1+\overline{P}^*_2)^2 =$ $= \left[\overline{P}(A^*)\right]^2 c^2 - \frac{\left[E(A^*)\right]^2}{c^2} = \left[\overline{P}(A)\right]^2 =$ $= -\frac{\left[E^*_0(A)\right]^2}{c^2} = -(m_1+m_2)^2c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = \dots$ $\overline{P}_1\overline{P}_2 = \overline{P}^*_1\overline{P}^*_2$	$\overline{P}^*_c = \overline{P}^*_c(\vec{p}^*_c, \frac{E^*_c}{c})$ $\overline{P}^*_r = \overline{P}^*_r(\vec{p}^*_r, \frac{E^*_r}{c})$ $(\overline{P}^*_{c,r})^2 = (p^*_{c,r})^2 - \frac{(E^*_{c,r})^2}{c^2} = -m^2_{c,r}c^2$ $\left[\overline{P}(B^*)\right]^2 = (\overline{P}^*_c+\overline{P}^*_r)^2 = \left[\overline{P}(B)\right]^2 =$ $= \left[\overline{P}(B^*)\right]^2 c^2 - \frac{\left[E(B^*)\right]^2}{c^2} =$ $= -\frac{E_0(B^*)}{c^2} = -(m_c+m_r)^2c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = \dots$ $\overline{P}_c\overline{P}_r = \overline{P}^*_c\overline{P}^*_r$

Since T.4.3 is created mostly using mathematical analogies and generally known methodology from 4-vector relativistic relations in the Minkowski space, (without paying too much attention to whether all details regarding newly introduced concepts about “virtual interaction participants” are already fully correct and defendable) it would be necessary to double-check and test all critical relations listed there, in order to be able to draw final and relevant conclusions (practically all relevant results, expressions and relations starting from (4.2), (4.3), (4.5) – (4.8) until (4.5-1) and (4.5-2) should be compared with similar, identical, or equivalent results and relations that could be developed from T.4.3, and should be mutually compatible, or if not, should be corrected and made compatible). An early and still non-finalized “experimental” attempt, without taking into account 4 vectors in the Minkowski space, to develop a similar concept, as T.4.3 is presently paving, has been initialized by formulating expressions for energies and moments given in (4.5-1) and (4.5-2), but the real remaining task and challenge in this situation, which would give the correct picture about unity of rectilinear and rotational motions, would be to make all colons (of energies, moments, velocities etc.) found in T.4.3 mutually compatible and correctly formulated in all details, what could still not be the case. Presently, the most important activity here has been to establish the concept of matter wave creation regarding real and virtual interaction participants in different systems of reference, and if we continue such process all of possibly missing or incorrect mathematical details would not escape to be arranged later (*in other words the significance of the concept that is presently being introduced here is much higher than still unfinished mathematical works around it*). We should also not forget that interacting (real and virtual) objects would mutually create certain force fields that should be in an agreement with generalized Newton-Coulomb force expressions given from (2.4) until (2.4-3); see the second chapter of this paper (Gravitation). We should not exclude the possibility of having “exotic” interactions between real ($\mathbf{m_1}$, $\mathbf{m_2}$), and virtual ($\mathbf{m_r}$, $\mathbf{m_c}$) objects (that are presently still hypothetical and brainstorming options).

From the point of view of the total energy conservation (comparing the situation in a Laboratory system given under A and B), the two-body situation from Fig. 4.1.2 could be described as,

$$E_{total} = E = E(A) = E_0(A) + E_k(A) = (m_1 + m_2)c^2 + (\gamma_1 - 1)m_1c^2 + (\gamma_2 - 1)m_2c^2 =$$
$$= E(B) = E_0(B) + E_k(B) = m_cc^2 + E_{0r} + (\gamma_c - 1)m_cc^2 + E_{kr} = m_cc^2 + m_rc^2 + (\gamma_c - 1)m_cc^2 + E_{kr} =$$
$$= \gamma_1m_1c^2 + \gamma_2m_2c^2 = \gamma_cm_cc^2 + E_r = \gamma_cMc^2 \Rightarrow$$
$$M_{total} = M = m_c + \frac{E_r}{\gamma_cc^2} = \frac{\gamma_1m_1 + \gamma_2m_2}{\gamma_c},\ (m_r = \frac{m_1m_2}{m_1 + m_2},\ m_c = m_1 + m_2),$$
$$E_r = E_{0r} + E_{kr} = m_rc^2 + E_{kr} = \gamma_c(M - m_c)c^2 = (\gamma_1m_1 + \gamma_2m_2 - \gamma_cm_c)c^2, E_{0r} = m_rc^2,$$
$$E_{kr} = \left[\gamma_c(M - m_c) - m_r\right]c^2 = \left[\gamma_c(\frac{M - m_c}{m_r}) - 1\right]m_rc^2 = (\gamma_1m_1 + \gamma_2m_2 - \gamma_cm_c - m_r)c^2,$$
$$E_c = \gamma_cm_cc^2 = E_{0c} + E_{kc} = m_cc^2 + (\gamma_c - 1)m_cc^2, E_{0c} = m_cc^2, E_{kc} = (\gamma_c - 1)m_cc^2.$$

Since situations A and B are describing two of mutually (dynamically) equivalent states, it should be valid,

$$E_0 = Mc^2 = \left(\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c}\right)c^2 = \left(m_c + \frac{E_r}{\gamma_c c^2}\right)c^2 = \left(m_c + \Delta m\right)c^2 = E_0(A) \; = \; E_0(B),$$

$$E_0(A) = E_{01} + E_{02} + \Delta E_A = m_1 c^2 + m_2 c^2 + \Delta E_A = m_c c^2 + \Delta E_A = Mc^2,$$

$$E_0(B) = E_{0r} + E_{0c} + \Delta E_B = m_r c^2 + m_c c^2 + \Delta E_B = Mc^2,$$

$$\Delta E_A = \left[\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c} - (m_1 + m_2)\right]c^2 = c^2 \Delta M_A,$$

$$\Delta E_B = \left[\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c} - (m_r + m_c)\right]c^2 = c^2 \Delta M_B,\ \Delta m = \frac{E_r}{\gamma_c c^2}.$$

$$E_k = (\gamma_c - 1)Mc^2 = (\gamma_c - 1)(m_c + \frac{E_r}{\gamma_c c^2})c^2 = (\gamma_c - 1)(\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c})c^2 = E_k(A) \; = \; E_k(B),$$

$$E_k(A) = E_{k1} + E_{k2} + \delta E_A = (\gamma_1\text{-}1)m_1 c^2 + (\gamma_2\text{-}1)m_2 c^2 + \delta E_A,$$

$$E_k(B) = E_{kc} + E_{kr} + \delta E_B = (\gamma_c\text{-}1)m_c c^2 + E_{kr} + \delta E_B = \left[(\gamma_1 m_1 + \gamma_2 m_2) - (m_r + m_c)\right]c^2 + \delta E_B,$$

$$\delta E_A = (\gamma_c - 1)(\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c})c^2 - \left[(\gamma_1\text{-}1)m_1 + (\gamma_2\text{-}1)m_2\right]c^2 = c^2 \delta M_A,$$

$$\delta E_B = (\gamma_c - 1)(\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c})c^2 - \left[(\gamma_1 m_1 + \gamma_2 m_2) - (m_r + m_c)\right]c^2 = c^2 \delta M_B.$$

(4.5-3)

Here we are paving or testing the concept that in near zone of interaction, at least certain time, interacting objects would create a virtually united object $\mathbf{m_c}$ with energy coupling and energy-exchange events (here presented with: $\Delta E_A, \Delta E_B, \delta E_A, \delta E_B, c^2 \Delta m$). The another message in formulating (4.5-3), and later (4.5-4), regardless if in some future mathematical revision (4.5-3) and (4.5-4) would be corrected, is to show that the part of the motional energy of $\mathbf{m_r}$ is being "effectively injected" into the "effective rest mass" of the central mass, as $m_c + \Delta m \; = \; m_c \; + \frac{E_r}{\gamma_c c^2}$. Motional energy associated to reduced-mass $\mathbf{m_r}$ is dynamically equivalent to energy of rotation (where effectively $\mathbf{m_r}$ is rotating around $\mathbf{m_c}$). Externally (from the Laboratory system) we can see only a linear particles motion, as a motion of their common center of mass (since particular rotational motions would be "mathematically captured" by internal content of the equivalent rest mass; -For additional conceptual clarification see chapter 2, equations (2.11.1) until (2.11.9) and T.2.4, T.2.5 and T.2.6). For instance, if certain particle is spinning around its own axis and performing a rectilinear motion at the same time it looks obvious that its total motional energy should have two different components: $E_{rot.} + E_{linear-motion} = E_{rot.} + E_k$. In order to follow the message from (4.5-3) and to be more explicit, we could say that the same particle without having any element of rotation (or spinning) would have the total and kinetic energy equal to: $E_{tot.} = \gamma mc^2$, $E_k = (\gamma - 1)mc^2$, and if elements of rotation are present the total and kinetic energy would become

$E_{tot.} = \gamma(m_0 + \frac{E_{rot.}}{c^2})c^2$, $E_k = (\gamma - 1)(m_0 + \frac{E_{rot.}}{c^2})c^2$, $\mathbf{m} = \mathbf{m_0} + \frac{\mathbf{E_{rot.}}}{\mathbf{c^2}}$. In other words, here, all elements of rotational motion are treated as certain equivalent contribution to the rest mass. In cases when we have a number of particles passing from one complex motional state (state 1) to the other (state 2), where particles could have linear and rotational motion components, the same situation would be presentable as given in (4.5-4),

$$\begin{aligned}
&\overline{P}^2 = p_1^2 - \frac{E_1^2}{c^2} = p_2^2 - \frac{E_2^2}{c^2} = -m^2c^2 = \text{Invariant} \quad , \\
&\vec{L} = J\vec{\omega} = \vec{L}_1 = J_1\vec{\omega}_1 = \vec{L}_2 = J_2\vec{\omega}_2 \quad (= \text{total orbital momentum conservation}), \\
&p_{1/2} = \gamma_{1/2} m_{1/2} v_{1/2} = \gamma_{1/2}(m_{0\text{-}1/2} + \frac{E_{rot\text{-}1/2}}{c^2})v_{1/2} \quad , \quad E_{rot\text{-}1/2} = E(\vec{L}_{1/2}) \qquad (4.5\text{-}4) \\
&E = \gamma_{1/2} m_{1/2} c^2 = \gamma_{1/2}(m_{0\text{-}1/2} + \frac{E_{rot\text{-}1/2}}{c^2})c^2, \\
&m_{1/2} = m_{0\text{-}1/2} + \frac{E_{rot.\text{-}1/2}}{c^2} \quad .
\end{aligned}$$

In all cases given by expressions in T.4.3 and (4.5-1)-(4.5-3), the real and initial interaction participants (m_1, m_2*) have only linear motion moments (no rotation, no spinning), and by applying the law of orbital moments conservation it should be clear that the sum of all (initial) orbital moments before interaction will stay equal to the sum of all orbital moments appearing after interaction (in this case equal zero). There is only a transitory period in the near zone of interaction, when two interacting or mutually approaching bodies (*m_1, m_2*) effectively create certain additional elements of rotation such as:* m_r *rotates around* m_c *or* m_1 *and* m_2 *both rotate around their common center-of-mass point, and such additional elements of rotation should also be balanced (producing that their total, vectorial orbital moment in every moment during the interaction equals zero). In other words, if the initial total orbital moment of (*m_1, m_2*) equaled zero (measured from the Laboratory System, before the interaction started), it is clear that in the transitory, near zone of interaction we should have only interaction products or participants with mutually balanced orbital moments that as vectors cancel each other. This is extremely important to consider if we want to understand the nature of rotation associated to the Center of Mass System (that is at the same time the source of de Broglie matter waves). In cases when initial particles (*m_1, m_2*) have non-zero orbital moments and spin attributes, like in (4.5-4), the same situation becomes much more complex and mathematically richer (because we need to apply the Orbital Moments Conservation Law and find all possible distributions and redistributions of orbital moments and spinning during the process of interaction, and after interaction).* ***This time we did not address the possibility that between two initial masses*** m_1, m_2 ***(entering interaction) exist some kind of electromagnetic or other binding energy coupling (such as*** U_{12} ***in (4.5-1)), what would make previous mathematical elaboration more complex, but without diminishing conclusions regarding motional or rotational energy transformation into a total, equivalent rest mass. Here could also be a big part of the answer about understanding hidden or "dark matter" of our universe (what is basically a question of proper mathematical interpretation of known conservation laws).***

Effectively, any two-body situation in the process of interaction evolving creates a kind of transitory, compound system where resulting (and equivalent) central rest mass m_c *is increased for the rest mass amount of* Δm*, becoming* $m_c + \Delta m$ *(where* $\Delta m = \frac{E_r}{\gamma_c c^2}$*).*

This way, the "rotation-like motion" of a reduced mass m_r *around* m_c *is effectively taken into account by the amount of* Δm*, and new, transitory compound system is presented only as a linear motion of the mass* $m_c+\Delta m$ *with the velocity* $\mathbf{v_c}$*. If* Δm *eventually became a real particle with a rest mass, the final rest mass increase or reduction (after the interaction is ended) would depend on many other factors, still not introduced here, in order to give the advantage to a simplified, global and conceptual thinking (without too many details). The mass* Δm *is presented only in the function to show how rotation related motional energy component could be "mathematically injected in, or extracted" from the rest mass. This situation would become more challenging if interacting particles with masses* m_1 *and* m_2 *present one real particle (with non-zero rest mass) and a photon (for instance), or if both of them are photons, because here we are paving the way to a new understanding of how particles are created (or disintegrated), and where the place of rotation in such a process is (regardless that what we have here could not be only an ordinary kind of rotation, but it is a state that can have orbital and magnetic moments). In fact de Broglie, matter waves are created inside the interaction zone between* m_c *and* m_r*, and such matter waves present the "communicating channel" for all energy exchanges and mass transformations that would happen there. The parameters of mentioned matter waves-like de Broglie wavelength and frequency are also products of the same interacting zone between* m_c *and* m_r*, since both of them (*m_c *and* m_r*) could effectively and dynamically be presented as having certain elements of rotational motions (for instance, as rotating around their common center of inertia, having orbital moments: see also Fig.4.1 and equations (4.3)).*

4.1.3.1. Example 2: X-ray Spectrum and Reaction Forces

As an illustration regarding the extension of the particle-wave duality concept, in this example we shall analyze the generation of x-rays in an x-ray tube. Let us imagine that there is a potential difference **U** between two stable metal electrodes (in a x-ray tube). If the potential difference between electrodes is sufficiently high, this will pull and accelerate electrons from the negative electrode towards the positive electrode. In the moment when the electron leaves the negative electrode (since there is an electrical voltage and field between electrodes, and the electron has its mass and charge), both electrodes will "feel" certain reactive force in the form of a small "electromechanical" shock/s, and certain transient electric current (or current pulse) will be measured in the external electrode circuit. Also, at the moment when the flying electron strikes the surface of the positive electrode, this will again create certain electromechanical shock, or waving perturbation inside the positive electrode, and x-rays will be radiated from the impact surface (of course, negative electrode will also "feel" the impact event", because there is an electrical field between the positive and the negative electrode and certain amount of current will flow). The acoustical activity (mechanical vibrations in electrodes) will also be generated when the electron leaves or strikes an electrode, because the electron behaves as a particle that has its mass, spin and charge. This situation is sufficiently complex to explain the nature and appearance of de Broglie matter waves.

Since in this process, time-wise, effectively we have three distinct time intervals with sets of different (waves and particles) energy and momentum states, in order to make a difference between them (in a time scale), we will introduce the following indexing: index "**0**" - will characterize all states of rest (or electrical non-activity) before the electron leaves the negative electrode (and before voltage between electrodes is switched on), index "**1**" will characterize all states covering the time interval when the electron flies between the two electrodes, and index "**2**" will characterize all states (in both electrodes and between them) after the electron strikes the positive electrode. For marking electron states we shall use index "**e**", for negative electrode states, index "**ne**", for positive electrode states index "**pe**" and for x-rays index "**x**", adding indexes **0**, **1** or **2** for indicating which time interval we are taking into account. For kinetic energy we will use the index "**k**", and to indicate that certain state is a kind of wave, vibration or oscillation (that is in fact a state of motional or kinetic energy), we will use the symbol "**~**".

We will apply the energy and momentum conservation laws, assuming that before the electron was accelerated by an electric field, we had only the electron in its (relative) state of rest (inside the negative electrode with energy: $\mathbf{E_{eo}} = \mathbf{mc^2}$), and when electrical voltage was switched on, we got a moving electron under the influence of electrical field (which will give the motional energy to the same electron of: $E_{electric\,field} = -eU = E_{ek1} = (\gamma_1 - 1)mc^2$).

We can also consider that the electron and both electrodes of the x-rays tube, in the state of rest (just until the moment the electron leaves the negative electrode, before the voltage was switched on, in the time interval marked with "**0**") had negligible amounts of (internal, average, equilibrium state) wave energy and wave momentum ($\tilde{\mathbf{E}}_{\mathbf{eo}} \approx 0$, $\tilde{\mathbf{p}}_{\mathbf{eo}} \approx 0$, $\tilde{\mathbf{E}}_{\mathbf{neo}} \approx 0$, $\tilde{\mathbf{p}}_{\mathbf{neo}} \approx 0$, $\tilde{\mathbf{E}}_{\mathbf{peo}} \approx 0$, $\tilde{\mathbf{p}}_{\mathbf{peo}} \approx 0$). Also, it is obvious that the negative and the positive electrode (both being relatively big masses in the permanent state of rest) cannot have (macroscopically) any kinetic energy, or momentum ($\mathbf{E_{neko}} = \mathbf{0}$, $\mathbf{p_{neo}} = \mathbf{0}$, $\mathbf{E_{nek1}} = \mathbf{0}$, $\mathbf{p_{ne1}} = \mathbf{0}$, $\mathbf{E_{nek2}} = \mathbf{0}$, $\mathbf{p_{ne2}} = \mathbf{0}$, $\mathbf{E_{peko}} = \mathbf{0}$, $\mathbf{p_{peo}} = \mathbf{0}$, $\mathbf{E_{pek1}} = \mathbf{0}$, $\mathbf{p_{pe1}} = \mathbf{0}$, $\mathbf{E_{pek2}} = \mathbf{0}$, $\mathbf{p_{pe2}} = \mathbf{0}$), providing that they are well fixed to the walls of an x-rays tube. This is important to underline, since we have already established (in this paper) that every kinetic energy of a particle automatically corresponds to the same amount of its wave energy, and since the electrodes do not move (looking externally from the position of the Laboratory System), there is no wave energy belonging to them, too. Since the electrodes are parts of a closed electric circuit, it is clear that certain amount of wave energy could be created internally, inside the electrodes, since the same electrodes act as a carrier medium for electric currents (or electric waves and oscillations), and a carrier for mechanical vibrations, and such internal electrode states having certain content of wave energy will be marked using the symbol "**~**".

Let us clarify the same situation more precisely. Usually, when we analyze a moving particle (in a free space), its kinetic energy is equal to its wave energy $\mathbf{E_{ek}} = (\gamma - \mathbf{1})\mathbf{mc}^2 = \tilde{\mathbf{E}} = \mathbf{hf}$, which is the case regarding an electron in the state "**1**", flying between two electrodes (accelerated by an electric field). When the same electron strikes the positive electrode (being absorbed in the state "**2**"), we shall say that the electron as a particle is stopped (loosing its kinetic energy and momentum: $(\mathbf{E_{ek2}} = \tilde{\mathbf{E}}_{e2}) \cong \mathbf{0}$, $(\mathbf{p_{e2}} = \tilde{\mathbf{p}}_{e2}) \cong \mathbf{0}$), but the positive electrode itself becomes the carrier of certain transient electric current pulse, and carrier of certain mechanical vibration, being characterized with non-zero internal wave states $(\tilde{\mathbf{E}}_{pe2}, \tilde{\mathbf{p}}_{pe2}) = (\tilde{\mathbf{E}}_{ne2}, \tilde{\mathbf{p}}_{ne2})$. Of course, a similar situation regarding internal electrode states, when the electron flies between them, will also make $(\tilde{\mathbf{E}}_{pe1}, \tilde{\mathbf{p}}_{pe1}) = (\tilde{\mathbf{E}}_{ne1}, \tilde{\mathbf{p}}_{ne1})$, because both electrodes are permanently a part of the externally closed electric circuit. In order to make this situation even clearer, in the following table all (particle and wave) energy and momentum states of electrodes, electron and x-ray photons are classified.

	States just before electron left negative electrode (indexing: 0)	**States after electron left negative electrode, before striking positive electrode (indexing: 1)**	**States just after electron stroke positive electrode (indexing: 2)**
electron (index: e)	$(\mathbf{E_{ek0}} = \tilde{\mathbf{E}}_{e0}) \cong \mathbf{0}$, $(\mathbf{p_{e0}} = \tilde{\mathbf{p}}_{e0}) \cong \mathbf{0}$	$\mathbf{E_{ek1}} = \tilde{\mathbf{E}}_{e1}$ $\mathbf{p_{e1}} = \tilde{\mathbf{p}}_{e1}$	$(\mathbf{E_{ek2}} = \tilde{\mathbf{E}}_{e2}) \cong \mathbf{0}$, $(\mathbf{p_{e2}} = \tilde{\mathbf{p}}_{e2}) \cong \mathbf{0}$
negative electrode (index: ne)	$(\mathbf{E_{neko}} = \tilde{\mathbf{E}}_{neo}) = \mathbf{0}$, $(\mathbf{p_{neo}} = \tilde{\mathbf{p}}_{neo}) = \mathbf{0}$	$\mathbf{E_{nek1}} = 0, \mathbf{p_{ne1}} = 0$ $\tilde{\mathbf{E}}_{ne1}, \tilde{\mathbf{p}}_{ne1}$	$\mathbf{E_{nek2}} = 0, \mathbf{p_{ne2}} = 0$ $\tilde{\mathbf{E}}_{ne2}, \tilde{\mathbf{p}}_{ne2}$
positive electrode (index: pe)	$(\mathbf{E_{peko}} = \tilde{\mathbf{E}}_{peo}) = \mathbf{0}$, $(\mathbf{p_{peo}} = \tilde{\mathbf{p}}_{peo}) = \mathbf{0}$	$\mathbf{E_{pek1}} = 0, \mathbf{p_{pe1}} = 0$ $\tilde{\mathbf{E}}_{pe1}, \tilde{\mathbf{p}}_{pe1}$	$\mathbf{E_{pek2}} = 0, \mathbf{p_{pe2}} = 0$ $\tilde{\mathbf{E}}_{pe2}, \tilde{\mathbf{p}}_{pe2}$
x-ray photons	**n/a**	**n/a**	$\tilde{\mathbf{E}}_{x2} = \mathbf{hf_x}, \tilde{\mathbf{p}}_{x2} = \mathbf{hf_x / c}$

We are now in the position to generalize and formulate explicitly another aspect of particle-wave duality regarding the internal wave energy content (not discussed in earlier chapters of this paper), practically summarizing above-mentioned facts, as follows:

Certain moving macro-body or particle (which is not a single and elementary particle) can be characterized by its kinetic energy (in a Laboratory System of coordinates), and at the same time the same kinetic energy can be conceptually presented in two different ways, such as $E_{ek} = (\gamma - 1)mc^2 = \tilde{E} = hf$, (producing experimentally, directly or indirectly verifiable effects of de Broglie matter waves, relative to its Laboratory System). If the same body is in a state of relative rest (not moving macroscopically), its kinetic and wave energy (relative to the Laboratory System) are again mutually equal, and equal zero, $E_{ek} = \tilde{E} = \tilde{E}_{external} = 0$ (looking externally). Since the same macro-body (electrodes in this example) presents a complex material structure, it can serve (internally) as the carrier of electric currents, and mechanical signals, meaning that inside the body we could also have certain kind of wave propagation, or certain wave energy content, which is exactly the case found in this example. This is the reason why a total motional energy of certain body should be presented as the sum of its (external) kinetic or wave energy (if the body moves relative to its Laboratory System), and its internally captured wave energy, if somehow this body is excited and becomes the carrier of mechanical, electrical and/or any other kind of signals (apart from counting rest mass energy as its internal wave energy content).

For the purpose of mathematical modeling, the appearance of any wave energy (and action-reaction forces) will be generally related to the case/s of sudden changes of electron's motional energy (meaning that, the first time when the electron leaves the negative electrode, and the second time when the electron strikes the positive electrode, we can expect some transient electric current waiving and acoustic perturbation and/or radiation effects on/in electrodes, or in the space around them). We also know that when the electron strikes the positive electrode, x-ray photon/s will be emitted from the positive electrode surface ($\tilde{E}_{x2} = hf_x$, $\tilde{p}_{x2} = hf_x/c$), and at the same time the external electrical circuit between the two electrodes will indicate presence of corresponding, transient current pulse (here represented by internal electrode states with corresponding wave energies and momentum: $\tilde{E}_{ne1}, \tilde{p}_{ne1}$, $\tilde{E}_{ne2}, \tilde{p}_{ne2}$, $\tilde{E}_{pe1}, \tilde{p}_{pe1}$, $\tilde{E}_{pe2}, \tilde{p}_{pe2}$).

*[♣ **COMMENTS & FREE-THINKING CORNER:** Going a little bit further in making generalizations, we can see that in all of above mentioned particle wave events or interactions, we deal with closed circuits of energy flow. For instance, an x-ray tube creates (at least) two of such, mutually coupled, closed circuits: one of them is electrical circuit, where external voltage **U** is connected to electrodes, causing the flow of electrons between electrodes, and the second circuit is a photonic one, starting (or branching) from the point where the electrons strike the positive electrode and start generating x-ray photons. Photons propagate using the external space as a carrier, and again, in some way, a closed circuit of electromagnetic energy flow goes back to the x-ray tube (not necessarily in the form of the electromagnetic waves). All particle and wave interactions and wave motions, current/s and different signal propagations are, in one or the other way, a part of a local or wider area, closed circuits of the energy flow. We are used to such concepts in Electric Circuit Theory, but in reality, this concept can be (analogically) extended and applied to all kind of motions, oscillations, waving, and to all particle and wave interactions known in physics. Operating with (correctly established) wave functions can help to generalize the energy flow analysis (in the frames of closed circuit analysis), regardless of the energy origin. Any theoretical analysis of certain interaction,*

where particle wave duality is involved, without understanding (or taking into account) how this interaction closes its energy flow circuit indicates that this interaction is still not completely explained. Action and reaction forces and concepts of inertial forces should be in a direct connection with channels that create closed circuits of the energy flow. Without closed circuit energy flow concepts (see Fig. 4.1.3), our theories also "float in the foggy space of uncertainty and probability", presenting very much locally valid statistical modeling, or limited set data fitting.

Whenever we talk about closed circuits of some fluid, electricity, particles etc. (basically about** the **flow of some entities carrying energy), implicitly we should understand that** the **flow of one sort of matter is usually coupled with inertia and/or induction effects of its complementary (conjugated) matter couple. This situation is analog to a flow of electrically charged particles that is accompanied by electrical field, and characterized by electrical current, causing** the **appearance of a complementary magnetic field, including "inertia" effects explained by Faraday, Maxwell and Lorenz laws of electromagnetic induction, etc., and in reality, also** the **Newton law of inertia, judging by analogy, belongs to the same generally valid concept of universally applicable Inertia and Induction laws.

For illustrating <u>closed circuit concept of an energy flow</u>, let us imagine that a particle, which initially in its state of rest has a mass $\mathbf{m}$, is moving under an action of certain active force F_a; see Fig. 4.1.3. In the space around the moving particle we could have a flow of other particles, and presence of different fields (waves and forces), making our moving particle affected, irradiated and internally excited in a number of ways, increasing its internal energy, temperature or its rest mass, which is the situation always present in real particle motions, and should be in some way taken into account in order to supplement our conceptual understanding of particle-wave duality. To have a simpler framework (regarding the situation presented in Fig.4.1.3), we will for the time being neglect the possible presence of external and internal rotational elements in the particle motion (such as orbital moments of any kind) and consider that our particle is moving by dominant influence of an active force F_a. The energy balance in such a case has to account for the existence of the particle initial rest mass and motional (or environmental) contribution to particle rest mass caused by all possible external influences that increase the particle internal energy. In other words, the moving particle, beside its principal and closed energy flow circuit, also has certain energy flow (or exchange) as a result of couplings with its environment, which in many practical situations (regarding calculations) could be neglected, but should not be completely forgotten.

$\mathbf{F_a} = \mathbf{dp}/\mathbf{dt}$ *(=) Principal, active force that makes particle moving (externally)*

$\tilde{\mathbf{F}}_{\text{int.}}$ *(=) Forces of external energy flow that internally excite rest mass states, such as: heating, different external radiations, vibrations, flux of elementary particles, etc.*

$\mathbf{E_0} = \mathbf{m_0 c^2}$ *(=) Real, minimal level of particle rest energy*

$\tilde{E}_{\text{int.}} = \tilde{m}_{\text{int.}} c^2 = \int_{[r]} \tilde{F}_{\text{int.}} dr$ *(=) Energy of rest mass (internally) excited states, caused by some external influence/s*

$\mathbf{E}_0 + \tilde{\mathbf{E}}_{\text{int.}} = \mathbf{mc}^2 = (\mathbf{m}_0 + \tilde{\mathbf{m}}_{\text{int.}})\mathbf{c}^2$ *(=) Total particle rest energy*

$\mathrm{E}_{\text{total}} = \mathrm{E}_0 + \mathrm{E}_k = \gamma\, \mathrm{mc}^2 = \gamma\, (\mathrm{m}_0 + \tilde{\mathrm{m}}_{\text{int.}})\mathrm{c}^2$ *(=) Total particle energy in motion*

$\mathrm{E}_k = \tilde{\mathrm{E}} = (\gamma - 1)\mathrm{mc}^2 = (\gamma - 1)(\mathrm{m}_0 + \tilde{\mathrm{m}}_{\text{int.}})\mathrm{c}^2 = \int_{[r]} \mathrm{F_a dr}$ *(=) Motional particle energy*

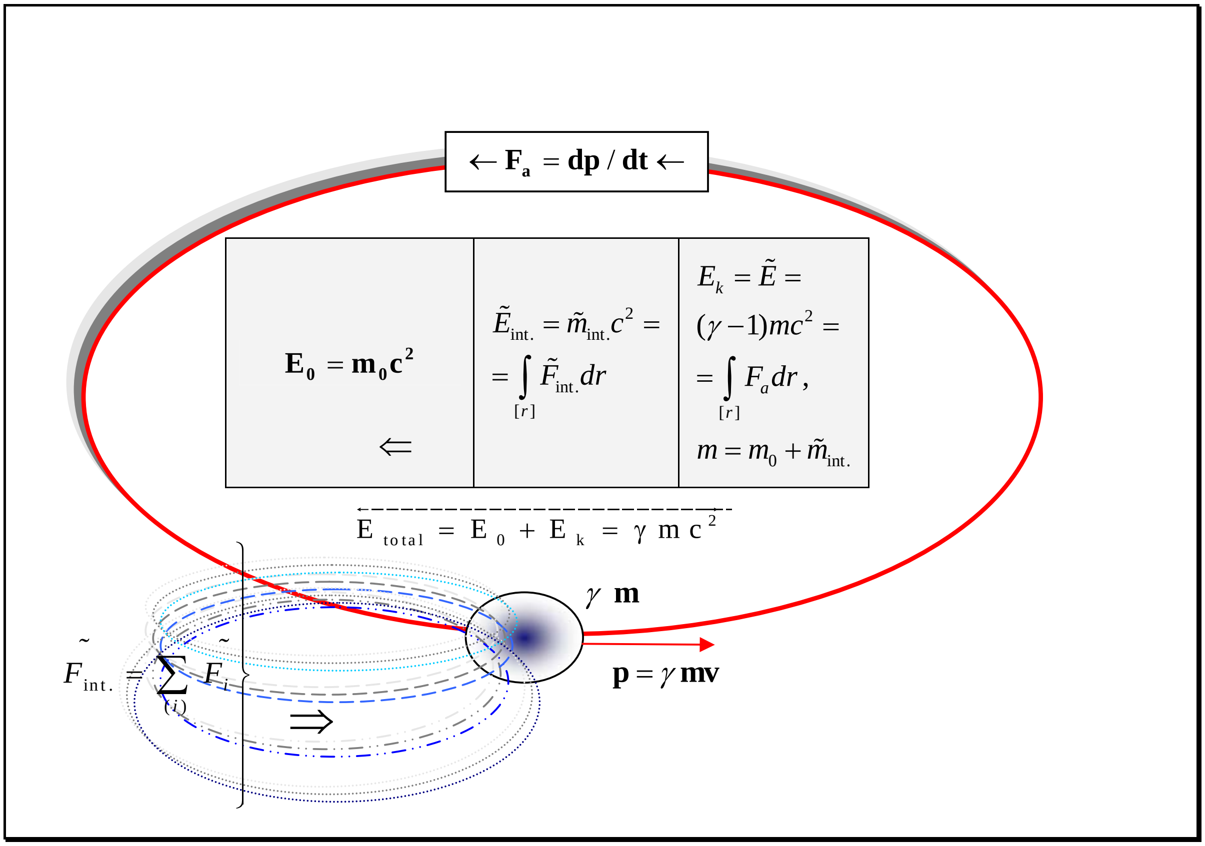


Fig. 4.1.3. An Illustration of the Closed Circuit Energy Flow ♣]

Let us go back to the previously analyzed example of x-ray radiation. Obviously, in the situation when analyzing x-ray radiation we have a sufficient number of tangible, measurable and visible waving and radiation (electrical and acoustic) events, and consequently, we cannot characterize de Broglie electron matter waves only as "phantom probability waves", since here we always have a closed electrical circuit where we are in an easy (and fully deterministic) position to know, see, calculate and measure what and where really waives and produces electrical currents, voltages and photons), in real time.

Taking into account the differences between electron group and phase velocities, when an electron flies between two electrodes, the effects of associated retarded potentials will make this situation a little bit analytically more complex.

Let us now calculate the outgoing kinetic energy $\mathbf{E_{ek1}}$ and speed $\mathbf{v_{e1}}$ of a single electron, $\mathbf{m} = \mathbf{m_e}$, $\mathbf{q_e} = -\mathbf{e}$ (just at the moment of leaving the surface of a negative electrode; time interval "**1**"). The total energy conservation law applied in this case will give,

$$\left\{E_{eo} = E_{ek1} + E_{eo} + E_{electric field}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} mc^2 = (\gamma_1 - 1)mc^2 + mc^2 - eU, \\ \gamma_1 = 1 + eU/mc^2 = (1 - v_{e1}{}^2/c^2)^{-1/2} \Rightarrow v_{e1} = c\left[1 - 1/(1 + eU/mc^2)^2\right]^{1/2}, \\ E_{ek1} = (\gamma_1 - 1)mc^2 = eU = \gamma_1 m v_{e1}{}^2 / \left[1 + (1 - v_{e1}{}^2/c^2)^{-1/2}\right], \; p_{e1} = mv_{e1} \end{array}\right\}$$

In reality, we know that when the electron leaves the negative electrode (since the electron has certain mass, moment, spin and charge), the negative electrode will "feel" small electrical and mechanical shock, and certain amount of energy ($\delta\tilde{\mathbf{E}} \geq \mathbf{0}$) will be dissipated in such (transient) process (in a closed electrical circuit), reducing outgoing electron speed (in fact, the outgoing electron speed will be: $v_{e1} \leq c\left[1 - 1/(1 + eU/mc^2)^2\right]^{1/2}$). Applying again the law of total energy conservation, we can take into account this correction on the following way:

$$\left\{\left[E_{eo} = E_{ek1} + E_{eo} + E_{electric field} + \delta\tilde{E}\right], \left[E_{ek1} = (\gamma_1 - 1)mc^2 = \tilde{E}_{e1} = \tilde{p}_{e1} \cdot u_{e1} = -eU\right]\right\} \Rightarrow$$

$$\left\{\begin{array}{l} mc^2 - \delta\tilde{E} = (\gamma_1 - 1)mc^2 + mc^2 - eU, \\ \gamma_1 = 1 + (eU - \delta\tilde{E})/mc^2 = (1 - v_{e1}{}^2/c^2)^{-1/2} \\ v_{e1} = c\left\{1 - 1/\left[1 + (eU - \delta\tilde{E})/mc^2\right]^2\right\}^{1/2} \leq c\left\{1 - 1/\left[1 + eU/mc^2\right]^2\right\}^{1/2}, \\ E_{ek1} = (\gamma_1 - 1)mc^2 = -eU - \delta\tilde{E} = \gamma_1 m v_{e1}{}^2 / \left[1 + (1 - v_{e1}{}^2/c^2)^{-1/2}\right] \leq -eU, \; p_{e1} = mv_{e1} = \tilde{p}_{e1} \end{array}\right\}$$

Since negative and positive electrodes are electrically connected by external voltage source, making closed electrical circuit, any (wave energy or electrical current) perturbation in one electrode will coincidently produce similar effect in the opposite electrode ($(\tilde{\mathbf{E}}_{\mathbf{pe1}}, \tilde{\mathbf{p}}_{\mathbf{pe1}}) = (\tilde{\mathbf{E}}_{\mathbf{ne1}}, \tilde{\mathbf{p}}_{\mathbf{ne1}}), (\tilde{\mathbf{E}}_{\mathbf{pe2}}, \tilde{\mathbf{p}}_{\mathbf{pe2}}) = (\tilde{\mathbf{E}}_{\mathbf{ne2}}, \tilde{\mathbf{p}}_{\mathbf{ne2}})$). Obviously, in this case, energy and momentum conservation laws should be applied in a very general way, taking into account (internal electrical and acoustical) states in electrodes, flying-electron energy, and energy of x-ray radiation from the positive electrode.

In order to have a more complete energy conservation picture of this process we should apply the principal relations (4.2), between (total) kinetic and wave energy and their momentum, $\Delta E_k = -\Delta\tilde{E}$ and $\Delta\mathbf{p} = -\Delta\tilde{\mathbf{p}}$. Let us first apply (4.2) between the states **(0 → 1),** when the electron was in the state of rest (on the negative electrode surface) and just after it left the negative electrode,

$$\Delta E_k = -\Delta\tilde{E} \Leftrightarrow \Delta\left[\sum_{(i)} E^i{}_k\right] = -\Delta\left[\sum_{(i)} \tilde{E}^i\right] \Rightarrow$$

$$\left[(E_{ek1} + \tilde{E}_{nek1} + \tilde{E}_{pek1}) - (E_{eko} + E_{neko} + E_{peko})\right] = -\left[(\tilde{E}_{e1} + E_{ne1} + \tilde{E}_{pe1}) - (E_{eo} + E_{neo} + E_{peo})\right] \Rightarrow$$

$$E_{ek1} = -(E_{e1} + E_{ne1} + E_{pe1}) = (\gamma_1 - 1)mc^2 = -eU - \delta E \leq -eU \;, \; \delta E \geq 0$$

$$\Delta p = -\Delta\tilde{p} \Leftrightarrow \Delta\left[\sum_{(i)} p^i\right] = -\Delta\left[\sum_{(i)} \tilde{p}^i\right] \Rightarrow$$

$$\left[(p_{e1} + p_{nek1} + p_{pe1}) - (p_{eo} + p_{neo} + p_{peo})\right] = -\left[(\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1}) - (\tilde{p}_{eo} + \tilde{p}_{neo} + \tilde{p}_{peo})\right] \Rightarrow$$

$$p_{e1} = -(\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1}) = mv_{e1}$$

Since electrical circuit between electrodes, when the electron passes from the negative to the positive electrode, is always closed, all internal waving phenomena (or currents) in one electrode will be at the same time present in the opposite electrode. By applying again the principal relations between kinetic and wave energy and their momentum, $\mathbf{\Delta E_k = -\Delta\tilde{E}}$ and $\mathbf{\Delta p = -\Delta\tilde{p}}$, between the states **(1 → 2),** when electron was flying between two electrodes, and just after it stroke the positive electrode, we get:

$$\Delta E_k = -\Delta\tilde{E} \Leftrightarrow \Delta\left[\sum_{(i)} E^i{}_k\right] = -\Delta\left[\sum_{(i)} \tilde{E}^i\right] \Rightarrow$$

$$\left[(E_{ek2} + E_{nek2} + E_{pek2}) - (E_{ek1} + E_{nek1} + E_{pek1})\right] =$$
$$= -\left[(\tilde{E}_{e2} + \tilde{E}_{ne2} + \tilde{E}_{pe2} + \tilde{E}_{x2}) - (\tilde{E}_{e1} + \tilde{E}_{ne1} + \tilde{E}_{pe1})\right] \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} 0 \le \tilde{E}_{x2} = hf_x = E_{ek1} - (\tilde{E}_{ne1} + \tilde{E}_{pe1}) - (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \le E_{ek1} \\ 0 \le \tilde{E}_{x2} - E_{ek1} = hf_x - E_{ek1} = -(\tilde{E}_{ne1} + \tilde{E}_{pe1}) - (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \le 0 \\ -(\tilde{E}_{ne1} + \tilde{E}_{pe1}) \le (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \\ E_{ke1} - (\tilde{E}_{ne1} + \tilde{E}_{pe1}) \le hf_x + (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \end{array}\right\}$$

$$\Delta p = -\Delta\tilde{p} \Leftrightarrow \Delta\left[\sum_{(i)} p^i\right] = -\Delta\left[\sum_{(i)} \tilde{p}^i\right] \Rightarrow$$

$$\left[(p_{e2} + p_{ne2} + p_{pe2}) - (p_{e1} + p_{nek1} + p_{pe1})\right] = -\left[(\tilde{p}_{e2} + \tilde{p}_{ne2} + \tilde{p}_{pe2} + \tilde{p}_{x2}) - (\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1})\right] \Rightarrow$$

$$\left[0 - p_{e1}\right] = -\left[(0 + \tilde{p}_{ne2} + \tilde{p}_{pe2} + \tilde{p}_{x2}) - (\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1})\right], p_{e1} = \tilde{p}_{e1} \Rightarrow$$

$$\tilde{p}_{x2} = hf_x / c = p_{e1} - (\tilde{p}_{ne2} + \tilde{p}_{pe2}) - (\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1}) = -(\tilde{p}_{ne2} + \tilde{p}_{pe2}) - (\tilde{p}_{ne1} + \tilde{p}_{pe1})$$

$$c\tilde{p}_{x2} = hf_x = \tilde{E}_{x2} = -c(\tilde{p}_{ne2} + \tilde{p}_{pe2}) - c(\tilde{p}_{ne1} + \tilde{p}_{pe1}) =$$
$$= E_{ek1} - (\tilde{E}_{ne1} + \tilde{E}_{pe1}) - (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \le E_{ek1} = (\gamma_1 - 1)mc^2 = -eU - \delta\tilde{E} \le -eU$$

$$0 < f_x \le \frac{E_{ek1}}{h} = \frac{(\gamma_1 - 1)mc^2}{h} = \frac{-eU}{h}.$$

Eventually, when the electron (as a particle) strikes the positive electrode, its final kinetic energy ($\mathbf{E_{ek1}}$) will be (partially or fully) transformed into radiation of x-rays (and certain part of the same energy would also create transient electric current and mechanical oscillations in electrodes circuit). The maximal frequency of radiated x-ray photons will be,

$$\left(f_x\right)_{max.} = -eU/h = \frac{(\gamma_1 - 1)mc^2}{h} = \frac{E_{ek1}}{h} > \frac{m_e v_{el}{}^2}{2}.$$

Obviously, we got the well-known frequency of x-rays $(\mathbf{f_x})_{max.} = -\mathbf{eU}/\mathbf{h}$, explaining this way that (**only and exclusively**) the relativistic motional energy of a particle $\mathbf{E_{ek1}}$, which is fully equal to the particle wave energy $\tilde{\mathbf{E}}_{\mathbf{e1}}$, is (fully or partially) radiated in the form of x-rays, $\mathbf{hf_x}$. If accelerated electrons have sufficiently high striking speeds, there is no way to show that x-ray energy could be calculated using classical mechanics kinetic energy expression $\mathbf{m_e v_e}^2/\mathbf{2}$, which indirectly indicates that traditional (non-relativistic) Schrödinger equation would also be inapplicable to this case (since in Schrödinger's equation, particle kinetic energy is treated as $\mathbf{m_e v_e}^2/\mathbf{2}$). In fact, we also see that contemporary quantum mechanical concept (or model) of a particle, which includes its rest mass and rest energy as an integral part of its wave packet, is unacceptable (at least in this case), since we can calculate and measure that only relativistic motional energy is transformed into x-rays and waving perturbations in electrodes (and all of them clearly belong to de Broglie or matter waves, being easily measurable and with deterministic nature).

The typical x-ray spectrum has a form of continuous spectral distribution because of many reasons such as: negative electrode of x-ray tube is heated in order to facilitate electrons emission (modulating the speed of electrons), external electrical circuit presents a resistive and reactive electric impedance, producing certain energy dissipation and oscillating-current effects, there are also associated acoustic phenomena in electrodes, and such process in reality is also different in many other (x-ray tube design) details in comparison with idealized case of this example. The only common valid conclusion for all x-ray devices is that the maximal experimentally measured x-ray frequency is exactly equal to the frequency calculated in this example $\mathbf{f_x} \leq -\mathbf{eU}/\mathbf{h}$, confirming that de Broglie matter waves present only a form of kinetic energy of ordinary vibrations (of electromagnetic, mechanical and/or any other nature), without any participation of rest masses.

[♣ <u>COMMENTS & FREE-THINKING CORNER:</u> Until here, all attempts are made to prove that matter wave energy (of de Broglie waves) belongs to kinetic or motional energy of particles, quasiparticles including different aspects of waves and oscillations, and that the rest mass does not belong directly to mater wave energy (but, within certain limits also presents an absorber or emitter of a wave energy). We also know that going deeper into the matter and particle structure, we gradually find more complex field and wave structures, which only conditionally present particles (with possible content of rest mass), and again, analyzing them structurally, we find new energy content in the form of some other waves and fields inside (somehow self-stabilized in a closed form of standing waves). What should be the rest mass, if internal building constituents of every particle are waves and fields, or motional energy in a form of stationary, standing waves, or other kind of self-resonant states? Most probably, this is the place for understanding intrinsic, self-sustaining, rotational field nature of elementary wave ingredients of our universe. Simply, somehow, all elementary matter structures, called elementary particles (electrons, protons, neutrons, etc.) create stable, closed, limited-space domains (like toroid, rotating rings etc.) of internally rotating, stationary and standing wave field formations. Once, when such self-sustaining, space-limited vortex domain is created, it behaves like an elementary particle (or if wave "sublimating and solidifying" process is not fully finished, we have a quasi-particle, a photon, etc.). Since in our universe it is natural that such closed, rotating wave structures can be created, remaining stable during certain time (or during a very long time), and since inside such structures there is always certain energy content, we are able to associate the rest mass to such objects (knowing the exact proportionality between mass and energy, from the Relativity Theory). The visible external signs or marks indicating that such rotating structures are the reality of our world are orbital moments and spin characteristics of all elementary particles (as well as de Broglie matter waves, spontaneous radioactivity, different fields and forces). Of course, this is still an intuitive and speculative concept, but sufficiently good as a starting platform for understanding the meaning of the rest mass. Nevertheless, it was a conceptual mistake of quantum mechanical matter waves and wave function modeling to include unconditionally the stable rest mass of a particle into a matter wave modeling, since only a space-time variable and non-stationary energy flow creates free-propagating matter waves. ♣]

4.2. INTERACTION MODELING

Before we enter the mathematics of wave functions, let us establish the last limits of traditional analysis regarding conservation laws and interactions where mater wave phenomenology is involved. The most general case of (arbitrary) interactions between particles and waves (in any mutual combination of participants) is presented in the table T.4.2.1.

T.4.2.1.

Elements entering interaction	Interaction field	Remaining elements after interaction
Particles: $\left\{E^i{}_k,\ p^i\right\},\ E^i{}_0$ $i \in [1, 2, 3, \ldots P]$	**?!?!**	**Particles**: $\left\{E^m{}_k,\ p^m\right\},\ E^m{}_0$ $m \in [1, 2, 3, \ldots M]$
Waves: $\left\{\tilde{E}^j,\ \tilde{p}^j\right\}$ $j \in [1, 2, 3, \ldots Q]$		***Waves***: $\left\{\tilde{E}^n,\ \tilde{p}^n\right\}$ $n \in [1, 2, 3, \ldots N]$

Energy and momentum conservation in cases from T.4.2.1 can be presented as follows:

$$
\begin{aligned}
&E_{total} = E = E_k + E_0 = \sum_{(i)} (E^i{}_k + E^i{}_0) + \sum_{(j)} \tilde{E}^j = \sum_{(m)} (E^m{}_k + E^m{}_0) + \sum_{(n)} \tilde{E}^n \Leftrightarrow \\
&\Leftrightarrow \left(\sum_{(i)} E^i{}_k - \sum_{(m)} E^m{}_k\right) + \left(\sum_{(i)} E^i{}_0 - \sum_{(m)} E^m{}_0\right) = -\left[\sum_{(j)} \tilde{E}^j - \sum_{(n)} \tilde{E}^n\right] \Leftrightarrow \\
&\Leftrightarrow \Delta E = \Delta E_k + \Delta E_0 = -\Delta \tilde{E}\ , \\
&(E_{total} = E = \gamma mc^2, E_k = (\gamma - 1)mc^2,\ E_0 = mc^2,\ \tilde{E} = E_k = hf = pu)
\end{aligned}
\tag{4.8-1}
$$

$$
\begin{aligned}
&\sum_{(i)} p^i + \sum_{(j)} p^j = \sum_{(m)} p^m + \sum_{(n)} p^n \Leftrightarrow \\
&\Leftrightarrow \sum_{(i)} p^i - \sum_{(m)} p^m = -\left[\sum_{(j)} \tilde{p}^j - \sum_{(n)} \tilde{p}^n\right] \Leftrightarrow \\
&\Leftrightarrow \Delta p = -\Delta \tilde{p}
\end{aligned}
\tag{4.8-2}
$$

$$
\Delta E_k = \sum_{(i)} E^i{}_k - \sum_{(m)} E^m{}_k = \Delta\left[\sum_{(i,m)} E^{i,m}{}_k\right],\ \Delta E_0 = \sum_{(i)} E^i{}_0 - \sum_{(m)} E^m{}_0 = \Delta\left[\sum_{(i,m)} E^{i,m}{}_0\right]
$$

$$
\Delta \tilde{E} = \sum_{(j)} \tilde{E}^j - \sum_{(n)} \tilde{E}^n = \Delta\left[\sum_{(j,n)} \tilde{E}^{j,n}\right]
$$

$$
\Delta p = \sum_{(i)} p^i - \sum_{(m)} p^m = \Delta\left[\sum_{(i,m)} p^{i,m}\right],\quad \Delta \tilde{p} = \sum_{(j)} \tilde{p}^j - \sum_{(n)} \tilde{p}^n = \Delta\left[\sum_{(j,n)} \tilde{p}^{j,n}\right]
$$

If the objects entering an interaction have orbital moments (both external and macro moments, $\mathbf{L} = \mathbf{L}_{external}$, and internal, intrinsic, or spin orbital moments, $\mathbf{L}_0 = \mathbf{L}_{internal}$), we can present the conservation of orbital moments in a similar way as we did for energy and momentum conservation. By summarizing all conservation laws in a condensed form, it will be:

$$\left.\begin{cases}\Delta E = \Delta E_k + \Delta E_0 = -\Delta\tilde{E} \\ \Delta p = -\Delta\tilde{p} \\ \Delta\mathbf{L} = \Delta\mathbf{L}_{\text{external}} + \Delta\mathbf{L}_{\text{internal}} = -\Delta\tilde{\mathbf{L}} \\ \ldots\ldots\ldots\ldots \end{cases}\right\} \Leftrightarrow \left\{\Delta X = -\Delta\tilde{Y}\right\} \qquad (4.8\text{-}3)$$

If we consider the case of a single object in the phase of transformation, we can address the meaning of inertial (or reaction) forces transforming (4.8-3) into (4.8-4), as for instance:

$$\{\Delta \to d\} \Rightarrow \left.\begin{cases} dE = dE_k + dE_0 = -d\tilde{E} \\ dp = -d\tilde{p} \\ dL = dL_{\text{external}} + dL_{\text{internal}} = -d\tilde{L} \\ \ldots\ldots\ldots\ldots \end{cases}\right\} \Leftrightarrow \left\{dX = -d\tilde{Y}\right\} \Rightarrow$$
$$\Rightarrow \left\{dX = -d\tilde{Y}\right\} / dt \Leftrightarrow \frac{dX}{dt} = -\frac{d\tilde{Y}}{dt} \Leftrightarrow \{\text{action} = \text{reaction in opposite direction}\} \qquad (4.8\text{-}4)$$

4.2.1. Elastic Collisions

Let us now analyze an ideally elastic collision of two objects (for instance between two particles, or between one particle and a photon, similar to the case of Compton effect). Since rest masses of interacting particles do not change, here we only have certain exchange of kinetic energies between collision participants. Since the elastic collision should not be dependent on time-axis direction, we will create the kinetic energy balance in case when particles $\mathbf{m_1}, \mathbf{m_2}$ enter the collision, producing $\mathbf{m_3}, \mathbf{m_4}$ (T.4.3.1), and when (reversing the time-axis direction and assuming that we could have the total process reversibility in all its aspects) particles $\mathbf{m_3}, \mathbf{m_4}$ go backwards, producing $\mathbf{m_1}, \mathbf{m_2}$, (T.4.4).

T.4.3.1.

Elements entering interaction	**Interaction field**	**Remaining elements after interaction**
<u>Object-1</u> $\{E_{k1}, p_1 = m_1v_1\}$ or $\{\tilde{E}_1, \tilde{p}_1\}$	**<u>?!?!</u>**	**<u>Object-3</u>** $\{E_{k3}, p_3 = m_3v_3\}$ or $\{\tilde{E}_3, \tilde{p}_3\}$
<u>Object-2</u> $\{E_{k2}, p_2 = m_2v_2\}$ or $\{\tilde{E}_2, \tilde{p}_2\}$		**<u>Object-4</u>** $\{E_{k4}, p_4 = m_4v_4\}$ or $\{\tilde{E}_4, \tilde{p}_4\}$

T.4.4. (The same interaction as in T.4.3.1, but time-reversed)

Elements entering interaction	Interaction field	Remaining elements after interaction
Object-1 $\{E_{k1},\ p_1 = m_1 v_1\}$ or $\{\tilde{E}_1,\ \tilde{p}_1\}$	**?!?!**	**Object-3** $\{E_{k3},\ p_3 = m_3 v_3\}$ or $\{\tilde{E}_3,\ \tilde{p}_3\}$
Object-2 $\{E_{k2},\ p_2 = m_2 v_2\}$ or $\{\tilde{E}_2,\ \tilde{p}_2\}$		**Object-4** $\{E_{k4},\ p_4 = m_4 v_4\}$ or $\{\tilde{E}_4,\ \tilde{p}_4\}$

If we now apply the energy conservation law on T.4.3.1 and T.4.4, for non-relativistic velocities, it will be:

$$\left\{\begin{matrix} E_{k1}+E_{k2}=E_{kr-12}+E_{kc-12}=E_{k3}+E_{k4} \\ E_{k3}+E_{k4}=E_{kr-34}+E_{kc-34}=E_{k1}+E_{k2} \end{matrix}\right\} \Rightarrow \left\{E_{kr-12}+E_{kc-12}=E_{kr-34}+E_{kc-34}\right\} \Rightarrow$$

$$\left\{\begin{matrix} \frac{m_1 v^2{}_1}{2}+\frac{m_2 v^2{}_2}{2}=\frac{m_1 m_2}{m_1+m_2}\frac{(\vec{v}_1-\vec{v}_2)^2}{2}+\frac{(m_1\vec{v}_1+m_2\vec{v}_2)^2}{2(m_1+m_2)}=\frac{m_3 v^2{}_3}{2}+\frac{m_4 v^2{}_4}{2} \\ \frac{m_3 v^2{}_3}{2}+\frac{m_4 v^2{}_4}{2}=\frac{m_3 m_4}{m_3+m_4}\frac{(\vec{v}_3-\vec{v}_4)^2}{2}+\frac{(m_3\vec{v}_3+m_4\vec{v}_4)^2}{2(m_3+m_4)}=\frac{m_1 v^2{}_1}{2}+\frac{m_2 v^2{}_2}{2} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} E_{kr}=\frac{m_1 m_2}{m_1+m_2}\frac{(\vec{v}_1-\vec{v}_2)^2}{2}=\mu_{r(1-2)}\frac{v^2_{r(1-2)}}{2}=\frac{m_3 m_4}{m_3+m_4}\frac{(\vec{v}_3-\vec{v}_4)^2}{2}=\mu_{r(3-4)}\frac{v^2_{r(3-4)}}{2}, \\ E_{kc}=\frac{(m_1\vec{v}_1+m_2\vec{v}_2)^2}{2(m_1+m_2)}=\frac{(m_3\vec{v}_3+m_4\vec{v}_4)^2}{2(m_3+m_4)}=m_c\frac{v_c^2}{2}=\frac{\vec{P}\vec{v}_c}{2},\ E_{kr}=\mu_r\frac{v_r^2}{2}=\frac{p_r v_r}{2}, \\ v_c=\frac{\vec{P}}{m_c},\ m_c=m_1+m_2=m_3+m_4,\ \mu_r=m_r=\frac{m_i m_j}{m_i+m_j} \\ \vec{P}=\vec{p}_1+\vec{p}_2=\vec{p}_3+\vec{p}_4=m_1\vec{v}_1+m_2\vec{v}_2=m_3\vec{v}_3+m_4\vec{v}_4,\ p_r=\mu_r v_r \\ v^2_{r(1-2)}=(\vec{v}_1-\vec{v}_2)^2,\ v^2_{r(3-4)}=(\vec{v}_3-\vec{v}_4)^2, \\ \frac{m_1 m_2}{m_1+m_2}(\vec{v}_1-\vec{v}_2)^2=\frac{m_3 m_4}{m_3+m_4}(\vec{v}_3-\vec{v}_4)^2,\ \frac{d\vec{p}_1+d\vec{p}_2}{m_1+m_2}=\frac{d\vec{p}_3+d\vec{p}_4}{m_3+m_4}=\frac{d\vec{P}}{m_c} \end{matrix}\right\}$$

or, equivalent expressions in case of relativistic velocities will be:

$$\Rightarrow \left\{ \begin{array}{l} E_{kr} = \dfrac{\gamma_1\gamma_2 m_1 m_2}{\gamma_1 m_1 + \gamma_2 m_2}\left[\dfrac{(\vec{v}_1 - \vec{v}_2)^2}{1+\sqrt{1-v^2_{r(1-2)}/c^2}}\right] = \dfrac{\gamma_3\gamma_4 m_3 m_4}{\gamma_3 m_3 + \gamma_4 m_4}\left[\dfrac{(\vec{v}_3 - \vec{v}_4)^2}{1+\sqrt{1-v^2_{r(3-4)}/c^2}}\right] = \\ = \dfrac{\gamma_r \mu_r v_r^2}{1+\sqrt{1-v_r^2/c^2}} = E_{k1} + E_{k2} - E_{kc} = E_{k3} + E_{k4} - E_{kc} = \int_{(1)}^{(2)} \vec{F}_r d\vec{r}_{12} = \int_{(3)}^{(4)} \vec{F}_r d\vec{r}_{34}, \\ v^2_{r(1-2)} = (\vec{v}_1 - \vec{v}_2)^2, \quad v^2_{r(3-4)} = (\vec{v}_3 - \vec{v}_4)^2, \ \vec{F}_r = \dfrac{d\vec{p}_{r(1-2)}}{dt} = \dfrac{d\vec{p}_{r(3-4)}}{dt}, \\ E_{kc} = \dfrac{P_c v_c}{1+\sqrt{1-v_c^2/c^2}} = \dfrac{P_c^2}{(1+\sqrt{1-v_c^2/c^2})\gamma_c m_c} = \dfrac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}} = \\ = E_{k1} + E_{k2} - E_{kr} = E_{k3} + E_{k4} - E_{kr} \\ \vec{v}_c = \dfrac{(\sum \vec{p})c^2}{\sum E} = \dfrac{\vec{P}}{\gamma_c m_c}, \quad \dfrac{\sum E}{c^2} = \gamma_c m_c = \gamma_1 m_1 + \gamma_2 m_2 = \gamma_3 m_3 + \gamma_4 m_4 \\ \vec{P} = \vec{p}_1 + \vec{p}_2 = \vec{p}_3 + \vec{p}_4 = \gamma_1 m_1 \vec{v}_1 + \gamma_2 m_2 \vec{v}_2 = \gamma_3 m_3 \vec{v}_3 + \gamma_4 m_4 \vec{v}_4 = \gamma_c m_c \vec{v}_c = \vec{P}_c, \\ E_{ki} = \dfrac{\gamma_i m_i v^2{}_i}{1+\sqrt{1-v_i^2/c^2}} = \dfrac{p_i v_i}{1+\sqrt{1-v_i^2/c^2}} = \dfrac{p_i^2}{(1+\sqrt{1-v_i^2/c^2})\gamma_i m_i} = p_i u_i = \tilde{E}_i \end{array} \right\}$$

(4.8-5)

As we can see from all energy and momentum conservation relations valid for Classical Mechanical case (or for low velocities), and for cases with relativistic velocities (4.8-5), it is relatively complicated to use such expressions and find general solutions for elastic impacts. The other opportunity is to use the energy conservation in its differential form, since mathematically we have the same and very simple expression, both for low and high velocity movements (and instead of kinetic energies we can also take total energies including state of rest elements), as for instance:

$$\begin{array}{l} d\{E_{k1} + E_{k2} = E_{kr} + E_{kc} = E_{k3} + E_{k4}\} \Leftrightarrow d\{E_1 + E_2 = E_{kr} + E_{kc} + E_{oc} = E_3 + E_4\} \Leftrightarrow \\ \Leftrightarrow v_1 dp_1 + v_2 dp_2 = v_r dp_r + v_c dp_c = v_3 dp_3 + v_4 dp_4, \\ dE_{kr} = \vec{v}_r d\vec{p}_r = (\vec{v})^2_{r(1-2)} d(\gamma_r \mu_{r(1-2)}) + \vec{p}_{r(1-2)} d\vec{v}_{r(1-2)} = (\vec{v})^2_{r(3-4)} d(\gamma_r \mu_{r(3-4)}) + \vec{p}_{r(3-4)} d\vec{v}_{r(3-4)}, \\ v^2_{r(1-2)} = (\vec{v}_1 - \vec{v}_2)^2, \ v^2_{r(3-4)} = (\vec{v}_3 - \vec{v}_4)^2, \\ E_i = E_{0i} + E_{ki}, E_{0i} = \text{const.}, dE_{0i} = 0, \ p_i = \gamma_i m_i v_i, \ \gamma_i = (1 - v_i^2/c^2)^{-0.5}, \ p_{ri} = \gamma_i \mu_i v_{ri}, \end{array} \quad (4.8\text{-}6)$$

Doing that way we can also address the particular dynamic forces acting in every phase of impacts,

$$\begin{array}{l} \{v_1 dp_1 + v_2 dp_2 = v_r dp_r + v_c dp_c = v_3 dp_3 + v_4 dp_4\}/dt \Rightarrow \\ \vec{v}_1 \dfrac{d\vec{p}_1}{dt} + \vec{v}_2 \dfrac{d\vec{p}_2}{dt} = \vec{v}_r \dfrac{d\vec{p}_r}{dt} + \vec{v}_c \dfrac{d\vec{p}_c}{dt} = \vec{v}_3 \dfrac{d\vec{p}_3}{dt} + \vec{v}_4 \dfrac{d\vec{p}_4}{dt} \Rightarrow \\ \vec{v}_1 \vec{F}_1 + \vec{v}_2 \vec{F}_2 = \vec{v}_r \vec{F}_r + \vec{v}_c \vec{F}_c = \vec{v}_3 \vec{F}_3 + \vec{v}_4 \vec{F}_4 \end{array} \quad (4.8\text{-}7)$$

For instance, only the mutual forces between moving masses $\mathbf{m_1}$ and $\mathbf{m_2}$, and $\mathbf{m_3}$ and $\mathbf{m_4}$, in their Center of Mass System (including Newton-Coulomb attractive forces) will be:

$$\vec{F}_{r(1-2)} = \frac{d\vec{p}_{r(1-2)}}{dt} = -\vec{F}_{r(2-1)} = \frac{d\vec{p}_{r(2-1)}}{dt}\ ,\ \vec{F}_{r(3-4)} = \frac{d\vec{p}_{r(3-4)}}{dt} = -\vec{F}_{r(4-3)} = \frac{d\vec{p}_{r(4-3)}}{dt},$$
$$E_{kr} = \int_{(1)}^{(2)} \vec{F}_{r(1\text{-}2)} d\vec{r}_{12} = \int_{(3)}^{(4)} \vec{F}_{r(3\text{-}4)} d\vec{r}_{34}.$$

The force acting in the center of mass (from the point of view from Laboratory System) where the concentrated mass $\mathbf{m_c}$ is, will be,

$$\vec{F}_c = \frac{\mathbf{d\vec{p}_c}}{\mathbf{dt}} = \frac{\mathbf{d\vec{P}}}{\mathbf{dt}} = \vec{F}_1 + \vec{F}_2 ,$$

And, of course, external forces acting on each particle (from the point of view from Laboratory System), will be, respectively,

$$\vec{F}_1 = \frac{\mathbf{d\vec{p}_1}}{\mathbf{dt}}, \vec{F}_2 = \frac{\mathbf{d\vec{p}_2}}{\mathbf{dt}}, \vec{F}_3 = \frac{\mathbf{d\vec{p}_3}}{\mathbf{dt}}, \vec{F}_4 = \frac{\mathbf{d\vec{p}_4}}{\mathbf{dt}} .$$

From (4.8-5)-(4.8-7) we can also conclude that the most important "transient laboratory place" where one interaction happens is the Center of Mass System, regarding transformations of reduced mass $\boldsymbol{\mu_r}$ and relative velocity $\mathbf{v_r}$,

$$\begin{aligned}
&\vec{v}_r \frac{d\vec{p}_r}{dt} = \vec{v}_r \vec{F}_r = (\vec{v})^2_{r(1-2)} \frac{d(\gamma_{r(1-2)}\mu_{r(1-2)})}{dt} + \vec{p}_{r(1-2)} \frac{d\vec{v}_{r(1-2)}}{dt} = \\
&= (\vec{v})^2_{r(3-4)} \frac{d(\gamma_{r(3-4)}\mu_{r(3-4)})}{dt} + \vec{p}_{r(3-4)} \frac{d\vec{v}_{r(3-4)}}{dt} \Leftrightarrow \\
&\Leftrightarrow \frac{dE_{kr}}{dt} = \vec{v}_r \vec{F}_r = (\vec{v})^2_{r(1-2)} \frac{d(\gamma_{r(1-2)}\mu_{r(1-2)})}{dt} + \vec{p}_{r(1-2)} \vec{a}_{r(1-2)} = \\
&= (\vec{v})^2_{r(3-4)} \frac{d(\gamma_{r(3-4)}\mu_{r(3-4)})}{dt} + \vec{p}_{r(3-4)} \vec{a}_{r(3-4)} \\
&\vec{p}_r = \gamma_r \mu_r \vec{v}_r,\ \vec{a}_{r(i-j)} = \frac{d\vec{v}_{r(i-j)}}{dt}\ (=)\,\text{acceleration}.
\end{aligned} \tag{4.8-8}$$

In reality, reduced mass $\gamma_r\mu_r$ is a kind of virtual object, or kind of de Broglie matter wave packet (placed in the space between interacting objects), and presents concentrated field or wave energy in process of continuous transformation ($\mathbf{dE_{kr}} = \mathbf{\vec{v}_r d\vec{p}_r} = \mathbf{\vec{v}_r \vec{F}_r dt} = c^2 d(\gamma_r \mu_r v_r) = \mathbf{hdf_r} = -\mathbf{d\tilde{E}_r}$), created by the mutually approaching objects, which has at least two different field or force components, $dE_{kr} = (\vec{v})^2_r d(\gamma_r\mu_r) + \vec{p}_r d\vec{v}_r$. Here is the part of the explanation why and how a single object (electron, or photon, etc.) coincidently passes two slits "making interference and diffraction with itself" on the opposite side of the diffraction plane (without involving any of Quantum Mechanical miracles). Also, in cases when energy $\mathbf{E_{kr}}$ reaches certain threshold level/s, we experience the creation of new particles (not initially entering into reaction), but this case is outside the presently analyzed framework of elastic collisions.

It is very important to notice that in cases of ideally plastic collisions, when initial objects form only one object (after collision), the energy E_{kr} will again be represented (as a transient wave or field energy) only in the close time and space vicinity, just before the act of a collision, and fully absorbed, or injected into internal rest mass, or state of rest energy of the newly created (single) object, which remains after plastic collision (represented by m_c). This should be a kind of direct wave energy to mass transformation example ($\mu_r \to m_c$), which has never been seen or explained from that point of view (in traditional analyses of collisions).

*[♣ COMMENTS & FREE-THINKING CORNER: The relativity theory also shoved or implicated that there is a simple relation of direct proportionality between any mass and total energy that could be produced by fully transforming that mass into radiation $\mathbf{E_0 = mc^2}$, $\mathbf{E_{tot.}} = \gamma\,\mathbf{mc^2} = \mathbf{E_0} + \mathbf{E_k} = E_0 + (\gamma - 1)mc^2 = \sqrt{\mathbf{E_0^2 + p^2c^2}}$. The most important conceptual understanding of frequency dependant matter wave energy, which is fully equivalent to particle motional, or kinetic energy, is related to the fact that total photon energy can be expressed as a product between Planck's constant and frequency of the photon wave packet, $\mathbf{E_f = hf}$, and consequently (since there is known proportionality between mass and energy), the photon momentum was correctly found as $\mathbf{p_f = hf / c = m_f c}$ (and proven applicable and correct in analyzes of different interactions). Since a photon has certain energy, we should be able to present this energy in two different ways, as for instance: $E_f = hf = \sqrt{E_{0f}^2 + p_f^2c^2} = p_f c = E_{kf}$. In reality, since photon rest mass equals zero, there is only photon kinetic energy $\mathbf{E_f = hf = p_f c = E_{kf} = m_f c^2}$, and in number of applications this concept (and all equivalency relations for photon energy and momentum) showed to be correct. Going backwards, we can apply the same conclusion, or analogy, to any real particle (which has a rest mass), accepting that particle kinetic energy is presentable as the product between Planck constant and characteristic particle's matter wave frequency $E_k = (\gamma - 1)mc^2 = \tilde{E} = hf$. Doing that we are able to find the frequency of de Broglie matter waves as, $f = E_k / h = (\gamma - 1)mc^2 / h = \tilde{E}/h$. Now, we can find the phase velocity of matter waves as, $u = \lambda f = \frac{h}{p} f = \frac{E_k}{p} = \frac{(\gamma - 1)mc^2}{\gamma mv} = \frac{v}{1 + \sqrt{1 - v^2/c^2}}$. The relation between phase and group velocity of a matter wave packet is also known in the form, $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}$, and combining two latest forms of phase and group velocities we can get: $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = u(1 + \sqrt{1 - v^2/c^2})$, implicating validity of the following differential relations: $d\tilde{E} = d\left[(\gamma - 1)mc^2\right] = mc^2 d\gamma = \mathbf{hdf = vdp = d(pu)}$, and practically confirming mathematical consistency of all above introduced equivalency and analogy based relations (named in this paper as **PWDC** = Particle-Wave Duality Code). Since the above given equivalency relations are found valid only if we use the wave packet model (as a replacement for a particle in motion), consequently, we have an argument more to say that matter waves (or wave functions) should exist as forms of harmonic, modulated sinusoidal signals (naturally satisfying the rules of Fourier signal and spectrum analysis).*

It is of essential importance to notice that a rest mass (or rest energy) does not belong to matter wave energy (opposite to many current presentations in modern physics books, regarding matter wave properties). Analyzing Compton Effect and many other elementary interactions known in Quantum Mechanics, can easily prove this statement (that only kinetic or motional energy presents the mater wave energy), as follows. ♣]

4.2.2. Example 3: Collision photon-particle (Compton Effect)

Let us now apply the energy and momentum conservation laws (T.4.2.1, T.4.3.1, T.4.4 and equations (4.2), (4.3), (4.8-3), (4.8-5)), in the case when a photon (elastically) collides with an electron, which is in a state of rest (see Fig.4.2).

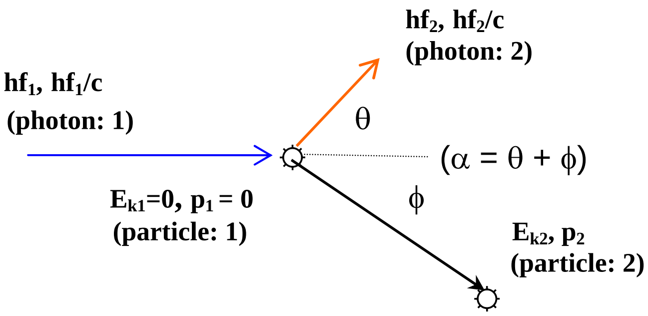


Fig. 4.2 Photon-Particle collision

After an elastic collision, the incident photon **(hf_1, hf_1/c)** loses a part of its energy being transformed into a new photon **(hf_2, hf_2/c)**, and the electron that was in the state of rest (**$E_{k1}=0$, $p_1=0$**) gets certain motional energy (**E_{k2}, p_2**). All particle and photon states before collision will be marked using index "1" and after collision using index "2", as presented in the Fig.4.2 and in the following table T.4.5.

T.4.5.

	States long before collision (indexing: 1)	States just after collision (indexing: 2)
Photon wave energy	$\tilde{E}_{f1} = hf_1, \tilde{p}_{f1} = hf_1 / c$	$\tilde{E}_{f2} = hf_2, \tilde{p}_{f2} = hf_2 / c$
Electron kinetic energy	$E_{k1} = 0, p_1 = 0$	$E_{k2} = (\gamma - 1)mc^2 = p_2 u_2 = \tilde{E}_2 \cong mv_2^2 / 2,$ $p_2 = \gamma m v_2 = \tilde{p}_2, \gamma = (1 - v_2^2 / c^2)^{-0.5} \cong$ $\cong mv_2$
Total kinetic energy	hf_1	$hf_2 + E_{k2}$
Total energy & momentum (electron & photon)	$mc^2 + hf_1$ $\overrightarrow{\frac{hf_1}{c}} = \frac{hf_1}{c}\vec{e}_1$	$\gamma mc^2 + hf_2$ $\vec{p}_2 + \overrightarrow{\frac{hf_2}{c}} = \vec{p}_2 + \frac{hf_2}{c}\vec{e}_2$
Total energy & momentum, only for electron	$E_1' = mc^2$ $P_1' = 0$	$E_2' = hf_1 - hf_2 + mc^2 = \gamma mc^2 \cong$ $\cong mc^2 + mv_2^2 / 2$ $\vec{P}_2' = \overrightarrow{\frac{hf_1}{c}} - \overrightarrow{\frac{hf_2}{c}} = \gamma m\vec{v}_2 \cong m\vec{v}_2$
Differential energy balance	$dE_{k1} + d\tilde{E}_{f1} = dE_{kc} + d\tilde{E}_{kr} = dE_{k2} + d\tilde{E}_{f2}$	

We can now connect *the* electron's total energy and momentum $\mathbf{E_2' = hf_1 - hf_2 + mc^2}$ and $\vec{\mathbf{P}}_2' = \overrightarrow{\frac{\mathbf{hf_1}}{\mathbf{c}}} - \overrightarrow{\frac{\mathbf{hf_2}}{\mathbf{c}}}$, using the relativistic relation $\mathbf{(E'_2)^2 - (\vec{P}'_2)^2 c^2 = m^2c^4}$, and find Compton wavelength shift, as follows,

$$\mathbf{(hf_1 - hf_2 + mc^2)^2 - (\frac{hf_1}{c}\vec{e}_1 - \frac{hf_2}{c}\vec{e}_2)^2 c^2 = m^2c^4},$$
$$\mathbf{2h^2 f_1 f_2 (1 - \cos\theta) = 2h(f_1 - f_2)mc^2},$$
$$\frac{1}{f_2} - \frac{1}{f_1} = \frac{h}{mc^2}(1-\cos\theta),\ \lambda_1 = c/f_1, \lambda_2 = c/f_2, x = \frac{hf_1}{mc^2},$$
$$hf_2 = \frac{hf_1}{1 + x(1-\cos\theta)},\ \lambda_2 = \lambda_1\left[1 + x(1-\cos\theta)\right],$$
$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta) = \lambda_{\text{Compton}}(1-\cos\theta),$$
$$\cos\theta = -1, \theta = \pi \Rightarrow (\Delta\lambda)_{\max.} = \frac{2h}{mc}$$

For the total motional energy of the electron (just) after collision we can get,

$$E_{k2} = (\gamma - 1)mc^2 = hf_1 - hf_2 = hf_e = hf_1 \frac{x(1-\cos\theta)}{1 + x(1-\cos\theta)} = hf_1 \frac{2x \cdot \cos^2\phi}{(1+x)^2 - x^2 \cdot \cos^2\phi} =$$
$$= p_2 u_2 = \frac{p_2 v_2}{1 + \sqrt{1 - v_2{}^2/c^2}} = \frac{m_2 v_2^2}{(1 + \sqrt{1 - v_2{}^2/c^2})\sqrt{1 - v_2{}^2/c^2}} = E_{ke} = \tilde{E}_e,$$
$$\cos\theta = -1, \theta = \pi \Rightarrow (E_{k2})_{\max.} = hf_1 \frac{2x}{1+2x}$$

Traditional analysis of Compton Effect does not take into account any wave energy or wave momentum present in the transition zone of the collision process. The objective of this example is to show that there is certain (hidden) phenomenology, which deals with inertial action-reaction forces, here directly related to the wave energy and wave momentum existing in a close time and space vicinity of a collision event. Let us first apply conservation laws (4.5), (4.8-3) and (4.8-5), in order to get all important energy members,

$$\mathbf{E_{tot.} = hf_1 + mc^2 = E_{0C} + E_{kc} + E_{kr} = hf_2 + \gamma\, mc^2},$$
$$\mathbf{hf_1 = E_{kc} + E_{kr} = hf_2 + (\gamma - 1)\, mc^2 = hf_2 + E_{k2}},$$
$$\mathbf{E_{0C} = m_c c^2,\ E_c = E_{0C} + E_{kc} = \gamma_c m_c c^2},$$
$$\mathbf{\tilde{p}_{f1} = \frac{hf_1}{c}\vec{e}_1 = \frac{hf_2}{c}\vec{e}_2 + \vec{p}_2,\ |\vec{e}_1| = |\vec{e}_2| = 1}$$

$$\vec{v}_c = \frac{(\sum \vec{p})c^2}{\sum E} = \frac{(\frac{hf_1}{c}\vec{e}_1)c^2}{hf_1 + mc^2} = \frac{\frac{hf_1}{c}\vec{e}_1}{m + \frac{hf_1}{c^2}} = \frac{c\vec{e}_1}{1 + \frac{mc^2}{hf_1}} = \frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2}} c\vec{e}_1 = \frac{\vec{v}_2 + \frac{hf_2}{\gamma mc^2} c\vec{e}_2}{1 + \frac{hf_2}{\gamma mc^2}} \Rightarrow$$

$$\Rightarrow \cos\alpha = \frac{\left[\frac{hf_1(1 + \frac{hf_2}{\gamma mc^2})}{hf_1 + mc^2}\right]^2 - \left(\frac{hf_2}{\gamma mc^2}\right)^2 - (\frac{v_2}{c})^2}{2\frac{v_2}{c}\frac{hf_2}{\gamma mc^2}}$$

$$\gamma_c m_c = \frac{\sum E}{c^2} = \frac{hf_1 + mc^2}{c^2} = m + \frac{hf_1}{c^2} = \frac{hf_2 + \gamma mc^2}{c^2} = \gamma m + \frac{hf_2}{c^2},$$

$$m_c = m\sqrt{1 + 2\frac{hf_1}{mc^2}} = m\left[1 + \frac{hf_2}{mc^2}\sqrt{1 - v_2^2/c^2}\right]\sqrt{\frac{1 - v_c^2/c^2}{1 - v_2^2/c^2}}, \quad E_{oc} = m_c c^2$$

$$\gamma_c = \frac{1}{\sqrt{1 - \frac{v_c^2}{c^2}}} = \frac{1 + \frac{hf_1}{mc^2}}{\sqrt{1 + 2\frac{hf_1}{mc^2}}} = \frac{1 + x}{\sqrt{1 + 2x}}, \gamma = \frac{1}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

$$\vec{p}_c = (\sum \vec{p}) = \frac{\sum E}{c^2}\vec{v}_c = (m + \frac{hf_1}{c^2})\vec{v}_c = \gamma_c m_c \vec{v}_c =$$

$$= \frac{\gamma mc^2 + hf_2}{c^2}\vec{v}_c = \gamma m(\vec{v}_2 + \frac{hf_2}{\gamma mc^2}c\vec{e}_2) = \vec{p}_2 + \frac{hf_2}{c}\vec{e}_2 = \frac{hf_1}{c}\vec{e}_1 = \tilde{p}_{f1},$$

$$p_2 = p_e = \frac{1}{c}\sqrt{E_{k2}(E_{k2} + 2mc^2)} = \frac{1}{c}\sqrt{E_{k2}(E_{k2} + \frac{2hf_1}{x})} =$$

$$= \frac{2hf_1}{c}\frac{(1 + x)\cos\phi}{(1 + x)^2 - x^2\cos^2\phi}, x = \frac{hf_1}{mc^2}.$$

$$E_{kc} = (\gamma_c - 1)m_c c^2 = hf_1 - E_{kr} = \frac{p_c v_c}{1 + \sqrt{1 - v_c^2/c^2}} = \frac{p_c^2}{\gamma_c m_c(1 + \sqrt{1 - v_c^2/c^2})} =$$

$$= hf_1\frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2} + \sqrt{1 + 2\frac{hf_1}{mc^2}}} = \left[1 + \frac{hf_1}{mc^2} - \sqrt{1 + 2\frac{hf_1}{mc^2}}\right]mc^2 = p_c u_c = hf_c =$$

$$= \frac{(\frac{\mathbf{hf_2}}{\mathbf{c}}\vec{\mathbf{e}}_2 + \gamma \mathbf{m}\vec{\mathbf{v}}_2)^2}{(\mathbf{1} + \sqrt{\mathbf{1} - \mathbf{v}_c^2/\mathbf{c}^2})(\gamma\,\mathbf{m} + \frac{\mathbf{hf_2}}{\mathbf{c}^2})} = (\frac{\gamma\,\mathbf{mc}^2}{\mathbf{1} + \sqrt{\mathbf{1} - \mathbf{v}_c^2/\mathbf{c}^2}})\frac{(\frac{\mathbf{hf_2}}{\gamma\,\mathbf{mc}^2})^2 + \frac{\mathbf{v}_2^2}{\mathbf{c}^2} + \mathbf{2}\frac{\mathbf{v}_2}{\mathbf{c}}(\frac{\mathbf{hf_2}}{\gamma\,\mathbf{mc}^2})\cos\alpha}{\mathbf{1} + \frac{\mathbf{hf_2}}{\gamma\,\mathbf{mc}^2}}$$

$$E_{kc(1)} = E_{kc(2)} \Rightarrow$$

$$\left\{ m\sqrt{1+2\frac{hf_1}{mc^2}(\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}}c\vec{e}_1)^2} \cong m\left[1+\frac{hf_2}{mc^2}\sqrt{1-v_2^2/c^2}\right]\sqrt{\frac{1-v_c^2/c^2}{1-v_2^2/c^2}(\frac{\vec{v}_2+\frac{hf_2}{\gamma mc^2}c\vec{e}_2}{1+\frac{hf_2}{\gamma mc^2}})^2} \right\} \Leftrightarrow$$

$$\Leftrightarrow \left\{ \sqrt{1+2\frac{hf_1}{mc^2}(\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}}c\vec{e}_1)^2} \cong \left[1+\frac{hf_2}{mc^2}\sqrt{1-v_2^2/c^2}\right]\sqrt{\frac{1-v_c^2/c^2}{1-v_2^2/c^2}(\frac{\vec{v}_2+\frac{hf_2}{\gamma mc^2}c\vec{e}_2}{1+\frac{hf_2}{\gamma mc^2}})^2} \right\}$$

$$E_{kr} = hf_1 - E_{kc} = hf_2 + E_{k2} - E_{kc} = \frac{p_r v_r}{1+\sqrt{1-v_r^{\,2}/c^2}} = hf_r =$$

$$= \frac{p_r^2}{\gamma_r \mu_r (1+\sqrt{1-v_r^{\,2}/c^2})} = p_r u_r = \frac{m(\frac{hf_1}{c^2})}{m+\frac{hf_1}{c^2}}\left[\frac{(\vec{0}-\vec{c})^2}{1+\sqrt{1-c^2/c^2}}\right] = mc^2\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} =$$

$$= \frac{\gamma m(\frac{hf_2}{c^2})}{\gamma m+\frac{hf_2}{c^2}}\left[\frac{(\vec{v}_2-\vec{c})^2}{1+\sqrt{1-v_{r2}^2/c^2}}\right] = (\frac{mc^2}{1+\sqrt{1-v_{r2}^2/c^2}})\frac{\frac{hf_2}{mc^2}(1+\frac{v_2^2}{c^2}-2\frac{v_2}{c}\cos\alpha)}{1+\frac{hf_2}{\gamma mc^2}} =$$

$$= hf_2 + \frac{p_2 v_2}{1+\sqrt{1-v_2^{\,2}/c^2}} - (\frac{\gamma mc^2}{1+\sqrt{1-v_c^2/c^2}})\frac{(\frac{hf_2}{\gamma mc^2})^2+\frac{v_2^2}{c^2}+2\frac{v_2}{c}(\frac{hf_2}{\gamma mc^2})\cos\alpha}{1+\frac{hf_2}{\gamma mc^2}},$$

$$v_r^2 = v_{r1}^2 = (0-\vec{c})^2 = c^2,\ v_{r2}^2 = (\vec{v}_2-\vec{c})^2\ ,$$

$$E_{kr(1)} = E_{kr(2)} \Rightarrow \left\{ \frac{m\frac{hf_1}{c^2}}{m+\frac{hf_1}{c^2}}(0-c)^2 \cong \frac{\gamma m\frac{hf_2}{c^2}}{\gamma m+\frac{hf_2}{c^2}}\left[\frac{(\vec{v}_2-\vec{c})^2}{1+\sqrt{1-(\vec{v}_2-\vec{c})^2/c^2}}\right] \right\} \Leftrightarrow$$

$$\Leftrightarrow \left\{ \begin{array}{l} v_2 << c \Rightarrow \dfrac{f_1}{\left(1+\dfrac{hf_1}{mc^2}\right)} \cong \dfrac{f_2}{\left(1+\dfrac{hf_2}{mc^2}\right)}, \\ v_2\begin{pmatrix}\approx\\ \leq\end{pmatrix}c \Rightarrow \dfrac{f_1}{\left(1+\dfrac{hf_1}{mc^2}\right)} \cong \dfrac{f_2}{\left(1+\dfrac{hf_2}{\gamma mc^2}\right)}\left[\dfrac{(\vec{v}_2-\vec{c})^2}{2c^2}\right] \cong \dfrac{1}{2}f_2\dfrac{(\vec{c}-\vec{v}_2)^2}{c^2} \end{array} \right\}$$

$$\alpha = \theta + \phi = \angle(\vec{v}_2, \vec{e}_2),\ \gamma = (1-v_2^2/c^2)^{-0.5},\ \gamma_c = (1-v_c^2/c^2)^{-0.5},\ \gamma_r = (1-v_r^2/c^2)^{-0.5}.$$

(Relativistic addition of velocities in some of the above given examples is neglected, for the sake of higher mathematical simplicity.)

Now, we are in the position to find all wave elements of the electron after its elastic impact with a photon, and basically we see that only the electron's kinetic energy presents its wave energy (meaning that the electron rest mass, or state of rest energy is not a part of matter wave energy). We shall also find that when electron and incident photon get close enough, then their local Center of Mass System becomes a dominant place where de Broglie matter waves would be "players of great importance" for the results of an interaction.

$$E_{k2} = hf_1 - hf_2 = h(f_1 - f_2) = \frac{p_2 v_2}{1+\sqrt{1-v_2^2/c^2}} = (\gamma_2 - 1)mc^2 = hf_e = p_2 u_2 =$$

$$= hf_1 \frac{(hf_1/mc^2)(1-\cos\theta)}{1+(hf_1/mc^2)(1-\cos\theta)} = hf_1 \frac{x(1-\cos\theta)}{1+x(1-\cos\theta)} = E_{ke} \Rightarrow$$

$$\lambda_e = \frac{h}{p_2} = \frac{v_2}{(1+\sqrt{1-v_2^2/c^2})(f_1 - f_2)} = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left[\frac{v_2/c}{1+\sqrt{1-v_2^2/c^2}}\right],$$

$$f_e = \frac{E_{k2}}{h} = \frac{(\gamma_2 - 1)mc^2}{h} = \frac{p_2 v_2}{h(1+\sqrt{1-v_2^2/c^2})} = f_1 - f_2 = \Delta f,$$

$$u_2 = \lambda_e f_e = u_e = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} = \frac{E_{k2}}{p_2} =$$

$$= \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}(f_1 - f_2)\left[\frac{v_2/c}{1+\sqrt{1-v_2^2/c^2}}\right] \cong \frac{\bar{\lambda}^2}{\Delta\lambda}\Delta f\left[\frac{v_2/c}{1+\sqrt{1-v_2^2/c^2}}\right] \Rightarrow \bar{\lambda}^2 \frac{\Delta f}{\Delta\lambda} \cong c.$$

$$E_{kr} = hf_1 - E_{kc} = hf_1 - hf_c = hf_2 + E_{k2} - E_{kc} = hf_2 + hf_e - hf_c = hf_r = mc^2 \frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}}$$

$$f_r = f_1 - f_c = f_2 + f_e - f_c = \left(\frac{mc^2}{h}\right)\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} = \frac{f_1}{1+\frac{hf_1}{mc^2}},$$

$$\mathbf{f_e = f_r + f_c - f_2 = \left(\frac{mc^2}{h}\right)\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} + f_c - f_2 = \frac{f_1}{1+\frac{hf_1}{mc^2}} + f_c - f_2 = f_1 - f_2},$$

$$\mathbf{(mc^2 >> hf_1) \Rightarrow f_r \cong f_1,\ (mc^2 << hf_1) \Rightarrow f_r \cong \frac{mc^2}{h}}$$

$$E_{kc} = hf_c = hf_1 - E_{kr} = hf_1 - hf_r = \left[1 + \frac{hf_1}{mc^2} - \sqrt{1 + 2\frac{hf_1}{mc^2}}\right] mc^2 = \left[1 + x - \sqrt{1+2x}\right] mc^2 \ ,$$

$$f_c = f_1 - f_r = f_1 - (\frac{mc^2}{h}) \frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2}} = \frac{mc^2}{h}\left[1 + \frac{hf_1}{mc^2} - \sqrt{1 + 2\frac{hf_1}{mc^2}}\right] = f_1 \frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2}} = f_r \frac{hf_1}{mc^2} \ ,$$

$$(mc^2 >> hf_1) \Rightarrow f_c \cong f_1 \cong f_r \ ; \ (mc^2 << hf_1) \Rightarrow f_c \cong f_1 - \frac{mc^2}{h} \cong f_1 - f_r \ .$$

The virtual objects μ_r and $\mathbf{m_c}$ in the Center of Mass System, applying the same logic regarding motional energy of de Broglie matter waves, should have characteristic frequencies $\mathbf{f_r}$ and $\mathbf{f_c}$, and since this is the case of an elastic collision, μ_r and $\mathbf{m_c}$ will eventually separate into a moving electron, $\gamma\mathbf{m}$ and a scattered photon $\mathbf{hf_2/c^2}$ (having characteristic frequencies $\mathbf{f_e}$ and $\mathbf{f_2}$). Wavelengths, frequencies and phase and group velocities of such virtual objects (μ_r and $\mathbf{m_c}$) vary (during transitory phase of interaction), before they get stable and final frequency values $\mathbf{f_e}$ and $\mathbf{f_2}$. Consequently, $\mathbf{f_e}$ and $\mathbf{f_2}$ are somehow directly related, or proportional to frequencies $\mathbf{f_r}$ and $\mathbf{f_c}$ (since what we know and calculate as $\mathbf{f_e}$ and $\mathbf{f_2}$ are only their final values, when interaction is completely ended). In order to continue the analysis of this situation, we can (mathematically) test several possibilities, as for instance:

$\mathbf{f_e} = \mathbf{f_r}$, $\mathbf{f_2} = \mathbf{f_c}$, or $\mathbf{f_e} = \mathbf{a} \cdot \mathbf{f_r}$, $\mathbf{f_2} = \mathbf{b} \cdot \mathbf{f_c}$, or $f_e = \alpha(f_r), f_2 = \beta(f_c)$,…

Practically, we assume that the Center of Mass (in a sufficiently close space-time vicinity of the impact) will become the “secret place” where electron’s de Broglie matter wave frequency, $\mathbf{f_e}$, and the frequency of scattered photon, $\mathbf{f_2}$, will be synthesized (from $\mathbf{f_r}$ and $\mathbf{f_c}$), generating the following results:

$$\left\{f_e = f_r \ , \ f_2 = f_c\right\} \Rightarrow \left\{ \begin{array}{l} \frac{f_c}{f_r} = \frac{hf_1}{mc^2} = \frac{f_c}{f_e} \\ \frac{f_e}{f_r} = 1 + \frac{f_c}{f_r} - \frac{f_2}{f_r} = \frac{f_1}{f_r} - \frac{f_2}{f_r} = 1 \end{array} \right\} ,$$

$$f_r = f_e = f_1 - f_2 = \frac{mc^2}{h}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right] = \frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2} c = \frac{\lambda_{Compton}}{c} f_1 f_2 = \frac{hf_1 f_2}{mc^2},$$

$$f_c = f_2 = f_1 - f_r = f_1 - f_e = f_1 - \frac{mc^2}{h}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right] = f_1(1 - \frac{hf_1}{mc^2}),$$

$$\lambda_c = \frac{h}{p_c} = \frac{c}{f_1} = \lambda_1 = \lambda_2 - \lambda_{Compton}(1 - \cos\theta),$$

$$u_c = f_c \lambda_c = c\left\{1 - \frac{mc^2}{hf_1}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right]\right\} = c(1 - \frac{hf_2}{mc^2}),$$

$$\lambda_r = \frac{h}{p_r} = \lambda_1 + \frac{h}{mc} = \lambda_1 + \lambda_{Compton} = \lambda_c + \lambda_{Compton}\text{ , } (p_r = \frac{\frac{hf_1}{c}}{1+\frac{hf_1}{mc^2}}),$$

$$u_r = f_r\lambda_r = \frac{mc^2}{h}\left[\sqrt{1+2\frac{hf_1}{mc^2}} - 1\right](\lambda_1 + \frac{h}{mc}) = c\left[\sqrt{1+2\frac{hf_1}{mc^2}} - 1\right](1+\frac{mc^2}{hf_1}) =$$

$$= c\left[\sqrt{1+2x} - 1\right](1+x) = c\frac{hf_2}{mc^2}(\frac{hf_1}{mc^2} - 1) = c\frac{hf_2}{mc^2}(x-1),$$

$$\lambda_e = \frac{h}{p_2} = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left[\frac{v_2 / c}{1+\sqrt{1-v_2^2/c^2}}\right] = \frac{u_e}{c}(\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}),$$

$$u_e = u_2 = \lambda_e f_e = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left[\frac{v_2 / c}{1+\sqrt{1-v_2^2/c^2}}\right](f_1 - f_2) = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}}\text{ ,}$$

$$(f_r = f_e) \Rightarrow \cos\theta = \frac{(\frac{hf_1}{mc^2}+1)\left[\frac{hf_1}{mc^2} - \sqrt{1+2\frac{hf_1}{mc^2}}\right]+1}{\frac{hf_1}{mc^2}\left[\frac{hf_1}{mc^2} - \sqrt{1+2\frac{hf_1}{mc^2}}+1\right]} = \frac{(x+1)\left[x - \sqrt{1+2x}\right]+1}{x\left[x - \sqrt{1+2x}+1\right]}\text{ .}$$

Continuing the same (mathematical) testing, by elimination of unacceptable results (one of them is given above), we should be able to find the most significant and exact relations between all frequencies, wavelengths and velocities of interaction participants. The objective in the above illustrated analysis is to show that mutually interacting objects create transitory, (time and space) variable phenomena, where dominant (interaction decisive) frame is the local Center of Mass System.

One of possibilities to analyze such situations is also to use differential forms of energy and momentum conservation laws, such as, $\mathbf{d\tilde{E}_i} = \mathbf{c^2 d(\gamma\tilde{m}_i)} = \mathbf{hdf_i} = \mathbf{v_i d\tilde{p}_i}$ $= \mathbf{d(\tilde{p}_i u_i)} = -\mathbf{dE_{ki}} = -\mathbf{c^2 d(\gamma m_i)} = -\mathbf{v_i dp_i} = -\mathbf{d(p_i u_i)}$, and in the process of integration we should be able to take care about specific boundary conditions (extending the same procedure to the effects of rotation, electromagnetic fields etc.).

The most promising strategy in addressing such and similar problems is to understand that the incident photon changes its frequency (loses its initial energy), and the particle (or electron), which was in the state of rest, gets more and more motional energy (in the transitional process when they approach each other). <u>*We can also conceptualize this situation as a kind of Doppler effect, where*</u> *the* <u>*incident photon gradually reduces its frequency (*$\mathbf{f_1 \to f_2}$*), or reduces its energy, and de Broglie electron's matter wave gradually increases its frequency (*$\mathbf{0 \to f_e}$*), in relation to the center of mass speed,* $\mathbf{v_c}$*. Similar conclusion/s should also be valid for any other type of collision (see* ***[6]****).*</u>

In fact, the principal message here is to show that every collision event (elastic or plastic) creates certain (oscillatory, dynamic and transient) field perturbation around collision participants, producing de Broglie matter waves. The energy of matter waves is only a form of kinetic energy composed of particular kinetic energies of interaction participants.

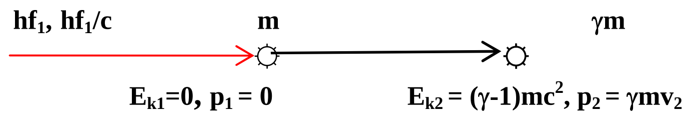
What we traditionally analyze as different collision types (found in all physics books) are mostly situations verifiable very long before and very long after the collision happens, when we see and measure only steady or stationary states. For instance, the above analyzed Compton effect is traditionally explained based on energy and momentum conservation, taking into account only the initial situation long before the impact, and the situation long after the impact happens (neglecting the transitory process between them), as for instance:

$$E_{tot.} = hf_1 + mc^2 = hf_2 + \gamma_2 mc^2 = hf_2 + E_{k2} + mc^2,$$
$$\frac{\mathbf{hf_2}}{\mathbf{c}}\vec{\mathbf{e}}_2 + \vec{\mathbf{p}}_2 = \frac{\mathbf{hf_1}}{\mathbf{c}}\vec{\mathbf{e}}_1,$$

and in this paper an attempt is made to show that such strategy is insufficient to fully describe Compton's and similar interactions.

The process which is opposite to (or inverse of) Compton Effect is the continuous spectrum of x-rays (of photons) emission, caused by impacts of electrons (accelerated in electrical field between two electrodes) with anode as their target. The emission of x-ray photons starts when the electrons are abruptly stopped on the anode surface. If the final, impact electron speed is non-relativistic, $\mathbf{v} << \mathbf{c}$*, the maximal frequency of the emitted x-rays is found in the relation:* $\mathbf{hf}_{max.} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{mv}^2$*, and in cases of relativistic electron velocities, we have* $\mathbf{hf}_{max.} = (\gamma - \mathbf{1})\mathbf{mc}^2$ (and both of them are experimentally confirmed to be correct). *If we now consider electrons (before the impact happen) as matter waves, where the electron matter wave energy corresponds only to a kinetic or motional energy, without rest mass energy content, we will be able to find de Broglie, matter wave frequency of such electrons (just before their impact with the anode). With impact realization the electrons are fully stopped, and the energy content of their matter waves is fully transformed and radiated in the form of x-ray photons (or into another form of waves), whose frequency corresponds to the matter wave frequency of electrons in the moment of the impact. This equality of the frequencies of radiated x-ray photons and electron matter-waves (in the moment of impact) explains us the essential nature of electron matter waves (eliminating the possibility that the rest mass belongs to matter wave energy content).*

Let us now analyze the simplified case (of Compton's effect) that could be also placed between the Photoelectric and Compton Effect. We can imagine that an incident photon is "fully absorbed" by its target, the free electron that was in the state of rest before the impact with a photon (and, of course, the electron will get certain kinetic energy after *the* impact). This situation will be presented, as before, graphically and with the table of input-output energy and momentum states, as follows:



(State: 1; -only incident photon and standstill electron) **(Particle: 2; -only excited, moving electron)**

Fig.4.2.1 Direct Photon-Particle collision, when the incident photon is fully absorbed by the electron. (Photon: hf_1, hf_1/c; Particle: mass m)

	States long before collision (indexing: 1)	States just after collision (indexing: 2)
Photon	$\tilde{E}_{f1} = hf_1, \tilde{p}_{f1} = hf_1/c$	$\tilde{E}_{f2} = 0, \tilde{p}_{f2} = 0$
Electron	$E_{k1} = 0, p_1 = 0,$ $v_1 = 0$	$E_{k2} = (\gamma - 1)mc^2 = p_2 u_2,$ $p_2 = \gamma mv_2 = p_e$

After the collision, the incident photon **(hf_1, hf_1/c)** disappears and its energy and momentum before impact are transformed into a moving particle (an electron) that was in a state of rest before the collision. When the particle was in a state of rest we shall again assume that it didn't have any wave energy or wave momentum. *The meaning of the particle-wave duality in this situation is that a particle (just) after the collision will get certain kinetic energy ($E_{k2} = (\gamma\text{-}1)mc^2$, $p_2 = \gamma mv_2$).* All particle and photon states before the collision will be marked using index "1" and after the collision using index "2", as already presented in the Fig.4.2.1 and in the above table.

Let us first apply the energy and momentum conservation laws,

$$hf_1 + mc^2 = \gamma mc^2, hf_1 = (\gamma - 1)mc^2 = E_{k2},$$
$$\frac{hf_1}{c} = \gamma mv_2 = \gamma mv_e = p_2 = p_e$$

From the energy and momentum conservation laws we can find all other matter wave characteristics of the excited particle (excited electron) after the collision (of course, using the already known **PWDC** relations between group and phase velocity from (4.1)-(4.3)),

$$p_2 = \gamma mv_2 = \frac{hf_1}{c} = p_e, \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1} = \frac{h}{(\gamma - 1)mc}, f_2 = f_e = f_1 = \frac{(\gamma - 1)mc^2}{h},$$
$$u_2 = \lambda_2 f_2 = \frac{v_2}{1 + \sqrt{1 - v_2^2/c^2}} = \frac{v_2}{1 + 1/\gamma} = c, \gamma = (1 - \frac{v_2^2}{c^2})^{-\frac{1}{2}} \Rightarrow v_2 = c,$$
$$E_{k2} = (\gamma - 1)mc^2 = p_2 u_2 = \frac{p_2 v_2}{1 + \sqrt{1 - v_2^2/c^2}} = \tilde{E}_2 = hf_2 = hf_1,$$

Obviously, it is not easy to imagine that any photon can immediately accelerate an electron, from the state of rest to **c**, meaning that something is wrong in above analyzed example. Also, if we analyze the same case traditionally, when $\mathbf{hf_1 << mc^2}$, we have $hf_1 \approx \frac{1}{2}mv_2^{\ 2}, \ \frac{hf_1}{c} \approx mv_2, \gamma \approx 1 \Rightarrow v_2 \approx 2c$, which is again an impossible result, since the particle velocity reaches **2c**. The only logical conclusion is that we should consider that the interaction between the photon and the electron starts long before the physical impact (or unification between them) happens.

Let us now analyze the same situation in the Center of Mass System, in the time-space domain before the photon and the electron become a united object. We could also say that in this first phase of interaction, analyses of the elastic and plastic impact are identical (as long as we could say that we have two interacting objects). From the earlier analysis of the Compton Effect, we already know,

$$\mathbf{E_{kr} = hf_r = \frac{hf_1}{1+\frac{hf_1}{mc^2}} = hf_1 \frac{1}{1+x}, E_{kc} = hf_c = hf_r x = E_{kr} x = hf_1 \frac{x}{1+x}\ , x = \frac{hf_1}{mc^2}}.$$

We also know that the energy $\mathbf{E_{kr}}$, after plastic impact materializes, is injected (absorbed) in the total system mass, making the excited electron after the impact have a (temporarily) higher rest mass (higher than **m**), and a lower kinetic energy (lower than $\mathbf{hf_1}$). Consequently, after gradual realization of the plastic impact we have only one object (a moving and excited electron that has temporarily increased rest mass, **m*;** The electron stays excited until it radiates again the initially "injected photon"),

$$\gamma_c m_c = \frac{\sum E}{c^2} = \frac{hf_1 + mc^2}{c^2} = m + \frac{hf_1}{c^2} = m(1+x), \ \{= \gamma m \Rightarrow \gamma = 1+x, \ ?!\},$$

$$m_c = m^* = m\sqrt{1+2\frac{hf_1}{mc^2}} = m\sqrt{1+2x} = m\sqrt{\frac{1-v_c^2/c^2}{1-v_2^2/c^2}} = m + \Delta m, \ E_{oc} = m_c c^2$$

$$\gamma_c = \frac{1}{\sqrt{1-\frac{v_c^2}{c^2}}} = \frac{1+\frac{hf_1}{mc^2}}{\sqrt{1+2\frac{hf_1}{mc^2}}} = \frac{1+x}{\sqrt{1+2x}}, \gamma = \frac{1}{\sqrt{1-\frac{v_2^2}{c^2}}} = \sqrt{1+x^2}$$

$$\vec{p}_c = (\sum \vec{p}) = \frac{\sum E}{c^2}\vec{v}_c = (m + \frac{hf_1}{c^2})\vec{v}_c = \gamma_c m_c \vec{v}_c =$$

$$= \gamma m \vec{v}_c = \gamma m \vec{v}_2 = \vec{p}_2 = \frac{hf_1}{c}\vec{e}_1 = \tilde{p}_{f1}, \lambda_2 = \frac{h}{p_2} = \lambda_e = \frac{c}{f_1},$$

$$v_c = v_2 = \frac{hf_1}{\gamma mc} = c\frac{x}{\gamma} = cx\sqrt{1-v_2^2/c^2} \Rightarrow v_2 = \frac{cx}{\sqrt{1+x^2}},$$

$$u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} = \frac{cx}{2} = \lambda_2 f_2 = c\frac{f_2}{f_1} = c\frac{f_e}{f_1},$$

$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2}$$

Now we can summarize the particle and wave properties of the excited electron, just after again collision (before it radiates a photon).

$$v_2 = \frac{cx}{\sqrt{1+x^2}}, u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} = \frac{cx}{2} = \lambda_2 f_2 = \lambda_e f_e = c\frac{f_2}{f_1} = c\frac{f_e}{f_1},$$
$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2} = f_2$$
$$p_2 = mcx = \frac{hf_1}{c} = p_e, \lambda_e = \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1},$$
$$x = \frac{hf_1}{mc^2}, \gamma = \frac{1}{\sqrt{1-\frac{v_2^2}{c^2}}} = \sqrt{1+x^2}.$$

For instance, let us first analyze the case (1°) when again incident photon has a very low energy, meaning that the electron after the collision can be treated as a non-relativistic particle.

1°
$$(hf_1 = \text{Low} << mc^2) \Rightarrow (v_2 << c, \gamma \approx 1, x << 1) \Rightarrow$$
$$v_2 = \frac{cx}{\sqrt{1+x^2}} \cong cx, u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} \cong \frac{cx}{2} = \frac{v_2}{2}$$
$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2} = f_2$$
$$p_2 = mcx = \frac{hf_1}{c}, \lambda_e = \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1}.$$

The second case (2°) of interest is when the incident photon has a very high energy and when the electron after the collision can be treated as a relativistic particle:

2°
$$(hf_1 = \text{very high} >> mc^2) \Rightarrow (v_2 \approx c, \gamma \to \infty, x >> 1) \Rightarrow$$
$$v_2 = \frac{cx}{\sqrt{1+x^2}} \approx c, u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} \approx c,$$
$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2} = f_2$$
$$p_2 = mcx = \frac{hf_1}{c}, \lambda_e = \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1}.$$

In both situations (non-relativistic 1°, and relativistic 2°), we are able to calculate all particle and wave characteristics of the excited electron (after the collision), and the results look very realistic (or at least not directly contradictory to the known conservation laws), contrary to the results of the traditional analysis of the same situation.

The situation under 2° looks similar to conditions causing Cherenkov Effect: the accelerated (or excited) electron starts radiating photons behind (creating the back cone of its wave energy).

4.2.3. Example 4: Doppler Effect

Doppler Effect describes the frequency difference between the emitting source signal and received signal, when the emitter and the receiver are mutually in relative motion.

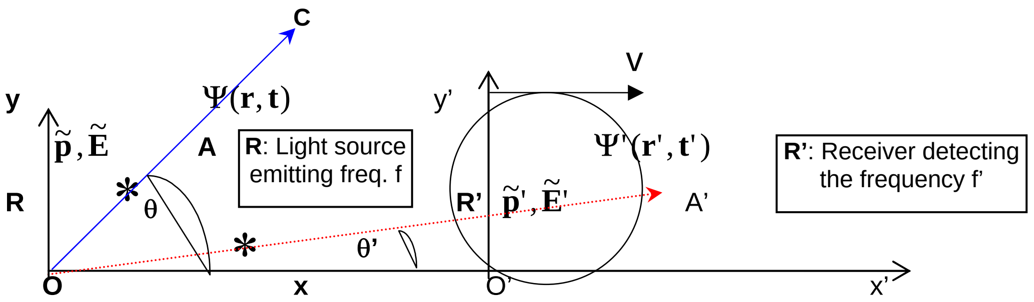
Let us imagine that the light source is in the referential system R: (Oxy), and signal receiver is in the system R': (O'x'y'), ant that relative speed between them is v. In the center O of the system R is the light signal source, and in the center O' of the system R' is the receiver of the same light signal. O emits monochromatic plane wave that has the frequency f, directed along **OA** (angle θ against Ox axis). Referential receiver system R' moves by uniform speed v relative to R, along their common axis Ox **-** O'x', and the received signal is detected by along **OA'** (angle**θ'** against O'x' axis).

Monochromatic plane wave in complex notation in the system R can be characterized by its wave function $\overline{\Psi}(\mathbf{r},\mathbf{t})$:

$\overline{\Psi}(r,\mathbf{t}) = \mathbf{a}(r,\mathbf{t})\mathbf{e}^{-j(\omega t - kr)} = \mathbf{a}(r,\mathbf{t})\mathbf{e}^{j(k_x x + k_y y + k_z z - \omega t)}$, where $\mathbf{k}_x, \mathbf{k}_y$ **i** $\mathbf{k}_z$ are components of the wave vector $\vec{\mathbf{k}}$. In the 4-dimensional Minkowski space, photon wave vector $\overline{\mathbf{K}}_4$ and its radial position vector $\overline{\mathbf{R}}_4$ are known as, $\overline{\mathbf{K}}_4 = \overline{\mathbf{K}}_4(\mathbf{k}_x, \mathbf{k}_y, \mathbf{k}_z, \frac{\omega}{\mathbf{c}}) = \overline{\mathbf{K}}_4(\vec{\mathbf{k}}, \frac{\omega}{\mathbf{c}})$ and $\overline{\mathbf{R}}_4 = \overline{\mathbf{R}}_4(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{ct}) = \overline{\mathbf{R}}_4(\vec{\mathbf{r}}, \mathbf{ct})$, making it possible to express the photon wave function using the product between $\overline{\mathbf{K}}_4$ and $\overline{\mathbf{R}}_4$:

$$\overline{\Psi}(\mathbf{r},\mathbf{t}) = \mathbf{a}(\mathbf{r},\mathbf{t})\mathbf{e}^{-j\overline{K}_4\overline{R}_4} = \mathbf{a}(\mathbf{r},\mathbf{t})\mathbf{e}^{j(\vec{k}\vec{r} - \omega t)} = \mathbf{a}(\mathbf{r},\mathbf{t})\mathbf{e}^{j(k_x x + k_y y + k_z z - \omega t)}$$

$$(\mathbf{j}^2 = -1,\ \mathbf{k} = \frac{2\pi}{\lambda},\ \omega = 2\pi\ \mathbf{f})$$



Doppler Effect Illustration

After applying Lorenz transformations on the above described case, creating relations between R and R', we will get:

$$\bar{K}'_4 = \bar{K}'(k'_x, k'_y, k'_z, \frac{\omega'}{c}),$$

$$k'_x = \gamma\ (k_x - \beta\frac{\omega}{c}),\quad k'_y = k_y,\quad k'_z = k_z,\quad \frac{\omega'}{c} = \gamma\ (\frac{\omega}{c} - \beta\ k_x),\quad (k_{x,y,z} = \frac{2\pi}{h}\tilde{p}_{x,y,z}),$$

$$c = \lambda f = \frac{\omega}{k} = \frac{\omega'}{k'},\quad \tilde{E} = \frac{h}{2\pi}\omega,\quad \tilde{E}' = \frac{h}{2\pi}\omega',\quad \gamma = (1-\beta^2)^{-\frac{1}{2}},\ \beta = \frac{v}{c}.$$

$$\mathbf{k_x = k\cos\theta = \frac{2\pi}{c} f\cos\theta},$$

The Doppler frequency difference between light signals in R and R' can be found from:

$$f' = f\frac{\tilde{E}'}{\tilde{E}} = \gamma\ f(1-\beta\ \cos\theta),\quad \lambda' = \frac{c}{f'} = \lambda\frac{\tilde{p}}{\tilde{p}'} = \frac{\lambda}{\gamma\ (1-\beta\ \cos\theta)}.$$

This situation looks like adding (or reducing) certain frequency shift Δf to the source frequency $\mathbf{f}$, or adding (or reducing) certain amount of motional (wave) energy $\Delta\tilde{E}$ to the source photon energy. We can imagine that between R and R' certain intermediary wave-coupling state materializes, realizing the mentioned energy difference. We can also associate the same wave coupling state to certain wave moment $\tilde{\mathbf{p}}^*$ and mass $\tilde{\mathbf{m}}^*$. Since the source and the receiver photon frequency, and relative speed between R and R' are known, we are able to calculate all characteristics of the wave coupling state (see the table with all results, below).

The message of the above given Doppler Effect analysis is that this is not only an observation-related phenomena, but much more, it is the case of real wave interactions and energy and momentum conservation rules. Also, the same case can be analogically applied to any mass movement, explaining the nature of the particle-wave duality from a larger perspective than presently known (or saying differently, every relative motion between minimum 2 particles, or quasiparticles should create similar wave coupling state/s).

	Source Photon in R	Differential Wave Coupling State Between R' and R	Detected Photon in R'
Wave energy	$\tilde{\mathbf{E}} = \mathbf{hf}$	$\tilde{E}^* = \Delta\tilde{E} = \tilde{E}' - \tilde{E} = h(f' - f) = h\Delta f = hf^* =$ $= (\gamma - 1)\tilde{m}^* c^2 =$ $= hf\left[\gamma(1-\beta\cos\theta) - 1\right]$	$\tilde{\mathbf{E}}' = \mathbf{hf'}$
Moment	$\tilde{\mathbf{p}} = \frac{\mathbf{hf}}{\mathbf{c}}$	$\vec{p}^* = \frac{\overrightarrow{hf'}}{c} - \frac{\overrightarrow{hf}}{c} = \gamma\, \tilde{m}^* \vec{v}$ $\tilde{p}^* = \gamma\, \tilde{m}^* v =$ $= \frac{hf}{c}\left[-\cos\theta \pm \sqrt{\gamma^2(1-\beta\cos\theta)^2 - \sin^2\theta}\right]$	$\tilde{\mathbf{p}}' = \frac{\mathbf{hf'}}{\mathbf{c}}$
Frequency	**f**	$f^* = \Delta f = f' - f = \frac{\Delta\tilde{E}}{h} = \frac{\tilde{E}^*}{h} =$ $= f\left[\gamma(1-\beta\cos\theta) - 1\right]$	$f' = f' = f\frac{\tilde{E}'}{\tilde{E}} =$ $= \gamma f(1 - \beta\cos\theta)$
Wave length	$\lambda = \frac{c}{f}$	$\lambda^* = \frac{h}{\tilde{p}^*} =$ $= \frac{c}{f}\left[-\cos\theta \pm \sqrt{\gamma^2(1-\beta\cos\theta)^2 - \sin^2\theta}\right]$	$\lambda' = \frac{c}{f'} =$ $\lambda\frac{\tilde{p}}{\tilde{p}'} = \frac{\lambda}{\gamma\ (1-\beta\cos\theta)}$
Group Velocity	**c**	$v = \frac{\partial\tilde{E}^*}{\partial\tilde{p}^*}$	**c**
Phase Velocity	**c**	$u = \frac{v}{1 + \sqrt{1 - \frac{v^2}{c^2}}} = \lambda^* f^* = \frac{\tilde{E}^*}{\tilde{p}^*}$	**c**
Effective Mass	$\tilde{\mathbf{m}} = \frac{\tilde{\mathbf{E}}}{\mathbf{c}^2} = \frac{\mathbf{hf}}{\mathbf{c}^2}$	$\tilde{m}^* = \frac{\Delta\tilde{E}}{(\gamma-1)c^2} = \frac{h\Delta f}{(\gamma-1)c^2} = \frac{\tilde{p}^*}{\gamma v} =$ $= \frac{hf\left[\gamma(1-\beta\cos\theta) - 1\right]}{(\gamma-1)c^2} = \frac{hf^*}{(\gamma-1)c^2} =$ $= \frac{hf\left[-\cos\theta \pm \sqrt{\gamma^2(1-\beta\cos\theta)^2 - \sin^2\theta}\right]}{\gamma v c}$	$\tilde{\mathbf{m}}' = \frac{\tilde{\mathbf{E}}'}{\mathbf{c}^2} = \frac{\mathbf{hf'}}{\mathbf{c}^2}$

The principal objective in all above given examples was to show that only and exclusively kinetic or motional energy creates, or belongs to de Broglie matter waves, contrary to the mainstream of contemporary understanding in Physics, regarding the same situation (where in most cases the total energy, including rest mass energy is included in matter wave energy).

We can also notice that all of the above-presented mathematical analysis has its limit, since this way we are only able to treat relatively simple situations concerning particle-wave duality. In fact here we are dealing with mixed particle and field interactions and we are faced with necessity to introduce operations with wave functions.

After establishing the wave function concept, and after formulating generally valid wave equations (as follows), such and similar problems can be much more completely treated, but even at this level (previously presented) there is already enough (mathematical) substance to qualitatively understand the concept of particle-wave duality (of this paper) and its relation to inertial and reaction forces.

4.3. WAVE FUNCTION AND GENERALIZED SCHRÖDINGER EQUATION

Wave motions, oscillations and fields are phenomenological reality of our universe. We know how to create them, how to describe them mathematically and how to make modeling of different Physics related wave phenomenology connecting general wave concepts with material properties. We also see that our universe (from our point of view) has different states than waves: particles. In many experimental situations we know that interacting particles become the sources of waves, or are building parts of waves. Waves propagate with and out of particles, and particles create wave-like movements when moving in large groups, etc. We also know (directly or indirectly) that different wave formations and oscillations are present (on number of ways) inside atoms and other particles. Consequently, we know that we need to master (theoretically and empirically) the wave motions and oscillations in their relations to particles, and this is a big part of what modern science and technology are busy with. By accepting the necessity and convenience of operating with wave function/s, we are touching the foundations of modern Quantum Mechanics (or Physics in general). In this paper we are considering existence of real and (directly or indirectly) measurable matter (or de Broglie), energy-carrying waves, contrary to the Orthodox Quantum Mechanics where wave functions and de Broglie waves are treated exclusively as mathematical tools or artificial "probability waves" (or using number of other exotic formulations like probability or possibility distributions in the form of waves). Later on it will be clarified how and why it was possible to associate the probability nature to quantum world (i.e. we shall see that probability concept for explaining the essence of Quantum Mechanics is ontologically wrong, but it works well in practice, in the frames, mathematical modeling and assumptions of Orthodox Quantum Mechanics, and we shall demonstrate why it works well). The question how to recognize what and where is the material carrier of de Broglie waves is often unanswered, and in this paper we shall just assume that in every particular situation certain carrier-medium (presently known or unknown) always exist, since assuming that nothing like that exists is against common-sense logic (like giving the full legitimacy to mysticism and metaphysics).

Quantum Mechanics treats de Broglie mater waves and all interactions in quantum world using the abstract concept of probability wave function/s (meaning that de Broglie matter waves are considered as not directly measurable). Contrary, here favored (active power) wave function, based on a kinetic energy or kinetic power flow (earlier introduced in (3.5)) is a kind of deterministic and natural wave function compared to the probabilistic Quantum Mechanics wave function (meaning that, in this paper, de Broglie matter waves will be treated almost as any other known wave phenomena in electromagnetism, acoustics, fluid mechanics etc.). Moreover, it will be shown that on formal and mathematic level there is no contradiction between probabilistic and normalized, dimensionless, power or motional energy carrying wave function. Such (deterministic) wave function $\Psi(\mathbf{t})$ should describe the associated and space-time evolving energy or field of de Broglie waves inside and around moving particle/s in connection with corresponding differential energy balance (4.7). Mathematically (in order to satisfy Parseval's identity, related to signal energy equivalence between time and frequency domains of the same signal; -see (4.13)-(4.17)) is extremely convenient to treat the square of a wave function, $\Psi^2(\mathbf{t})$, as the active power function $\mathbf{P(t)}$, or a wave power since,

$$\left\{ \mathbf{d\tilde{E} = hdf = dE_k = c^2 d(\gamma m) = vdp = d(pu) = -c^2 d\tilde{m} = -vd\tilde{p} = -d(\tilde{p}u)} \right\} / \mathbf{dt}$$

$$\Leftrightarrow \left\{ \mathbf{P(t) = \Psi^2(t) = \frac{d\tilde{E}}{dt} = h\frac{df}{dt} = c^2\frac{d(\gamma m)}{dt} = \frac{d(pu)}{dt} = v\frac{dp}{dt} = ... = [W]} \right\}, \tag{4.9-0}$$

where de Broglie wave group energy is $\tilde{E} = \int P(t)dt = \int \Psi^2(t)dt = \tilde{p}u = hf = E_k$. All wave functions and *Schrödinger-like wave equation/s* developed (later on) in this paper will be based on (4.9-0), *and also fully merged with **P**article-**W**ave **D**uality **C**ode found in (4.1)-(4.3).*

The primary objective of this paper is only to modify and re-establish the foundations and most relevant step-stones of (new) Particle-Wave Duality (or better to say Particle-Wave Unity) Theory, and not to deal with already known associated mathematics that is habitual in Quantum Mechanics and good for dealing with number of wave equations. In fact, the habitual mathematical background and mathematical processing that is already well established in Quantum Mechanics could stay fully applicable to all new or modified forms of wave equations that will be developed in this paper, and nothing of that matter will be changed. ***The message underlined here will be to show that for developing all wave functions and wave equations (known in modern Physics), even equations more general than known in quantum theory (and applicable to all wave phenomena in our universe), we do not need quantum mechanics' and probabilistic assumptions. We only need to respect all conservation laws of physics, merging them properly with the Particle-Wave reality described here as PWDC (see (4.1)-(4.3)), and to place all of that into a much more generally valid mathematical framework of Analytic Signal concepts (based on Hilbert transformation: see about Analytic Signals later).***

Regardless of significant differences in treating the wave function (in this paper and in Orthodox Quantum Mechanics); it is easy to show that the Schrödinger's wave equation, as one of the very important differential equations in microphysics, formally (mathematically) keeps the same form for the probability and/or energy carrying waves.

We also know that often we do not have the complete or good enough answer about what kind of substance is the real carrier of de Broglie waves (or also carrier of electromagnetic waves), but this does not affect our mathematical modeling of such wave phenomena. It is better to say that in this paper we should (for the time being) forget, or put aside all questions about correct interpretation of the wave function and its material carrier/s, and discuss the same subject a bit later, when this situation naturally becomes clearer (and when some basic dilemmas become obsolete).

Let us briefly mention (in three steps: **A**), **B**) and **C**)) the starting points and results in process of developing Schrödinger-like equation/s, **based on a wave function concept introduced in (4.9-0)**, apart from any Quantum Theory background:

A) By extending an arbitrary wave function $\Psi(t)$, which could be generally valid and applicable to any kind of wave, particles and fields motion phenomena (regardless if it would have probabilistic or deterministic nature), into its equivalent complex form, $\Psi(t) \to \Psi(x,t) \to \overline{\Psi}(x,t)$ (using the **Complex Analytic Signal** model, first time introduced by Dennis Gabor; -see [7] and [8]), we shall have:

$$\overline{\Psi}(\mathbf{x,t}) = \Psi(\mathbf{x,t}) + \mathbf{j}\hat{\Psi}(\mathbf{x,t}) = \mathbf{a(x,t)e}^{\mathbf{j}\varphi(\mathbf{x,t})} = \frac{\mathbf{1}}{(\mathbf{2}\pi)^{\mathbf{2}}} \iint_{(-\infty,+\infty)} \mathbf{U}(\omega,\mathbf{k})\mathbf{e}^{\mathbf{-j(\omega t\text{-}kx)}}\mathbf{dkd}\omega =$$

$$= \frac{\mathbf{1}}{\pi^{\mathbf{2}}} \iint_{(\mathbf{0},+\infty)} \mathbf{U}(\omega,\mathbf{k})\mathbf{e}^{\mathbf{-j(\omega t\text{-}kx)}}\mathbf{dkd}\omega = \frac{\mathbf{1}}{\pi^{\mathbf{2}}} \iint_{(\mathbf{0},+\infty)} \mathbf{A}(\omega,\mathbf{k})\mathbf{e}^{\mathbf{-j(\omega t\text{-}kx+\Phi(\omega,k))}}\mathbf{dkd}\omega\ ,\ \mathbf{j}^{\mathbf{2}} = -\mathbf{1}, \tag{4.9}$$

$$\mathbf{U}(\omega,\mathbf{k}) = \mathbf{U_c}(\omega,\mathbf{k}) - \mathbf{j}\ \mathbf{U_s}(\omega,\mathbf{k}) = \iint_{(-\infty,+\infty)} \overline{\Psi}(\mathbf{x,t})\ \mathbf{e}^{\mathbf{j(\omega t\text{-}kx)}}\mathbf{dtdx} = \mathbf{A}(\omega,\mathbf{k})\mathbf{e}^{\mathbf{-j\Phi(\omega,k)}}\ ,$$

where $\hat{\Psi}(x,\mathrm{t})$ is the Hilbert transformation of $\Psi(\mathbf{x,t}) = \mathbf{a(x,t)}\cos\varphi(\mathbf{x,t})$, or $\hat{\Psi}(x,\mathrm{t}) = H\left[\Psi(x,t)\right] = \mathbf{a(x,t)}\sin\varphi(\mathbf{x,t})$.

In fact, it will be shown later that Complex Analytic Signal (4.9) presents more important (more rich and more productive) generic wave function formulation, than any other form of traditionally known Schrödinger or other wave equations. Starting from (4.9) we can develop all variants of Schrödinger-like and a number of other wave equations known in Physics (see attachments regarding Analytic Signal).

B) By applying multiple derivation to (4.9), (see very similar procedure applied in **[5],** pages: 175-179), we will be able to get (4.9-1):

$$\begin{aligned}
&-\frac{\omega}{k^2}\Delta\overline{\Psi} = \omega\overline{\Psi} = \frac{-1}{\omega}\frac{\partial^2\overline{\Psi}}{\partial t^2} = -j\frac{\omega}{k}\nabla\overline{\Psi} = j\frac{\partial\overline{\Psi}}{\partial t}, \overline{\Psi} = \Psi + j\hat{\Psi} = \Psi + jH\left[\Psi\right],\\
&-\frac{\omega}{k^2}\Delta\Psi = \omega\Psi = \frac{-1}{\omega}\frac{\partial^2\Psi}{\partial t^2} = \frac{\omega}{k}\nabla\hat{\Psi} = -\frac{\partial\hat{\Psi}}{\partial t},\\
&-\frac{\omega}{k^2}\Delta\hat{\Psi} = \omega\Psi = \frac{-1}{\omega}\frac{\partial^2\hat{\Psi}}{\partial t^2} = -\frac{\omega}{k}\nabla\Psi = \frac{\partial\Psi}{\partial t}
\end{aligned} \tag{4.9-1}$$

C) <u>Now, by implementing the **P**article-**W**ave **D**uality **C**ode, (4.1)-(4.3) into (4.9-0) and (4.9-1)</u>, $\tilde{m} = \gamma m, p = \gamma m v = \tilde{m}v,\ \tilde{E} = \hbar\omega = \tilde{m}vu = pu,\ u = \frac{\omega}{k},\ \lambda = \frac{h}{\tilde{p}} = \frac{h}{p}$, and taking into account that de Broglie waves phenomenology is effectively presentable (directly or indirectly, or analogically) as a motion of a particle (quasi-particle, or a particle-energy equivalent) $\mathbf{m}$ in an energy field of some bigger particle, $\mathbf{M} >> \mathbf{m}$ (which creates potential field with energy $\mathbf{U_p}$), we shall be able to develop the following, generalized Schrödinger's equation (and later, many other wave equations):

$$-\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} = \mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial\ \mathbf{t}} = \tilde{\mathbf{E}}\,\overline{\Psi} = \tilde{\mathbf{p}}\mathbf{u}\overline{\Psi} = \frac{-\hbar^2}{\tilde{\mathbf{E}}}\frac{\partial^2\overline{\Psi}}{\partial\ \mathbf{t}^2} = -\mathbf{j}\hbar\mathbf{u}\nabla\overline{\Psi}\ ,\ \ (\mathbf{U_p} = \mathbf{0}),$$

$$\left[\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \tilde{\mathbf{E}} - \mathbf{U_p},\ \mathbf{U_p} \neq \mathbf{0}\right] \Rightarrow \left[\overline{\Psi} \to \overline{\Psi}'\ ,\ \tilde{\mathbf{m}} \to \tilde{\mathbf{m}}'...\right] \Rightarrow$$

$$-\frac{\hbar^2}{\tilde{\mathbf{m}}'}(\frac{\mathbf{u}}{\mathbf{v}})'\Delta\overline{\Psi}' = \mathbf{j}\hbar\frac{\partial\overline{\Psi}'}{\partial\ \mathbf{t}} = (\tilde{\mathbf{E}} - \mathbf{U_p})\overline{\Psi}' = \frac{-\hbar^2}{\tilde{\mathbf{E}} - \mathbf{U_p}}\frac{\partial^2\overline{\Psi}'}{\partial\ \mathbf{t}^2} = -\mathbf{j}\hbar\mathbf{u}'\nabla\overline{\Psi}'.$$

$$\left[\tilde{\mathbf{m}}' \to \tilde{\mathbf{m}}\ \ ,\overline{\Psi}' \to \overline{\Psi}\right] \Rightarrow$$

$$\left\{\begin{array}{l} -\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \mathbf{U_p}\,\overline{\Psi} = \tilde{\mathbf{E}}\overline{\Psi} = \tilde{\mathbf{p}}\mathbf{u}\overline{\Psi} = \mathbf{j}\hbar\,\frac{\partial\overline{\Psi}}{\partial\ \mathbf{t}} + \mathbf{U_p}\,\overline{\Psi} = \frac{-\hbar^2}{\tilde{\mathbf{E}}-\mathbf{U_p}}\cdot\frac{\partial^2\overline{\Psi}}{\partial\ \mathbf{t}^2} + \mathbf{U_p}\,\overline{\Psi} \\ (\frac{\tilde{\mathbf{E}}-\mathbf{U_p}}{\hbar})\overline{\Psi} = \mathbf{j}\frac{\partial\overline{\Psi}}{\partial\ \mathbf{t}} = \frac{-\hbar}{\tilde{\mathbf{E}}-\mathbf{U_p}}\cdot\frac{\partial^2\overline{\Psi}}{\partial\ \mathbf{t}^2}\ ,\quad (\frac{\tilde{\mathbf{E}}-\mathbf{U_p}}{\hbar})^2\,\overline{\Psi} = -\frac{\partial^2\overline{\Psi}}{\partial\ \mathbf{t}^2}\ , \\ \frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}}-\mathbf{U_p})\,\overline{\Psi} = (\frac{\tilde{\mathbf{E}}-\mathbf{U_p}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial\ \mathbf{t}^2} = \frac{\partial\overline{\Psi}}{\partial\ \mathbf{t}} + \mathbf{u}\nabla\overline{\Psi} = \mathbf{0} \end{array}\right\},\ (4.10)$$

or the same equation can be transformed (or abbreviated) into operators form:

$$\left(\mathrm{H} = -\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta + \mathrm{U_p}\ \mathbf{(=)\ Hamiltonian}\right) \Rightarrow \left\{\mathrm{H}\overline{\mathbf{\Psi}} = \tilde{\mathbf{E}}\overline{\mathbf{\Psi}} = \mathbf{j}\hbar\frac{\partial}{\partial\,\mathrm{t}}\overline{\mathbf{\Psi}} + \mathrm{U_p}\overline{\mathbf{\Psi}} = \mathbf{...}\right\}$$

$$\Rightarrow \tilde{\mathbf{E}} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar\frac{\partial}{\partial\,\mathrm{t}} + \mathrm{U_p} \Leftrightarrow -\mathbf{j}\hbar\mathrm{u}\nabla\ , \qquad (4.11)$$

$$\left(\tilde{\mathbf{p}}_{\mathrm{i}}\overline{\mathbf{\Psi}} = -\mathbf{j}\hbar\nabla\overline{\mathbf{\Psi}}\right) \Rightarrow \tilde{\mathbf{p}}_{\mathbf{i}} \Leftrightarrow -\mathbf{j}\hbar\nabla \Leftrightarrow \frac{1}{\mathrm{u}}(\mathbf{j}\hbar\frac{\partial}{\partial\,\mathrm{t}} + \mathbf{U_p}) \Leftrightarrow \frac{1}{\mathrm{u}}\mathrm{H}\,.$$

From general wave propagation analyses we are also familiar with the following forms of wave equations (equivalent to (4.10) and (4.11)):

$$\begin{array}{l} \frac{\mathrm{u}^2}{\tilde{\mathbf{E}}}\left[\Delta\overline{\mathbf{\Psi}} - \frac{\tilde{\mathbf{E}}}{\tilde{\mathbf{E}}-\mathbf{U_p}}\cdot\frac{1}{\mathbf{u}^2}\frac{\partial^2\overline{\mathbf{\Psi}}}{\partial\,\mathbf{t}^2}\right] = \frac{\mathbf{j}}{\hbar}\frac{\partial\overline{\mathbf{\Psi}}}{\partial\,\mathrm{t}} + \frac{1}{\tilde{\mathbf{E}}-\mathbf{U_p}}\cdot\frac{\partial^2\overline{\mathbf{\Psi}}}{\partial\,\mathbf{t}^2} = \\ = \frac{-\,\mathrm{ju}}{\hbar}\nabla\overline{\mathbf{\Psi}} + \frac{1}{\tilde{\mathbf{E}}-\mathbf{U_p}}\frac{\partial^2\overline{\mathbf{\Psi}}}{\partial\,\mathbf{t}^2} = \frac{\tilde{\mathbf{E}}-\mathbf{U_p}}{\hbar^2}\overline{\mathbf{\Psi}} + \frac{1}{\tilde{\mathbf{E}}-\mathbf{U_p}}\frac{\partial^2\overline{\mathbf{\Psi}}}{\partial\,\mathbf{t}^2} = 0, \end{array} \qquad (4.12)$$

also assuming a merge of (4.12) with **PWDC** relations found in (4.1), (4.2) and (4.3).

In the cases of non-relativistic velocities, after replacing relation between group and phase velocity with its approximate form $\mathrm{v} \cong 2\mathrm{u}$, generalized Schrödinger's equation $\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\mathbf{\Psi}} + (\tilde{\mathbf{E}} - \mathrm{U_p})\overline{\mathbf{\Psi}} = 0$, (4.10)-(4.12), becomes equal to the traditionally known (non-relativistic) Schrödinger's equation, $\frac{\hbar^2}{2\tilde{\mathbf{m}}}\Delta\overline{\mathbf{\Psi}} + (\tilde{\mathbf{E}} - \mathrm{U_p})\overline{\mathbf{\Psi}} = 0$.

The above-presented forms of generalized Schrödinger's equation take into account only motional (wave or kinetic) energy as a particle wave energy, and this fact should be noticed as the biggest difference between traditional interpretation of Schrödinger's equation (which usually takes a total particle energy into account) and all forms, (4.10)-(4.12), developed here.

We can also notice (and demonstrate) that Dirac's relativistic modification of Schrödinger's equation is (or should be) automatically included in equations (4.10)-(4.12), since the energy $\tilde{\mathrm{E}}$ ***and ratio*** $\mathbf{u}/\mathbf{v}$ ***are already treated as relativistic or Lorenz-transformations velocity-dependent functions (see (4.1)-(4.3)).***

In (4.10) we made wave energy level translation for the amount of the potential field energy $\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \tilde{\mathbf{E}} - \mathbf{U_p}, \mathbf{U_p} \neq 0$, and as the result we got the familiar looking (traditional) Schrödinger's equation (since when, $\mathbf{v} << \mathbf{c} \Rightarrow \mathbf{u/v} = \mathbf{1/2}$),

$$\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} - \mathrm{U_p})\overline{\Psi} = 0 \Rightarrow \frac{\hbar^2}{\mathbf{2}\tilde{\mathbf{m}}}\Delta\overline{\Psi} + (\tilde{\mathbf{E}} - \mathrm{U_p})\overline{\Psi} = 0 .$$

Let us now create another wave energy level translation to "capture" the total (relativistic) energy of a particle: $\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \mathbf{E_{total}} = \tilde{\mathbf{E}} + \mathbf{E_0} + \mathbf{U_p},\ \mathbf{U_p} \neq 0,\ \mathbf{E}_0 = \mathbf{const.}$ (understanding that a total energy equals to the sum of motional energy $\tilde{\mathbf{E}}$, state of rest energy $\mathbf{E_0}$, and energy of surrounding potential field, $\mathbf{U_p}$), and let us apply this energy translation on the (already modified, or energy shifted) wave equation (4.10), as for instance,

$$\left\{\begin{matrix} \left[-\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \mathbf{U_p}\overline{\Psi} = \tilde{\mathbf{E}}\overline{\Psi} = \tilde{\mathrm{p}}\mathrm{u}\overline{\Psi} = \mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial \mathrm{t}} + \mathbf{U_p}\overline{\Psi} = \frac{-\hbar^2}{\tilde{\mathbf{E}} - \mathbf{U_p}}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} + \mathbf{U_p}\overline{\Psi}\right], \\ \left[\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \mathrm{E_{total}} = \tilde{\mathbf{E}} + \mathbf{E_0} + \mathbf{U_p}\right] \Rightarrow \left[\overline{\Psi} \to \overline{\Psi}', \tilde{\mathbf{m}} \to \tilde{\mathbf{m}}'...\right] \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} -\frac{\hbar^2}{\tilde{\mathbf{m}}'}(\frac{\mathbf{u}}{\mathbf{v}})'\Delta\overline{\Psi}' + \mathbf{U_p}\overline{\Psi}' = \mathbf{j}\hbar\frac{\partial\overline{\Psi}'}{\partial \mathrm{t}} + \mathbf{U_p}\overline{\Psi}' = \mathrm{E_{total}}\overline{\Psi}' = \\ = \frac{-\hbar^2}{\mathrm{E_{total}} - \mathbf{U_p}}\frac{\partial^2\overline{\Psi}'}{\partial \mathbf{t}^2} + \mathbf{U_p}\overline{\Psi}' = -\mathrm{j}\hbar\mathrm{u}'\nabla\overline{\Psi}' \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{matrix} \left[\begin{matrix} -\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \mathbf{U_p}\overline{\Psi} = \mathrm{E_{total}}\overline{\Psi} = \mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial \mathrm{t}} + \mathbf{U_p}\overline{\Psi} = \\ = \frac{-\hbar^2}{\mathrm{E_{total}} - \mathbf{U_p}}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} + \mathbf{U_p}\overline{\Psi} = -\mathrm{j}\hbar\mathrm{u}\nabla\overline{\Psi} \end{matrix}\right], \\ (\frac{\mathrm{E_{total}} - \mathbf{U_p}}{\hbar})\overline{\Psi} = \mathbf{j}\frac{\partial\overline{\Psi}}{\partial \mathrm{t}} = \frac{-\hbar}{\mathrm{E_{total}} - \mathbf{U_p}}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2},\ (\frac{\mathrm{E_{total}} - \mathbf{U_p}}{\hbar})^2\overline{\Psi} = -\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2}, \\ \frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\mathrm{E_{total}} - \mathbf{U_p})\overline{\Psi} = (\frac{\mathrm{E_{total}} - \mathbf{U_p}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \frac{\partial\overline{\Psi}}{\partial \mathrm{t}} + \mathrm{u}\nabla\overline{\Psi} = 0 \end{matrix}\right\}, \quad (4.10\text{-}1)$$

or the same equation can be transformed into another operators form:

$$\left(\mathrm{H} = -\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta + \mathrm{U_p}\ (=)\ \mathbf{Hamiltonian}\right) \Rightarrow \left\{\mathrm{H}\overline{\Psi} = \mathbf{E}_{\mathrm{total}}\overline{\Psi} = \mathbf{j}\hbar\frac{\partial}{\partial \mathrm{t}}\overline{\Psi} + \mathrm{U_p}\overline{\Psi} = ...\right\}$$

$$\Rightarrow \mathbf{E}_{\mathrm{total}} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar\frac{\partial}{\partial \mathrm{t}} + \mathrm{U_p} \Leftrightarrow -\mathbf{j}\hbar\mathrm{u}\nabla, \quad (4.11\text{-}1)$$

$$\tilde{\mathbf{p}}_{\mathbf{i}} \Leftrightarrow -\mathbf{j}\hbar\nabla - \frac{\mathrm{E_0} + \mathrm{U_p}}{\mathrm{u}} \Leftrightarrow \frac{1}{\mathrm{u}}(\mathbf{j}\hbar\frac{\partial}{\partial \mathrm{t}} - \mathbf{E_0}) \Leftrightarrow \frac{1}{\mathrm{u}}(\mathrm{H} - \mathrm{E_0} - \mathrm{U_p}).$$

We can also make the same energy translation, $\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \mathbf{E}_{\mathbf{total}} = \tilde{\mathbf{E}} + \mathbf{E}_{\mathbf{0}} + \mathbf{U}_{\mathbf{p}}$, and apply it to the very first wave equation from (4.10), that is not energy translated,

$$\left\{\begin{array}{l} \left[-\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} = \mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial t} = \tilde{\mathbf{E}}\overline{\Psi} = \tilde{\mathrm{p}}\mathrm{u}\overline{\Psi} = \frac{-\hbar^2}{\tilde{\mathbf{E}}}\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = -\mathrm{j}\hbar\mathrm{u}\nabla\overline{\Psi}, \mathbf{U}_{\mathbf{p}} = 0\right], \\ \left[\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \mathrm{E}_{\mathrm{total}} = \tilde{\mathbf{E}} + \mathbf{E}_{\mathbf{0}} + \mathbf{U}_{\mathbf{p}}, \mathrm{U}_{\mathrm{p}} \neq 0\right] \Rightarrow \left[\overline{\Psi} \to \overline{\Psi}', \tilde{\mathbf{m}} \to \tilde{\mathbf{m}}'...\right] \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{-\frac{\hbar^2}{\tilde{\mathbf{m}}'}(\frac{\mathbf{u}}{\mathbf{v}})'\Delta\overline{\Psi}' = \mathbf{j}\hbar\frac{\partial\overline{\Psi}'}{\partial t} = \mathrm{E}_{\mathrm{total}}\overline{\Psi}' = \frac{-\hbar^2}{\mathrm{E}_{\mathrm{total}}}\frac{\partial^2\overline{\Psi}'}{\partial \mathbf{t}^2} = -\mathrm{j}\hbar\mathrm{u}'\nabla\overline{\Psi}'\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \left[-\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} = \mathrm{E}_{\mathrm{total}}\overline{\Psi} = \mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial t} = \frac{-\hbar^2}{\mathrm{E}_{\mathrm{total}}}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = -\mathrm{j}\hbar\mathrm{u}\nabla\overline{\Psi}\right], \\ (\frac{\mathrm{E}_{\mathrm{total}}}{\hbar})\overline{\Psi} = \mathbf{j}\frac{\partial\overline{\Psi}}{\partial t} = \frac{-\hbar}{\mathrm{E}_{\mathrm{total}}}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2}, \ (\frac{\mathrm{E}_{\mathrm{total}}}{\hbar})^2\overline{\Psi} = -\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2}, \\ \frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} - \mathrm{E}_{\mathrm{total}}\overline{\Psi} = 0, \ (\frac{\mathrm{E}_{\mathrm{total}}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = 0, \frac{\partial\overline{\Psi}}{\partial t} + \mathrm{u}\nabla\overline{\Psi} = 0 \end{array}\right\}, \qquad (4.10\text{-}2)$$

or, again, the same equation can be transformed into new operators form:

$$\left(\mathrm{H} = -\frac{\hbar^2}{\tilde{\mathrm{m}}}(\frac{\mathrm{u}}{\mathrm{v}})\Delta + \mathrm{U}_{\mathrm{p}} \ (=) \ \text{Hamiltonian}\right) \Rightarrow$$

$$\Rightarrow \left\{\mathrm{H}\overline{\Psi} = \mathbf{E}_{\mathrm{total}}\overline{\Psi} = \mathbf{j}\hbar\frac{\partial}{\partial t}\overline{\Psi} = -\mathbf{j}\hbar\mathrm{u}\nabla\overline{\Psi}\right\}$$

$$\Rightarrow \mathbf{E}_{\mathrm{total}} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar\frac{\partial}{\partial t} \Leftrightarrow -\mathbf{j}\hbar\mathrm{u}\nabla\overline{\Psi}, \qquad (4.11\text{-}2)$$

$$\tilde{\mathbf{p}}_{\mathbf{i}} \Leftrightarrow -\mathbf{j}\hbar\nabla - \frac{\mathrm{E}_0 + \mathrm{U}_{\mathrm{p}}}{\mathrm{u}} \Leftrightarrow \frac{1}{\mathrm{u}}(\mathbf{j}\hbar\frac{\partial}{\partial t} - \mathrm{E}_0 - \mathbf{U}_{\mathbf{p}}) \Leftrightarrow \frac{1}{\mathrm{u}}(\mathrm{H} - \mathrm{E}_0 - \mathbf{U}_{\mathbf{p}}).$$

Now we can summarize above results (regarding different energy translations found in (4.10), (4.10-1), (4.10-2) and (4.10-3)) and see that all of them produce mutually equivalent and correct Schrödinger-like (or Dirac's) equations:

$$\begin{array}{l} \frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \tilde{\mathbf{E}}\overline{\Psi} = 0, \ (\frac{\tilde{\mathbf{E}}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = 0, \frac{\partial\overline{\Psi}}{\partial t} + \mathrm{u}\nabla\overline{\Psi} = 0, \mathbf{U}_{\mathbf{p}} = 0; \\ \frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} - \mathbf{U}_{\mathbf{p}})\overline{\Psi} = 0, \ (\frac{\tilde{\mathbf{E}} - \mathbf{U}_{\mathbf{p}}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = 0, \frac{\partial\overline{\Psi}}{\partial t} + \mathrm{u}\nabla\overline{\Psi} = 0; \\ \frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\mathrm{E}_{\mathrm{total}} - \mathbf{U}_{\mathbf{p}})\overline{\Psi} = 0, \ (\frac{\mathrm{E}_{\mathrm{total}} - \mathbf{U}_{\mathbf{p}}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = 0, \frac{\partial\overline{\Psi}}{\partial t} + \mathrm{u}\nabla\overline{\Psi} = 0; \\ \frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \mathrm{E}_{\mathrm{total}}\overline{\Psi} = 0, \ (\frac{\mathrm{E}_{\mathrm{total}}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = 0, \frac{\partial\overline{\Psi}}{\partial t} + \mathrm{u}\nabla\overline{\Psi} = 0. \end{array} \qquad (4.10\text{-}3)$$

It is worth mentioning that based on (4.10-3) we could ask ourselves what Dirac's wave equation really predicted, and how much this prediction (matter-antimatter particles) was based (only) on Dirac's equation. In other words, the prediction was luckily successful, but not too much (and exclusively) related to what Dirac took as the starting and strongest platform for making such prediction. Much worse than that is that generations of scientists after Dirac, involved in similar fields of research, just continued non-critically repeating what Dirac (and his well-obeying followers) said in his shiny moments of brainstorming and inspiration.

We can also summarize above results regarding different energy translations and corresponding operator forms of Schrödinger-like equations (found in (4.11), (4.11-1) and (4.11-2)) and see that all of them produce mutually different operators,

$$\left\{ \begin{array}{l} \tilde{\mathbf{E}} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar \dfrac{\partial}{\partial t} + \mathrm{U_p} \Leftrightarrow -\mathbf{j}\hbar \mathrm{u} \nabla \text{ ,} \\ \tilde{\mathbf{p}}_\mathbf{i} \Leftrightarrow -\mathbf{j}\hbar \nabla \Leftrightarrow \dfrac{1}{\mathrm{u}} (\mathbf{j}\hbar \dfrac{\partial}{\partial t} + \mathbf{U_p}) \Leftrightarrow \dfrac{1}{\mathrm{u}} \mathrm{H} \end{array} \right\}, \qquad (4.11)$$

$$\left\{ \begin{array}{l} \mathbf{E}_{\mathrm{total}} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar \dfrac{\partial}{\partial t} + \mathrm{U_p} \Leftrightarrow -\mathbf{j}\hbar \mathrm{u} \nabla \text{ ,} \\ \tilde{\mathbf{p}}_\mathbf{i} \Leftrightarrow -\mathbf{j}\hbar \nabla - \dfrac{\mathrm{E_0} + \mathrm{U_p}}{\mathrm{u}} \Leftrightarrow \dfrac{1}{\mathrm{u}} (\mathbf{j}\hbar \dfrac{\partial}{\partial t} - \mathbf{E_0}) \Leftrightarrow \dfrac{1}{\mathrm{u}} (\mathrm{H} - \mathrm{E_0} - \mathrm{U_p}) \end{array} \right\}, \qquad (4.11\text{-}1)$$

$$\left\{ \begin{array}{l} \mathbf{E}_{\mathrm{total}} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar \dfrac{\partial}{\partial t} \Leftrightarrow -\mathbf{j}\hbar \mathrm{u} \nabla \text{ ,} \\ \tilde{\mathbf{p}}_\mathbf{i} \Leftrightarrow -\mathbf{j}\hbar \nabla - \dfrac{\mathrm{E_0} + \mathrm{U_p}}{\mathrm{u}} \Leftrightarrow \dfrac{1}{\mathrm{u}} (\mathbf{j}\hbar \dfrac{\partial}{\partial t} - \mathrm{E_0} - \mathbf{U_p}) \Leftrightarrow \dfrac{1}{\mathrm{u}} (\mathrm{H} - \mathrm{E_0} - \mathbf{U_p}) \end{array} \right\}. \qquad (4.11\text{-}2)$$

The most important conclusion from (4.10-3) and (4.11), (4.11-1) and (4.11-2) is that some of the very basic elements and step-stones of modern Quantum Mechanics and particle-wave dualism concept could/should be revised, as proposed in this paper *(since different forms of Schrödinger equations can be simply developed without knowing that Quantum Theory ever existed). None of the equations from (4.10-3) gives us the (generally valid) right to treat the total energy, or total particle mass (including the rest energy and rest mass) as a 1:1 equivalent to a total particle wave energy.* Only motional or kinetic energy create de Broglie matter waves. Also, from (4.11), (4.11-1) and (4.11-2) it looks that there are no common sense and universally applicable operators that could uniquely present Schrödinger's equation, since every time when we create an energy level shift, operators also change their forms. Later on it will be shown that more universally valid operators for all Schrödinger-like equations can be found, but in a bit different form than presently seen in Quantum Mechanics: see (4.22)-(4.28).

Contrary to (4.10-3), the official Quantum Mechanics (regarding Schrödinger's equation) effectively generated the following results (treating the particle energy, either as kinetic energy, as known in Classical Mechanics, or using the relativistic expression for the total particle energy (Dirac), including potential energy, in both cases, and approximating $\mathbf{u}/\mathbf{v} = ½$**,** for $\mathbf{v} << \mathbf{c}$; **-** see [**9**] and (4.10-3)),

$$-\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \mathbf{U_p}\overline{\Psi} = \frac{-\hbar^2}{\tilde{\mathbf{E}}}\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial t} = -j\hbar u\nabla\overline{\Psi} = \tilde{\mathbf{E}}\overline{\Psi},$$

$$\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} - \mathbf{U_p})\overline{\Psi} = 0\,,\ (\frac{\tilde{\mathbf{E}}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = 0\,,\ \frac{\partial\overline{\Psi}}{\partial t} + u\nabla\overline{\Psi} = 0\,,$$

$$\begin{aligned}&-\frac{(\hbar u)^2}{\tilde{E}}\left[\Delta\overline{\Psi} - \frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}\right] + U_p\overline{\Psi} = \\ &= j\hbar\frac{\partial\overline{\Psi}}{\partial t} + \frac{\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2} = -j\hbar u\nabla\overline{\Psi} + \frac{\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2} = \tilde{E}\overline{\Psi} + \frac{\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2} = 0,\ v \cong 2u.\end{aligned} \tag{4.10-4}$$

Starting from (4.10-4) we can easily construct the following operator forms of traditional Schrödinger equation (most of them found in today's Quantum Mechanics):

$$\begin{aligned}&\left(H = -\frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta + U_p\ (=)\ \text{Hamiltonian}\right) \Rightarrow \left\{H\overline{\Psi} = j\hbar\frac{\partial}{\partial t}\overline{\Psi} = -j\hbar u\nabla\right\},\\ &\tilde{E} \Leftrightarrow H \Leftrightarrow j\hbar\frac{\partial}{\partial t} \Leftrightarrow -j\hbar u\nabla\ ,\ \tilde{p}_i \Leftrightarrow -j\hbar\nabla \Leftrightarrow j\frac{\hbar}{u}\frac{\partial}{\partial t} \Leftrightarrow \frac{1}{u}H.\end{aligned} \tag{4.11-3}$$

The differences between (4.10-3) and (4.10-4), as well as between (4.11), (4.11-1), (4.11-2) and (4.11-3), are sufficiently small and not so evident and clear, that it could be difficult to find what is really wrong with (4.10-4) and (4.11-3), but even such small differences have been sufficient to produce a lot of challenges and waving in the Physics of 20th century (especially when rest mass or rest energy is taken as a part of matter waves energy).

Now we are in the position to explicitly formulate the principal differences between the wave energy concepts in the traditionally known Schrödinger's equation (4.10-4) and generalized, Schrödinger-like equations, (4.10-3), developed in this paper:

Traditional (present) particle-wave duality concept (valid for equations (4.10-4)),

$$\left[\begin{array}{l}\tilde{E}_{\text{traditional}} = E = hf = \hbar\omega \equiv \begin{cases} E_{\text{total-Classic}} = \dfrac{p^2}{2m} + U_p = E_k + U_p\ ,\text{or} \\ E_{\text{total-Relativistic}} = \sqrt{c^2p^2 + (E_0)^2} + U_p = E_k + E_0 + U_p \end{cases} \\ uv = c^2;\ v \le c\ \Rightarrow u \ge c;\ v << c \Rightarrow u \cong v/2,\end{array}\right]$$

Corrected particle-wave duality in this paper (valid for equations (4.10-3)),

$$\left[\begin{array}{l} \tilde{\mathbf{E}}_{\text{this-paper}} = \tilde{\mathbf{E}} = \mathbf{hf} = \hbar\omega = \tilde{\mathbf{p}}\mathbf{u} = (\textbf{particle} - \textbf{wave energy}) = \mathbf{E_k} = (\gamma - 1)\mathbf{mc}^2 = \\ = \begin{cases} \mathbf{E}_{\text{total-Classic}} - \mathbf{U_p} = \dfrac{\mathbf{p}^2}{\mathbf{2m}} = \mathbf{pu} = \mathbf{E_k}\,,\ \ \textbf{or} \\ \mathbf{E}_{\text{total-Relativistic}} - \mathbf{E_0} - \mathbf{U_p} = \sqrt{\mathbf{c}^2\mathbf{p}^2 + (\mathbf{E_0})^2} - \mathbf{E_0} = \mathbf{E_k} = \mathbf{pu} = \dfrac{\mathbf{pv}}{1 + \sqrt{\mathbf{1} - \mathbf{v}^2/\mathbf{c}^2}} \end{cases} \\ \mathbf{d}\tilde{\mathbf{E}} = \mathbf{hdf} = \mathbf{vd}\tilde{\mathbf{p}} = \mathbf{d}(\tilde{\mathbf{p}}\mathbf{u}) = -\mathbf{dE_k} = -\mathbf{vdp} = -\mathbf{d}(\mathbf{pu}), \\ \mathbf{v} = \mathbf{u} - \lambda(\mathbf{du}/\mathbf{d}\lambda),\ \ (\mathbf{v}/\mathbf{u}) = \mathbf{1} + \sqrt{\mathbf{1} - \mathbf{v}^2/\mathbf{c}^2}\ \ \Rightarrow\ \ \mathbf{0} \le \mathbf{2u} \le \sqrt{\mathbf{uv}} \le \mathbf{v} \le \mathbf{c}. \end{array}\right] \quad \textbf{(4.10-5)}$$

Finally, when the best data (and ideas) fitting and cross-correlation of different concepts, and the best mathematical modeling was established (in the form of today's Orthodox Quantum Mechanics, see [**9**]), we could say that (now) we have well-operating and abstract mathematical theory (*which mathematically united all similarities and bridged, or compensated all differences (4.10-5), mentioned above, even without noticing them from the same point of view as described in this paper*). Sometimes, in the absence of sufficient conceptual argumentation, in many books regarding modern Quantum Mechanics we can find the statements (based on correct experimental achievements), that countless number of predictions and applications (or calculations using Schrödinger's equation in the frames of Orthodox Quantum Mechanics) have been justifying certain (ad-hock) mathematical concepts, or equations and models that are not fully and step-by-step developed starting from very basic elements. *This way, indirectly and deductively, Orthodox Quantum Mechanics established its strongest supporting arguments, which are also supporting here (step-by-step) developed equations.*

The next important point is to clarify the meaning of the complex wave function (4.9), favored here, that is not fully equivalent to the usual complex wave function known in Quantum Mechanics. *In fact, (in this paper) only a real wave function, $\Psi(\mathbf{x},\mathbf{t})$, is directly representing de Broglie matter wave/s*, but because of number of mathematical conveniences, we transform a real (and arbitrary) wave function into its complex replacement $\Psi(\mathbf{x},\mathbf{t}) \to \overline{\Psi}(\mathbf{x},\mathbf{t})$, known in mathematics as Analytic Signal, where we can easily find and represent all signal elements (amplitude, phase and frequency, both in time and frequency domains) which are equivalent (or similar) to any simple harmonic (sinusoidal) function $\Psi = a\,\mathbf{sin}\,\omega t$, as for instance:

$$\overline{\Psi}(\mathbf{x},\mathbf{t}) = \Psi(\mathbf{x},\mathrm{t}) + j\hat{\Psi}(x,t) = \mathbf{a}(\mathbf{x},\mathbf{t})\mathbf{e}^{\mathbf{j}\varphi(\mathbf{x},\mathbf{t})} = \frac{\mathbf{1}}{\pi^2}\iint_{(\mathbf{0},+\infty)} \mathbf{A}(\omega,\mathbf{k})\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}-\mathbf{kx}+\Phi(\omega,\mathbf{k}))}\mathbf{dkd}\omega\,,$$

$$\hat{\Psi}(x,\mathrm{t}) = \mathrm{H}\left[\Psi(\mathrm{x},\mathrm{t})\right],\ \mathrm{a}(\mathrm{x},\mathrm{t}) = \sqrt{\Psi^2 + \hat{\Psi}^2}\,,$$

$$\varphi(x,t) = \operatorname{arctang}\left[\hat{\Psi}(x,t)/\Psi(x,t)\right],\ \ \omega(\mathrm{t}) = 2\pi\mathrm{f}(\mathrm{t}) = \partial\varphi/\partial\mathrm{t}\,,$$

$$\mathbf{U}(\omega,\mathrm{k}) = \mathbf{U_c}(\omega,\mathrm{k})\ \textbf{-}\,\mathbf{j}\,\mathbf{U_s}(\omega,\mathrm{k}) = \iint_{(-\infty,+\infty)} \overline{\Psi}(\mathbf{x},\mathbf{t})\,\mathbf{e}^{\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}\mathrm{dtdx} = \mathbf{A}(\omega,\mathrm{k})\mathbf{e}^{-\mathbf{j}\Phi(\omega,\mathrm{k})}\,.$$

In order to fully understand all advantages of Analytic Signal forms it is useful to refer to relevant chapters of Signal Analysis (see [**7**] and [**8**]). Here we can briefly say that Analytic Signal Model (beside many other advantages) produces explicit definitions (and expressions) of immediate and time-evolving amplitude, phase and frequency functions of any signal (or arbitrary wave function), both in time and frequency domains (even in a joint time-frequency domain). This is not possible to find by using Orthodox Quantum Mechanics complex wave function (since precise phase functions have no use, or meaning in today's Quantum Mechanics, because "probability philosophy" takes care only about statistical distributions and resulting mean effects of certain process). *After calculating with complex wave functions in the form of Analytic Signals, the final result can easily be transformed (back) into real wave function,* $\overline{\Psi}(\mathbf{x},\mathbf{t}) = \mathbf{a}(\mathbf{x},\mathbf{t})\mathbf{e}^{\mathbf{j}\varphi(\mathbf{x},\mathbf{t})} \rightarrow \Psi(\mathbf{x},\mathbf{t}) = \mathbf{a}(\mathbf{x},\mathbf{t})\cos\varphi(\mathbf{x},\mathbf{t})$*, that is the right solution, relevant for wave functions and equations favored in this paper (describing the square root of active power and giving information regarding field components of certain wave phenomena).* Contrary to Analytic Signal wave function (which could be either harmonic or an arbitrary and non-harmonic function), in Quantum Mechanics it is essential that the wave function is very much artificially formulated (from the very beginning and later on; -see [**9**]) as a complex and harmonic function, and relevant results should be transformed into real functions finding their absolute values (also using specific complex operators for every specific case). In many other aspects, comparing: (**a**) -typical Fourier signal analysis (applied on wave functions), (**b**) -Quantum Mechanics operations with wave functions, and (**c**) -Analytic Signal wave functions, we can notice so many (mathematical) similarities between them, that one should have a passionate and profound attention to small details in order to extract all finesses, differences and advantages of the Analytic Signal model (see [**7**] and [**8**]). *When we come back to physics, we can distinguish two different, but mutually not-contradictory understandings of the wave function, as for instance: a) In Orthodox Quantum Mechanics we say that probability (distribution) of finding the particle (including its rest mass and its total energy constituents) in a certain space-element is given by the square of (the Schrödinger's) wave function, and here, regarding the same particle, b) For Analytic Signal Wave Function we talk about the square of the wave function that represents active power distribution of de Broglie, or Matter-Wave Field (*$\mathbf{P} = \Psi^2 = \mathbf{d}\tilde{\mathrm{E}} / \mathbf{dt}$*) associated with the particle in the same space domain (but not taking into account the particle's rest mass, or rest energy).*

*[♣ **COMMENTS & FREE-THINKING CORNER:** There are some more mathematical possibilities (here only briefly mentioned) to be exploited in relation to Analytic Signal and general wave equations, as for instance:*

1° Schrödinger equation (in this paper) is created taking into account the complex wave function $\overline{\Psi}(\mathbf{x},\mathbf{t}) = \Psi(\mathbf{x},\mathbf{t}) + \mathbf{j}\hat{\Psi}(\mathbf{x},\mathbf{t}) = \mathbf{a}(\mathbf{x},\mathbf{t})\mathbf{e}^{\mathbf{j}\varphi(\mathbf{x},\mathbf{t})}$*, and we can also try to create other forms of similar equations dependant only on* $\Psi(\mathbf{x},\mathbf{t})$ *or* $\hat{\Psi}(\mathrm{x},\mathrm{t})$*.*

2° The next interesting possibility would be to separate (or develop) wave equations to become dependant only on amplitude, $\mathrm{a}(\mathbf{x},\mathbf{t})$*, or phase function,* $\varphi(\mathbf{x},\mathbf{t})$*, in order to master all problems regarding phase and group velocity and energy transfer.*

3° Since the wave function Ψ *and active power* $\mathbf{P}$ *are closely related we can also formulate all wave equations to be dependant only on active power function (*$\mathbf{P} = \Psi^2 = \mathbf{d}\tilde{\mathrm{E}} / \mathbf{dt}$*).*

4° The Analytic Signal modeling of the wave function can easily be installed in the framework of the Fourier Integral Transform, which exists on the basis of simple harmonic functions cosωt,

$$\Psi(t) = a(t)\cos\varphi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})e^{j2\pi ft}df = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\{\overline{H}[\cos 2\pi ft]\}df = F^{-1}\left[U(\frac{\omega}{2\pi})\right],$$

$$U(\frac{\omega}{2\pi}) = A(\frac{\omega}{2\pi})e^{j\Phi(\frac{\omega}{2\pi})} = \int_{-\infty}^{\infty}\Psi(t)e^{-j2\pi ft}dt = \int_{-\infty}^{\infty}\Psi(t)\{\overline{H}^*[\cos 2\pi ft]\}dt = F[\Psi(t)],\ \omega = 2\pi f,$$

where the meaning of symbols is:

***F** (=) Direct Fourier transform,*

***F**$^{-1}$ (=) Inverse Fourier transform,*

$\overline{H} = 1 + jH$ *(=) Complex Hilbert transform, $j^2 = -1$,*

$\overline{H}^* = 1 - jH$ *(=) Conjugate complex Hilbert transform.*

$$\overline{H}[\cos\omega t] = e^{j\omega t},\quad H[\cos\omega t] = \sin\omega t,$$
$$\overline{H}^*[\cos\omega t] = e^{-j\omega t},\quad H[\sin\omega t] = -\cos\omega t,$$
$$e^{\pm j\omega t} = (1 \pm jH)[\cos\omega t],$$
$$\overline{H}[\Psi(t)] = \overline{\Psi}(t) = \Psi(t) + jH[\Psi(t)] = \Psi(t) + j\hat{\Psi}(t)\ ,$$
$$\overline{H}^*[\Psi(t)] = \overline{\Psi}^*(t) = \Psi(t) - jH[\Psi(t)] = \Psi(t) - j\hat{\Psi}(t).$$

*The further generalization of the Fourier integral transformation can be realized by convenient replacement of its simple harmonic functions basis **cosωt** by some other (compatible) signal basis **$\alpha(\omega,t)$**. Now, the general wave function (in the generalized framework of Fourier transform) can be represented as,*

$$\Psi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\{\overline{H}[\alpha(\omega,t)]\}df = F^{-1}\left[U(\frac{\omega}{2\pi})\right],$$

$$U(\frac{\omega}{2\pi}) = \int_{-\infty}^{\infty}\Psi(t)\{\overline{H}^*[\alpha(\omega,t)]\}dt = F[\Psi(t)]\ .$$

Let us imagine that $\alpha(t)$ presents the finite energy elementary signal that carries the energy of a single energy quantum. Now Planck's wave energy expression should correspond to:

$$\int_{[t]}[\alpha(t)]^2 dt = \frac{1}{2\pi}\int_{[0,\infty]}[B(\omega)]^2 d\omega = h\,\overline{f} = hf,\quad f = \frac{\omega}{2\pi},\ h = const..$$

5° Another possibility is to treat the wave function (4.9) as the composition of two waves propagating in mutually opposed directions,

$$\overline{\Psi}(\mathbf{x,t}) = \Psi^{+}(\mathbf{x,t}) + \mathbf{j}\hat{\Psi}^{+}(\mathbf{x,t}) + \Psi^{-}(\mathbf{x,t}) + \mathbf{j}\hat{\Psi}^{-}(\mathbf{x,t}) = \mathbf{a}(\mathbf{x,t})\mathbf{e}^{\mathbf{j}\varphi(\mathbf{x,t})} =$$

$$= \frac{\mathbf{1}}{(\mathbf{2}\pi)^2} \iint\limits_{(-\infty,+\infty)} \mathbf{U}^{+}(\omega,\mathbf{k})\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}\mathbf{dkd}\omega + \frac{\mathbf{1}}{(\mathbf{2}\pi)^2} \iint\limits_{(-\infty,+\infty)} \mathbf{U}^{-}(\omega,\mathbf{k})\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}+\mathbf{kx})}\mathbf{dkd}\omega =$$

$$= \frac{\mathbf{1}}{\pi^2} \iint\limits_{(\mathbf{0},+\infty)} \mathbf{U}^{+}(\omega,\mathbf{k})\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}-\mathbf{kx})}\mathbf{dkd}\omega + \frac{\mathbf{1}}{\pi^2} \iint\limits_{(\mathbf{0},+\infty)} \mathbf{U}^{-}(\omega,\mathbf{k})\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}+\mathbf{kx})}\mathbf{dkd}\omega =$$

$$= \frac{\mathbf{1}}{\pi^2} \iint\limits_{(\mathbf{0},+\infty)} \mathbf{A}^{+}(\omega,\mathbf{k})\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}-\mathbf{kx}+\Phi(\omega,\mathbf{k}))}\mathbf{dkd}\omega + \frac{\mathbf{1}}{\pi^2} \iint\limits_{(\mathbf{0},+\infty)} \mathbf{A}^{-}(\omega,\mathbf{k})\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}+\mathbf{kx}+\Phi(\omega,\mathbf{k}))}\mathbf{dkd}\omega,\ \mathbf{j}^2 = -\mathbf{1},$$

$$\mathbf{U}^{+/-}(\omega,\mathbf{k}) = \mathbf{U}^{+/-}{}_{\mathbf{c}}(\omega,\mathbf{k}) - \mathbf{j}\,\mathbf{U}^{+/-}{}_{\mathbf{s}}(\omega,\mathbf{k}) = \mathbf{A}^{+/-}(\omega,\mathbf{k})\mathbf{e}^{-\mathbf{j}\Phi(\omega,\mathbf{k})}\ .$$

There are numerous interesting possibilities to continue representing and analyzing wave functions using different signal basis functions (to be analyzed some other time). ♣*]*

4.3.1. Wave Energy and Mean Values

Up to present we only demonstrated applicability and compatibility of Planck's wave energy relation, $\tilde{E}_i = hf_i$, with Energy and Momentum conservation laws, as well as with de Broglie matter wavelength, $\lambda_i = h / p_i$, without precisely showing what really makes those relations correct. The answer on the most simple question how and why only one specific frequency (multiplied by Planck's constant) can represent the motional energy of a wave group (or what means that frequency) should be found. The first intuitive and logical starting point could be to imagine that this is just the mean frequency, $\bar{\mathbf{f}}$, of the corresponding matter-wave group calculated regarding its energy (since wave group, or wave packet, or de Broglie matter wave is composed of infinity of elementary waves, covering certain, not too wide, frequency interval: $\mathbf{0} \leq \mathbf{f}_{\min.} \leq \bar{\mathbf{f}} \leq \mathbf{f}_{\mathbf{max.}} < \infty$).

$$\tilde{E} = \int_{-\infty}^{+\infty} P(t)dt = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty} \left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)dt = \tilde{p}u = h\bar{f}\ . \qquad (4.13)$$

As known from Signal and Spectrum analysis, time and frequency domains of wave function (4.9) can be connected using Parseval's identity, thus the energy of de Broglie matter wave can also be presented as:

$$\tilde{E} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left|U(\omega)\right|^2 d\omega = \frac{1}{\pi}\int_{0}^{\infty} \left[A(\omega)\right]^2 d\omega = \tilde{p}u = h\bar{f}\,. \qquad (4.14)$$

Now (by definition) we can find the mean frequency as,

$$\bar{f} = \frac{\frac{1}{\pi}\int_0^{\infty} f\cdot\left[A(\omega)\right]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\right]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty} f\cdot\left[A(\omega)\right]^2 d\omega}{\tilde{E}}, \qquad (4.15)$$

and replace it into the wave energy expression, (4.14),

$$\tilde{E} = \frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega = h\overline{f} = h\frac{\frac{1}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega}{\tilde{E}} \Rightarrow \tilde{E}^2 = \frac{h}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega\,. \qquad (4.16)$$

Using one of the most general formulas valid for all definite integrals (and applying it to (4.16)), we can prove that the wave energy (of a de Broglie wave group) should be equal to the product between Planck's constant and mean frequency of the wave group in question, as follows:

$$\left\{\begin{array}{l} \int_a^b f(x)\cdot g(x)dx = f(c)\int_a^b g(x)dx\,, a < c < b\,, g(x) \geq 0\,, \\ f(x) \text{ and } g(x) - \text{continuous in } [a \leq x \leq b], \\ f(x) = f\,, g(x) = [A(\omega)]^2 > 0\,, x = \omega \in (0\,, \infty) \end{array}\right\} \Rightarrow$$

$$\Rightarrow\left[\tilde{E}^2 = \frac{h}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega = \frac{h}{\pi}\cdot\overline{f}\cdot\int_0^{\infty}[A(\omega)]^2 d\omega = h\overline{f}\cdot\tilde{E}\right] \Rightarrow \tilde{E} = h\overline{f}\,. \qquad (4.17)$$

If Planck's energy of wave group deals with mean frequency (of that wave group), the same should be valid for de Broglie wavelength, as well as for its phase and group velocities (meaning that all of them should be treated as mean values describing the motion of an effective center of inertia, or center of gravity of that wave group). Consequently we do not need to mark them, as it was the case with mean frequency (since we know that all of them should anyhow be mean values $\overline{\mathbf{f}} = \mathbf{f}$, $\overline{\lambda} = \lambda$, $\overline{\mathbf{u}} = \mathbf{u}$, $\overline{\mathbf{v}} = \mathbf{v}$). *The next important conclusion is that* <u>*all forms of wave equations, (4.10-3), effectively deal only with mean values (regarding energy, momentum, frequency, wavelength, velocities...).*</u> For instance, group and phase velocity can also be found as mean values in the following way:

$$\overline{v}_g = \overline{v} = \frac{\frac{1}{\pi}\int_0^{\infty} v\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty}\frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty}\frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{d\tilde{E}}{d\tilde{p}}, \qquad (4.17\text{-}2)$$

$$\overline{v}_f = \overline{u} = \frac{\frac{1}{\pi}\int_0^{\infty} u\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty}\frac{\omega}{k}\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty}\frac{\omega}{k}\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{\tilde{E}}{\tilde{p}}. \qquad (4.17\text{-}1)$$

*[♣ **COMMENTS & FREE-THINKING CORNER:** We could additionally test the Planck's radiation law, regarding photon energy* $\mathbf{E}=\mathbf{hf}=\frac{\mathbf{h}}{2\pi}\omega$. *It is well proven that a photon has the wave energy equal to the product between Planck's constant h and photon's frequency f. Photon is a wave phenomena and it should be presentable using certain time-domain wave function* $\psi(\mathbf{t})=\mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})$, *expressed in the form of an Analytic signal. Since the Analytic signal presentation gives the chance to extract immediate signal amplitude* $\mathbf{a}(\mathbf{t})$, *phase* $\varphi(\mathbf{t})$, *and frequency* $\omega(\mathbf{t})=\frac{\partial\varphi(\mathbf{t})}{\partial\mathbf{t}}=2\pi\mathbf{f}(\mathbf{t})$, *let us extend and test the meaning of Planck's energy when: instead of constant photon frequency (valid for a single photon) we take the mean wave frequency,* $\Omega=\langle\omega(\mathbf{t})\rangle$, *of the time-domain wave function* $\psi(\mathbf{t})$.

$$\mathbf{E}=\mathbf{hf}=\frac{\mathbf{h}}{2\pi}\omega$$

$$\mathbf{E}=\int_{[\mathbf{T}]}\psi^2(\mathbf{t})\mathbf{dt}=\int_{[\mathbf{T}]}\left[\mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})\right]^2\mathbf{dt}=\int_{[\mathbf{T}]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}\ (=)\ \mathbf{hf}=\frac{\mathbf{h}}{2\pi}\langle\omega\rangle$$

$$\psi(\mathbf{t})=\mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})=-\mathbf{H}\left[\hat{\psi}(\mathbf{t})\right],\ \hat{\psi}(\mathbf{t})=\mathbf{a}(\mathbf{t})\sin\varphi(\mathbf{t})=\mathbf{H}\left[\psi(\mathbf{t})\right]$$

$$\mathbf{a}(\mathbf{t})=\sqrt{\psi^2(\mathbf{t})+\hat{\psi}^2(\mathbf{t})}$$

$$\varphi(\mathbf{t})=\mathbf{arctg}\frac{\hat{\psi}(\mathbf{t})}{\psi(\mathbf{t})}$$

$$\omega(\mathbf{t})=\frac{\partial\varphi(\mathbf{t})}{\partial\mathbf{t}}=2\pi\mathbf{f}(\mathbf{t})\Rightarrow\quad\Omega=\langle\omega(\mathbf{t})\rangle=\frac{\mathbf{1}}{\mathbf{T}}\int_{[\mathbf{T}]}\omega(\mathbf{t})\mathbf{dt}\ (=)\ \frac{\frac{\mathbf{1}}{\mathbf{T}}\int_{[\mathbf{T}]}\mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}}{\int_{[\mathbf{T}]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}}$$

$$\mathbf{E}=\mathbf{hf}=\frac{\mathbf{h}}{2\pi}\omega\Rightarrow\mathbf{E}=\frac{\mathbf{h}}{2\pi}\Omega\Rightarrow$$

$$\mathbf{E}=\frac{\mathbf{h}}{2\pi}\frac{\mathbf{1}}{\mathbf{T}}\frac{\int_{[\mathbf{T}]}\mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}}{\int_{[\mathbf{T}]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}}(=)\frac{\mathbf{h}}{2\pi}\frac{\mathbf{1}}{\mathbf{T}}\int_{[\mathbf{T}]}\omega(\mathbf{t})\mathbf{dt}=\int_{[\mathbf{T}]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}\ \Rightarrow$$

$$\Rightarrow\frac{\left[\int_{[\mathbf{T}]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}\right]^2}{\frac{\mathbf{1}}{\mathbf{T}}\int_{[\mathbf{T}]}\mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}}=\frac{\mathbf{h}}{2\pi}=\mathbf{Const.},\quad\mathbf{or}\quad\frac{\int_{[\mathbf{T}]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}}{\frac{\mathbf{1}}{\mathbf{T}}\int_{[\mathbf{T}]}\omega(\mathbf{t})\mathbf{dt}}=\frac{\mathbf{h}}{2\pi}=\mathbf{Const.}$$

Depending on how we calculate the mean frequency, we should be able to prove at least one of the above given relations (see the last line), or to find the family of wave functions which describe photon in a time domain, or in any case, to see how universal Planck's energy law could be regarding energy of arbitrary wave functions. ♣]

4.3.2. Inertial and Reaction Forces

Now we can formulate the starting platform for establishing unified force/fields theory (at least valid for all dynamic interactions between different motional states of particles, quasiparticles and similar objects). De Broglie waves undoubtedly describe specific, coupling (waiving or oscillating) field structure inside and around interacting objects, manifesting specific forces and fields between them. If de Broglie, or matter wave function represents active power distribution, we should be able to determine a force field/s distribution in the space of definition of wave function, $\Psi(t,r)$ based on the idea first introduced in the second chapter (see equations from (2.5.1) to (2.9) and (4.3)), regarding the unity of linear and rotational motions:

$$\begin{aligned} & dE = c^2 d(\gamma m) = mc^2 d\gamma = vdp + \omega dL = \Psi^2 dt, \quad p = \gamma mv, \quad \gamma = (1 - v^2/c^2)^{-0.5} \\ & \Psi^2 = \frac{dE}{dt} = v\frac{dp}{dt} + \omega\frac{dL}{dt} = \frac{dx}{dt}\frac{dp}{dt} + \frac{d\alpha}{dt}\frac{dL}{dt} = vF + \omega\tau = mc^2\frac{d\gamma}{dt} = \text{Power} = [W] \end{aligned} \tag{2.5.1}$$

The objective behind (2.5.1) is to establish the concept that any complex particle (regarding its total energy content) is composed of linear motion components, $\mathbf{vdp}$ and rotational motion components, $\omega d\mathbf{L}$ and that both of them are intrinsically involved in creating total particle energy ($dE = mc^2 d\gamma = vdp + \omega d\mathbf{L}$). The mutual relation between linear and rotational motion components (related to the same moving particle) could be "analogically visualized" as a relation between current and voltage components inside of a closed, oscillating R-L-C circuit (where electric and magnetic field mutually communicate by currents and voltages trough inductive and capacitive elements). The wave function behind such modeling should have two wave components,

$$\begin{aligned} & \Psi^2 = \frac{d\tilde{E}}{dt} = v\frac{dp}{dt} + \omega\frac{dL}{dt} = vF + \omega\tau = -v\frac{d\tilde{p}}{dt} - \omega\frac{d\tilde{L}}{dt} = -v\tilde{F} - \omega\tilde{\tau} = \\ & = vF_{linear} + \omega F_{angular} = \Psi^2_{linear} + \Psi^2_{angular} = mc^2\frac{d\gamma}{dt} \\ & \left\{ \begin{aligned} & \Psi^2_{linear} = vF_{linear} \Leftrightarrow F_{linear} = \frac{dp}{dt} = -\frac{d\tilde{p}}{dt} = \frac{1}{v}\Psi^2_{linear} \\ & \Psi^2_{angular} = \omega F_{angular} \Leftrightarrow F_{angular} = \frac{dL}{dt} = -\frac{d\tilde{L}}{dt} = \frac{1}{\omega}\Psi^2_{angular} \end{aligned} \right\} \Rightarrow F_q = \frac{1}{V_q}\Psi^2_q \\ & v = \frac{d\tilde{E}}{d\tilde{p}} = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = -\frac{\lambda^2}{2\pi}\frac{d\omega}{d\lambda}, \; u = \frac{\tilde{E}}{\tilde{p}} = \lambda f = \lambda\frac{\omega}{2\pi}, \; \tilde{E} = hf, \end{aligned} \tag{4.18}$$

where $\mathbf{v}$ and $\mathbf{u}$ are group and phase velocity of de Broglie wave packet. Practically, for any specific situation, it would be necessary to find the wave function, $\Psi(t,r)$, solving general Schrödinger's equation (4.10)-(4.12), and then to determine the linear and radial force field structure (see also generalization of force law/s formulated by (4.30), (4.31) and (4.37)).

Nuclear (and other known or still undiscovered) forces should also be presentable as a consequence of specific force field distribution/s, similar to (4.18) or (4.37).

In addition to (4.18), we could evoke the ideas of the philosopher R. Boskovic, who was the first to explain the (need of) existence of certain oscillatory force field (attractive and repulsive forces) in a narrow zone of impact between two objects (see [6]).

If the above concept proves logical, or at least specifying fruitful brainstorming ideas and shows good directions, the road to a unified and general field theory will be largely paved (see also (4.26), (4.29), (4.30), (4.31), (5.15) and (5.16) to get an idea how the particle-wave duality, universal force law and Schrödinger-like equations can additionally be upgraded).

[♣ COMMENTS & FREE-THINKING CORNER: It is very important to notice that (based on (4.2) and (4.18)) the particle-wave duality concept is extended to any situation where motional energy (regardless of its origin) is involved, indicating that motional energy is immanently coupled with appearance of de Broglie matter waves creating inertia-like reaction forces in the form of waves (where reaction and inertial waving forces can have gravitational, mechanical, rotational, electromagnetic... or some other nature). The most significant relation, found in (4.2), leading to such conclusion is the connection between wave and kinetic energy, $d\tilde{E} = hdf = vd\tilde{p} = hvdf_s = d(\tilde{p}u) = dE_k = vdp = d(pu)$***, which is compatible to de Broglie wavelength*** $\lambda = h/\tilde{p}$***, and to relations expressing group and phase velocities*** $v = d\omega / dk = dE/dp$, $u = \omega / k = \tilde{E}/\tilde{p}$, $v = u - \lambda du/d\lambda$***. Using similar conclusion process (based on analogies), we could also develop the following particle-wave duality relations*** $\omega d\tilde{L} = -\omega dL$, $ud\tilde{q}_{electric} = -udq_{electric}$, $id\tilde{q}_{magnetic} = -\mathbf{idq_{magnetic}}$ ***, (where*** $\tilde{\mathbf{q}}_{\mathbf{electric}}, \mathbf{q}_{\mathbf{electric}}$ ***and*** $\tilde{\mathbf{q}}_{\mathbf{magnetic}}, \mathbf{q}_{\mathbf{magnetic}}$ ***are electric and magnetic charges). From the point of view of cosmology and "expanding universe" we should also conclude that what we see as an expansion (driving force, or positive energy: characterized by*** $\mathbf{dE_k}$, $\mathbf{dp}$, $\mathbf{dL}$,... ***) should always be balanced with something what we, most probably, do not see (named here as reaction energy or inertial forces,*** $-\mathbf{d\tilde{E}}, -\mathbf{d\tilde{p}}, -\mathbf{d\tilde{L}}, \dots$ ***). It looks that here we are dealing with forces described by Newton action-reaction, or inertia law (extended to rotation, electromagnetism etc.). In (4.2) we also find that after integration of differential relations connecting particle and wave aspect of motion, we can get very useful finite differences relations, such as:*** $\Delta E_k = \Delta\tilde{E}$, $\Delta p = -\Delta\tilde{p}$, $\Delta L = -\Delta\tilde{L}$, $\Delta q = -\Delta\tilde{q}$, $\Delta\dot{p} = -\Delta\dot{\tilde{p}}$, $\Delta\dot{L} = -\Delta\dot{\tilde{L}}$, $\Delta\dot{q} = -\Delta\dot{\tilde{q}}, \dots$ *(see the end of the chapter 5 of this paper, where advantages of using <u>Central Differences</u> are presented). Modern physics also addresses the same problem (analyzing origins of Inertia) using the terminology of "transient mass fluctuations, electromagnetic radiation reaction forces, inertial reaction forces" etc.*

This paper initially started by establishing the wide analogy platform between different physical entities. For instance (see T.1.1 to T.1.6 and T.3.1 to T.3.3), there is a multilevel analogy between velocity and voltage (or potential difference), and between electric charge and momentum, which could be directly and imaginatively applied (in fact tested) in the equation (4.2) that connects group and phase velocity $v_g = v$, $v_p = u$ *(producing that group velocity,* $v_g = v$*, is analog to "**group voltage**" (=)* u_g*, or* $v_g = v \Leftrightarrow u_g$*, and phase velocity,* $v_p = u$*, is analog to "**phase voltage**" (=)* u_p*", or* $v_p = u \Leftrightarrow u_p$*,*

$$\left\{\begin{array}{l}(v = v_{group} = v_g \Leftrightarrow u_g), (u = v_{phase} = v_p \Leftrightarrow u_p), (q \Leftrightarrow p, \tilde{q} \Leftrightarrow \tilde{p}), \\ v_g = v_p - \lambda \frac{dv_p}{d\lambda} = v_p + \tilde{p}\frac{dv_p}{d\tilde{p}} = \frac{d\tilde{E}}{d\tilde{p}}, \ v_p = \frac{\tilde{E}}{\tilde{p}}\end{array}\right\} \Rightarrow$$

$$\Rightarrow u_g = u_p - \lambda' \frac{du_p}{d\lambda'} = u_p + \tilde{q}\frac{du_p}{d\tilde{q}} = \frac{d\tilde{E}}{d\tilde{q}}, u_p = \frac{\tilde{E}}{\tilde{q}}. \qquad (4.19)$$

*Using analogies in the similar way as in (4.19) we can (hypothetically) extend "**group phase**" concept to magnetic field and rotational motion (connecting "**magnetic group voltage**" and "**magnetic phase voltage**" as well as "**angular group**" and "**angular phase**" velocity) etc., (see also (3.4), (3.5), T.3.1 - T.3.3 for understanding the meaning of magnetic voltages and currents). It is another question (still not answered here) to prove if the "**group phase**" concept (4.19), based only on analogies, is universally applicable, and how to integrate it into positive knowledge of today's relevant physics (see also (5.15) and (5.16)). One of the possibilities that should be analyzed is to connect "**group phase**" concept to Retarded Lorentz Potentials (known in Maxwell Electromagnetic Theory). ♣]*

4.3.3. Probability and Conservation Laws

Obviously, it looks possible to develop majority of wave equations (known in Quantum Mechanics and wider, (4.10)-(4.10-3)... (4.12)) without (exclusively) associating the probability nature to wave function/s. It is also clear that we shall not sacrifice the flexibility and positive results of Quantum Mechanics by modifying (or correcting) the meaning of wave function from probability to "active power" wave function. By simple normalization we can always make "active power" wave function, (4.9), dimensionless and use it almost in the same way as it is currently used in Quantum Mechanics. Since here we modeled (or explained) de Broglie waves as the phenomenon that (beside other aspects) unites linear motion with rotation, (see (4.3)-(4.4) and Fig.4.1), the generalized Schrödinger equation, (4.10)-(4.12), (4.10-3), presents an additional (deductive) support to the hypothesis of this paper stating that new field/s and forces caused by rotation, (see (2.2)), should be introduced (or recognized) in physics.
In order to understand the profound backgrounds regarding how Orthodox Quantum Mechanics successfully established a wave function as a probability function, let us start from conservation laws as the strongest platform in physics. The majority of conservation laws in physics are covered by the law of energy conservation and several laws of different vector value conservation (such as different momentum conservation laws). Since total input energy, $\mathbf{E}_{\mathbf{inp.}}$, of one isolated system (passing trough certain internal transformation) will always stay equal to its total output energy, $\mathbf{E}_{\mathbf{outp.}}$, measured before and after realizing the transformation, and since the same is valid for all other important vector parameters of that system, $\vec{\mathbf{A}}_{\mathbf{inp.}} = \vec{\mathbf{A}}_{\mathbf{outp.}}$ (its moments, for instance), we can easily formulate the following normalized forms of such conservation laws:

$$\left\{\begin{array}{l}E_{tot.} = E_{inp.} = \sum_{(i)} E_i = E_{outp.} = \sum_{(j)} E_j \\ \vec{A}_{tot.} = \vec{A}_{inp.} = \sum_{(i)} \vec{A}_i = \vec{A}_{outp.} = \sum_{(j)} \vec{A}_j = \left|\vec{A}_{tot.}\right| \cdot \vec{a}_0 = A \cdot \vec{a}_0\end{array}\right\} \Rightarrow$$

$$\Rightarrow\begin{Bmatrix} 1=\frac{E_{inp.}}{E_{tot.}}=\sum_{(i)}\frac{E_i}{E_{tot.}}=\frac{E_{outp.}}{E_{tot.}}=\sum_{(j)}\frac{E_j}{E_{tot.}} \\ 1=\frac{\left|\vec{A}_{inp.}\right|}{A}=\left|\sum_{(i)}\frac{\vec{A}_i}{A}\right|=\frac{\left|\vec{A}_{outp.}\right|}{A}=\left|\sum_{(j)}\frac{\vec{A}_j}{A}\right|=\left|\vec{a}_0\right| \end{Bmatrix}\Leftrightarrow\begin{Bmatrix} 1=\sum_{(i)}e_i=\sum_{(j)}e_j \\ 1=\left|\sum_{(i)}\vec{a}_i\right|=\left|\sum_{(j)}\vec{a}_j\right| \end{Bmatrix}. \tag{4.20}$$

Quantum Mechanics succeeded to model the most convenient wave function, that can represent, or replace equations of normalized energy and vector conservation laws, (4.20), of a system in certain transformation, $\left(\sum_{(i)}e_i\,,\sum_{(j)}e_j\right)=\frac{E_{i,j}}{E_{tot.}}$, $\left(\left|\sum_{(i)}\vec{a}_i\right|,\left|\sum_{(j)}\vec{a}_j\right|\right)=\frac{\left|\vec{A}_{i,j}\right|}{A}$, by convenient finite or infinite summation forms (for instance using Fourier Integral Transformation, and framework of Probability theory and Statistics). This practically replaces finite (or discrete) summation elements $\sum_{(i)}\mathbf{e_i}\,,\sum_{(j)}\mathbf{e_j}\,,\left|\sum_{(i)}\vec{\mathbf{a}}_\mathbf{i}\right|,\left|\sum_{(j)}\vec{\mathbf{a}}_\mathbf{j}\right|$ by equivalent integral summation forms that have infinite number of elements (which now present spectral and/or probability components of such wave function). *Always when we have an infinite number of elements of certain system (even artificially constructed by applying certain abstractions and generalizations) this would be very much sufficient argument to apply laws of Probability and Statistics to that system, and to be sure to get very certain results.* Conveniently formulating or modeling the wave function as a probability-like function ($|\Psi|^2$), Quantum Mechanics **"invented"** the rules of the following replacement/s (or transformation/s):

$$\begin{Bmatrix} \sum_{(i)}e_i=\sum_{(j)}e_j=1 \\ \left|\sum_{(i)}\vec{a}_i\right|=\left|\sum_{(j)}\vec{a}_j\right|=1 \end{Bmatrix}\Leftrightarrow\begin{Bmatrix} \int|\Psi|^2\,dq=1\,,dq=dx_1\cdot dx_2\cdot...\cdot dx_n \\ \Psi=\Psi(x_1,x_2,...x_n)=\sum_m\alpha_m\Psi_m\,,\ \alpha_m=\text{const.} \end{Bmatrix} \tag{4.21}$$

The remaining part of the procedure was to establish all necessary regulations, transformations and definitions (belonging to the content of modern Quantum Mechanics) that will satisfy, support and defend above presented steps ((4.20) and (4.21)). Even the statement that $|\Psi|^2$ represents probability distribution of a certain energy state could be considered only as a **probability behaving function**, since its intrinsic and hidden origin is only related to satisfying basic conservation laws of physics. *Since the probability of certain multi-component event also obeys the law of total probability conservation (sum over all single probabilities is equal to one), this can be considered as an equivalent formulation of (4.21). The rest of the modeling work (of Quantum Theory) has been to make its wave function fully mathematically compatible (or not-contradictory) to all other physics related items, initially not taken into account.* ***Basically, the simple mathematical structure of several conservation laws (known in Physics before Quantum Theory was established) was maximally unified, normalized and generalized, and than replaced by another, isomorphic mathematical structure taken from Statistics, Probability and Signal Analysis (conveniently merging all of them, and where merging was not very much evident, simple and natural, new rules have been invented and legalized as the rules of victorious Quantum Theory).*** *Enormously big assistance in such legalizing,* *merging and modeling process, Quantum Theory could take from the Amalie Nether theorem (formulated 1905):*

-For every continuous symmetry of the laws of physics, there must be a conservation law.
-For every conservation law, there should exist a continuous symmetry.
Consequently, the most logical to do (and naturally applicable), has been to construct or complete the missing chapters of the Quantum Theory by (artificially) completing already recognizable symmetries, knowing that in this way we are also in agreement with all known or not well-known conservation laws that should be in the background of symmetries in question. Unfortunately Nether theorem and consciousness that it can be used for here described purpose (final modeling of Quantum Theory) has been taken very seriously much later than it was published, but still it has been very useful to finalize, generalize and test the building of already known Orthodox Quantum Theory. Later, the next "*multi-level of confusion*" in the same field has been unintentionally introduced by people involved in popularization and simplified explanations of what modern Quantum Theory should mean from different aspects of Reality. Eventually a "mass of uncritical and well obeying, good-will followers took everything of such Quantum Theory as the eternal truth of god's messages and orders". By the way, it is also important to say, that contemporary Quantum Theory really works well, and it is also honest what many of the founders of Quantum Theory say that nobody understands why it works well, and there is really a need and time to rectify such situation.

In fact, (the opinion of the author of this paper is that) only correct mathematics belonging to Signal and Spectrum Analysis was largely sufficient to formulate majority of rules, theorems, definitions and models (also) belonging to modern Quantum Mechanics (of course when integrated with **PWDC**, (4.1)-(4.3) and with other conservation laws). Probability and Statistics has been, in addition, a complementary and very convenient modeling framework and cower structure that gave the final form to the book of Quantum Mechanics (because (4.21) looks like summation of probabilities, and all rules of Probability and Statistics should anyhow be satisfied). *Since all forms of wave equations, (4.10-3), operate only with mean values (regarding energy, momentum, frequency, wavelength, velocities...-see (4.17)), there is only a small theoretical step to be made to formulate the complete Quantum Mechanics using the language and rules of Probability Theory and Statistics, what really happened in founding the Orthodox Quantum Mechanics. It should also be clear that operating with mean values is necessary, but not sufficient argument to say that Microphysics ontologically and intrinsically deals only with Probability and Statistics.*

One of the problems in (4.21) to be solved was/is in the fact that normalized energy members $\sum_{(i)} \mathbf{e}_i = \sum_{(j)} \mathbf{e}_j = \mathbf{1}$ present summations of state of rest (static, or constant) energy members, $\mathbf{e}_{oi}$ and $\mathbf{e}_{oj}$, and their remaining motional energies, $\mathbf{e}_{mi}$ and $\mathbf{e}_{mj} \Rightarrow$ $\sum_{(i)} \mathbf{e}_i = \sum_{(i)} (\mathbf{e}_{oi} + \mathbf{e}_{mi}) = \sum_{(j)} \mathbf{e}_j = \sum_{(j)} (\mathbf{e}_{oj} + \mathbf{e}_{mj}) = \mathbf{1}$. Quantum Mechanics pretends to represent all of such energy members (static = rest energy, and dynamic = motional energy) on the same way (using wave function/s, as a wave groups or wave packets), but ***in this paper we support the platform that proper wave function can exclusively represent only motional (dynamic) energy content, and that state of***

rest and motional energy members should be separately treated as: $\sum_{(i)} \mathbf{e}_{oi} = \sum_{(j)} \mathbf{e}_{oj}, \sum_{(i)} \mathbf{e}_{mi} = \sum_{(j)} \mathbf{e}_{mj}$. Also, after normalizing vector components, (4.21); $\left|\sum_{(i)} \vec{\mathbf{a}}_i\right| = \left|\sum_{(j)} \vec{\mathbf{a}}_j\right| = \mathbf{1}$, we additionally sacrifice (loose) their motional, or time-space evolving, signal phase information (loosing the chance to deal with the concept of active and reactive power and/or energy, and to analyze optimal active power transfer, like in electronics). In order to find a solution to such and similar problems, Quantum Mechanics simply introduced different (and mathematically practical) complex functions, normalization or renormalization rules, Feynman diagrams, Operators' algebra etc., without modifying the foundations of Orthodox Quantum Mechanics, making this situation more complex and conceptually fogy (but still mathematically operational and applicable to Physics).

*[♣ **COMMENTS & FREE-THINKING CORNER:** The official history of modern Quantum Theory is not presented in the same way as here, neither its historical development was following the structure presented with (4.20)-(4.21), but now (opinion of the author of this paper is that) the answer to the question why Quantum Theory has been so successful should be very much obvious: this is just the consequence of correct merging of conservation laws of Physics with mathematical rules and models of Signal Analysis, Probability and Statistics, also effectively merged with effects of rotation complemented to rectilinear motion, here formulated as **PWDC**. Probability associated to Quantum Mechanics is only a useful, neither essential nor wrong theoretical and modeling tool (fully compatible with the meaning of normalized conservation laws), relatively easy applicable whenever possible and logical (and, of course, we should agree that the nature of certain phenomena doesn't become exclusively probabilistic just because we use Statistics and Probability to make mathematical modeling of that phenomena). Certain phenomenology that looks to us to have a deep and intrinsic probabilistic nature (in a 4-dimensional space-time world) could easily lose its probabilistic image if we succeed to present and analyze the same phenomenology in a multi-dimensional space (with more than 4 dimensions), or if we find the better conceptual and modeling framework to handle the same problems. Some of the strange behaviors of de Broglie matter waves and light (or electromagnetic waves) maybe indirectly tell us that we are possibly dealing with oscillating and wave phenomena of some strange and for us still undetectable media (but we should not take such arguments and thinking too seriously). We are able to measure final effects and results of matter particle interactions with this (or inside of this) undetectable media, but (often) we really do not see what is oscillating, regarding de Broglie waves. We could also say that some kind of multi-dimensional (revitalized) ether or fluid should exist out of our possibility to detect it, where matter forms from our world are able to make interactions, propagation and waving (such as light or electromagnetic waves propagation in "empty space"). Effectively, we should conclude that there is no empty space in our universe, but often we do not know what exactly exist in the space where we "see nothing". Regardless of our inability to (directly) see and detect such strange media, or fluid, we are able to make mathematical modeling that is exactly predicting results of particle-wave interactions in it (of course when final results are projected back to our 4-dimensional word). This way, judging from our world, it could look to us that we are dealing with probabilities that something would happen (and often it really happens, exactly as predicted using Schrödinger's equation...), confirming only that we have practical and sufficiently well operating mathematical models for handling matter waves phenomenology (but obviously we do not have a complete conceptual picture of the same reality). Being more imaginative, we could assume existence of this (invisible and strange) fluid-like media that behaves similar to other fluids, where we can make (and mentally visualize) wave propagation and perturbations, vortices, diffraction and interference effects... Any particle moving inside such strange fluid will always create associated (de Broglie) wave phenomena around itself, like stone thrown in water. ♣]*

4.3.4. Mater Waves and Unified Field Theory

By simple algebraic transformations of different forms of Schrödinger equations (4.10.3), valid for different levels of energy translation, we can easily obtain the wave equation that has exactly the same form as electromagnetic waves equation $\Delta\Psi = \Delta(\mathcal{E},\mathcal{H}) = \frac{\mathbf{1}}{\mathbf{u}^2}\frac{\partial^2(\mathcal{E},\mathcal{H})}{\partial\,\mathbf{t}^2}$, $\{(\mathcal{E},\mathcal{H}) =$ (*electric and magnetic field*)$\}$, as well as several more of similar wave equations valid for other energy levels, as for instance,

$$\left\{\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \tilde{\mathbf{E}}\overline{\Psi} = 0\,,\ (\frac{\tilde{\mathbf{E}}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial\,\mathbf{t}^2} = 0\,,\ \mathbf{U_p} = 0\right\} \Rightarrow \underline{\Delta\overline{\Psi} = \frac{1}{\mathrm{u}^2}\frac{\partial^2\overline{\Psi}}{\partial\,\mathbf{t}^2}},$$

$$\left\{\frac{\hbar^2}{\tilde{\mathrm{m}}}(\frac{\mathrm{u}}{\mathrm{v}})\Delta\overline{\Psi} + (\tilde{\mathrm{E}}\text{-}\mathbf{U_p})\overline{\Psi} = 0\,,(\frac{\mathrm{E}\text{-}\mathbf{U_p}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial\,\mathrm{t}^2} = 0\right\} \Rightarrow \Delta\overline{\Psi} = (\frac{\tilde{\mathrm{E}}}{\mathrm{E}\text{-}\mathbf{U_p}})\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial\,\mathrm{t}^2},$$

$$\left\{\frac{\hbar^2}{\tilde{\mathrm{m}}}(\frac{\mathrm{u}}{\mathrm{v}})\Delta\overline{\Psi} + (\mathrm{E_{total}} - \mathrm{U_p})\overline{\Psi} = 0, (\frac{\mathrm{E_{total}} - \mathrm{U_p}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial\,\mathrm{t}^2} = 0\right\} \Rightarrow$$

$$\Rightarrow \Delta\overline{\Psi} = (\frac{\tilde{\mathrm{E}}}{\mathrm{E_{total}} - \mathrm{U_p}})\frac{1}{\mathrm{u}^2}\frac{\partial^2\overline{\Psi}}{\partial\,\mathrm{t}^2}, \tag{4.22}$$

$$\left\{\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \mathrm{E_{total}}\overline{\Psi} = 0\,,\ (\frac{\mathrm{E_{total}}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial\,\mathbf{t}^2} = 0\right\} \Rightarrow \Delta\overline{\Psi} = (\frac{\tilde{\mathbf{E}}}{\mathbf{E_{total}}})\frac{1}{\mathrm{u}^2}\frac{\partial^2\overline{\Psi}}{\partial\,\mathbf{t}^2}\,.$$

From (4.22), especially from $\Delta\overline{\Psi} = \frac{1}{\mathrm{u}^2}\frac{\partial^2\overline{\Psi}}{\partial\,\mathbf{t}^2} \Leftrightarrow \Delta(\mathcal{E},\mathcal{H}) = \frac{\mathbf{1}}{\mathbf{u}^2}\frac{\partial^2(\mathcal{E},\mathcal{H})}{\partial\,\mathbf{t}^2}$, *it is obvious that electromagnetic waves (in a free space,* $U_p = 0$*) are just one of visible (and non-probabilistic) manifestations of de Broglie matter waves (when rest mass doesn't exist* $\Rightarrow \tilde{\mathbf{E}} = \mathbf{E_{total}} = \mathbf{E_k}$*). There are also forms of (4.22) where rest mass should be involved, and where we can recognize other forms of matter waves. We also see that famous Schrödinger equation is nothing else but just another form of well-known (classical) wave equation, valid in electromagnetic theory, mechanics, acoustics, fluid dynamics... Going backwards to some of the earlier chapters of this paper, we shall be able to defend initial hypothetical statements (of this paper) that New Theory of Gravitation should be constructed following analogy with Faraday-Maxwell Electromagnetic Theory (of course, first by upgrading both of them to become more compatible for unification).*

The way to establish a Unified Field Theory (suggested in this paper) will go back to the presentation of analogies found in the beginning of this paper. In order to give an idea how to relate Schrödinger equation/s (4.22) to Gravitation (and to any other field), we should remember that the square of the wave function in this paper presents the active power function $\Psi^2(\mathbf{t},\mathbf{r}) = \mathbf{P}(\mathbf{t},\mathbf{r}) = \mathbf{d}\tilde{\mathbf{E}}/\mathbf{dt} = -\mathbf{dE_k}/\mathbf{dt}$. Since we are already used to express power functions as products between corresponding current and voltage (in electro technique), or force and velocity (in mechanics) etc. (***see the first chapter of this paper dealing with analogies***), this will directly enable us to develop new forms of wave equations (valid for Gravitation and other fields), formally similar to Schrödinger and classical wave equations (4.22), but dealing with velocities, forces, currents, voltages... as for instance,

$$
\Psi^2(t,r) = P(t,r) = \frac{d\tilde{E}}{dt} (=) \text{Active Power } (=)
$$

$$
(=) \left\{ \begin{array}{lll}
i(t)\cdot u(t) & (=) & [\text{Current} \cdot \text{Voltage}], \text{ or} \\
f(t)\cdot v(t) & (=) & [\text{Force} \cdot \text{Velocity}], \text{ or} \\
\tau(t)\cdot \omega(t) & (=) & [\text{Orb.} - \text{moment} \cdot \text{Angular velocity}], \text{or} \\
(\vec{E} \times \vec{H})\cdot \vec{S} & (=) & \left[\overrightarrow{\text{Pointyng Vector}}\right] \cdot \overrightarrow{\text{Surface}} \\
----- & (=) & --------- \\
s_1(t)\cdot s_2(t) & (=) & [(\text{signal}-1)\cdot(\text{signal}-2)]
\end{array} \right\} \qquad (4.23)
$$

(see also (4.0.82), chapter 4.0).

*[♣ **COMMENTS & FREE-THINKING CORNER:** We can also profit from the well-developed methodology (in electronics) related to optimal active power transfer, to real, imaginary and complex power, to real and complex impedances etc., applying by analogy, models, structures and conclusions developed in Electronics, to Classical Mechanics, Gravitation, Quantum Mechanics...*

Here it will be made an attempt to connect arbitrary Power Function (product between current and voltage, or product between force and velocity, or product between any other relevant, mutually conjugated functions creating power) to a Wave function as it is known in Quantum Mechanics. Energetically analyzed, any wave propagation in time and frequency domain can be mutually (time frequency) correlated using Parseval's identity. Consequently, the immediate (time domain) Power signal can be presented as the square of the wave function $\Psi^2(\mathbf{t})$. *Analysis of the optimal power transfer can be extended to any wave propagation field (and to arbitrary shaped signals). Thus, we profit enormously (in booth, directions) after unifying traditional concepts of Active, Reactive and Apparent power with the* $\Psi^2(\mathbf{t})$ *wave function mathematics, based on Analytic Signal methodology.*

In Quantum Mechanics the wave function $\Psi^2(\mathbf{t})$ *is conveniently modeled as a probability function, but effectively it behaves like normalized and dimensionless Power function, and here it will be closely related to Active Power, or power delivered to a load expressed in Watts as its units:*

$$
\Psi^2(\mathbf{t}) = \mathbf{P(t)} = \mathbf{S(t)}\cos\theta\mathbf{(t)} = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{ui} + \hat{\mathbf{u}}\hat{\mathbf{i}}) = \mathbf{Q(t)}\cdot\mathbf{cotan}\,\theta\mathbf{(t)}\ \mathbf{(=)}[\mathbf{W}]\ .
$$

The power reflected from a load, or Reactive Power, can be given as:

$$
\mathbf{Q(t)} = \mathbf{S(t)}\sin\theta\mathbf{(t)} = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{u}\hat{\mathbf{i}} - \hat{\mathbf{u}}\mathbf{i}) = \Psi^2(\mathbf{t})\cdot\tan\theta(\mathbf{t}) = \mathbf{P}(\mathbf{t})\cdot\tan\theta(\mathbf{t})(=)\ [\mathbf{VAR}]
$$

Electric Power and Energy transfer analysis (especially for arbitrary voltage and current signal forms) can be related to Wave function analysis if we establish the Wave function (or more precisely, the square of the wave function) in the following way:

$$P(t) = \Psi^2(t) = [a(t)\cos\varphi(t)]^2 = \text{Wave function}\ ,\ t \in [T],$$
$$\Psi(t) = a(t)\cos\varphi(t),\ \hat{\Psi}(t) = a(t)\sin\varphi(t),$$
$$\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = \Psi(t) + jH[\Psi(t)] = a(t)e^{j\varphi(t)} = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} U(\omega)e^{-j\omega t}d\omega =$$
$$= \frac{1}{\pi^2}\int_{0}^{+\infty} U(\omega)e^{-j\omega t}d\omega = \frac{1}{\pi}\int_{(0,+\infty)} A(\omega)e^{-j\omega t}d\omega,$$
$$U(\omega)\ =\ U_c(\omega)\ \ -\ j\ U_s(\omega)\ \ =\ \int_{(-\infty,+\infty)} \overline{\Psi}(t)\ e^{j\omega t}dt = A(\omega)e^{-j\Phi(\omega)}\ ,$$
$$U_c(\omega) = A(\omega)\cos\Phi(\omega),\ U_s(\omega) = A(\omega)\sin\Phi(\omega),$$
$$a(t) = \sqrt{\Psi(t)^2 + \hat{\Psi}(t)^2}\ ,\ \ \varphi(t) = \mathrm{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}\ ,$$
$$A^2(\omega) = U^2{}_c(\omega) + U^2{}_s(\omega),\ \ \Phi(\omega) = \mathrm{arctg}\frac{U_s(\omega)}{U_c(\omega)}\ ,$$
$$T\cdot\langle P(t)\rangle = \int_{-\infty}^{+\infty} P(t)dt = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty} \left|\overline{\Psi}(t)\right|^2 dt =$$
$$= \frac{1}{2}\int_{-\infty}^{+\infty} \left|\Psi(t) + j\hat{\Psi}(t)\right|^2 dt = T\cdot\langle \hat{P}(t)\rangle = \int_{-\infty}^{+\infty} \hat{P}(t)dt =$$
$$= \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |U(\omega)|^2 d\omega = \frac{1}{\pi}\int_{0}^{\infty} [A(\omega)]^2 d\omega,$$

As we can see, every single wave function has at least two wave components (since to create power function it is essential to make the product of two relevant, mutually conjugated signals, like current and voltage, velocity and force, or some other equally important couple of signals: see (4.18)):

$$\Psi^2(t) = P(t) = S(t)\cos\theta(t) = \frac{1}{2}(ui + \hat{u}\hat{i}) = \Psi_1{}^2(t) + \Psi_2{}^2(t),\ \Psi_1^2 = \frac{ui}{\sqrt{2}},\ \Psi_2^2 = \frac{\hat{u}\hat{i}}{\sqrt{2}},$$

and it shouldn't be too big success to causally explain quantum mechanical diffraction, superposition and interference effects, when a "single wave object" and/or a single particle (like an electron, or photon) passes the plate with (at least) two small, diffraction holes, because in reality there isn't a single object (there are always minimum 2 mutually conjugated wave elements and their mixed products, somehow energetically coupled with their environment, extending the number of interaction participants). What looks to us like strange quantum interaction, or interference of a single wave or particle object with itself, in fact, presents an interaction of at least 2 wave entities with some other, third object ($\Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t)$ *). Somehow Nature always creates complementary and conjugated couples of important elements (signals, particles, energy states…) belonging to every kind of matter motions. We can also say that every object (or energy state) in our universe has its non-separable and conjugated image (defined by Analytic Signal concepts). Consequently, the quantum mechanical wave function and wave energy should represent only a motional energy (or power) composed of minimum two mutually coupled wave functions (* $\Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t)$ *).*

Here applied mathematics, regarding wave functions $\Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t)$*, after making appropriate normalization/s and generalizations, would start looking as applying Probability Theory laws, like in the contemporary Quantum Theory. Consequently, modern Quantum Theory could also be treated as the generalized mathematical modeling of micro world phenomenology, by conveniently unifying all conservation laws of physics in a joint, dimensionless, mutually well-correlated theoretical platform, creating new mathematical theory that is different by appearance, but in reality isomorphic to remaining Physics. Also, a kind of generalized analogy with Norton and Thevenin's theorems (known in Electric Circuit Theory) should also exist (conveniently formulated) in all other fields of Physics and Quantum Theory, since the cause or source of certain action produces a certain effect, and vice versa, and such events are always mutually coupled. ♣]*

4.3.5. Wave Function and Euler-Lagrange-Hamilton Formalism

One of the biggest achievements of Classical Mechanics is Euler-Lagrange-Hamilton formalism derived from Calculus of Variations. In this methodology we usually formulate the most appropriate Lagrange function, or **Lagrangian (=) L**, and apply Euler-Lagrange equations on it, in order to find all elements of certain complex motion. Without going too far in discussing Euler-Lagrange formalism, just analyzing different forms of Schrödinger wave equations (4.22), we can conclude that **Lagrangian** can also have different (floating energy level) forms, as for instance,

$$\left\{\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \tilde{\mathbf{E}}\overline{\Psi} = 0 \text{ , } \mathbf{U_p} = 0 \text{, } \Delta\overline{\Psi} = \frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial\, \mathbf{t^2}}\right\} \Rightarrow$$

$$\Rightarrow \left\{\underline{L = \tilde{E}} = -\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\frac{\Delta\overline{\Psi}}{\overline{\Psi}} \Leftrightarrow E_k \text{ , } \frac{1}{c^2} \le \frac{1}{u^2} < \infty\right\},$$

or

$$\left\{\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} - \mathbf{U_p})\overline{\Psi} = 0 \text{ , } \Delta\overline{\Psi} = (\frac{\tilde{\mathbf{E}}}{\tilde{\mathbf{E}} - \mathbf{U_p}})\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial\, \mathbf{t^2}}\right\} \Rightarrow$$

$$\Rightarrow \left\{\underline{L = \tilde{\mathbf{E}} - \mathbf{U_p}} = -\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\frac{\Delta\overline{\Psi}}{\overline{\Psi}} = (\frac{\tilde{\mathbf{E}}}{\Delta\overline{\Psi}})\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial\, \mathbf{t^2}} \Leftrightarrow \mathbf{E_k} - \mathbf{U_p} \text{ , } \frac{1}{c^2} \le (\frac{\tilde{\mathbf{E}}}{L})\frac{1}{u^2} < \infty\right\},$$

or

$$\left\{\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (E_{total} - \mathbf{U_p})\overline{\Psi} = 0 \text{, } \Delta\overline{\Psi} = (\frac{\tilde{\mathbf{E}}}{\mathbf{E_{total}} - \mathbf{U_p}})\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial\, \mathbf{t^2}}\right\} \Rightarrow$$

$$\Rightarrow \left\{\underline{L = \mathbf{E}_{total} - \mathbf{U_p}} = -\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\frac{\Delta\overline{\Psi}}{\overline{\Psi}} = (\frac{\tilde{\mathbf{E}}}{\Delta\overline{\Psi}})\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial\, \mathbf{t^2}} \text{, } \frac{1}{c^2} \le (\frac{\tilde{\mathbf{E}}}{L})\frac{1}{u^2} < \infty\right\}, \qquad (4.24)$$

or

$$\left\{\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + E_{total}\overline{\Psi} = 0 \text{ , } \Delta\overline{\Psi} = (\frac{\tilde{\mathbf{E}}}{\mathbf{E_{total}}})\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial\, \mathbf{t^2}}\right\} \Rightarrow$$

$$\left\{\underline{L = E_{total}} = -\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\frac{\Delta\overline{\Psi}}{\overline{\Psi}} = (\frac{\tilde{\mathbf{E}}}{\Delta\overline{\Psi}})\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial\, \mathbf{t^2}} \text{, } \frac{1}{c^2} \le (\frac{\tilde{\mathbf{E}}}{L})\frac{1}{u^2} < \infty\right\}.$$

Based on (4.24) and (4.10-3) we can formulate the unique and even more general form/s of relativistic Schrödinger (or Dirac's) wave equation/s, which replace all previously formulated wave equations,

$$\left\{\begin{array}{l}\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi}+L\overline{\Psi}=0\,;\,\Delta\overline{\Psi}=(\frac{\tilde{\mathbf{E}}}{L})\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2}=jk(\frac{\tilde{\mathbf{E}}}{L})\nabla\overline{\Psi}\,;\,(\frac{L}{\hbar})^2\overline{\Psi}+\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2}=0,\,\tilde{\mathbf{E}}\Leftrightarrow\mathbf{E}_{\mathbf{k}}\\ \frac{L}{\hbar}\overline{\Psi}=j\frac{\partial\overline{\Psi}}{\partial t}=-\frac{\hbar}{L}\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2};\,\frac{\partial\overline{\Psi}}{\partial t}+u\nabla\overline{\Psi}=0;\overline{\Psi}=\overline{\Psi}(t,r),\,j^2=-1\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{H\overline{\Psi}=0\,,H=\frac{\tilde{E}}{k^2}\Delta+L=\frac{\tilde{E}}{k^2}\Delta-\frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2}=\frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2}+L=j\hbar\frac{\partial}{\partial t}-L\right\}$$

$$\frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta=\frac{(\hbar u)^2}{\tilde{E}}\Delta=\frac{\tilde{E}}{k^2}\Delta=j(\frac{\tilde{E}^2}{kL})\nabla=\frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2}\,,\Delta=jk(\frac{\tilde{E}}{L})\nabla,\ \tilde{\mathbf{E}}-\mathbf{U}_{\mathbf{p}}\leq\mathbf{L}<\infty,$$

$$S=\int_{t_1}^{t_2}L(q_i,\dot{q}_i...,t)dt=\text{extremum}\,, \tag{4.25}$$

where Lagrangian L *should be considered as the floating (and variable:* $\mathbf{L}=\mathbf{E}_{tot.}-\mathbf{U}_{\mathbf{p}}$, $\mathbf{0}\leq\tilde{\mathbf{E}}\leq\mathbf{E}_{tot.}$, $\mathbf{U}_{\mathbf{p}}\geq0$*) energy level, and* $\tilde{\mathbf{E}}-\mathbf{U}_{\mathbf{p}}$ *as its lowest point or minimal energy level. For instance, in the case of electromagnetic waves in a free space (*$\mathbf{U}_{\mathbf{p}}=0$*), floating energy level (Lagrangian) is equal to the wave energy* $\mathbf{L}=\tilde{\mathbf{E}}$ *. This is (most probably) related to the fact that photons do not have a rest mass (but when strong* γ *photon penetrates sufficiently close to an atom, it can transform its energy into an electron-positron pair, because there is a certain potential field involved in the reaction).* *<u>By performing an energy translation on the scale (or axis) of a floating energy level</u>* $\tilde{\mathbf{E}}-\mathbf{U}_{\mathbf{p}}\leq\mathbf{L}<\infty$*<u>, we should be able "to materialize" all elementary particles and other energy forms of our universe. It should also be highlighted that all of the wave equations in (4.25) are mutually compatible or describe the same wave phenomena. The solutions to such equations are essentially dependant on the solutions of Euler-Lagrange equations applied on the relevant Lagrangian L.</u>* It is also interesting to notice that in (4.25) the factor $(\frac{\tilde{\mathbf{E}}}{L})\frac{1}{u^2}$ should determine a kind of wave (phase) speed. If we just limit this speed to be $\frac{1}{c^2}\leq(\frac{\tilde{\mathbf{E}}}{L})\frac{1}{u^2}<\infty$, we shall get $(\frac{\tilde{\mathbf{E}}}{\mathbf{L}})\geq(\frac{\mathbf{u}}{\mathbf{c}})^2$ and $-\mathbf{L}\leq\frac{\tilde{\mathbf{E}}-\mathbf{L}}{1-\frac{\mathbf{u}^2}{\mathbf{c}^2}}\leq\frac{\mathbf{U}_{\mathbf{p}}}{1-\frac{\mathbf{u}^2}{\mathbf{c}^2}}$, opening the following possibilities:

A) If we imagine that $(\frac{\tilde{\mathbf{E}}}{\mathbf{L}})\frac{\mathbf{1}}{\mathbf{u}^2}=\frac{\mathbf{1}}{\mathbf{v}^2}$ determines the group velocity of the same wave group (because there is not a big choice of possible velocities of a wave packet, since $\mathbf{u}$ is already its phase velocity), we get $(\frac{u}{v})^2=(\frac{\tilde{\mathbf{E}}}{L})$, and since from (4.2) we already know the relation between group and phase velocity, we shall easily obtain the following relations: $\{\mathbf{0}\leq(\frac{\mathbf{u}}{\mathbf{c}})^2\leq(\frac{u}{v})^2=(\frac{\tilde{\mathbf{E}}}{\mathbf{L}})=1/\left[1+\sqrt{1-\frac{v^2}{c^2}}\right]^2\leq(\frac{\mathbf{c}}{\mathbf{v}})^2\leq1\}$

$\Rightarrow \frac{L}{4} \leq (\tilde{E} = L / \left[1+\sqrt{1-\frac{v^2}{c^2}}\right]^2) \leq \mathbf{L}$. This mathematical exercise (regarding group and phase velocity) should be considered, at present, more as a brainstorming proposal in order to initiate the search for the real meaning of the velocity involved in $(\frac{\tilde{\mathbf{E}}}{\mathbf{L}})\frac{\mathbf{1}}{\mathbf{u^2}} = \frac{\mathbf{1}}{\mathbf{v^2}}$, than as the final answer regarding this situation (since here we are dealing with non-uniform, variable and accelerated movements, $u, v \neq const.$).

B) The second, more interesting brainstorming option (regarding wave equations from (4.25)) is that for every (pure) wave motion could be valid,

$$\{(\frac{\tilde{\mathbf{E}}}{L})\frac{1}{u^2} = \frac{1}{u^2},\ \tilde{\mathbf{E}} = L\} \Rightarrow \Delta\overline{\Psi} = (\frac{\tilde{E}}{L})\frac{1}{u^2}\frac{\partial^2 \overline{\Psi}}{\partial t^2} = \frac{1}{u^2}\frac{\partial^2 \overline{\mathbf{\Psi}}}{\partial \mathbf{t^2}},\ \Delta\overline{\mathbf{\Psi}} + k^2\overline{\mathbf{\Psi}} = 0,$$

$$H = \tilde{E}(\frac{\Delta}{k^2}+1) = \frac{\tilde{E}}{k^2}\Delta - \frac{\hbar^2}{\tilde{E}}\frac{\partial^2}{\partial t^2} = \frac{\hbar^2}{\tilde{E}}\frac{\partial^2}{\partial t^2} + \tilde{E} = j\hbar\frac{\partial}{\partial t} - \tilde{E},$$

because this way we simply create well-known wave equation/s valid in different domains of physics.

C) We also know that almost complete Special Relativity Theory is constructed based on Lorentz transformations where maximal speed (of light), $\mathbf{c} = \mathbf{const.}$, should be invariant to relative frame motion (or always the same regardless of reference frame), and where space-time interval in Minkowski space ($(ds)^2 = (ds')^2$) is locally not changing with coordinate transformations, what makes the following wave equation invariant (regarding coordinate/s transformation/s),

$$\Delta\overline{\mathbf{\Psi}} - \frac{1}{c^2}\frac{\partial^2 \overline{\mathbf{\Psi}}}{\partial \mathbf{t^2}} = \Delta'\overline{\mathbf{\Psi}} - \frac{1}{c^2}\frac{\partial^2 \overline{\mathbf{\Psi}}}{\partial \mathbf{t'^2}} = 0 \Leftrightarrow (ds)^2 = (ds')^2.$$

In our case (concerning results from this paper) we should be able to prove that the following wave equation (found in (4.25)) is in the same way (as in the Special Relativity Theory) invariant to Lorentz transformations,

$$\left\{\Delta\overline{\Psi} - (\frac{\tilde{E}}{L})\frac{1}{u^2}\frac{\partial^2 \overline{\Psi}}{\partial t^2} = \Delta'\overline{\Psi} - (\frac{\tilde{E}}{L})'\frac{1}{u'^2}\frac{\partial^2 \overline{\Psi}}{\partial t'^2} = 0\right\} \Leftrightarrow \left\{(\frac{\tilde{E}}{L})\frac{1}{u^2} = (\frac{\tilde{E}}{L})'\frac{1}{u'^2} = \frac{1}{c^2}\right\} \Rightarrow$$

$$\Rightarrow \left\{\tilde{E}c^2 = Lu^2 = \tilde{p}uc^2 = hfc^2\right\} \Rightarrow \Delta\overline{\Psi} - (\frac{\tilde{E}}{L})\frac{1}{u^2}\frac{\partial^2 \overline{\Psi}}{\partial t^2} = \Delta\overline{\Psi} - \frac{1}{c^2}\frac{\partial^2 \overline{\Psi}}{\partial t^2} = 0, \tag{4.25-1}$$

what should obviously be the case, since (combining already known data and above given Lorentz invariant conditions) we can get the following expressions (equivalent to data from (4.1)-(4.3) and T.4.1.):

$$\left\{\begin{array}{l} L=\frac{\tilde{p}c^2}{u}=\tilde{E}(\frac{c}{u})^2=hf(\frac{c}{u})^2=E_{tot.}-U_p=\sqrt{c^2p^2+(E_0)^2}=\frac{pv}{1+\sqrt{1-v^2/c^2}}+E_0=pu+E_0, \\ E_{tot.}=\sqrt{c^2p^2+(E_0)^2}+U_p=E_k+E_0+U_p=pu+E_0+U_p=\frac{pv}{1+\sqrt{1-v^2/c^2}}+E_0+U_p, \\ \tilde{p}=(\frac{u}{c})\sqrt{p^2+(\frac{E_0}{c})^2}=\frac{\tilde{E}}{u}=mu\sqrt{(\frac{\gamma v}{c})^2+1}=\frac{mv}{(1+\sqrt{1-v^2/c^2})\sqrt{1-v^2/c^2}}=\frac{mu}{\sqrt{1-v^2/c^2}} \\ \tilde{p}=\frac{\tilde{E}}{u}=p=\gamma mv=\frac{E_k}{u},\ E_0=mc^2 \end{array}\right\}.$$

(4.25-2)

From (4.25) we can find Lagrangian **L** and apply Euler-Lagrange-Hamilton equations on it, taking into account all possible coordinates and relevant field parameters of certain wave function (4.23), as for instance:

$$\overline{\Psi}=\overline{\Psi}(q_i,\dot{q}_i,...,t),\ S=\int_{t_1}^{t_2}L(q_i,\dot{q}_i,...,t)dt=\text{extremum},\ H\overline{\Psi}=0,$$

$$L=-\frac{\tilde{E}}{k^2}\frac{\Delta\overline{\Psi}}{\overline{\Psi}}=(\frac{\tilde{\mathbf{E}}}{\Delta\overline{\Psi}})\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t^2}}=j\frac{\hbar}{\overline{\Psi}}\frac{\partial\overline{\Psi}}{\partial t}=...,\ \frac{\hbar^2}{\tilde{\mathbf{m}}}(\frac{\mathbf{u}}{\mathbf{v}})\Delta=\frac{(\hbar u)^2}{\tilde{E}}\Delta=\frac{\tilde{E}}{k^2}\Delta=\frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2},$$

$$H=\frac{\tilde{E}}{k^2}\Delta+L=\frac{\tilde{E}}{k^2}\Delta-\frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2}=\frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2}+L=j\hbar\frac{\partial}{\partial t}-L,\ k=\frac{2\pi}{\lambda}=\frac{2\pi}{h}\tilde{p},$$

$$\tilde{\mathbf{E}}-\mathbf{U_p}\le\mathbf{L}<\infty,\ L\in\left\{(\tilde{E}-U_p),...\ \tilde{E},...\ (E_{total}-U_p),...\ E_{total}...\right\},$$

$$\tilde{E}=-k^2L\frac{\overline{\Psi}}{\Delta\overline{\Psi}}=Lu^2\Delta\overline{\Psi}/\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\hbar k^2}{\Delta\overline{\Psi}}\frac{\partial\overline{\Psi}}{\partial t}=j\hbar u^2\frac{\Delta\overline{\Psi}}{\overline{\Psi}}\frac{\partial\overline{\Psi}}{\partial t},$$

$$\tilde{E}=\int_{-\infty}^{+\infty}\Psi^2(\omega t-kx)dt=\int_{-\infty}^{+\infty}\left|\overline{\Psi}(\omega t-kx)\right|^2dt\Rightarrow$$

$$\frac{d\tilde{E}}{dt}=2(\omega-k\frac{dx}{dt})\int_{-\infty}^{+\infty}\Psi(\omega t-kx)dt,\ \frac{\partial\tilde{E}}{\partial x}=-2k\int_{-\infty}^{+\infty}\Psi(\omega t-kx)dt\Rightarrow$$

$$\frac{d\tilde{E}}{dt}=\frac{\omega-k\frac{dx}{dt}}{-k}\cdot\frac{\partial\tilde{E}}{\partial x}=(v-u)\cdot\frac{\partial\tilde{E}}{\partial x}\Rightarrow\frac{d\tilde{p}}{dt}=(1-\frac{u}{v})\cdot\frac{\partial\tilde{E}}{\partial x}=\frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\cdot\frac{\partial\tilde{E}}{\partial x}. \quad (4.26)$$

In simpler situations when Lagrangian presents a function of certain coordinates and their first derivatives, $L=L(q_i,\dot{q}_i,t)$, Euler-Lagrange equations will generate,

$$\left.\begin{cases} \mathbf{L} = \mathbf{L}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \mathbf{t}), \overline{\Psi} = \overline{\Psi}(q_i, \dot{q}_i ..., t) \ , \ \tilde{E} = \int \left\| \overline{\Psi} \right\|^2 \cdot dt, \\ \mathbf{S} = \int_{t_1}^{t_2} \mathbf{L}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \mathbf{t}) \mathbf{dt} = \mathbf{extremum}, \ H\overline{\Psi} = 0, \\ H = \frac{\tilde{E}}{k^2}\Delta + L = \frac{\tilde{E}}{k^2}\Delta - \frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2} = \frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2} + L = j\hbar\frac{\partial}{\partial t} - L \end{cases}\right\} \Rightarrow \tag{4.27}$$

$$\Rightarrow \frac{d}{dt}\left[\frac{\partial L(q_i, \dot{q}_i, t)}{\partial \dot{q}_i}\right] - \frac{\partial L(q_i, \dot{q}_i, t)}{\partial q_i} = 0.$$

In all other situations when Lagrangian presents more complex function, Euler-Lagrange equations will be,

$$\left.\begin{cases} \mathbf{L} = \mathbf{L}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \dddot{\mathbf{q}}_i, ..., \mathbf{q}_i^{(n)}, \mathbf{t}), \overline{\Psi} = \overline{\Psi}(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t) \ , \ \tilde{E} = \int \left\| \overline{\Psi} \right\|^2 \cdot dt, \\ \mathbf{S} = \int_{t_1}^{t_2} \mathbf{L}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \dddot{\mathbf{q}}_i, ..., \mathbf{q}_i^{(n)}, \mathbf{t}) \mathbf{dt} = \mathbf{extremum}, \ H\overline{\Psi} = 0, \\ H = \frac{\tilde{E}}{k^2}\Delta + L = \frac{\tilde{E}}{k^2}\Delta - \frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2} = \frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2} + L = j\hbar\frac{\partial}{\partial t} - L \end{cases}\right\} \Rightarrow$$

$$\Rightarrow \frac{\partial \int L dt}{\partial q_i} - \frac{d}{dt}\left[\frac{\partial \int L dt}{\partial \dot{q}_i}\right] + \frac{d^2}{dt^2}\left[\frac{\partial \int L dt}{\partial \ddot{q}_i}\right] - ... + (-1)^n \cdot \frac{d^n}{dt^n}\left[\frac{\partial \int L dt}{\partial q_i^{(n)}}\right] = 0. \tag{4.28}$$

The Orthodox Quantum Mechanics developed a big part of its mathematical apparatus exploiting the achievements of Euler-Lagrange and Hamilton Theory, basically accepting restraints of Classical Mechanics. Euler-Lagrange equations have been extended to Hamilton equations, producing the rules of Poisson brackets, leading to "Evolution Equation", to Schrödinger's Equation, to Operators Mechanics etc. The simple and seducing mathematical symmetries and elegant forms of Euler-Lagrange and Hamilton's equations, discovered more than a century ago, fully modeled at that time to describe Classical, Newton Mechanics, still influence the physics in its foundations (but often their applicability is limited only to non-relativistic motions, $v << c$). It is really the time to transform and upgrade this excellent concept and methodology (based on Calculus of Variations) to the new level of a more general and more applicable physics theory (without blindly and exclusively accepting the restrictions of Classical Mechanics). By using Relativistic form of Lagrangian, here we are proposing how it would be possible to extend applicability of Euler-Lagrange equations, (4.26)-(4.28). This way, a new theory would replace traditionally known Euler-Lagrange and Hamilton's Theory (and, of course, there is a lot of work to be done in that direction before we create such new theory).

In all of the above given equations, (4.25)-(4.28), we do not have the real restriction to treat (motional and wave) energy and Lagrange function/s only from the point of view of Classical Mechanics, as usual in Classical and Quantum Mechanics (since here we already use relativistic compatible expressions for energy). Of course, when using only Classical Mechanics formulas for mass, momentum and energy, system of Euler-Lagrange equations generates very attractive and symmetrical mathematical expressions and structures, but this should not be the only reason to sacrifice domain of applicability of Euler-Lagrange concept (which is universally valid, since it is the result of Hamilton's Principle derived from Calculus of Variations, applied on Lagrangian). If once physics finds a better theory to replace Einstein's Relativity Theory, the Euler-Lagrange concept will still hold valid, except that mass, momentum and energy would be differently described.

Obviously, equations found in (4.25)-(4.28) are an excellent demonstration of essential connections between Classical, Relativistic and Quantum Mechanics with Particle-Wave Duality Theory presented in this paper. They are universally valid for any particle-wave duality aspect of motion, and applicable to Gravitation, Electromagnetism, Linear and Rotational motions, coupled Action and Reaction forces etc. (See also T4.1, T5.2, (4.29)-(4.31), and Uncertainty Relations (5.15) and (5.16) that complement the above statement). In the background of (4.25)-(4.28), summarizing the results of this paper, we can find a perfect merge and integration of:

1° "Particle-Wave Duality Code" and coupled Action-Reaction forces, given by (Relativity Theory compatible) relations found in (4.1), (4.2) and (4.3),

2° with generally valid wave function (and its differential equation: Schrödinger equation) given in the form of Analytic Signal (4.9),

3° respecting the principle that only motional energy creates de Broglie mater waves, summarized by the second part of (4.10-5),

4° also respecting Energy and Momentum conservation laws,

5° and placing all of them (above-mentioned) inside the frames of universally valid Calculus-of-Variation Principles (from Euler-Lagrange-Hamilton mechanics).

Since the Orthodox Quantum Mechanics is still the leading and officially accepted theory of micro world, it would be useful to underline the most important differences and similarities between here presented particle-wave duality concept and particle-wave duality in Orthodox Quantum Mechanics (or in Schrödinger's wave mechanics), following above-listed step-stones (from 1° to 5°).

1° Orthodox Quantum Mechanics (OQM) also made necessary integration of "Particle-wave Duality Code" with its wave function and wave equation, but not using all options found in (4.1)-(4.3), and staying on the ground of Classical Mechanics (regarding particle velocity, energy etc.). Especially significant negative aspect of OQM can be found in its imprecise phase and group velocity treatment. Also, coupled Action-Reaction forces and effects of intrinsic rotation (or torsion fields) are not addressed in OQM.

2° The wave function and wave equation in OQM is very much artificially assembled, and not created following the general model of representing almost any arbitrary function by Analytic Signal (in comparison with the wave function (4.9)), followed by absence of the (immediate time-space frequency) signal phase information, but it is eventually completed and modeled to serve its place properly. In many other aspects (except missing phase function), OQM wave function behaves almost as a complex Analytic Signal wave function, and produces the same type of Schrödinger equation (as the wave function (4.9)).

3° In OQM the wave function represents the totality of a particle, including its rest energy and rest mass states, summarized by the first part of (4.10-5). In many cases this does not present a serious problem (except that analysis of relativistic particles is not well supported), since after every differentiation (when creating wave equations) we loose the freestanding constant members (rest mass and rest energy, for instance).

4° Instead of directly respecting Energy and Momentum conservation laws, OQM creates an equivalent platform, operating with dimensionless probability functions, which are (or can be) effectively created by simple normalization of relevant energy and momentum conservation expressions. Of course, OQM does not explicitly claim that in its background we should find the shadows of well known conservation laws. Since OQM wave function doesn't have the information about real (immediate, time-space-frequency dependant) signal phase of de Broglie matter wave which it should represent, in order to avoid possible mistakes, it was very convenient (for OQM) to accept the modeling frame borrowed from Statistics and Probability theory (mixed with appropriate Signal and Spectrum Analysis knowledge). This is intrinsically compatible with all conservation laws known in Physics. This way, OQM dimensionless wave function effectively compensated the part of "its missing body", and became very operational, but only in averaged statistical terms, and in the frames of (re)invented mathematics of OQM.

5° Euler-Lagrange-Hamilton mechanics and variation principles cannot be directly represented and satisfied in the structure of OQM, because in OQM we operate with dimensionless probability-compatible wave functions. However, indirectly OQM naturalized a big part of mathematical apparatus exploiting the achievements and models of non-relativistic Euler-Lagrange and Hamilton Theory, basically accepting restraints of Classical Mechanics.

----

*[♣ **<u>COMMENTS & FREE-THINKING CORNER:</u>** The opinion of the author of this paper regarding Quantum Mechanical "probabilistic philosophy" and probabilistic wave function (including a lot of assumptions) is that this has been just one of the acceptable mathematical modeling possibilities (not the best and not the most general). It obviously has a lot of supporting background, because of its **"in-average compatibility"** with Energy and Momentum (and other) conservation laws, as well as compatibility with generally valid mathematics (from Signal and Spectrum Analysis and Probability theory). Regarding all associated "mathematical cosmetics", assumptions and proper fitting into positive knowledge of Physics (based on de Broglie, Heisenberg and Schrödinger contributions regarding particle-wave duality), a sufficiently well and still one of the best known operating structure of Orthodox Quantum Mechanic was established. **Effectively, this has been a long-lasting triumph and at the same time the biggest conceptual, common sense, and philosophical obstacle (almost dead-end street) of modern physics regarding future (more significant) advances in the same field. We should be ready to start general and large scale refinement of modern physics, especially of Orthodox Quantum Mechanics (what is one of the messages of this paper). In other words we could say that existing Quantum Mechanics effectively blocks (or misleads) creation of new and more advanced concepts in Physics (regardless of its important contribution to the modern physics). Under more significant advances in Physics, here we should understand creation of a Unified and Open Architecture Field-Theory (which unites all known natural forces and creates open links for integration of forces to be discovered; -possibly a kind of Superstring theory).** ♣]*

4.3.6. Further Extensions of Wave Function and Universal Field Theory

It is very important to notice that the wave function presented in this paper is different than Quantum Mechanic's wave function. Generally valid in this paper is that we can associate the wave function to any form of motional energy (or particularly to linear motion, to Rotation, to motion in Electric field and to motion in Magnetic field, or to any of their combinations). If there are some other forms of motional energy presently unknown or not mentioned, for each one of them it should exist one associated wave function. However, what looks probable regarding plurality of motional energy components is that the real motional energy nature could have its profound origins only in some of different manifestations of an electromagnetic field.

In order to develop a more general (and Relativity compatible) forms of wave and Euler-Lagrange equations we shall use the expression for kinetic energy in a differential form, that is (mathematically) the same in Classical and Relativistic Mechanics (instead of using expression for non-relativistic kinetic energy), as for instance:

$$\mathbf{dE_k = vdp = d\tilde{E} = hdf},$$

since in Classical Mechanics for kinetic energy we have,

$$\{E_k = mv^2/2 = pv/2, p = mv, \ m = \text{const.}\} \Rightarrow \underline{dE_k} = mvdv = vd(mv) = \underline{vdp},$$

and we can get the same result in Relativistic Mechanics,

$$\left\{E_k = \frac{\gamma mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = (\gamma-1)mc^2, \ \gamma = 1/\sqrt{1-\frac{v^2}{c^2}}, p = \gamma mv\right\} \Rightarrow \underline{dE_k = vdp}.$$

It is also obvious that differential of kinetic energy, from the point of view of Classical Mechanics can be found as $dE_k = pdv \ (= vdp)$, but we eliminate this possibility since in Relativistic Mechanics we can get only $dE_k = vdp$. This way we indirectly implement compatibility of any form of kinetic energy (and wave functions developed based on motional energy) with Lorentz transformations and Euler-Lagrange equations.

In a situation when particle only performs the rotation, its kinetic (rotational and relativistic) energy, by analogy with the above given example (see T.3.1-T.3.3 and T.4.3.1), can be expressed as a function of its angular momentum and angular velocity, $dE_k = \omega d(J\omega) = \omega dL$.

The square of the wave function in this paper is just the active (motional) power, and using analogies developed before we can summarize several forms of possible wave functions, presented in the following table:

T.4.3.1

Linear motion	$\mathbf{dE_k = vdp = -\Psi^2 \cdot dt}$, $\Psi^2 = \mathbf{v} \cdot \tilde{\mathrm{F}}$	v = **velocity**, $\tilde{\mathrm{F}}$ = **force**	p = **momentum**, $\mathbf{dp = -d\tilde{p}}$, $\tilde{\mathrm{F}} = \mathbf{d\tilde{p} / dt}$
Rotation	$\mathbf{dE_{kr}} = \omega \mathbf{dL} = -\Psi^2 \cdot \mathbf{dt}$, $\Psi^2 = \omega \cdot \tilde{\tau}$	ω = **angular velocity**, $\tilde{\tau}$ = **torque**	L = **angular momentum**, $dL = -d\tilde{L}$, $\tilde{\tau} = \mathbf{d\tilde{L} / dt}$
Electric field	$dE_{ke} = udq_e = -\Psi^2 \cdot dt$, $\Psi^2 = u \cdot \tilde{i} = i_{mag.} \cdot \tilde{u}_{mag.}$	u = **electric voltage**, $\tilde{i}$ = **electric current**	q_e = **electric charge**, $dq_e = -d\tilde{q}_e$, $\tilde{i} = d\tilde{q}_e / dt$
Magnetic field	$dE_{km} = id\Phi = -\Psi^2 \cdot dt$, $\Psi^2 = u_{mag.} \cdot \tilde{i}_{mag.} = i_{el.} \cdot \tilde{u}_{el.}$	$u_{mag.}$ = **magn. voltage**, $\tilde{i}_{mag.}$ = **magn. current**	Φ = **magnetic flux**, $d\Phi = -d\tilde{\Phi}$, $\tilde{i}_{mag.} = d\tilde{\Phi} / dt$

where: $(u = u_{el.}, \tilde{u} = \tilde{u}_{el.}) \equiv (i_{mag.}, \tilde{i}_{mag.})$; $(i = i_{el.}, \tilde{i} = \tilde{i}_{el.}) \equiv (u_{mag.}, \tilde{u}_{mag.})$;
$q = q_{el.} = \Phi_{el.}, \tilde{q} = \tilde{q}_{el.} = \tilde{\Phi}_{el.}, q_{mag.} = \Phi_{mag.} = \Phi, \tilde{q}_{mag.} = \tilde{\Phi}_{mag.} = \tilde{\Phi}$.

In the case when all of the above energy elements are present in the same motional situation, we shall have:

$$\begin{aligned} dE_{k-total} &= \sum_i dE_k(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, \ldots, q_i^{(n)}, t) = vdp + \omega dL + udq_e + id\Phi + \ldots = \\ &= -vd\tilde{p} - \omega d\tilde{L} - ud\tilde{q}_e - id\tilde{F} - \ldots = \Psi^2{}_{total} dt = \sum_i \Psi^2(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, \ldots, q_i^{(n)}, t)dt = \\ &= \sum_i \Psi^2(q_i, v, \omega, u, i, p, L, q_e, \Phi, \ldots, t)dt = d\tilde{E}_{total}\ . \end{aligned} \tag{4.29}$$

We could again apply the Euler-Lagrange formalism, (4.26)-(4.28) on (4.29) and analyze much more complex field situations, this way approaching the fields' unification objective from a more general platform.

For instance, we could now generalize the meaning of (multi-component, linear) **force**, extending by analogy the force expression (4.18), (respecting also (4.19) and (4.26)-(4.29)), to the following form:

$$\begin{aligned} &\tilde{\mathbf{F}}(\mathbf{t}, \mathbf{r}) = \frac{\mathbf{1}}{\mathbf{v}} \Psi^2(\mathbf{t}, \mathbf{r}) \Rightarrow \tilde{\mathbf{F}}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \ldots, \mathbf{q}_i^{(n)}, \mathbf{t}) = \frac{\mathbf{1}}{\dot{\mathbf{q}}_i} \Psi^2(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \ldots, \mathbf{q}_i^{(n)}, \mathbf{t}), \\ &\tilde{\mathbf{F}} = \sum_{(i)} \alpha_i \tilde{\mathbf{F}}_i = \sum_{(i)} \frac{\alpha_i}{\dot{\mathbf{q}}_i} \Psi_i^{\,2} = \sum_{(i)} \frac{\alpha_i}{\dot{\mathbf{q}}_i} \frac{\mathbf{d}\tilde{\mathbf{E}}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \ldots, \mathbf{q}_i^{(n)}, \mathbf{t})}{\mathbf{dt}} = \\ &= \sum_{(i)} \alpha_i \frac{\dot{E}_i}{\dot{q}_i} = \sum_{(i)} \alpha_i \frac{\dot{E}_{ki}}{\dot{q}_i}, \quad \alpha_i \in \{\text{Const...}\} \forall_i, \end{aligned} \tag{4.30}$$

$$\tilde{\mathbf{E}} = -\mathbf{k}^2 \mathbf{L} \frac{\overline{\Psi}}{\Delta\overline{\Psi}} = \mathbf{L}\mathbf{u}^2 \Delta\overline{\Psi} / \frac{\partial^2 \overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j} \frac{\hbar \mathbf{k}^2}{\Delta\overline{\Psi}} \frac{\partial \overline{\Psi}}{\partial \mathbf{t}} = \mathbf{j}\hbar \mathbf{u}^2 \frac{\Delta\overline{\Psi}}{\overline{\Psi}} \frac{\partial \overline{\Psi}}{\partial \mathbf{t}}\ .$$

It looks that generalized force law (4.30) represents only dynamical (transitory, or time-dependant) forces, but we can easily see that this is not correct, since every component of motional (or kinetic) energy E_{ki} plus certain constant energy $\mathbf{E_{0i}}$ can create the total energy $E_{total\text{-}i}$,

$$E_{total\text{-}i} = E_{ki} + E_{0i}\,,\; E_{0i} = \text{Constant.} \Rightarrow \frac{dE_{ki}}{dt} = \frac{dE_{total\text{-}i}}{dt} \Rightarrow$$
$$\Rightarrow \tilde{F} = \sum_{(i)} \alpha_i \tilde{F}_i = \sum_{(i)} \alpha_i \frac{\dot{\tilde{E}}_i}{\dot{q}_i} = \sum_{(i)} \alpha_i \frac{\dot{E}_{ki}}{\dot{q}_i} = \sum_{(i)} \alpha_i \frac{\dot{E}_{total\text{-}i}}{\dot{q}_i}, \alpha_i = \text{const.}. \tag{4.31}$$

The generalized force law (4.31) should be considered (presently) more as a discussion example in a direction of creating more practical force formula, since the force must be considered as a vector. Constants α_i should take care about dimensional agreements and generalized coordinates $\dot{q}_i$ should belong to the set of generalized velocities, $q_i \in \{v_i, \omega_i, ...\}$.

The most important "secret" (hidden) in the force law (4.31) should be that certain motional (time-dependant and stationary) energy and field components are somehow trapped (blocked, or permanently captured) inside every constant energy level E_{0i}. *For instance, every magnet (electromagnet or permanent magnet) should be created as the result of certain current flow, but in the case of permanent magnets we do not see such current flow. This is because it is permanently captured inside small, atomic-size magnet domains, created by circulation of electrons around their cores. Consequently, the ultimate structure of matter (or of our universe) is that we have frozen (permanently captured, stationary and stable) energy states (or levels)* E_{0i}*, and free energy levels* E_{ki}. *Whenever we are able to penetrate (experimentally or theoretically) into internal structure of such constant energy states* E_{0i}*, we find different motional energy states, and sometimes a number of other relatively stable energy states, or particles. Again, it is obvious that the most important forces creating such (internal) energy structure should be the forces in some close relation to the rotation. Now it becomes clearer that the real sources of Gravity should be hidden in certain more sophisticated entity than only related to a particle mass.*

Almost a perfect symmetry between electric and magnetic voltages and currents is established as the result of presentations based on analogies (see T.3.1-T.3.3 and T.4.3.1). Of course, this symmetry exists as an achievement of Maxwell-Faraday Electromagnetic Theory, and here it becomes evident just because of the way it is presented. *It is also clear (after reading the entire contents of this paper) that presently we do not have the same level of symmetry regarding Linear motion and Rotation, and that the missing link should be related to de Broglie matter waves theory, meaning that every rectilinear motion should have some associated effects of rotation and waving (since every form of rotation is also a source of harmonic oscillations). Consequently, the new theory has to be established in order to cover all aspects of here presented analogies and particle-wave duality phenomenology.*

4.3.7. Mater-Wave 4-Vectors in Minkowski Space and Elements for New Topology

The principal idea here is that when creating New Topology Basis we should take into account only coordinates, motional elements and/or degrees of freedom which are really (and always) contributing to the total energy of certain system (see also (5.16)). Based on (4.30) and (4.31) we can extract the elements for the New (Universal Field) Topology if we determine the total velocity $\mathbf{V}_{\Sigma}$ *and its path element* $\mathbf{dX}_{\Sigma}$ *caused by complex (multi-component) force* $\mathbf{F}_{\Sigma}$*, or by complex momentum* $\mathbf{P}_{\Sigma}$*, as for instance:*

$$
\begin{aligned}
&\mathsf{F}_{\Sigma} = \sum_{(i)} \alpha_i \frac{\dot{\mathbf{E}}_{ki}}{\dot{\mathbf{q}}_i} = \sum_{(i)} \alpha_i \frac{\dot{\mathbf{E}}_{\text{total-i}}}{\dot{\mathbf{q}}_i} = \\
&= \sum_{(i)} \frac{\alpha_i}{\dot{\mathbf{q}}_i} \frac{\mathbf{dE}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \dddot{\mathbf{q}}_i, \dots, \mathbf{q}_i^{(n)}, \mathbf{t})}{\mathbf{dt}} = \frac{\mathbf{dP}_{\Sigma}}{\mathbf{dt}} = \dot{\mathbf{P}}_{\Sigma} = \frac{\mathbf{1}}{\mathbf{V}_{\Sigma}} \frac{\mathbf{dE}_{\Sigma}}{\mathbf{dt}} \Rightarrow \\
&\tilde{\mathsf{F}}_{\Sigma} = -\mathsf{F}_{\Sigma} = -\sum_{(i)} \alpha_i \frac{\dot{\mathbf{E}}_{ki}}{\dot{\mathbf{q}}_i} = -\sum_{(i)} \alpha_i \frac{\dot{\mathbf{E}}_{\text{total-i}}}{\dot{\mathbf{q}}_i} = \sum_{(i)} \alpha_i \frac{\dot{\tilde{\mathbf{E}}}_i}{\dot{\mathbf{q}}_i} = \\
&= -\sum_{(i)} \frac{\alpha_i}{\dot{\mathbf{q}}_i} \frac{\mathbf{dE}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \dddot{\mathbf{q}}_i, \dots, \mathbf{q}_i^{(n)}, \mathbf{t})}{\mathbf{dt}} = -\frac{\mathbf{dP}_{\Sigma}}{\mathbf{dt}} = -\dot{\mathbf{P}}_{\Sigma} = \dot{\tilde{\mathbf{P}}}_{\Sigma} \Rightarrow \\
&V_{\Sigma} = \frac{dE_{\Sigma}}{dP_{\Sigma}} = \frac{d\left[\sum_{(i)} E_i\right]}{dP_{\Sigma}} = \frac{\dot{E}_{\Sigma}}{\mathsf{F}_{\Sigma}} = \frac{\left(\sum_{(i)} E_i\right)^{\cdot}}{\mathsf{F}_{\Sigma}} = \frac{dX_{\Sigma}}{dt}, \qquad (4.32) \\
&\mathbf{dP}_{\Sigma} = \sum_{(i)} \frac{\alpha_i}{\dot{\mathbf{q}}_i} \mathbf{dE}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \dddot{\mathbf{q}}_i, \dots, \mathbf{q}_i^{(n)}, \mathbf{t}), \\
&\mathbf{E}_i = \mathbf{E}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i, \dddot{\mathbf{q}}_i, \dots, \mathbf{q}_i^{(n)}, \mathbf{t}), \\
&\mathbf{dX}_{\Sigma} = \mathbf{V}_{\Sigma}\mathbf{dt} = \frac{\sum_{(i)} \mathbf{dE}_i}{\mathbf{dP}_{\Sigma}} \mathbf{dt} = \frac{\sum_{(i)} \dot{\mathbf{E}}_i}{\mathsf{F}_{\Sigma}} \mathbf{dt} = \frac{\sum_{(i)} \mathbf{dE}_i}{\mathsf{F}_{\Sigma}} = \frac{\sum_{(i)} \mathbf{dE}_i}{\sum_{(i)} \frac{\alpha_i}{\dot{\mathbf{q}}_i} \mathbf{dE}_i} \mathbf{dt} = \frac{\sum_{(i)} \mathbf{dE}_i}{\sum_{(i)} \alpha_i \frac{\mathbf{dE}_i}{\mathbf{dq}_i}}.
\end{aligned}
$$

With (4.32) only the principal idea and starting platform for creating New Topology is formulated (but to finalize this task will request much more efforts; -see also (5.15)-(5.17)).

In the contemporary physics (especially in Relativity Theory) we use (under Lorentz transformations) covariant forms of 4-space vectors in Minkowski space, as the most elegant and most general synthesis of Energy and Momentum conservation laws. For instance, 4-space covariant vectors of particle velocity (here we can also say group velocity when we are using wave packet or wave group model) and momentum are known as:

$$
\begin{aligned}
&\overline{V}_4 = \overline{V}\left[\gamma\vec{v}, \gamma c\right] = \text{invariant}, \\
&\overline{P}_4 = m\overline{V}_4 = \overline{P}\left[\vec{p} = \gamma m\vec{v}, \frac{E}{c} = \gamma mc\right] = \overline{P}(\vec{p}, \frac{E}{c}) = \text{invariant}.
\end{aligned} \qquad (4.33)
$$

The wave energy and wave momentum can be connected in a similar way, but only as differential forms of 4-space wave momentum,

$$
\begin{aligned}
&d\tilde{P}_4 = -d\overline{P}_4 = d\tilde{P}(\tilde{p}, \frac{\tilde{E}}{c}) = \tilde{P}(d\tilde{p}, \frac{d\tilde{E}}{c}) = \tilde{P}(\frac{d\tilde{E}}{v}, \frac{d\tilde{E}}{c}) = \tilde{P}(\frac{\Psi^2}{v}dt, \frac{\Psi^2}{c}dt),\\
&d\tilde{E} = hdf = vd\tilde{p} = d(\tilde{p}u) = dE_k = dE = vdp = d(pu) = \Psi^2 dt, \qquad (4.34)\\
&E_k = pu, E = \gamma mc^2, \tilde{E} = \tilde{p}u.
\end{aligned}
$$

A wave 4-vector and 4-space phase velocity should also be formulated as differential forms,

$$
\begin{aligned}
&d\overline{K}_4 = \frac{2\pi}{h}d\tilde{P}_4 = \overline{K}(d\vec{k}, \frac{d\omega}{c}) \quad (= \text{ wave vector}),\\
&d\overline{U}_4 = \overline{U}(d\vec{u}, \frac{\vec{v}}{c}d\vec{u}) \quad (= \text{ phase velocity}),\\
&\overline{V}_4 d\overline{K}_4 = \overline{V}_4\frac{2\pi}{h}d\tilde{P}_4 = 0,\ k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}\tilde{p}, \qquad (4.35)\\
&v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{d\tilde{p}}, u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{\tilde{p}}.
\end{aligned}
$$

We can also determine differential forms of 4-space vectors of wavelength and frequency, as for instance:

$$
\begin{aligned}
&\mathbf{d\overline{\Lambda}_4 = \overline{\Lambda}(d\lambda, \frac{v}{c}d\lambda)} \qquad (= \textbf{ wavelength}),\\
&\mathbf{d\bar{f}_4 = \bar{f}(df, \frac{v}{c}df) = \frac{1}{h}d\tilde{E}_4 = \frac{1}{h}(d\tilde{E}, \frac{v}{c}d\tilde{E}) = \vec{v}d\tilde{P}_4} \qquad (= \textbf{ frequency}). \qquad (4.36)
\end{aligned}
$$

It is obvious that differential forms of 4-space vectors found in (4.34)-(4.36) can't be considered as Lorentz-covariant 4-vectors. However, we can easily combine them with regular Lorentz-covariant 4-vectors and get correct results (after applying integration or usual arithmetic operations). Also, if we apply simple integration *omitting* constants of integration (or boundary conditions), 4-vector forms (4.34)-(4.36) will not be transformed into proper Lorentz-covariant 4-vectors, as for instance

$$
\begin{aligned}
&\tilde{P}_4 \neq (\tilde{p}, \frac{\tilde{E}}{c}),\ \overline{K}_4 = \frac{2\pi}{h}\tilde{P}_4 \neq (\vec{k}, \frac{\omega}{c}),\ \overline{U}_4 \neq (\vec{u}, \frac{\vec{v}}{c}\vec{u}) \neq (\gamma\vec{u}, \gamma c),\\
&\overline{\Lambda}_4 \neq (\lambda, \frac{v}{c}\lambda),\ \bar{f}_4 \neq (f, \frac{v}{c}f).
\end{aligned}
$$

From (4.34) we can also develop the force 4-vectors which are Lorentz-covariant in Minkowski space,

$$\tilde{F}_4 = \gamma \frac{d\tilde{P}_4}{dt} = \left[\gamma \frac{d\tilde{p}}{dt}, \frac{\gamma}{c}\frac{d\tilde{E}}{dt}\right] = \left[\gamma \tilde{F}, \gamma \frac{\vec{v}}{c}\tilde{F}\right] = \left[\frac{\gamma}{v}\Psi^2, \frac{\gamma}{c}\Psi^2\right] =$$
$$= -\gamma \frac{d\overline{P}_4}{dt} = -\overline{F}_4 = -\left[\gamma \frac{d\vec{p}}{dt}, \frac{\gamma}{c}\frac{dE}{dt}\right] = -\left[\gamma \vec{F}, \gamma \frac{\vec{v}}{c}\vec{F}\right],$$
$$\Psi^2 = \frac{d\tilde{E}}{dt} = -\frac{dE}{dt} = -\frac{d\sum_{(i)} E_i}{dt} = -\frac{\sum_{(i)} dE_i}{dt},$$
$$\tilde{F}_4^{\,2} = \overline{F}_4^{\,2} = \text{invariant}, \tilde{F}_4 \cdot \overline{V}_4 = -\overline{F}_4 \cdot \overline{V}_4 = \tilde{F}'_4 \cdot \overline{V}'_4 = -\overline{F}'_4 \cdot \overline{V}'_4 = 0 \ . \tag{4.37}$$

All binary or two-object interactions can be treated in the most general way by applying (4.33)-(4.37). Newton law of action and reaction (regarding inertial forces) is also expressed by (4.37), $\overline{F}_4 = -\tilde{F}_4$. In order to expose a more general view of force law (4.37), we should treat such force as being multi-component (complex) force, as presented in (4.32), modeling it towards creating Lorentz-compatible 4-vectors in the Minkowski space.

Implicitly, from force 4-vector found in (4.37) we can also determine what should mean Lorentz-covariant wave function Ψ^2 in Minkowski space. Moreover, we can establish higher compatibility between Quantum Mechanics and Relativity Theory, instead of making "mathematical experiments and arbitrary function fit" in order to upgrade the existing Quantum Mechanics wave function.

[♣ ***COMMENTS & FREE-THINKING CORNER:*** *Using analogies (see T.3.1-T.3.3) we could "invent" possible (and hypothetical) directions of further extensions of 4-space vectors (in Minkowski space) towards rotation, electromagnetism etc. Let us reformulate the 4-vector of velocity (4.33) to become a direct consequence of 4-vector of momentum,*

$$\overline{V}_4 = \frac{1}{m}\overline{P}_4 = (\frac{\vec{p}}{m}, \frac{E}{mc}) = (\frac{\vec{p}}{m}, \frac{pc}{mv}) = (\frac{\gamma m\vec{v}}{m}, \frac{\gamma mc^2}{mc}) = \overline{V}[\gamma\vec{v}, \gamma c] = \text{invariant.} \tag{4.38}$$

In terms of rotation (based on analogies), instead of linear velocity **v**, *we have angular velocity* ω, *and instead of speed* **c** *we should have maximal angular speed* ω_c. *By replacing all values from (4.38) with their analogies in rotational motion we will get:*

$$\overline{\Omega}_4 = \frac{1}{J}\overline{L}_4 = (\gamma\frac{\vec{L}}{J}, \gamma\omega_c) = \overline{\Omega}[\gamma\vec{\omega}, \gamma\omega_c] = \text{invariant},$$
$$\overline{\Omega}_4^{\,2} = -\omega_c^{\,2}, \gamma = (1\text{-} v^2/c^2)^{-1/2} = (1\text{-}\omega^2/\omega_c^{\,2})^{-1/2}. \tag{4.39}$$

In terms of magnetic field (using conclusions based only on analogies), instead of **v** *we have electric current (or "magnetic voltage")* $i_{el.}$. *Instead of speed* **c** *we would have certain maximal electric current* $i_{el.-c}$. *By replacing all values from (4.38) with their analogies in magnetic field we would formally get:*

$$\overline{I}_{el.-4} = \overline{I}_{el.}[\gamma i_{el.}, \gamma i_{el\text{-}c.}] = \text{invariant},$$
$$(\overline{I}_{el.-4})^2 = -(i_{el\text{-}c.})^2, \gamma = (1 - v^2/c^2)^{-1/2} = \left[1 - (i_{el.}/i_{el\text{-}c.})^2\right]^{-1/2}. \tag{4.40}$$

*In terms of electric field (using analogies), instead of **v** we have electric voltage (or "magnetic current") $u_{el.}$, and instead of speed **c** we would have maximal electric voltage $u_{el.-c}$. By replacing all values from (4.38) with their analogies in electric field we will get:*

$$\bar{U}_{el.-4} = \bar{U}_{el.}\left[\gamma\, u_{el.}, \gamma\, u_{el-c.}\right] = \text{invariant},$$
$$(\bar{U}_{el.-4})^2 = -(u_{el-c.})^2, \; \gamma = (1 - v^2/c^2)^{-1/2} = \left[1 - (u_{el.}/u_{el-c.})^2\right]^{-1/2}. \tag{4.41}$$

We could continue similar mathematical experiments (creating analogies of possible 4-space vectors) towards many other values found in T.3.1-T.3.3. The principal question, whether something like that really produces logical and correct results (or results that later can be transformed into more realistic formulas), could be analyzed some other time.

Another important conceptual difference between our usual modeling regarding mechanical movements and electromagnetic family of phenomena (where currents and voltages are involved) is that in most of the situations regarding electromagnetic phenomenology we know or understand that electric circuits are by their nature kind of fully-closed circuits (Kirchoff's and Ohm's laws etc.). Regarding (mechanical) particles motions we are often talking about (somehow free hanging) linear and/or rotational motions without defining what should be their closed mechanical circuits. In reality, both electrical and mechanical phenomena, models and motions should have similar, closed circuits nature. This is certainly in relation to Lorenz transformations and 4-vector rules, as presently formulated, and also a reason for extending Lorenz and 4-vectors framework, towards unified models that would take care about unity of linear and rotational motions (what is basically proposed here).

In the first three chapters of this paper we already established analogies between different "field charges" or values equivalent to linear and orbital moments. Let us try "analogically" to extend all such values to 4-vectors in Minkowski space. By analogy with the linear momentum 4-vector we would be able to formulate similar relation for an orbital moment, as for instance,

$$\bar{P}_4 = \bar{P}(\vec{p}\;,\;\frac{E}{c}) = \bar{P}(\vec{p}\;,\;\frac{E}{c^2}v\big|_{v\to c}) = \bar{P}(\gamma\, m\vec{v}\;,\;\gamma\, m\, c) \Leftrightarrow$$
$$\Leftrightarrow \bar{L}_4 = \bar{L}(\vec{L}, \frac{E}{\omega_c^2}\omega) = \bar{L}(\vec{L}, \frac{hf}{\omega_c^2}\omega\big|\omega \to \omega_c) = \bar{L}(\vec{L}, \frac{h}{2\pi}). \tag{4.33-1}$$

The orbital moment 4-vector is found by understanding that the particle (which has the ordinary, externally measurable, mechanical orbital moment $\vec{L}$), is created by fully absorbing (into its self-closed and stable vortex formation) the wave package which has its total or motional energy equal to $\mathbf{E} = \mathbf{E_k} = \mathbf{hf} \Rightarrow \frac{E}{\omega} = \frac{h}{2\pi}$. Presently, this looks possible, in the frames of the concepts presented in this paper, mostly for single and elementary particles such as electrons and protons (and for complex particles, composed of many atoms and other elementary particles, a similar concept should be upgraded). In reality, the same particle could have linear, orbital and electromagnetic moments (at the same time), and we would need to find a way to address such complex moments reality creating a single 4-vectors formulation.

Now we could propose the following table, T.4.3.2, just to stimulate creative curiosity and without insisting that newly proposed 4-vectors are correct.

T.4.3.2	[Q] = CHARGES / MOMENTS	Meaning	4-vectors: Moment / Force
Gravitation & Linear Motion	**p = mv**	**Linear moment**	$\overline{\mathbf{P}}_4 = \overline{\mathbf{P}}(\vec{\mathbf{p}}, \frac{\mathbf{E}}{\mathbf{c}})$ $\overline{F}_4 = \gamma \frac{d\overline{P}_4}{dt} = (\gamma\vec{F}, \frac{\gamma}{c}\frac{dE}{dt}) = (\gamma\vec{F}, \frac{\gamma}{c}\vec{F}\cdot\vec{v})$
Rotation	**L = Jω**	**Orbital moment**	$\overline{L}_4 = \overline{L}(\vec{L}, \frac{h}{2\pi})$ $\overline{M}_4 = \gamma \frac{d\overline{L}_4}{dt} = (\gamma\vec{M}, \frac{\gamma}{\omega_c}\frac{dE}{dt}) =$ $= (\gamma\vec{M}, \frac{\gamma}{\omega_c}\vec{M}\cdot\vec{\omega}) = (\gamma\vec{M}, \frac{\gamma}{c}\vec{M}\cdot\vec{v}_t)$
Electric Field	$\Phi_{el} = q_{el.}$	**Electric charge**	**? !** $\overline{\mathbf{Q}}_{el.-4} = \overline{\mathbf{Q}}(\mathbf{q}_{el.}, \mathbf{e}\cdot\frac{\mathbf{v}^2}{\mathbf{c}^2}) =$ $= \int \overline{\mathbf{I}}_{el.-4}\mathbf{dt}$
Magnetic Field	$\Phi_{mag} = q_{mag.}$	**Magnetic flux**	**? !** $\overline{\Phi}_{4-mag.} = \overline{\Phi}(q_{mag.}, \frac{h}{2e}\cdot\frac{v^2}{c^2})$ $= \int \overline{U}_{el.-4}dt$

Obviously, for scalar values of electric charge and magnetic flux it would not be directly applicable to place them in a format of the 4-vectors of Minkowski-space (as here presented), but just for the purpose of brainstorming (and maybe in relation to some future redefinition of such values, which would give them more of vectorial meanings), here is initiated the first step. ♣]

4.3.8. Mass, Particle-Wave Duality and Real Sources of Gravitation

By applying (or comparing) different analogies all over this paper we see that mass and gravitation somehow "avoid" being simply presentable, following the same patterns of other natural forces and their sources (or charges). It looks (based upon here established analogies in earlier chapters) that real sources of gravity (between particles with rest masses) should also be linear and orbital moments of particles (and maybe some other dynamical parameters), contrary to common opinion that only pure masses are proper gravity charges and primary sources of Gravitation (see also chapter 2: GRAVITATION, starting from equations (2.3) to (2.4-3)).

In order to understand better what means mass, let us summarize the most important expressions concerning energy of a particle (moving in a free space):

$$\begin{aligned} &\mathbf{E} = \mathbf{E}_{total} = \mathbf{E_0} + \mathbf{E_k} = \gamma\, \mathbf{mc}^2 = \sqrt{\mathbf{E}_0^2 + \mathbf{p}^2\mathbf{c}^2} = \mathbf{m}_{total}\mathbf{c}^2 = \mathbf{m}_t\mathbf{c}^2, \\ &\mathbf{E_k} = \mathbf{E} - \mathbf{E_0} = (\gamma - 1)\mathbf{mc}^2 = \mathbf{E}(1 \pm \sqrt{1 - \frac{\mathbf{p}^2\mathbf{c}^2}{\mathbf{E}^2}}) = \mathbf{m}_{motional}\mathbf{c}^2 = \mathbf{m}_m\mathbf{c}^2, \\ &\mathbf{E_0} = \mathbf{mc}^2 = \mathbf{m_0 c}^2, \end{aligned} \tag{4.41-1}$$

The most general understanding of mass concept is to present mass just as another form of particle energy (devised by the constant $\mathbf{c}^2$), where $\mathbf{m_t}$ is the total mass, $\mathbf{m_m}$ is the motional mass and $\mathbf{m} = \mathbf{m_o}$ is the rest mass:

$$\begin{aligned} &\mathbf{m}_t = \frac{\mathbf{E}}{\mathbf{c}^2} = \gamma\, \mathbf{m} = \mathbf{m} + \mathbf{m}_m = \frac{\mathbf{1}}{\mathbf{c}^2}\sqrt{\mathbf{E}_0^2 + \mathbf{p}^2\mathbf{c}^2} = \frac{\mathbf{E_0}}{\mathbf{c}^2}\sqrt{\mathbf{1} - \frac{\mathbf{p}^2\mathbf{c}^2}{\mathbf{E}_0^2}}, \\ &\mathbf{m}_m = \frac{\mathbf{E_k}}{\mathbf{c}^2} = (\gamma - \mathbf{1})\mathbf{m} = \mathbf{m}_t - \mathbf{m} = \frac{\mathbf{E}}{\mathbf{c}^2}(\mathbf{1} \pm \sqrt{\mathbf{1} - \frac{\mathbf{p}^2\mathbf{c}^2}{\mathbf{E}^2}}), \\ &\mathbf{m} = \mathbf{m_0} = \frac{\mathbf{E_0}}{\mathbf{c}^2}. \end{aligned} \tag{4.41-2}$$

This way, the same mass concept can be extended to all constituents of our universe, such as elementary particles and quasiparticles that do not have a rest mass (like photons), as well as to fields and waves around particles. We could also describe the mass like space-time-spread entity. Here minimal internal mass content captured by internal particle space (by particle geometry-boundaries) equals $\mathbf{m}$, and external mass content, spread in the space around the particle, equals $(\gamma - \mathbf{1})\mathbf{m}$, what effectively (for moving particle) makes the total (relativistic) particle mass equal $\mathbf{m}_t = \gamma\, \mathbf{m} = \mathbf{m} + (\gamma - \mathbf{1})\mathbf{m}$.

Let us now transform above given general relations for energy and mass into relations applicable to photons,

$$\tilde{\mathbf{E}} = \mathbf{E_f} = \mathbf{E} = \mathbf{E_{total}} = \mathbf{E_0} + \mathbf{E_k} = \mathbf{E_k} = \tilde{\mathbf{p}}\mathbf{c} = \mathbf{m_t c^2} = \tilde{\mathbf{m}}\mathbf{c^2},\ \tilde{\mathbf{m}} = \mathbf{m_t} = \frac{\tilde{\mathbf{E}}}{\mathbf{c^2}} = \frac{\tilde{\mathbf{p}}}{\mathbf{c}} = \mathbf{m_f},$$
$$\tilde{\mathbf{E}} = \mathbf{E_f} = \mathbf{E_k} = \mathbf{E} - \mathbf{E_0} = \mathbf{E} = \mathbf{E}(1 \pm \sqrt{1 - \frac{\mathbf{p^2 c^2}}{\mathbf{E^2}}}) = \mathbf{m_m c^2} = \tilde{\mathbf{m}}\mathbf{c^2}, \mathbf{m_m} = \tilde{\mathbf{m}} = \frac{\tilde{\mathbf{E}}}{\mathbf{c^2}} = \frac{\tilde{\mathbf{p}}}{\mathbf{c}},$$
$$\mathbf{E_0} = \mathbf{0},\ \ \mathbf{m} = \mathbf{m_0} = \mathbf{m_{f0}} = \frac{\mathbf{E_0}}{\mathbf{c^2}} = \mathbf{0}.$$

(4.41-3)

Now it is possible to introduce a new understanding of a rest mass as the form of motional (wave) energy, conveniently stabilized and captured into a rotating form of self-sustaining, stationary and standing waves, (as a toroid, rotating ring and/or similar forms), which only externally looks as a stable, non-moving particle.
If the total internal wave energy content of a certain particle is, $\mathbf{E_{k-int.}} = \tilde{\mathbf{E}} = \tilde{\mathbf{E}}_{int} = \tilde{\mathbf{m}}_{int.}\mathbf{c^2}$, then the particle rest mass should be $\mathbf{m_0} = \tilde{\mathbf{m}}_{int.} = \frac{\tilde{\mathbf{E}}_{int}}{\mathbf{c^2}}$. In this way we are in a position to treat equally internal rest mass, and externally spread (moving) mass, since externally spread mass is already defined as an equivalent to kinetic particle energy (devised by $\mathbf{c}^2$). The equality of mass treatments is based on the fact that in both cases (when we calculate mass) we are talking about kinetic or motional (or wave) energy, but:

1° For a rest mass we are using only the internal wave or internal motional energy, and
2° For a mass equivalent corresponding to “external” particle motion we will use “external” particle kinetic energy $\mathbf{m_m} = \frac{\mathbf{E_k}}{\mathbf{c^2}} = (\gamma - \mathbf{1})\mathbf{m}$.

Of course, this situation (regarding mass concept) becomes much more general and better unified if we treat the particle (external) kinetic energy also as a form of wave energy (what presents the proper meaning of the **P**article-**W**ave **D**uality of this paper). In case of an elementary mass particle, we will have,

$$\begin{aligned} m_0 &= \tilde{m}_{int.} = \frac{\tilde{E}_{int}}{c^2} = \frac{hf_{int.}}{c^2}, \\ m_m &= m_{ext.} = \frac{E_k}{c^2} = (\gamma-1)m = \frac{\tilde{E}_{ext.}}{c^2} = \frac{hf_{ext.}}{c^2} \\ E_k &= (\gamma-1)mc^2 = \tilde{E}_{ext.} = \tilde{E} = hf_{ext.} = hf \\ m_t &= \tilde{m}_{int.} + m_{ext.} = \frac{E}{c^2} = \frac{\tilde{E}_{int} + \tilde{E}_{ext.}}{c^2} = \frac{E_0 + E_k}{c^2} = \gamma m \end{aligned} \qquad (4.41\text{-}4)$$

The terminology “internal and external” motional energy is maybe not the best choice of names, but here it serves the need for simplified and faster explanation of the fact that what we name as particles only look like (to our instruments) as particles (detectable and measured externally), having internally their proper wave nature. Also similar wave nature exists “externally”, as a motional energy of particles (being fully connected or coupled with internal wave structure of a particle in motion). Rotation and wave packing is what makes the difference between particles with rest mass and freely propagating waves.

Finally, the mass concept (as here introduced) tells us that in the background of what makes our universe should only be a certain kind of kinetic, motional or wave energy, "**assembled in different packaging, free propagating and/or resonant formats**".

Particularly interesting consequences of previous analysis are in the extension of **PWDC** concept to the internal macro-particles (elementary mass) domains, meaning that:

A) A single particle has its "external" kinetic or wave energy content, $\mathbf{E_k} = (\gamma - 1)\mathbf{mc^2} = \tilde{\mathbf{E}}_{\mathbf{ext.}} = \mathbf{hf}_{\mathbf{ext.}} = \tilde{\mathbf{E}} = \mathbf{hf}$ *, and*

B) Can be in the same time "internally excited" on number of ways (for instance, by heating, or by passing alternate electrical currents and/or mechanical vibrations trough it), this way getting an elevated content of internal wave energy, $\tilde{\mathbf{E}}_{\text{int.}} = \mathbf{hf}_{\text{int.}}$. *Consequently, internally excited particle, would once become able to radiate the surplus of its "internal" wave energy (when necessary complex conditions were be satisfied), what really happens in a number of well-known experimental situations. Sometimes, the surplus of internally excited wave states is so high that a macro particle becomes spontaneous radiator of real (smaller) particles, such as electrons,* α*-particles etc., besides radiating photons (all of that leading to the conclusion that differences between real particles that have stable rest masses, and pure waves without rest mass content, is not in their essential nature, but only in their appearance, or in a way of "**energy packaging**").*

We should not forget that the ontological background of all discussions regarding different forms of energy, mass and waves should be relatively simple conceptual framework already presented on the Fig. 4.1.2 and with energy conservation (4.5-3). All higher-level and more sophisticated concepts should have their roots there.

*[♣ **COMMENTS & FREE-THINKING CORNER:** Let us now make an attempt to show how linear and orbital moments could create the rest mass. Let us analyze a stable and neutral particle (consisting of one or many atoms, or countless number of elementary entities) in the state of relative rest, which has a mass* $\mathbf{m}$*. We can also say that such particle consists of different molecules, atoms, electrons, nuclear and sub-nuclear particles and fields, all of them moving, circulating, spinning or oscillating inside the particle shell. Externally, it seems that the particle is stable, electrically neutral and in the state of rest. Consequently, total linear and orbital particle moments,* $\vec{P}_{\text{int.-total}}, \vec{L}_{\text{int.-total}}$ *(looking externally, outside of the particle shell), should be close to zero, or should have negligibly small values (if we make a vectorial sum of all internal particle moments), as for instance,*

$$\vec{P}_{\text{int.-total}} = \sum_{(i)} \vec{p}_i = \sum_{(i)} m_i \vec{v}_i \rightarrow 0 \,, \; \vec{L}_{\text{int.-total}} = \sum_{(i)} \vec{L}_i = \sum_{(i)} \mathbf{J}_i \vec{\omega}_i \rightarrow 0 \,. \qquad (4.42)$$

For the particle in the state of rest we could also estimate its center-of-mass linear speed, and center-of-inertia angular speed, $\vec{v}_c$, $\vec{\omega}_c$ *that should again be negligibly small or close to zero values,*

$$\vec{v}_c = \frac{\vec{P}_{\text{int.-total}}}{\sum_{(i)} m_i} = \frac{\sum_{(i)} m_i \vec{v}_i}{\sum_{(i)} m_i} \rightarrow 0 \,, \; \vec{\omega}_c = \frac{\vec{L}_{\text{int.-total}}}{\sum_{(i)} \mathbf{J}_i} = \frac{\sum_{(i)} \mathbf{J}_i \vec{\omega}_i}{\sum_{(i)} \mathbf{J}_i} \rightarrow 0 \,. \qquad (4.43)$$

Now, combining (4.42) and (4.43) we can find the total mass and total moment of inertia of the particle in the state of rest as the result of an internal superposition and mutual interferences of all of its internal, linear and orbital moments,

$$\begin{aligned} &m = \sum_{(i)} m_i = \frac{\left|\vec{P}_{\text{int.-total}}\right|}{\left|\vec{v}_c\right|} = \frac{\left|\sum_{(i)} m_i \vec{v}_i\right|}{\left|\vec{v}_c\right|} = \frac{E_0}{c^2} = \frac{1}{c^2}\int_{[P]} \vec{v}d\vec{p} = \text{const.} \quad , \\ &\mathbf{J} = \sum_{(i)} \mathbf{J}_i = \frac{\left|\vec{L}_{\text{int.-total}}\right|}{\left|\vec{\omega}_c\right|} = \frac{\left|\sum_{(i)} \mathbf{J}_i \vec{\omega}_i\right|}{\left|\vec{\omega}_c\right|} = \sum_{(i)} m_i r_i^{\,2} = \frac{E_0}{\omega_c^{\,2}} = \frac{1}{\omega_c^{\,2}}\int_{[L]} \vec{\omega}\, d\vec{L} = Const. \end{aligned} \tag{4.44}$$

Effectively, in (4.44) we have divisions between two values; both of them negligibly small, but the results of such divisions are constant and realistically high numbers.

Just for the purpose of creating certain quantifiable and simple mathematical forms (dimensionally correct, at least), we could "invent" the following indicative relations (that would be later on most probably modified, but presently are good enough to show that stable particle in the state of rest is intrinsically composed of permanently moving and rotating internal states):

$$\begin{aligned} &E_o = J\omega_c^2 = mc^2 = \int_{[P\text{-int.}]} \vec{v}d\vec{p} = \int_{[L\text{-int.}]} \vec{\omega}\, d\vec{L} = hf_c \Rightarrow \omega_c = c\sqrt{\frac{m}{J}} = \frac{c}{r^*} = 2\pi f_c \Rightarrow \\ &\Rightarrow f_c = \frac{c}{2\pi}\sqrt{\frac{m}{J}},\ c = \lambda_c f_c \Rightarrow \lambda_c = 2\pi r^*. \end{aligned} \tag{4.44-1}$$

If we go back to the Newton law of gravitation between two masses (see (2.3), (2.4)-(2.4-3), we can conclude that there we explicitly deal with masses, but implicitly also with their linear and orbital moments, like in (4.44), and with many other dynamic properties of internal (and external) mass constituents. There are some other, more profound (macrocosmic and microcosmic) consequences of such active mass modeling, especially if in (4.44) we apply Minkowski space 4-vectors, (4.32)-(4.37), in order to establish a more complex active mass modeling (see also (2.3)-(2.4-3), (4.5-1) and (5.15)-(5.17)).

Under certain conditions we know that rest mass can be created combining the energy states (mater waves) that do not have their own rest masses, as for instance,

$$m = m_0 = \frac{1}{c^2}\sqrt{E_t^2 - P_t^2 c^2} \quad , \quad E_t = \sum_{(i)} E_{t-i} \quad , \quad \vec{P}_t = \sum_{(i)} \vec{P}_{t-i} \quad . \tag{4.44-2}$$

By analogy (presently without claiming that such result would be correct) we could create similar relation for a corresponding static moment of inertia,

$$J = J_0 = \frac{1}{\omega_c^2}\sqrt{E_t^2 - L_t^2\omega_c^2}\ ,\ E_t = \sum_{(i)} E_{t-i}\ ,\ \vec{L}_t = \vec{r}^* \times \vec{P}_t = \sum_{(i)} \vec{L}_{t-i}\ ,\left(\left|\vec{L}_{t-i}\right| = \frac{c^2}{r^*\omega_c^2}\right). \tag{4.44-3}$$

By combining (4.44-2) and (4.44-3) we will be able to get,

$$mc^2 + P_t^2 c^2 = J\omega_c^2 + L_t^2\omega_c^2 = E_t^2 = (\gamma\, mc^2) \quad . \tag{4.44-4}$$

Most probably that combining (4.44-4) and (4.33-1) we would be able to find (maybe after applying some more modifications) the most appropriate relations that show that linear and rotational motion elements are well synchronized and united (or coupled like mutually conjugated values).

Similar to (4.44), using analogies given in T.3.1-T.3.3, $\mathbf{m} \leftrightarrow \mathbf{C}, \mathbf{J} \leftrightarrow \mathbf{L}$*, we could later create new formulations of electric capacitance and magnetic inductance. This situation can be also compared to an atom structure: Externally, far enough, in the space around an atom, we could measure that atom is electrically neutral and in the state of relative rest, regardless of the fact that inside the atom structure everything is moving (and that there is no neutrality there). The next step is to conclude that total energy of active (or moving) atom constituents, conveniently super-imposed into stable and (externally) neutral atom create atom mass (where rest mass can be found using well-known Einstein relation* $E_{tot.} = mc^2$*). It is also clear that very particular dynamic "gearing and fitting all atom constituents should be satisfied in order to create an atom (and this gearing and fitting we are presently explaining and see as the quantum and/or discrete nature of matter; -see also Wilson-Sommerfeld rules, (5.4.1)).*

The same situation could also be analyzed using generalized Schrödinger equation, and equations of Relativistic Electrodynamics (producing more complex mathematical picture, than here presented), but important qualitative, phenomenological and conceptual aspects of particle-wave duality and action-reaction forces would not be clearly and simply identified as here presented.

We could also conclude that de Broglie matter waves (belonging to moving particles) should be very much present inside the atom structure, complementing the above mentioned dynamic gearing and fitting atom constituents. Similar concept can be extended to any other stable particle. The same statement differently formulated is that sources of de Broglie matter waves should be found inside the internal structure of particles, in the form of standing wave formation of internal particle structure (externally manifesting as orbital moments and spin attributes). When a particle changes its state of motion, intrinsic, internal wave formation (previously being in the state of stationary and standing waves structures) is becoming an active, de Broglie matter wave, producing externally measurable consequences, as for instance: photoelectric and Compton Effect, particles diffraction…

Every wave motion, wave group or packet of wave energy has its equivalent (wave) mass that can be found (combining (4.17-1) and (4.44)) as,

$$\tilde{m} = \frac{\tilde{E}}{\overline{uv}} = \frac{\left[\int_0^{\infty} [A(\omega)]^2 d\omega\right]^3}{\pi\left[\int_0^{\infty} \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega\right]\left[\int_0^{\infty} \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega\right]} \cdot$$

, or in case of electromagnetic waves,

$$\tilde{\mathbf{m}} = \frac{\tilde{\mathbf{E}}}{\mathbf{uv}} = \frac{\tilde{\mathbf{E}}}{\mathbf{c}^2} \tag{4.45}$$

Intuitively, we see that a stable particle (having stable rest mass) should have certain stationary waving structure (internally properly balanced), and we also know that in many interactions stable particles manifest particle-wave duality properties. Moreover, they could be disintegrated into pure wave energy constituents. It is also known that a convenient super-position of pure wave elements could produce a stable particle (electron-positron creation, for instance). Consequently, the general case of a stable particle should be that its internal constituents are composed of wave-mass elements in the form of (4.45), and maybe some other, more elementary, stable particle cores (whatever of them exist in every specific case). The proper internal and dynamic "gearing and fitting" of all particle constituents should produce a stable particle, which has stable rest mass (found by applying the rules of Relativity Theory: -connection between total energy, rest energy and particle momentum). Of course, there are many intermediary and mixed particle-wave objects, which sometimes behave more as particles or as waves.

*Obviously, particle-wave duality concept favored in this paper creates sufficiently clear frontier between stable particles (which have constant rest mass in its/their center of inertia reference system), and wave or particle-wave phenomena that belong only to different states of motion. Contrary to this position, we also know that internal structure of a stable particle is composed of wave and particle-wave constituents, which are properly "geared and fitted" producing (looking only externally) a stable particle (**based on standing waves, resonant field structures**). What is missing in such conceptual picture of particle-wave duality is to explain conditions when certain dynamic combination (super-position) of waves transforms into a stable particle; -for instance, when wave mass given by (4.45) will create a stable particle (**by closing an open and relatively free propagating waveform into itself, internally structured as certain standing waves, self-stabilized vortex-like field in resonance**). The cornerstones and frames for wave-to-particle transformation should be found in the following:*

1. ***Stable particle will be created when wave mass (4.45) is transformed into constant (externally measurable) rest mass,*** $\mathbf{m} \to \mathbf{m_0}$ ***which is time independent and localized in certain limited space (satisfying also (4.44)).***
2. ***Stable particle should have (in its Center of Inertia System) non-zero rest mass.***
3. ***Center-of-mass velocity of all internal wave particle constituents (in its Laboratory System) should be minimal or equal zero.***
4. ***Rest mass (created from a wave group or some kind of superposition of interaction between wave groups) can also be found by satisfying Minkowski-space 4-vector relation between total energy and momentum:*** $\mathbf{m} = \mathbf{m_0} = (\sqrt{\mathbf{E}^2_{\mathbf{tot.}} - \mathbf{P}^2_{\mathbf{tot.}}\mathbf{c}^2})/\mathbf{c}^2$***...*** *Here we can make another step regarding conceptualizing the wave packaging nature of stable particles that have non-zero rest masses. Let us again start from the general relation between mass, total energy and total momentum presented as,*

$$\mathbf{mc^2} = \mathbf{E_0} = \sqrt{\mathbf{E}^2_{\mathbf{tot.}} - \mathbf{P}^2_{\mathbf{tot.}}\mathbf{c}^2} = \sqrt{\mathbf{E}^2 - \mathbf{p}^2\mathbf{c}^2} = const. \Rightarrow$$
$$\Rightarrow \mathbf{E}_0^2 = \mathbf{E}^2 - \mathbf{p}^2\mathbf{c}^2 = (\mathbf{E} - \mathbf{pc}) \cdot (\mathbf{E} + \mathbf{pc}) = Const.$$

Let us now imagine that energy product member $(\mathbf{E} - \mathbf{pc})$ *has the spectral content* $\mathbf{U_1} = \mathbf{U(E - pc)}$, *and that the energy product member* $(\mathbf{E} + \mathbf{pc})$ *has the spectral content* $\mathbf{U_2} = \mathbf{U(E + pc)}$, *where the total particle mass (here represented by* $\mathbf{E}_0^2$*) has the resulting spectral content,* $\mathbf{U(E,pc)} = \mathbf{U_1} \cdot \mathbf{U_2} = \mathbf{U(E - pc)} \cdot \mathbf{U(E + pc)}$. *In other words, we are claiming that rest mass* $\mathbf{m}$ *is created as certain kind of super-position (or interaction) between two wave groups that have spectral functions* $\mathbf{U_1} = \mathbf{U(E - pc)}$ *and* $\mathbf{U_2} = \mathbf{U(E + pc)}$, *and corresponding time-space domain functions* $\overline{\Psi}_1(t,x)$ *and* $\overline{\Psi}_2(t,x)$.

Now relations between mutually corresponding time and frequency domains can be conceptually presented (carrying an over-simplified brainstorming message) as,

$$E_0^2 (\Leftrightarrow) \begin{Bmatrix} E_0 \cdot E_0 \\ \overline{\psi}(t); \overline{\psi}(t) \\ U(E,pc) \cdot U(E,pc) \end{Bmatrix} (\Leftrightarrow) \begin{Bmatrix} (E - pc) \cdot (E + pc) \\ \overline{\psi}_1(t,x);\ \overline{\psi}_2(t,x) \\ U(E - pc) \cdot U(E + pc) \end{Bmatrix} (\Leftrightarrow) \begin{Bmatrix} \text{Energy} \\ \text{time-space} \\ \text{frequency} \end{Bmatrix}$$
$$E = E(\omega, k)\ ,\ p = \frac{h}{2\pi} k = \frac{h}{\lambda},\ U = U(\omega, k),\ (E \mp pc) \Leftrightarrow (\omega\, t\ \mp kx)$$

$$\overline{\psi}_1(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U_1(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U(E-pc)e^{j\omega t}d\omega = a(t)e^{j\Phi\ (E-pc)}$$

$$\overline{\psi}_2(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U_2(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U(E+pc)e^{j\omega t}d\omega = a(t)e^{j\Phi\ (E+pc)} \qquad (4.46)$$

$$\overline{\psi}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U_1 \cdot U_2 e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U(E,pc)e^{j\omega t}d\omega = a(t)e^{j\Phi\ (E,pc)} =$$

$$= \int_{-\infty}^{+\infty} \psi_1(t)\cdot\psi_2(t-\tau)e^{j\omega t}d\omega = \int_{-\infty}^{+\infty} \psi_1(t-\tau)\cdot\psi_2(t)e^{j\omega t}d\omega.$$

It is almost obvious that both, $\mathbf{U_1 = U(E-pc)}$ *and* $\mathbf{U_2 = U(E+pc)}$ *should carry the same amount of energy (equal to* $\mathbf{E_0}$*), and that a stable rest mass is created as the time cross-section (or convolution) of the mutually phase shifted wave functions* $\psi_1(t)$ *and* $\psi_2(t)$*. In this way they capture the time variable wave content into a time stable packaging format. In other words, here we are in the good way to re-discover how waves of matter are able to create stable particles, and later (using similar methodology) we could also start thinking about how particles could be transformed into waves.*

Most probably that by continuing this process we can find some other important conditions for wave-to-particle transformation, related to solutions of Schrödinger-type wave equations. As we know, the Orthodox Quantum Mechanics does not respect too much the frontiers between waves and particles, treating all particles and particle-wave objects as waves (including a rest mass in the same basket), and associating the proper wave function to every quantum entity. This ***is basically correct in the framework of Quantum Theory, but still the difference should exist between waves that are unbounded and propagating relatively free, and when the same wave structure is internally self-closed, creating standing waves resonant field structure, spatially well localize****). This is one of the differences between the particle-wave duality explained here and official particle-wave duality found in today's Quantum Theory (since in this paper the rest mass and rest energy are excluded from the free propagating wave energy). Luckily (for Quantum Theory), in many practical situations, mentioned difference is even not noticed (or not too much relevant) since when creating differential wave equations (Schrödinger equation types) we are simply eliminating constant mass or constant energy members (differentiation of constant/s is zero).*

Here, the main intention has been to present sufficiently clear conceptual message about real (multi-components) nature of mass on a very simple way, indirectly underlying that Gravitation, as presently supported in Physics, could also be (or should be) differently established. It is also clear that one should invest much more work to make this mass concept more operational.

As we can see, the differentiation between mass and energy related to a particle (or wave group) in different states of motion should be much more systematically and more profoundly analyzed if we would like to capture by the same modeling the unity of linear and rotational motions. In order to start realizing such project, the first step should be to encounter all possible, mutually distinctive states of motion, as for example, listed in the T.4.3.3, and illustrated on the Fig.4.3.1. Eventually we should be able to specify at least 16 of energy mass states that are mutually different and also mutually related (16 states in the T.4.3.3, from 1.0.1 to 4.3.2), showing the basic relations and connections (or complementary nature) between linear motion and rotation.

T.4.3.3. The chart of possible particle states

Different motional states of a stable particle	1. Standstill state	2. Linear Motion	3. Rotational Motion	4. Combined Linear Motion and Rotation
From the point of view of different observers (see 1. and 2., below):	1.0. No external motion. Particle is in the state of rest, looking from the outside space.	2.0. Particle as a whole is only in linear motion relative to a certain external system of reference.	3.1. Particle as a whole is only rotating around certain point, relative to certain external system of reference (or center, which is in external space, not captured by the particle). No linear motion.	4.1. Particle as a whole is in linear motion and at the same time rotating relative to a certain external system of reference (or center, which is in external space, not captured by the particle).
			3.2. Particle as a whole is only rotating around itself (or around its center of gravity, or around one of its axes). No linear motion.	4.2. Particle as a whole is in linear motion and at the same time rotating around itself, relative to a certain (moving) system of reference that is inside the domain captured by the particle.
			3.3. Particle as a whole is rotating around itself (or around its center of gravity, or around one of its axes), and in the same time also rotating around certain point which is in external space, not captured by the particle (performing a multi-component rotational motion). No linear motion.	4.3. Particle as a whole is in linear motion and at the same time rotating relative to a certain external system of reference, and also rotating around itself, relative to a certain (moving) system of reference that is inside the domain captured by the particle.
Comments:	Only internal particle constituents, or matter waves are in complex motion (united rotation and linear motion inside the particle structure; -nothing of that being visible externally).	No externally visible rotation exists; -internal particle structure and internal matter-wave motions are not visible externally.		
1. The (virtual) observer who is placed inside the particle structure	1.0.1.	2.0.1.	3.1.1.	4.1.1.
			3.2.1.	4.2.1.
			3.3.1.	4.3.1.
2. An observer who is placed in the external particle space (external, independent system of reference, not captured by the particle domain).	1.0.2.	2.0.2.	3.1.2.	4.1.2.
			3.2.2.	4.2.2.
			3.3.2.	4.3.2.

In reality, the same energy state (particle, wave packet…) could have many mutually coupled levels of its (internally and externally), energetically atomized structure, where each level would have its own linear and rotational couple of motional components, symbolically visualized on the Fig.4.3.1 with 4 of such levels (see also chapter 6., MULTIDIMENSIONALITY, where an attempt is made to formulate similar concepts mathematically).

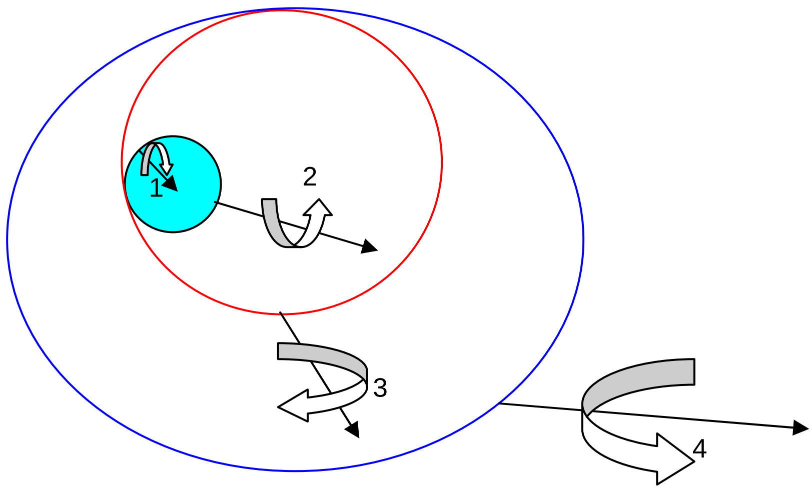


Fig.4.3.1. Symbolic visualization of multilevel linear-rotational motional couples ♣*]*

5. UNCERTAINTY RELATIONS AND ELEMENTARY DOMAINS

Before we start discussing and analyzing Uncertainty Relations in Physics, let us first say what kind of entities in our universe Uncertainty Relations address. Effectively, until present (allover this paper) we have been exploiting different analogies and complementary nature between corresponding *Original* and *Spectral* signal domains, indirectly describing the most important **Symmetries** of Physics. Briefly saying, **Uncertainty Relations** in Physics (and in this paper) deal with quantifiable relations between durations and/or signal intervals belonging to couples of mutually Original and Spectral domains (regardless if this is the world of atoms and micro-particles, or everything else belongs to a macro world of planets and galaxies).

In 1905, a mathematician named Amalie Nether proved the following theorem (regarding universal laws of Symmetries):

-For every continuous symmetry of the laws of physics, there must exist a conservation law.
-For every conservation law, there should exist a continuous symmetry.

Let us summarize the already known conservation laws and symmetries (in this paper initially introduced only in relation to analogies), by creating the table T.5.1.

T.5.1. Symmetries of the Laws of Physics

Original Domains ↔	↔ Spectral Domains
Time = t **Time Translational Symmetry**	**Energy =** $\tilde{E}$ **(or frequency =** f **, or mass =** m**)** **Law of Energy Conservation**
Displacement = $x = S\dot{\tilde{p}} = S\tilde{F}$, $(\tilde{F}=\text{force})$ **Space Translational Symmetry**	**Momentum =** $\tilde{p} = \tilde{m}\dot{x} = \tilde{m}v$ **Law of Momentum Conservation**
Angle = $\alpha = S_R\dot{L} = S_R\tau$ **Rotational Symmetry**	**Angular momentum =** $L = J\dot{\alpha} = J\omega$ **Law of Angular Momentum Conservation**
Electric Charge = $q_{el.} = \Phi_{el.} = C\dot{q}_{mag.} = Ci_{mag.}$ **Law of Total Electric Charge Conservation** **The Magnetic Charge-reversal Symmetry**	**Magn. Charge =** $q_{mag.} = \Phi_{mag.} = L\dot{q}_{el.} = Li_{el.}$ **The Electric Charge-reversal Symmetry** **"Total Magnetic Charge" Conservation**

(Here, mono-magnetic charge is not a free and self-standing entity)

Obviously (as we can see from T.5.1), couples of conjugate, Original Spectral domains, created using simple analogies, are also in agreement with the most important symmetries and conservation laws of our universe (see also T.1.8 from the first chapter of this paper: Analogies…).

There are many mathematical expressions of uncertainty relations between mutually coupled or conjugated interval lengths (in connection to entities presented in the table T.5.1), belonging to a certain signal, object or energy state, and related to different Fourier-type transformation domains of that signal.

Let us start from the simplest, already known mathematical forms of Uncertainty relations applied to a certain waveform, or wave packet, often found in Signal (or Spectrum) Analysis, Telecommunications Theory and in certain earlier works regarding Quantum Theory. We will consider, as the starting position in this analysis, that there is a wave packet model, which is by its definition or formation equivalent (only in a couple of most important aspects) to certain moving particle. The wave energy of the wave packet in question should be equal to the total motional (or kinetic) energy of its particle couple, the wave momentum of the wave packet will be equal to the particle momentum, and the group wave velocity will be equal to the particle (center-of-mass) velocity.

If we take into account only the absolute or total, one-dimensional wave packet durations in all of its domains (in time, frequency and space, as it is already explained in the chapter: 4.0 – "Wave functions wave velocities and uncertainty relations", starting from the expression (4.0.55)), uncertainty relations are given as:

$$\Delta \mathbf{x} \cdot \Delta \mathbf{p} = \Delta \mathbf{t} \cdot \Delta \mathbf{E} = \mathbf{h} \cdot \Delta \mathbf{t} \cdot \Delta \mathbf{f} \geq \mathbf{h}/2 \Leftrightarrow \Delta \mathbf{x} \cdot \Delta \tilde{\mathbf{p}} = \Delta \mathbf{t} \cdot \Delta \tilde{\mathbf{E}} = \mathbf{h} \cdot \Delta \mathbf{t} \cdot \Delta \mathbf{f} \geq \mathbf{h}/2 \,, \qquad (5.1)$$

where the meaning of the symbols is the same as introduced allover this paper ($\mathbf{x}$ - displacement, $\mathbf{p}, \tilde{\mathbf{p}}$ -particle and/or wave momentum, $\mathbf{E}, \tilde{\mathbf{E}}$ - particle and/or wave-packet motional energy, $\mathbf{f}$ -frequency, $\mathbf{t}$ -time and $\mathbf{h}$ -Planck's constant, see expressions (4.1) - (4.3)).

Since here we are still addressing an integral wave packet or particle (which should be mutually equivalent), it is clear that *in (5.1) all momentum and energy values are the total momentum and motional energy amounts, regardless of using delta difference symbols, effectively making* $\Delta \mathbf{p} = \Delta \tilde{\mathbf{p}} = \tilde{\mathbf{p}} = \mathbf{p}$ *and* $\Delta \tilde{\mathbf{E}} = \Delta \mathbf{E} = \tilde{\mathbf{E}} = \mathbf{E}_{\mathbf{k}}$. In order to avoid repeating lengthy introductions and explanations regarding relations (5.1), it is very much recommendable first to recover most of necessary background information (related to wave packets and introductory aspects of uncertainty relations) in the earlier chapter 4.0: "Wave functions wave velocities and uncertainty relations".

Here is also appropriate to mention that modern Quantum Theory modified a little bit the Uncertainty Relations (5.1) by often considering delta differences (Δ), or important signal durations as statistical, standard deviation intervals.

The leading objective in this paper is to establish a much larger conceptual framework regarding universally valid Uncertainty Relations than presently found in Physics (condensing and simplifying, as much as possible, associated mathematical complexity). Using electromechanical analogies (established in the earlier chapters of this paper), it is possible to additionally address and extend all Uncertainty Relations (5.1).

Taking data from T.1.2, T.3.1, T.3.2, T.3.3 and T.5.1, let us analogically create T.5.2, producing the results that will **dimensionally** give the mutually complementing energy-time product $\Delta\mathbf{t}\Delta\mathbf{E}$ for cases regarding electromagnetic fields and linear and rotational motions (of course, every time capturing a relatively isolated and compact, self-standing energy state, such as a motional particle or an equivalent wave packet):

T.5.2.

Electro-Magnetic Field	Linear Motion	Rotation
$u_{el.} = i_{mag.} = -\frac{d\Phi_{mag.}}{dt} = -\frac{dq_{mag.}}{dt}$ (=) El. volt. (=) Mag. current ↓	$\mathbf{v = \frac{dx}{dt}}$ **(=)Velocity** ↓	$\omega = \frac{d\alpha}{dt}$ **(=) Angular Velocity** ↓
$\mathbf{i_{el.} = u_{mag.} = +\frac{d\Phi_{el.}}{dt} = +\frac{dq_{el.}}{dt}}$ (=)**El. current** (=)**Mag. voltage**	$\mathbf{F = \frac{dp}{dt}}$ **(=)Force**	$\tau = \frac{dL}{dt}$ (=) Torque
$\mathbf{P = u_{el.} i_{el.} = i_{mag.} u_{mag.} = \frac{dE}{dt} = -\frac{dq_{mag.}}{dt} \cdot \frac{dq_{el.}}{dt}}$ **(=) Power(=)Ψ^2**	$\mathbf{P = vF = \frac{dx}{dt} \cdot \frac{dp}{dt} = \frac{dE}{dt}}$ **(=) Power (=) Ψ^2**	$P = \omega\tau = \frac{d\alpha}{dt} \cdot \frac{dL}{dt} = \frac{dE}{dt}$ (=) Power (=) Ψ^2
$\mathbf{\Phi_{mag.} = q_{mag.}}$ (=) **Magn. Flux / Charge**	**x** (=)Displacement	α **(=) Angle**
$\mathbf{\Phi_{el.} = q_{el.}}$ **(=) El. Flux / Charge**	**p (=) Momentum**	**L (=) Angular Momentum**
$\mathbf{\Delta E = P\Delta t}$ **(=) Energy**	$\mathbf{\Delta E = P\Delta t}$ **(=) Energy**	$\mathbf{\Delta E = P\Delta t}$ **(=) Energy**
$\mathbf{\Delta\Phi_{mag.}\Delta q_{el.} = \Delta q_{mag.}\Delta\Phi_{el.} = P(\Delta t)^2 = \Delta E\Delta t}$	$\mathbf{\Delta x\Delta p = P(\Delta t)^2 = \Delta E\Delta t}$	$\mathbf{\Delta\alpha\Delta L = P(\Delta t)^2 = \Delta E\Delta t}$

Combining (4.2) with relations found in the bottom line of T.5.2 it would be possible (by analogy, and considering that this way we are taking into account a complete and finite wave packet of certain kind) to extended uncertainty relations (5.1) for two more members,

$$\Delta q_{mag.} \cdot \Delta q_{el.} = \Delta\alpha \cdot \Delta L = h \cdot \Delta t \cdot \Delta f = \Delta x \cdot \Delta\tilde{p} = \Delta t \cdot \Delta\tilde{E} = \Delta s_1 \Delta s_2 \geq h/2,$$
$$\lambda = h/p = h/\Delta\tilde{p} \Rightarrow$$
$$\frac{\lambda}{2} \leq \frac{\Delta s_1 \Delta s_2}{\Delta\tilde{p}} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\Delta\tilde{p}} = \frac{\Delta\alpha \cdot \Delta L}{\Delta\tilde{p}} = \frac{h \cdot \Delta t \cdot \Delta f}{\Delta\tilde{p}} = \frac{\Delta x \cdot \Delta\tilde{p}}{\Delta\tilde{p}} = \frac{\Delta t \cdot \Delta\tilde{E}}{\Delta\tilde{p}} = \bar{v} \cdot \Delta t, \tag{5.2}$$

The above-given mathematical options (regarding de Broglie wavelength) seem a bit oversimplified, but we will see that the relations (5.2) and their consequences are correct. For instance, when we consider $\mathbf{\Delta q_{el.}}$ as an elementary (and minimal possible) electric charge of an electron, we will get: $(\mathbf{\Delta q_{el.}} = \Delta\Phi_{el.})_{min.} = \mathbf{e} \Rightarrow \Delta\Phi_{mag.} = \Delta\mathbf{q_{mag.}} > \mathbf{h/2e}$, where $\mathbf{h/2e}$ presents an elementary charge of a magnetic flux, or (dynamic) magnetic charge, $(\Delta\mathbf{q_{mag.}})_{min.} = \mathbf{h/2e}$. Similarly, we can also support the resonant-like quantification of rotational motion ($\Delta\alpha, \Delta L$), and most probably connect it to spin properties of elementary particles (or possibly find some other couple/s of conjugated variables, $\mathbf{s_1}$, $\mathbf{s_2}$ satisfying similar relation).

What is significant here is that both particles and waves are treated equally, applying the same Uncertainty Relations (the only difference is that particles should be considered as "frozen" or well packed, self-stabilized standing wave structures, where <u>Uncertainty is transformed into a Certainty</u>, or where an inequality sign, ≥ is transformed into an equality sign, =). Obviously, (based on (5.2)), we can conclude (maybe at this time still a bit prematurely) that metrics and energy formatting of nature, regarding its elementary parts (such as atoms and elementary particles), has come to certain, conditionally nondivisible (and minimal) units, optimal packaging and minimal domain intervals, which should satisfy the following "resonant gearing and fitting conditions", (5.3),

$$\begin{aligned}&\left(\Delta q_{mag.}\right)_{min.}\cdot\left(\Delta q_{el.}\right)_{min.}=\left(\Delta\alpha\right)_{min.}\cdot\left(\Delta L\right)_{min.}=h\cdot\left(\Delta t\right)_{min.}\cdot\left(\Delta f\right)_{min.}=\\&=\left(\Delta x\right)_{min.}\cdot\left(\Delta\tilde{p}\right)_{min.}=\left(\Delta t\right)_{min.}\cdot\left(\Delta\tilde{E}\right)_{min.}=\left(\Delta s_1\right)_{min.}\cdot\left(\Delta s_2\right)_{min.}=h/2\ ,\end{aligned}\tag{5.3}$$

<u>what in reality presents kind of resonant states and standing wave relations</u>, between each couple of mutually conjugated variables, where $(\Delta \mathbf{s}_1)_{min.}$ and $(\Delta \mathbf{s}_2)_{min.}$ symbolize all other, minimal and elementary, quantifiable interval lengths (which are also mutually conjugate variables). After transforming (5.3) into an equivalent (de Broglie mater waves) half-wavelength, we will have:

$$\begin{aligned}&\lambda=h/p=h/\Delta\tilde{p}=h/\left(\Delta\tilde{p}\right)_{min.}\Rightarrow\\&\frac{\lambda_{min.}}{2}=\frac{\left(\Delta q_{mag.}\right)_{min.}\cdot\left(\Delta q_{el.}\right)_{min.}}{\Delta\tilde{p}}=\frac{\left(\Delta\alpha\right)_{min.}\cdot\left(\Delta L\right)_{min.}}{\Delta\tilde{p}}=\frac{h\cdot\left(\Delta t\right)_{min.}\cdot\left(\Delta f\right)_{min.}}{\Delta\tilde{p}}=\\&=\frac{\left(\Delta x\right)_{min.}\cdot\left(\Delta\tilde{p}\right)_{min.}}{\Delta\tilde{p}}=\frac{\left(\Delta t\right)_{min.}\cdot\left(\Delta\tilde{E}\right)_{min.}}{\Delta\tilde{p}}=\frac{\left(\Delta s_1\right)_{min.}\cdot\left(\Delta s_2\right)_{min.}}{\Delta\tilde{p}}\ ,\end{aligned}\tag{5.4}$$

Intentionally choosing the half-wavelength as the most important elementary energy formatting and packaging unit of anything that is quantifiable in our universe ((5.2)-(5.4)), we are simply stating that quantification and standing waves are phenomenological and conceptual synonyms regarding different waves and resonance-related phenomena. Real manifestations of quantification are characteristic only for self-closed, space-atomized domains and energy exchanges between them, where the smallest domain unit is equal to half-wavelength, $\frac{\lambda}{2}$. Of course, the meaning of half-wavelength should be extended to all other aspects of structured waves and fields, as for instance to rotational or angular values and to corresponding electromagnetic values. By continuing developing the same concept, we would probably find that Euler-Lagrange-Hamilton formalism presents another framework to express **tendency to optimal and dynamic energy packaging of the matter in motion.**

Let us now address the (resonant) periodicity of motional states, fields and wave packets using the framework of analogies (established in the first and third chapter) and **Wilson-Sommerfeld action integrals** (see [9]), known from early ages of Quantum Mechanics. By "playing with analogies", based on data from **T.3.1** and **T.3.3**, (Chapter 3), combining them with "*elementary particle metrics"* (5.4), we could introduce even **wider meaning of de Broglie Wavelength/s, which will now become periodical and quantifiable matter wave intervals.**

For instance, we will be able to formulate analogically "***de Broglie, matter waves angular wavelength***", $\overleftrightarrow{\theta}=\frac{\mathbf{h}}{\mathbf{L}}$, and "***de Broglie electromagnetic charge/s wavelength/s***", $\overleftrightarrow{\Phi}_{\mathbf{mag.}}=\frac{\mathbf{h}}{\Phi_{\mathbf{el.}}}=\frac{\mathbf{h}}{\mathbf{q}},\overleftrightarrow{\Phi}_{\mathbf{el.}}=\mathbf{q}=\frac{\mathbf{h}}{\Phi_{\mathbf{mag.}}}$, as given in the **T.5.3**). The idea here is to show that micro-world quantification is not too far from standing waves, resonant energy packaging.

T.5.3

Waves Periodicity of Fields and Motional States	[DISPLACEMENTS] $[X] = [q_{mag.}, q_{el.}, x, \alpha]$	[CHARGES] $[Q] = [q_{el.}, q_{mag.}, p, L]$	*De Broglie, Periodicity Wave Intervals* $[\overleftrightarrow{X}][Q] = [h]$
Electric Field (and total electric charge conservation)	$\Phi_{mag} = L_{mag} i_{el} = q_{mag}$	$[C_{el.} u_{el.}^{1}]$ (=) $[q_{el.} u_{el.}^{0}]$ (=) $[q_{el.}]$ (=) $[\Phi_{el}]$	$\overleftrightarrow{\Phi}_{mag.} = \frac{h}{\Phi_{el.}} = \frac{h}{q}$
Magnetic Field (and total, or dynamic magnetic charge conservation)	$\Phi_{el.} = L_{el.} i_{mag.} = q_{el.}$	$[C_{mag.} u_{mag.}^{1}]$ (=) $[q_{mag.} u_{mag.}^{0}]$ (=) $[q_{mag.}]$ (=) $[\Phi_{mag}]$	$\overleftrightarrow{\Phi}_{el.} = q = \frac{h}{\Phi_{mag.}}$
Gravitation & Linear Motion (and linear momentum conservation)	$x = Sf$	$[mv^{1}]$ (=) $[pv^{0}]$ (=) $[p]$	$\overleftrightarrow{\lambda} = \frac{h}{p} = \lambda$
Rotation (and angular momentum conservation)	$\alpha = S_R M$	$[J\omega^{1}]$ (=) $[L\omega^{0}]$ (=) $[L]$	$\overleftrightarrow{\theta} = \frac{h}{L}$

In fact, what T.5.3, equations (5.2)-(5.4) and Wilson-Sommerfeld action integrals really describe (and predict) should be a kind of universal (resonant) wave periodicity, or kind of fields, waves and charges, energy and space related atomization. This way we are able to formulate fitting and gearing rules, expressing some of the very important Symmetries of our universe (or simply saying, we are expressing "**optimal mater waves and energy formatting and packaging rules**"). Here we can also formulate an extension of de Broglie particle-wave hypothesis. *Since a kind of Particle-Wave Duality, periodicity, waving and resonant quantization is associated to all linear motions (already known in the form of:* $\lambda = \mathbf{h}/\tilde{\mathbf{p}}$, $\tilde{\mathbf{E}} = \mathbf{hf}$ *), the same (by analogy, under similar conditions) should be applicable to rotational motions, to electromagnetic phenomena, to any kind of motional energy, and to all other fields and motions of Nature. In addition, all of such manifestations should exist coincidently and mutually well synchronized (often united, coupled inside of the same object, or belonging to the same energy state,*

$$\lambda = \frac{h}{\tilde{p}}, \overleftrightarrow{T} = \frac{h}{\tilde{E}}, \overleftrightarrow{\theta} = \frac{h}{L}, \overleftrightarrow{\Phi}_{el.} = \frac{h}{\Phi_{mag.}}, \overleftrightarrow{\Phi}_{mag.} = \frac{h}{\Phi_{el.}}, \overleftrightarrow{s}_{1,2} = \frac{h}{s_{2,1}}).$$

By analogy with (5.2) and using the newly formulated "*de Broglie angular wavelength*" from T.5.3, we could also have the following, extended uncertainty relation:

$$\vec{\theta} = h / L = h / \Delta\tilde{L} = h / \Delta L \Rightarrow$$
$$\vec{\theta} \leq \frac{\Delta s_1 \Delta s_2}{\Delta\tilde{L}} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\Delta\tilde{L}} = \frac{\Delta\alpha \cdot \Delta\tilde{L}}{\Delta\tilde{L}} = \frac{h \cdot \Delta t \cdot \Delta f}{\Delta\tilde{L}} = \frac{\Delta x \cdot \Delta\tilde{p}}{\Delta\tilde{L}} = \frac{\Delta t \cdot \Delta\tilde{E}}{\Delta\tilde{L}} = \overline{\omega} \cdot \Delta t \ .$$

If $\Delta\tilde{L} = n\frac{h}{2\pi}$, then $\vec{\theta} = h/L = h/\Delta\tilde{L} = h/\Delta L = \frac{2\pi}{n}$, $n \in [1 \ , \ 2 \ , \ 3 \ , \ ...] \Rightarrow$ (5.2.1)

$$\vec{\theta} = \frac{2\pi}{n} \leq \frac{\Delta s_1 \Delta s_2}{\Delta\tilde{L}} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\Delta\tilde{L}} = \frac{\Delta\alpha \cdot \Delta\tilde{L}}{\Delta\tilde{L}} = \frac{h \cdot \Delta t \cdot \Delta f}{\Delta\tilde{L}} = \frac{\Delta x \cdot \Delta\tilde{p}}{\Delta\tilde{L}} = \frac{\Delta t \cdot \Delta\tilde{E}}{\Delta\tilde{L}} = \overline{\omega} \cdot \Delta t \ ,$$

and something similar could also be formulated with reference to $\vec{\Phi}_{el.}$ **,** $\vec{\Phi}_{mag.}$ **,** $\vec{s}_i$ **...**

Wilson-Sommerfeld action integrals (see [9]), related to any periodical motion on a self-closed stationary orbit, applied over one period of the motion, present the kind of general quantifying rule (for all closed standing waves, which are energy carrying structures) that was also successfully used in supporting N. Bohr's Planetary Atom Model. By analogical extension of Wilson-Sommerfeld action integrals to all "**CHARGE**" elements found in T.5.3, we can formulate the following quantifying expressions (again between mutually conjugated variables) that are also in agreement with "de Broglie Wave Intervals" from T.5.3:

Metrics of Elementary Particles:

$$\left\{\begin{array}{l} \left|(2\Delta q_{mag.})_{min.}(\Delta q_{el.})_{min.}\right| = \left|(2\Delta\alpha)_{min.}(\Delta \mathbf{L})_{min.}\right| = \left|(2\Delta x)_{min.}(\Delta p)_{min.}\right| = \\ = \left|(2\Delta\tilde{E})_{min.}(\Delta t)_{min.}\right| = h\left|(2\Delta f)_{min.}(\Delta t)_{min.}\right| = \left|(2\Delta s_1)_{min.}(\Delta s_2)_{min.}\right| = h \\ \left\{\lambda = \frac{h}{\tilde{p}}, \ \tilde{E} = hf = h\frac{1}{\vec{T}} \Leftrightarrow \vec{T} = \frac{h}{\tilde{E}}\right\} \Rightarrow \left\{\vec{\theta} = \frac{h}{\mathbf{L}}, \ \vec{\Phi}_{el.} = \frac{h}{\Phi_{mag.}}, \ \vec{\Phi}_{mag.} = \frac{h}{\Phi_{el.}}, \vec{s}_1 = \frac{h}{s_2}\right\} \\ [X] = [q_{mag.}, q_{el.}, x, \alpha], \ [Q] = [q_{el.}, q_{mag.}, p, \mathbf{L}] \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} \vec{\Phi}_{mag.} \cdot \Phi_{el.} = \vec{\Phi}_{el.} \cdot \Phi_{mag.} = \vec{\lambda} \cdot p = \vec{\theta} \cdot \mathbf{L} = \vec{T} \cdot \tilde{E} = ... = \vec{s}_1 \cdot s_2 = h, \\ (\vec{\Phi}_{mag.} \cdot \Phi_{el.})_1 + (\vec{\lambda} \cdot p)_2 + (\vec{\theta} \cdot \mathbf{L})_3 + (\vec{T} \cdot \tilde{E})_4 + ... + (\vec{s}_1 \cdot s_2)_n = mh, \ m = 1, 2, 3... \end{array}\right\} \Rightarrow$$

$$\Rightarrow [\vec{X}][Q] = [h] \Rightarrow$$

Wilson-Somerfeld action integrals

$$\left\{\begin{matrix}\oint\limits_{C_n} p_\lambda d\lambda = n_\lambda h, \ \oint\limits_{C_n} L_\theta d\theta = n_\theta h, \\ (n_\lambda, n_\theta) = \text{integers } (=1,2,3,...)\end{matrix}\right\} \Rightarrow \left\{\begin{matrix}\oint\limits_{C_n} \Phi_{el.} d\Phi_{magn.} = n_{el.} h, \ \oint\limits_{C_n} \Phi_{mag.} d\Phi_{el.} = n_{mag.} h, \\ \oint\limits_{C_n} \tilde{E}_n dt = nh, \ (n_{el.}, n_{mag.}, n) = \text{integers}\end{matrix}\right\} \Rightarrow$$

(5 .4.1)

$$\Rightarrow \begin{matrix}\oint\limits_{C_n} [X] d[Q]^T = \oint\limits_{C_n} [Q] d[X]^T = \oint\limits_{C_n} \tilde{E} dt = n \cdot [X] \cdot [Q]^T = n \cdot [Q] \cdot [X]^T = nh, \ n = 1, 2, 3, ... \\ ([X] = [q_{mag.}, q_{el.}, x, \alpha], [Q] = [q_{el.}, q_{mag.}, p, L]).\end{matrix}$$

What we could additionally conclude from T.5.3 and Wilson-Sommerfeld action integrals (5.4.1) is that Uncertainty relations (recognized by using the symbol "≤") in the form (5.1)-(5.2), most probably exist only for cases of relatively unbounded, open-space, free propagating and transitory wave motions. On the contrary, stable and self-closed space structures, (5.3)-(5.4), like stationary and resonant energy states and standing wave structures inside atoms and elementary particles (where rest masses could be involved), effectively and "in average" alter this form of Uncertainty, *making it much more certain than uncertain*. In other words, instead of "≤", in cases related to localized energy states that have stable and non-zero rest masses, like atoms and certain elementary particles, we should have an equality sign "=", since only an integer number of half or full-wavelengths of any relevant matter wave entity that creates standing waves there, could exist in such well-defined and stable elementary structures (see T.5.4).

We can also say, regarding contemporary Physics, that everything related to unity, connections and coupling between electric and magnetic charges and fields is mathematically better formulated than analogous relations and connections between linear and rotational motions. In other words, most probably modern important definitions and parameters of linear and rotational motions (see also T.5.4) should be appropriately reformulated (or slightly modified), in order to get the same elegancy and symmetry as in the case of electric and magnetic fields and charges.

In order to extend the mentioned aspects of analogies and symmetry between electromagnetic and mechanical motions, let us summarize the already introduced concept of wavelength analogies and symmetries between mechanical and electromagnetic aspects of matter waves, by creating the table T.5.4. The terminology in T.5.4 is in some cases slightly modified (compared to what we could find in Physics books) in order to additionally expose previously mentioned analogies.

The next unifying (presently still hypothetical) step in this process will be to show (macroscopically and microscopically) existence of intrinsic coupling (as an analogy and symmetry) between magnetic phenomena and rotation, and electric phenomena and linear motions, starting, for instance, from the well known relation/s between orbital and magnetic moments valid in atom world (for electrons and protons).

T.5.4. Wavelength analogies in different frameworks

<table>
<tr><th>Mater Wave Analogies</th><th>Linear Motion</th><th>Rotation</th><th>Electric Field</th><th>Magnetic Field</th></tr>
<tr><td>Characteristic Charge</td><td>Linear Momentum
p</td><td>Orbital Momentum
$L = pR$</td><td>Electric Charge
$q_e = q$</td><td>“Magnetic Charge”
$q_m = \Phi$</td></tr>
<tr><td>Mater Wave Periodicity</td><td>Linear Path Periodicity
$\lambda = \frac{h}{p}$
(Linear Wavelength)</td><td>Angular Motion Periodicity
$\theta = \frac{h}{L}$
(Angular Wavelength)</td><td>“Electric Periodicity”
$\lambda_e = \frac{h}{q_e} = q_m$</td><td>“Magnetic Periodicity”
$\lambda_m = \frac{h}{q_m} = q_e$</td></tr>
<tr><td rowspan="2">Standing Waves on a circular self-closed zone</td><td>$n\lambda = 2\pi R$
$p = n\frac{h}{2\pi} \cdot \frac{1}{R}$</td><td>$n\theta = 2\pi$
$L = n\frac{h}{2\pi}$</td><td colspan="2" rowspan="2">$\lambda_e \lambda_m = q_e q_m = h$</td></tr>
<tr><td colspan="2">$\theta L = \lambda p = h$, $\theta = \frac{\lambda}{R} = \frac{2\pi}{n}$</td></tr>
</table>

(Periodicity – here adopted term for unifying all de Broglie wavelengths,
$q_m = \Phi$ is not a free and independent, self-standing magnetic charge)

In fact, understanding structure/s of elementary particles, based on (5.2.1), (5.4.1) and wavelengths analogies T.5.4 can reveal a very simple picture, as follows (see also the second chapter equations from (2.5.1) to (2.11)).

A) Since we already know that all elementary particles (and quasiparticles) have certain intrinsic and quantized spin or orbital moments (expressed in units $\frac{h}{2\pi} = \hbar$ multiplied by integers), this clearly tells us that certain kind of rotation is present there. Every elementary particle exists as a measurable entity (meaning that it can be localized with certain precision in certain time and space). *Since such particle wave entity manifests some kind of internal rotation, it is obvious that its elementary space domain should present kind of a rotating mass* $\boldsymbol{m}$ *(or rotating matter wave with an initial energy content of* $\boldsymbol{mc^2}$*), which could be geometrically conceptualized also as a thin and narrow toroidal, or ring form with distributed mass* $\boldsymbol{m}$ *(for instance). The condition, which makes such conceptualization defendable is that when the average radius of rotation* $\boldsymbol{R}$ *is much bigger than any other rotating particle or rotating ring dimensions, moment of inertia of such object is in all cases equal to* $J = mR^2$. Angular periodicity belonging to such object, which is analog to (here invented) angular matter wavelength, should be $\theta = \frac{h}{L}$ and, since we are describing space localized and relatively stable object, it is clear that only integer number of such angular periodicity sectors captures the total particle domain, $n\theta = 2\pi = n\frac{h}{L}$ with a consequence that orbital moment of the intrinsic rotation in question should be $L = n\frac{h}{2\pi} = n\hbar$, $n = 1, 2, 3\ldots$.

B) Now, the idea about self-closed circular path (of an elementary particle structure) is already getting stronger legitimacy, and we could (mathematically) imagine the same rotating object (elementary particle that is in relative rest regarding Laboratory System) as being an equivalent rotating mass or wave group that rotates around its center of inertia. Such rotating mass on its circular line path (observed internally) performs a kind of linear motion, which should be equivalently presentable as a matter wave which has its “ordinary” de Broglie matter wavelength equal to $\lambda = \frac{\mathbf{h}}{\mathbf{p}}$, and again, since we are describing the same rotating object as before, only an integer number of such wavelengths should cower the circular path perimeter,

$$\mathbf{n}\,\lambda = \mathbf{n}\frac{\mathbf{h}}{\mathbf{p}} = \mathbf{2}\ \pi\,\mathbf{R} \Leftrightarrow \mathbf{p} = \mathbf{n}\frac{\mathbf{h}}{2\pi}\cdot\frac{\mathbf{1}}{\mathbf{R}} = \mathbf{n}\hbar\cdot\frac{\mathbf{1}}{\mathbf{R}}\ ,\ \mathbb{L} = \mathbf{pR} = \mathbf{J}\omega = \mathbf{J}\frac{\mathbf{v}}{\mathbf{R}}\ . \qquad (5.4.2)$$

The “<u>rotating motional energy</u>” associated to the intrinsic orbital moment $\mathbb{L} = \mathbf{n}\,\hbar$, should be equal to the equivalent particle <u>linear motion energy</u> (on a self-closed circular path, where an equivalent mass content moves with its tangential velocity $\mathrm{v} = \omega\mathrm{R}$), which is associated to particle linear momentum $\mathbf{p} = \mathbf{n}\hbar\cdot\frac{\mathbf{1}}{\mathbf{R}}$ (because here we are describing two mathematical aspects of the same motion, belonging to the same, stable and space localized object). Such motional energy equivalence means that the following identities are automatically satisfied (when for linear and orbital moments we take the above-found and quantized values, $\mathbf{p} = \mathbf{n}\hbar\cdot\frac{\mathbf{1}}{\mathbf{R}}$, $\mathbb{L} = \mathbf{n}\,\hbar$):

$$\left\{\mathrm{pv} = \mathrm{L}\omega\,, \frac{\mathrm{mv}^2}{2} = \frac{\mathrm{J}\omega^2}{2}, \mathrm{vdp} = \omega\mathrm{dL}\,,\ \frac{\mathrm{dp}}{\mathrm{p}} = \frac{\mathrm{dL}}{\mathrm{L}}\right\} \text{and} \left\{\mathrm{p} = \mathrm{n}\hbar\cdot\frac{1}{\mathrm{R}}\quad,\ \mathbb{L} = \mathrm{n}\,\hbar\right\} \qquad (5.4.3)$$

$$\Leftrightarrow \{\mathrm{n}\ \hbar\cdot\frac{\mathrm{v}}{\mathrm{R}} = \mathrm{n}\ \hbar\omega,\ \mathrm{v} = \omega\mathrm{R}\ , \theta\,\mathrm{L} = \lambda\mathrm{p} = \mathrm{h},\ \theta = \frac{\lambda}{\mathrm{R}} = \frac{2\pi}{\mathrm{n}}\},$$

which is obviously the case.

C) The conceptual picture regarding basic structure of elementary particles, just presented, is of course oversimplified, but clear, simple, elegant and fascinating. Most probably, the mentioned elementary and rotating matter domains are internally composed of electromagnetic waves in certain form of stationary and standing wave structure, since majority of elementary particles have their magnetic moments, and many of them are in some ways sensitive to external electric and magnetic fields. What is characteristic for many elementary particles is that they have stable gyromagnetic ratios, meaning that both magnetic and orbital moments in question are mutually strongly coupled and coincidently present. Here, we also come close to understanding how nature creates elementary matter domains with rest masses, using waves or fields as a primary building substance. Also, spin numbers of bosons and fermions could be explained inside picture given here, since integers regarding linear and orbital periodicity can be different, as for instance, $\mathbf{p} = \mathbf{n}\hbar\cdot\frac{\mathbf{1}}{\mathbf{R}}$, $\mathbb{L} = m\ \hbar$, $\mathrm{n}, m \in [1,2,3...]$. By the nature of here described self-closed circular path (of an elementary particle) it is clear that in some cases integer m = $\mathbf{n}$, and in other cases m = $\mathbf{n}/\mathbf{2}$, or m = $\mathbf{n}/\mathbf{2k},$ $\mathrm{n}, m, \mathrm{k} \in [1,2,3...]$ making $\mathrm{n}\ \hbar\cdot\frac{\mathrm{v}}{\mathrm{R}} = \mathrm{m}\ \hbar\,\omega,\ \mathrm{v} = \frac{\mathrm{m}}{\mathrm{n}}\omega\mathrm{R} = \omega\mathrm{R}$ ... (see also Chapter 4.1, Fig.4.1 and equations under (4.3)).

*[♣ **COMMENTS & FREE-THINKING CORNER**:*

D) *The hybridized wave-mechanic picture of elementary particles (described above, and in the second chapter with equations from (2.5.1) to (2.11)), can additionally be upgraded and generalized. For instance, if we consider that some matter form (initially without having rest mass) aggregates in the mentioned elementary and rotating domains, it is clear that this should be certain kind of motional and wave energy, mathematically presentable as a wave packet, or as an equivalent rotating particle, or certain toroidal form of rotating distributed mass. Here we are guided by the idea that rotation that creates closed toroidal form should be the mechanism of initial rest mass aggregation (since we already know that elementary matter domains or elementary particles have their magnetic and mechanical orbital moments). Here we are claiming that the very first, original elementary matter substance belongs to certain waveforms and fields (most probably to electromagnetic fields and waves), and that later such matter substance, when being in a specific motion (with elements of rotation, creating standing waves on a closed path) is able to create stabilized forms that when externally observed could be treated as particles with a rest mass content. Let us consider that rotating form in question does not make any other linear motion (regarding the Laboratory System), or being more precise we can say that its center of mass linear velocity equals zero,* $v_c = 0$*, and that its rotational speed* ω *is constant. Here, the equivalent particle tangential velocity is* $v = \omega R$*. If we now express mentioned wave rotation as a linear motion of certain equivalent particle along its circular path, we will have the following differential form of its motional energy:*

$$\begin{aligned}
&dE_{motional} = dE = d\tilde{E} = dE_{linear\text{-}motion} = dE_{rotational-motion} = \\
&= \underline{vdp} = \omega R dp = \underline{\omega dL} = c^2 dm = hdf\ ,\ dL = Rdp = n(\frac{h}{2\pi})\frac{dp}{p}, \qquad (5.4.4)\\
&\begin{Bmatrix} mv^2 = J\omega^2, J = mR^2, v = \omega R,\ 2\pi R = n\cdot\lambda,\ \lambda = h/p, u = \lambda f \\ L = J\omega,\ \theta = h/L, q\cdot\theta = 2\pi = qh/L\ , (n,q) = 1,2,3,... \end{Bmatrix}
\end{aligned}$$

Again to underline, the condition which makes such conceptualization logical (like expressions in (5.4.4)) is that when the average radius of rotation of certain particle R is much bigger than any other rotating particle, equivalent rotating ring or torus dimensions, moment of inertia of such thin walls object is in all cases equal to $J = mR^2$*.*

E) *Now, we could imagine that initial rotating energy content (*$\omega dL = vdp = c^2 dm$*) starts making additional linear motion related to its Laboratory System (*$v_c dp$*), having certain non-zero center of mass velocity* $v_c \neq 0$*. Here we are already implicitly neglecting what is happening inside the rotating energy form, considering it (observing externally) a particle that has certain rest mass content* $m = \tilde{m}$*. The new motional energy picture, analog to (5.4.4), will be,*

$$\begin{aligned}
&dE = d\tilde{E} = v_c dp + \omega dL = c^2 d(\gamma m) \\
&\begin{Bmatrix} p = \gamma m v_c\ , \gamma = (1 - v_c^2/c^2)^{-0.5}, \\ v_c = u - \lambda\frac{du}{d\lambda}, \lambda = \frac{h}{p}, u = \lambda f = \lambda\frac{\omega}{2\pi} \end{Bmatrix} \qquad (5.4.5)\\
&\mathbf{E_{motional}} = \tilde{\mathbf{E}} = \mathbf{E_{linear\text{-}motion}} = \mathbf{E_{rotational-motion}} = \mathbf{pu} = \tilde{\mathbf{m}}\mathbf{uv} = \tilde{\mathbf{m}}\mathbf{c^2} = \mathbf{hf}
\end{aligned}$$

In addition, assuming that every linear motion is only a particular case (or approximation) of certain curvilinear or rotational motion (with sufficiently large radius of rotation), we could again make equivalency between two aspects of such motion, as in (5.4.4),

$$dE = d\tilde{E} = v_c dp \; + \; \omega dL = \omega_c dL \; + \; mc^2 = c^2 d(\gamma m)$$
$$dE = d\tilde{E} = v_c dp \; + \; \omega_c dL^* = c^2 d(\gamma m)$$
$$v_c dp \; = \omega_c dL \, , \; \omega dL = mc^2$$

(See also Chapter 4.1, Fig.4.1 and equations under (4.3))

F) *In cases of real elementary particles we should have some kind of rotational, pure wave motion (with zero rest mass) that creates standing waves, and this way becomes a stable elementary particle (for instance electron). Such self-closed wave object observed externally looks much more as a particle that has its rest mass (as a carrier of its total energy). Since internally observed, there is no rest mass, we will have the following total energy associated to such object,*

$$\begin{aligned} &\mathbf{E_{total}} = \mathbf{E_{motional}} = \tilde{\mathbf{E}} = \mathbf{E_{linear\text{-}motion}} = \mathbf{E_{rotational-motion}} = \mathbf{pu} = \tilde{\mathbf{m}}\mathbf{uv} = \tilde{\mathbf{m}}\mathbf{c^2} = \mathbf{hf} \\ &\mathbf{E^2_{total}} = \mathbf{E}_0^2 + \mathbf{p}^2\mathbf{c}^2 = \mathbf{p}^2\mathbf{c}^2 = (\tilde{\mathbf{m}}\mathbf{c^2})^2 \Leftrightarrow \mathbf{pc} = \tilde{\mathbf{m}}\mathbf{c^2} \Leftrightarrow \mathbf{p} = \tilde{\mathbf{m}}\mathbf{c} = \tilde{\mathbf{m}}\mathbf{v} \Leftrightarrow \mathbf{v} = \mathbf{c} = \mathbf{u} \end{aligned} \qquad (5.4.6)$$

The only realistic possibility, found in (5.4.6), is that the wave packet in question rotates with group (or tangential) velocity that is constant and equal to $\boldsymbol{c}$*. Now, from the equivalency* $\mathbf{pv} = \mathrm{L}\omega$*, replacing group velocity with* $\boldsymbol{c}$*, we have,*

$$\begin{aligned} &\left(\mathbf{pv} = \mathrm{L}\omega, \, \mathbf{v} = \mathbf{c} = \omega_c \mathbf{R}\, , \; \mathrm{L} = \mathbf{n}\frac{\mathbf{h}}{2\pi}\right) \Leftrightarrow \\ &\mathbf{pc} = \mathrm{L}\omega_c = \mathbf{n}\frac{\mathbf{h}}{2\pi}\omega_c = \mathbf{nhf_c} = \tilde{\mathbf{m}}\mathbf{c}^2 = \mathbf{E_{total}} = \mathbf{E_{motional}} = \mathbf{hf}\, , \\ &\mathrm{L} = \frac{\mathbf{pc}}{\omega_c} = \mathbf{n}\frac{\mathbf{h}}{2\pi}\, , \mathbf{f} = \mathbf{nf_c}\, , \omega_c = 2\pi\mathbf{f_c} = \mathbf{const.}\;\; \mathbf{c} = \mathbf{Const.}\, , \\ &\mathbf{f} = \mathbf{nf_c} = \frac{\tilde{\mathbf{m}}\mathbf{c}^2}{\mathbf{h}}\, , \; \mathbf{p} = \tilde{\mathbf{m}}\mathbf{c} = \frac{\mathbf{hf}}{\mathbf{c}} = \frac{\mathbf{nhf_c}}{\mathbf{c}}\, , \; \mathbf{n} = 1,2,3,... \end{aligned} \qquad (5.4.7)$$

We can find somewhat similar and fascinating concepts regarding structure of elementary particles in number of papers coming from Bergman, David L. and Lucas, Jr., Charles W. and their collaborators (see mentioned literature references at the end of this paperwork).

What is being explained here is that certain pure wave state, without any rest mass content (observed internally), under certain conditions transforms itself into a state that externally starts behaving as having rest mass. The basic laws governing behaviors and appearance of such matter state are as usual: Law of Total Energy Conservation, Law of Total Linear Momentum Conservation, and Law of Total Orbital Momentum Conservation. Let us imagine that matter state in question is a set of distributed matter elements (wave packet), which is locally isolated system, passing certain transformation. The following conservation laws can describe such locally isolated system,

$$\begin{aligned} &E_{total} = E = \sum_{(i)} E_i = \sum_{(j)} E_j = \gamma mc^2, \; \frac{\partial E}{\partial t} = 0, \\ &\vec{P}_{total} = \vec{P} = \sum_{(i)} \vec{p}_i = \sum_{(j)} \vec{p}_j = \gamma m\vec{v}_c, \; \frac{\partial P}{\partial t} = 0, \\ &\vec{L}_{total} = \vec{L} = \sum_{(i)} \vec{L}_i = \sum_{(j)} \vec{L}_j = J\vec{\omega}_c = \gamma mR^2\vec{\omega}_c, \; \frac{\partial L}{\partial t} = 0, \end{aligned} \qquad (5.4.8)$$

In addition, the total energy conservation should satisfy the following relations,

$$\left.\begin{cases} E_{total}^2 = E_0^2 + P^2c^2 = E^2, \\ dE = \omega_c dL + v_c dP = c^2 d(\gamma m) \end{cases}\right\} \Rightarrow$$

$$E = \int dE = \int \omega_c dL + \int v_c dP = \gamma mc^2,$$

$$\int \omega_c dL = m_0 c^2 + L^2 \omega_c^2 = mc^2, m_0 = const. \qquad (5.4.9)$$

$$\int v_c dP = (\gamma - 1)mc^2$$

$$\gamma = (1 - v_c^2/c^2)^{-0.5}$$

G) *Until here, we did not make any differentiation between mechanical or angular rotation* $\omega_m = \omega_{gm}$ *and wave angular speed* $\omega = 2\pi f$ *, since we had only circular wave motion (without rest mass).*

Let us imagine that certain particle (which could have non-zero rest mass) rotates, having tangential velocity $v = v_g = \omega_m R = \omega_{gm} R$ *, where the radius of rotation is* $\boldsymbol{R = const.}$ *and* $\boldsymbol{\omega_m = \omega_{gm}}$ *is the mechanical, angular particle velocity (number of full rotations per second), and let us find all particle and matter wave parameters associated to such movement.* ***Practically, the same concept of a wave packet, which has its group and phase velocity, in cases or rotational motions should be analogically extended to the rotating wave packet that has group and phase angular velocity, as for instance,***

$$\mathbf{v = v_g = R\omega_{gm} = \frac{d\omega}{dk} (=)\ group\ wave\ velocity (=) particle,\ linear\ velocity,}$$

$$\mathbf{\omega_{gm} = \frac{v}{R} = 2\pi f_{gm} (=) group\ angular\ velocity\ or\ frequency (=) particle\ angular\ velocity,}$$

$$\mathbf{u = v_f = R\omega = \frac{\omega}{k} = \lambda f (=)\ phase,\ wave\ velocity,\ Rk = 1,}$$

$$\mathbf{\omega = \omega_f = \frac{u}{R} = 2\pi f (=) angular\ wave\ frequency,} \qquad \mathbf{\frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = \frac{v}{u},} \qquad (5.4.10)$$

$$\mathbf{k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p (=)\ wave\ \ vector,\ \tilde{E} = hf,}$$

$$\left.\begin{cases} \left[\mathbf{v = u - \lambda \frac{du}{d\lambda} = u + k\frac{du}{dk} = \frac{d\omega}{dk}} \right] \cdot \mathbf{\frac{1}{R}} \Leftrightarrow \\ \mathbf{\frac{v}{R} = \frac{u}{R} - \lambda \frac{d(\frac{u}{R})}{d\lambda} = \frac{u}{R} + k \frac{d(\frac{u}{R})}{dk} = \frac{1}{R}\frac{d\omega}{dk}} \end{cases}\right\} \Rightarrow \left\{\begin{matrix} \mathbf{\omega_{gm} = \omega - \lambda \frac{d\omega}{d\lambda} = \omega + k\frac{d\omega}{dk} = \frac{1}{R}\frac{d\omega}{dk}} \\ \mathbf{\omega = \frac{\omega}{Rk} = 2\pi f} \end{matrix}\right\}$$

$$(0 \le v \le c) \Rightarrow 1 \le \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = \frac{v}{u} \le 2$$

$$(v << c) \Rightarrow v = 2u \Rightarrow \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = 2$$ *(See also (4.3), chapter 4.1; Mater Waves)*

$$(v \cong c) \Rightarrow (v = u) \cong c \Rightarrow \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = 1$$

(Indexing: m (=) mechanical, f (=) phase, g (=) group)

*Matter waves associated to any particle motion are practically defined by **PWDC** relations (equations (4.2)-(4.3), chapter 4.1), and not directly equal to particle mechanical-rotating parameters, and we should be very careful in making a difference between mechanical rotation (mechanical angular speed) and orbital frequency of associated matter waves. In cases of inter-atomic circular motions, where standing matter waves are an intrinsic structural property, it can be:*

$$2\pi \mathbf{R} = \mathbf{n}\lambda \Rightarrow \mathbf{Rk} = \mathbf{n} = \mathbf{1, 2, 3...} \Rightarrow \omega_{\mathbf{gm}} = \frac{\mathbf{1}}{\mathbf{R}}\frac{\mathbf{d}\omega}{\mathbf{dk}}, \frac{\omega_{\mathbf{gm}}}{\omega} = \frac{\mathbf{v}}{\mathbf{u}} = \frac{\mathbf{k}}{\omega}\frac{\mathbf{d}\omega}{\mathbf{dk}} \tag{5.4.11}$$

Usually, such precise differentiation between mechanical and mater wave parameters has never been made, influencing many ad hoc and exotic postulates and suspicious, or theory-correcting and supporting statements (regarding elementary particles and atom structure) got legitimacy in Orthodox Quantum Mechanics (in order to explain certain conflicting situations such as: gyromagnetic ratio, spin attributes, correspondence principle, orbital and magnetic moments etc.). In cases if radius related to rotation is not constant, the above given example should be appropriately adjusted (and accorded with Wilson-Sommerfeld rules: see (5.4.1)). It would be fruitful to elaborate more profoundly here mentioned ideas.

In other words, quantification is not a universal phenomenon (applicable without limits to all motions, fields, waves and charges), and it literally exists only when stable, elementary matter domains are created and mutually interacting (exchanging energy) in a number of ways. The nature and architecture of matter domains, regarding their gearing, fitting and packing (both internally and externally) is closely related to standing waves and resonant structures. The attempts to apply similar Quantum Theory concepts, and/or to quantify all fields and waves known in physics in the same way and in all cases of different motions, are most probably wrong objectives. In most of the cases regarding particle wave phenomenology we only see emanations of rules of signal analysis and synthesis (known in spectrum analysis and digital signal processing under different signal-sampling and signal-recovery techniques; - Shannon–Kotelnikov, for instance), sometimes mistakenly mixed or replaced (and even generalized) by somewhat mysterious and universal quantification (as official, contemporary Quantum Theory promotes). Most probably, Quantum Theory should change its name and some of its objectives if universal quantification shows to be not much present in our Universe as promoted. Real quantification (where integers and discrete intervals show their importance) is only a modus of energy packing inside stable objects (when available packaging space is limited). We will have manifestations of energy quantification also in situations when objects like atoms and elementary particles mutually exchange energy, but not all energy formats and all energy exchanges in our Universe have forms of quantifiable standing waves. For instance, in certain cases of transient, progressive and space-time evolving wave motions and field particle interactions, we would not be able to implement clear and simple energy quantification concepts.

The author's intention and priority here is to show new ways of understanding particle-wave duality, and to initiate or generate new concepts and models related to the same field (combining multi-level analogies, conservation laws and/or symmetries, and rules of optimal energy and mater wave packaging). The way this objective addressed here could be in some aspects incomplete or unprofessionally presented (or even partially erroneous). The author considers that the big part of the success would be if certain old dogmas and ad hock postulates of Quantum Theory start evolving and changing towards some of here proposed directions. Eventually, researchers involved in the same field, would start seriously considering number of different and new ideas that in some ways diverge from Orthodox Quantum Theory. Since Quantum Theory in many situations mathematically already works very well, our task will be to transform inexplicable, unnatural, heavily abstract, or postulated segments found there, into something what would be conceptually much more tangible and better integrated into the rest of Physics. ♣]

5.1. Uncertainty in Physics and Mathematics

In literature, the Uncertainty Principle is usually in connection with Heisenberg's Uncertainty Relations. For real, correct and full understanding of number of relations linked to Uncertainty Principles it is, for the time being, better to forget that Heisenberg made any invention regarding Uncertainty. In other words, we should know (or learn) that Uncertainty is not married almost exclusively with Quantum Theory and that it belongs to mathematics (meaning universally applicable and presenting mutually conjugated domain length relations). It is also the current case that Uncertainty, as presented in contemporary Physics (mostly in Quantum Theory), is often applied as a very useful supporting background for number of oversimplifications, mystifications, and justifications of number of logical, conceptual and methodological Uncertainties in Physics. *Here-upgraded concept of Uncertainties will show that this is a much richer and more diversified concept compared to one presented and applied in contemporary physics.*

A more general approach to Uncertainty relations (in connection to quantum manifestations of energy formats) should start from a finite wave function $\Psi(t)$. Here we shall implicitly treat the square of the wave function as power: $\Psi^2(\mathbf{t}) = \mathbf{Power} = \mathbf{dE}/\mathbf{dt}$, but we could also treat $\mathbf{\Psi(t)}$ as a dimensionless function without influencing results of the analysis that follows.

Let us designate with $\mathbf{T}$ and $\mathbf{F} = \Omega/\mathbf{2}\pi$ the absolute time and frequency durations of a wave function $\Psi(t)$ and its spectral function $\mathbf{A(f)}$, (4.9), where ($t \in [\mathbf{T}]$, $0 \leq \mathbf{T} < \infty$, $-\infty < \mathbf{t} < \infty$), and ($\mathbf{f} \in [\mathbf{F}]$, $0 \leq \mathbf{F} < \infty$, $0 \leq \mathbf{f} < \infty$, $\mathbf{2}\pi\mathbf{F} = \Omega, \mathbf{2}\pi\mathbf{f} = \omega$). We can also say that, $\mathbf{T}$ and $\mathbf{F}$ are absolute (or total) time and frequency lengths of $\Psi(t)$ and its spectrum $\mathbf{A}(f)$).

Here we are considering the wave function, $\Psi(t)$, as a presentable as an Analytical Signal form (first time introduced by Dennis Gabor, in connection with Hilbert transform, see **[7]** and **[8]**), as well as a finite, energy limited function, either in its time or frequency domain (or in both of them, if applicable with reasonable approximation). It is an advantage of Analytical Signals that they cover only natural domains of real time and frequency: $-\infty < \mathbf{t} < \infty$, $\mathbf{0} \leq \mathbf{f} < \infty$. This is opposite to the traditional Signal Analysis (Fourier analysis), where frequency can also take negative values, but in all other aspects Analytic Signals give the same or equivalent results, as in the case of Fourier Signal Analysis, including producing some additional, time-frequency dependent, dynamic and spectral signal properties which the Fourier analysis is not able to perform. Before we start further evolving of concepts of Uncertainties, it would be very much recommendable to go to the chapter 4.0 of this paperwork collection (Wave functions wave velocities and uncertainty relations) in order not to repeat already presented Uncertainties background.

Spectrum Analysis shows (without any doubts) general validity of the following Uncertainty relation (equivalent to (5.1)):

$$\mathbf{TF} > \frac{\mathbf{1}}{\mathbf{2}}, \mathbf{T}\Omega > \pi \quad . \tag{5.5}$$

If certain transformation (5.6) happens to a wave function $\Psi(t)$, changing its time and frequency lengths, **T** and **F**, for the amounts **Δt** and **Δf**, the same signal transformation will automatically influence all other space and energy related parameters of **Ψ(x, t)** to change, producing similar Uncertainty relations, as given in (5.1). Obviously, that effective physical signal length **L** and signal wave energy $\tilde{\mathbf{E}}$ also change, as for instance (results taken from the chapter 4.0):

$$\left\{\begin{array}{l} T \to T \pm \Delta t > 0, F \to F \pm \Delta f > 0, L \to L \pm \Delta x > 0, K \to K \pm \Delta k > 0 \\ \Delta\tilde{E} = h\ \Delta f = \bar{v}\Delta p = \tilde{F}\Delta x, \tilde{F} = \dfrac{\Delta p}{\Delta t} = \text{force}, \\ 0 < \delta\ t \cdot \delta f = \delta\ x \cdot \dfrac{\delta k}{2\pi} = \delta\ x \cdot \delta f_x < \dfrac{1}{2} < F \cdot T = \dfrac{1}{2\pi} \cdot K \cdot L = F_x \cdot L \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \tilde{E} \to \tilde{E} \pm \Delta\tilde{E}, \\ \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{\Delta\tilde{E}}{\Delta p} = \dfrac{\Delta\omega}{\Delta k} = \dfrac{\delta\ x}{\delta\ t} = \dfrac{\delta\omega}{\delta k} = \dfrac{\delta\tilde{E}}{\delta p} \end{array}\right\} . \qquad (5.6)$$

In cases when time and frequency changes are either positive or negative, we have:

$$T \cdot F > \frac{1}{2} \Leftrightarrow \left\{\begin{array}{l} (T + \Delta t) \cdot (F + \Delta f) > \dfrac{1}{2} \\ (T - \Delta t) \cdot (F - \Delta f) > \dfrac{1}{2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \left[T^2 - (\Delta t)^2\right] \cdot \left[F^2 - (\Delta f)^2\right] > \dfrac{1}{4} \\ T \cdot F + \Delta t \cdot \Delta f > \dfrac{1}{2} \\ T \cdot \Delta\ f + \Delta t \cdot F > 0 \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} 1 - (\dfrac{\Delta t}{T})^2 - (\dfrac{\Delta f}{F})^2 + (\dfrac{\Delta t}{T})^2 \cdot (\dfrac{\Delta f}{F})^2 > \dfrac{1}{4} \\ 1 + (\dfrac{\Delta t}{T}) \cdot (\dfrac{\Delta f}{F}) > \dfrac{1}{2T \cdot F}, T \cdot F > \dfrac{1}{2} \\ (\dfrac{\Delta t}{T}) + (\dfrac{\Delta f}{F}) > 0 \end{array}\right\} . \qquad (5.7)$$

Let us additionally conceptualize the same idea (about sudden signal duration changes) using the moving particle Energy-Momentum 4-vector from the Minkowski-space of Relativity Theory, by (mathematically) introducing mutually coupled changes of a total system energy and belonging total momentum, applying discrete, central differentiations method,

$$\begin{array}{l} \bar{P}_4 = \bar{P}\left[\vec{p} = \gamma m\vec{v}, \dfrac{E}{c} = \gamma mc\right], \bar{P}^2 = \vec{p}^2 - \dfrac{E^2}{c^2} = -\dfrac{E_0^2}{c^2}, E_0 = mc^2, E = \gamma E_0 \Rightarrow \\ \vec{p}^2 c^2 + E_0^2 = E^2, (p \to p \pm \Delta p) \Leftrightarrow (E \to E \pm \Delta E) \Rightarrow \end{array} \qquad (5.7.1\text{-}1)$$

$$\left\{\begin{array}{l}(p+\Delta p)^2 c^2+E_0^2=(E+\Delta E)^2 \\ (p-\Delta p)^2 c^2+E_0^2=(E-\Delta E)^2 \\ \overline{v}=\frac{\Delta x}{\Delta t}=\frac{\delta\ x}{\delta\ t}=\frac{\delta\ \omega}{\delta\ k}=\frac{\Delta\ \omega}{\Delta\ k}\end{array}\right\} \Leftrightarrow\left\{c^2 \cdot p\ \Delta p=E\ \Delta E \Leftrightarrow \frac{\Delta E}{\Delta p}=c^2 \frac{p}{E}=\overline{v}\right\} \Rightarrow$$
$$\Rightarrow \frac{\Delta x}{\Delta t}=\frac{\delta\ x}{\delta\ t}=\frac{\Delta E}{\Delta p}=c^2 \frac{p}{E}=\overline{v}=h \frac{\Delta f}{\Delta p}=\frac{\Delta \tilde{E}}{\Delta p}=\frac{\Delta \omega}{\Delta k}=\frac{\delta\ \omega}{\delta\ k} \leq c\ . \tag{5.7.1-2}$$

What we can see from (5.7.1-2) is that sudden energy momentum changes in a certain system (a moving particle, here) are directly related to its average center-of-mass velocity, which is at the same time equal to the system average group velocity. This now gives a new and more tangible meaning of Uncertainty Relations that should be analyzed well, before we make other conclusions. Before, we found that signal domain proportionality only dimensionally and quantitatively indicated that this could be the signal group velocity (or particle velocity), and now we can safely confirm that this is really the case.

The idea here is to show that so-called Uncertainty Relations are directly related to a velocity of matter wave propagation. The meaning of that is that at the same time when certain motional object or signal experiences sudden change of its energy related parameters, matter waves are automatically created, and this produces results captured by Uncertainty Relations (indirectly saying that there is no real Uncertainty in its old and traditional meaning). For instance, we can express the average group velocity $\overline{\mathbf{v}}$ associated to the transformations (5.7.1-2) as:

$$\left\{u=\frac{\omega}{k}, \omega=ku\right\} \Rightarrow \Delta\omega=(k+\frac{1}{2}\Delta k)(u+\frac{1}{2}\Delta u)-(k-\frac{1}{2}\Delta k)(u-\frac{1}{2}\Delta u)=$$
$$=k\Delta u+u\Delta k \Leftrightarrow \overline{v}=\frac{\Delta\omega}{\Delta k}=u+k\frac{\Delta u}{\Delta k} \Leftrightarrow \tag{5.7.1-3}$$
$$\Leftrightarrow\left\{v=u+k\frac{du}{dk}=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=\frac{dx}{dt}=\text{immediate group velocity}\right\}.$$

Such group velocity (expressed in terms of finite differences) is fully analog to its differential form where infinitesimal signal changes are involved. By merging average group velocity with Uncertainty Relations, we will again see that they are mutually compatible,

$$\left\{\begin{array}{l}\overline{v}=u+k\frac{\Delta u}{\Delta k}=\frac{\Delta\omega}{\Delta k}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta x}{\Delta t}=\text{average group velocity} \\ \text{and} \\ \left|\Delta x\ \Delta p\right|=\left|\Delta t\ \Delta\tilde{E}\right|=h\left|\Delta t\ \Delta f\right|>h/2, \Delta\tilde{E}=h\Delta\ f, \\ 0<\delta\ t\cdot\delta f=\delta\ x\cdot\delta f_x<\frac{1}{2}\leq F\cdot T=F_x\cdot L\leq\frac{1}{4\cdot\delta\ t\cdot\delta f}=\frac{1}{4\cdot\delta\ x\cdot\delta f_x}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \overline{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta E}{\Delta p} = h\frac{\Delta f}{\Delta p} = \frac{\Delta \tilde{E}}{\Delta p} = \frac{\Delta \omega}{\Delta k} = u + k\frac{\Delta u}{\Delta k} = \frac{\delta x}{\delta t} = \frac{\delta \omega}{\delta k} = \frac{dx}{dt} = \frac{d\omega}{dk}. \qquad (5.7.1\text{-}4)$$

A very interesting fact regarding the average group velocity "$\overline{\mathbf{v}}$" in (5.7.1-3) and (5.7.1-4), which could pass unnoticed, is that the full analogy between the expression for the average group velocity "$\overline{\mathbf{v}}$" and the expression for the immediate group velocity "$\mathbf{v}$" is not made as an approximation, automatically by formal and simple replacement of infinitesimal difference "d" with discrete, delta difference "Δ". The real development of the average group velocity (given here) is based on applying <u>symmetrical central differences</u> to basic definitions of group and phase velocity, and as we can see it is correct. This shows that there is a deterministic connection between the physics of continuum and physics of discrete or finite steps (to support better this statement it would be necessary to devote certain time to learn about properties of central, symmetrical differences).

Since we know all relations between group and phase velocity (of certain wave packet) in connection with signal wavelength and frequency, as given in (4.2), we can find the absolute motional parameter frames where all of signal parameters, caused by transformation (5.6), should be expected, as for instance:

$$\left\{ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{dE}{dp} = \frac{dx}{dt},\ \lambda = \frac{h}{p},\ dE = hdf = c^2 d(\gamma m),\ p = \gamma m v \right\} \Rightarrow$$

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta E}{\Delta p} = \frac{1}{\lambda_{max.} - \lambda_{min.}}\int_{\lambda_{min.}}^{\lambda_{max.}} (-\lambda^2\frac{df}{d\lambda})d\lambda = \frac{-1}{\Delta\lambda}\int_{f_{max.}}^{f_{min.}} \lambda^2 df = \frac{-h}{\Delta\lambda}\int_{f_{max.}}^{f_{min.}} \frac{1}{p^2}d(hf) =$$

$$= \frac{-h}{\Delta\lambda}\int_{[\Delta E]}\frac{1}{p^2}dE = \frac{-h}{\Delta\lambda}\int_{[\Delta v]}\frac{c^2}{(\gamma m v)^2}d(\gamma m) = \frac{-hc^2}{m\Delta\lambda}\int_{[\Delta v]}\frac{d\gamma}{\gamma^2 v^2} = \frac{-h}{m\Delta\lambda}\int_{[\Delta v]}\frac{dv}{v\sqrt{1-v^2/c^2}} =$$

$$= \frac{h}{2m\Delta\lambda}\left[\frac{1-\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right]_{V_{MIN.}}^{V_{MAX.}} =$$

$$= \frac{v_{min.}v_{max.}}{2\left[v_{max.}\sqrt{1-v_{min.}^2/c^2} - v_{min.}\sqrt{1-v_{max.}^2/c^2}\right]}\left[\frac{1-\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right]_{V_{MIN.}}^{V_{MAX.}} \leq$$

$$\leq \frac{v_{min.}v_{max.}}{2\left[v_{max.}\sqrt{1-v_{min.}^2/c^2} - v_{min.}\sqrt{1-v_{max.}^2/c^2}\right]} = \frac{h}{2m\Delta\lambda},$$

$$\Delta\lambda = \lambda_{max.} - \lambda_{min.} = \frac{h}{\gamma(v_{min.})mv_{min.}} - \frac{h}{\gamma(v_{max.})mv_{max.}} =$$

$$= \frac{h}{m}\left[\frac{v_{max.}\sqrt{1 - v_{min.}^2/c^2} - v_{min.}\sqrt{1 - v_{max.}^2/c^2}}{v_{min.}v_{max.}}\right] \leq \frac{h}{2m\overline{v}},$$

$$\lambda_{max.} = \frac{h}{m}\left[\frac{v_{max.}\sqrt{1-v_{min.}^2/c^2}}{v_{min.}v_{max.}}\right],\ \lambda_{min.} = \frac{h}{m}\left[\frac{v_{min.}\sqrt{1 - v_{max.}^2/c^2}}{v_{min.}v_{max.}}\right],$$

$$\overline{\lambda} = \frac{h}{\overline{p}} = \frac{h}{m\overline{v}}\sqrt{1-\overline{v}^2/c^2} \leq \frac{h}{m\overline{v}},\ \frac{\Delta\lambda}{\overline{\lambda}} \leq \frac{1}{2},$$

$$\lambda = \frac{h}{p} \Rightarrow p\Delta\lambda + \lambda\Delta p + \Delta p\Delta\lambda = 0,\ (1+\frac{\Delta p}{p})(1+\frac{\Delta\lambda}{\lambda}) = 1,\ \frac{\Delta\lambda}{\overline{\lambda}} = -\frac{\frac{\Delta p}{\overline{p}}}{1+\frac{\Delta p}{\overline{p}}} .$$

$$u_{min} = \lambda_{min.} f_{min.} = \frac{v_{min.}}{1+\sqrt{1-v_{min.}{}^2/c^2}},\ u_{max.} = \lambda_{max.} f_{max.} = \frac{v_{max.}}{1+\sqrt{1-v_{max.}{}^2/c^2}},$$

$$\overline{u} = \overline{\lambda}\,\overline{f} = \frac{h\overline{f}}{m\overline{v}}\sqrt{1-\overline{v}^2/c^2} = \frac{\overline{v}}{1+\sqrt{1-\overline{v}^2/c^2}},$$

$$f_{max.} = \frac{1}{\lambda_{max.}}\left[\frac{v_{max.}}{1+\sqrt{1-v_{max}^2/c^2}}\right] = \frac{m}{h}\left[\frac{v_{min.}v_{max.}}{(1+\sqrt{1-v_{max.}{}^2/c^2})\sqrt{1-v_{min}^2/c^2}}\right],$$

$$f_{min.} = \frac{1}{\lambda_{min.}}\left[\frac{v_{min.}}{1+\sqrt{1-v_{min}^2/c^2}}\right] = \frac{m}{h}\left[\frac{v_{min.}v_{max.}}{(1+\sqrt{1-v_{min}^2/c^2})\sqrt{1-v_{max}^2/c^2}}\right],$$

$$\overline{f} = \frac{m\overline{v}^2}{h(1+\sqrt{1-\overline{v}^2/c^2})\sqrt{1-\overline{v}^2/c^2}},$$

$$\Delta f = f_{max.} - f_{min.} \leq \frac{v_{max.}}{\lambda_{max.}} - \frac{v_{min.}}{2\lambda_{min.}} = \frac{m(v_{min.}v_{max.})}{h}(1-\frac{1}{2\sqrt{1-v_{max}^2/c^2}}),$$

$$\mathbf{\overline{E}_k = \tilde{E} = \frac{\overline{p}\overline{v}}{1+\sqrt{1-\overline{v}^2/c^2}} = \frac{m\overline{v}^2}{(1+\sqrt{1-\overline{v}^2/c^2})\sqrt{1-\overline{v}^2/c^2}} = h\overline{f},\quad m = \frac{E_{total}}{c^2}},$$

$$v <<< c \Rightarrow$$

$$\overline{v} \cong \frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})},\ \frac{\Delta v}{\overline{v}} \cong 2\frac{(v_{max.}-v_{min.})^2}{v_{min.}v_{max.}},$$

$$\overline{u} = \overline{\lambda}\,\overline{f} \cong \frac{1}{2}\overline{v} \cong \frac{v_{min.}v_{max.}}{4(v_{max.}-v_{min.})},$$

$$\Delta\lambda \cong \frac{h}{m}\left[\frac{v_{max.}-v_{min.}}{v_{min.}v_{max.}}\right],\ \overline{\lambda} \cong \frac{h}{m\overline{v}} \cong \frac{2h}{m}\left[\frac{v_{max.}-v_{min.}}{v_{min.}v_{max.}}\right] \cong 2\Delta\lambda,\ \frac{\Delta\lambda}{\overline{\lambda}} \cong \frac{1}{2}, \tag{5.7.2}$$

$$\Delta f \cong \frac{m(v_{min.}v_{max.})}{2h},\ \overline{f} \cong \frac{m}{2h}\overline{v}^2 \cong \frac{m}{2h}\left[\frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}\right]^2,\ \frac{\Delta f}{\overline{f}} \cong 4\frac{(v_{max.}-v_{min.})^2}{v_{min.}v_{max.}},$$

$$\overline{E}_k = \tilde{E} = \frac{m}{2}\left[\frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}\right]^2 = \frac{E_{total}}{2c^2}\left[\frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}\right]^2,\ m = \frac{E_{total}}{c^2}.$$

Particle-wave duality and Uncertainty relations will become even more challenging research subjects if we express the signal length variation from (5.6), in connection to signal wavelength variation as $\mathbf{\Delta x \cong const.} \times \Delta\lambda$, or if the total signal length presents the integer multiple of the average signal half-wavelength, $\mathbf{L = n\frac{\overline{\lambda}}{2}}$, $\mathbf{n = 1,2,3...}$

Most probably results and relations from (5.7.2) should be well applicable on explaining the spectral nature of the Black Body Radiation (and Planck's law) on a more general way than presently found in Physics books. In reality, whatever the meaning of results found in (5.7.2) is, in relation with the signal transformation (5.6) and (5.7), the message regarding understanding Uncertainty relations is that we should always make very precise definition and clarification of what we are talking about. For instance, looking only from mathematical point of view, everything regarding Uncertainty relations is crystal clear, since we know that we are talking about absolute or relative interval relations between signal lengths expressed in their time and frequency domains. When we say that our signal or wave function $\Psi(x, t)$ should represent a real physical object, we should master the methodology how to transfer mathematically correct and clear Uncertainty relations into real (dimensional) objects and parameters of the world of Physics (such as, velocity, momentum, energy, etc.). A part of confusion and misunderstandings regarding Uncertainty relations in Physics comes from very much arbitrary, non-consistent and almost non-existent methodology in connecting mathematical aspects of Uncertainty of wave function domains with real models of physics world, such that wave function should represent. *Uncertainty relations correctly treated from the point of view of mathematics are very clear and understandable, but Uncertainties in Common Sense Logic, Concepts and Methodology in Physics are not acceptable platforms.*

If we now go backwards from (5.6)-(5.7.2) to (5.1)-(5.5), we will understand that Uncertainty relations related to the world of Physics should be more precisely reformulated and generalized in the frames of meanings of (5.6)-(5.7.2), in order to get a more complete and more precise picture regarding real and generally valid Uncertainty relations and quantum nature in physics.

For instance, all relations and mathematical expressions, starting from (5.6) until (5.7.2) should also be applicable to any macro object, or even to the totality of our universe. Let us hypothetically imagine that with (5.6) and (5.7) we describe the total size and all other frames (or frontiers) of our universe, approximating it geometrically to an "expanding balloon". Since we know (or just presently it seems that this is correct) how the external diameter of that balloon should grow, based on Hubble's law (and number of observations), we could draw many interesting and challenging predictions and conclusions, just by correctly applying the belonging Uncertainty relations. We could also deductively conclude that certain step-stones and pillars of modern physics, presently accepted as unquestionable, should become questionable.

Understanding Uncertainty relations in physics (presently still on a mathematical level) is also related to our choice of signal duration intervals. Until here we have been using (or talking about) real, absolute or total signal interval lengths. Now we will once more extend already established Uncertainty Relations of absolute signal duration intervals, taking into consideration corresponding signal standard deviation intervals.

Since the Orthodox Quantum Mechanics mostly deals with statistical distributions and probabilities, interval lengths are represented by signal variance intervals, which are statistic or standard deviations of certain variables around their mean values. Consequently, mathematical expressions of basic Uncertainty Relations, when using variance intervals or statistical deviations, present another aspect of Uncertainty Relations (not mentioned before, but very much present in today's Quantum

Mechanics literature), and should be properly integrated into a chain of all other, already known Uncertainty Relations. The statistic concept of variance is used to measure the signal's energy spreading in time and frequency domains. For instance, for a finite wave function, we can define the following variances (see **[7]**, pages: 29-37, and **[8]**, pages: 273-277):

$$
\begin{aligned}
&(\sigma_t)^2 = \Delta^2 t = \frac{1}{\tilde{E}} \int_{-\infty}^{+\infty} (t - \langle t \rangle)^2 \left|\overline{\Psi}(t)\right|^2 dt = \int_{-\infty}^{+\infty} t^2 \frac{\left|\overline{\Psi}(t)\right|^2}{\tilde{E}} dt - \langle t \rangle^2 < T^2, \\
&(\sigma_\omega)^2 = \Delta^2 \omega = \frac{1}{\pi \tilde{E}} \int_{0}^{+\infty} (\omega - \langle \omega \rangle)^2 \left|A(\omega)\right|^2 d\omega = \frac{1}{\pi} \int_{0}^{+\infty} \omega^2 \frac{\left|A(\omega)\right|^2}{\tilde{E}} d\omega - \langle \omega \rangle^2 < (2\pi F)^2, \qquad (5.8) \\
&\omega = 2\pi f, \ \sigma_\omega = 2\pi \sigma_f, \ \tilde{E} = \left\|\overline{\Psi}(t)\right\|^2 = \int_{-\infty}^{+\infty} \left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\pi} \int_{0}^{+\infty} \left|A(\omega)\right|^2 d\omega,
\end{aligned}
$$

where mean time and mean frequency should be found as:

$$
\langle t \rangle = \frac{1}{\tilde{E}} \int_{-\infty}^{+\infty} t \left|\overline{\Psi}(t)\right|^2 dt, \ \langle \omega \rangle = \frac{1}{\pi \tilde{E}} \int_{0}^{+\infty} \omega \left|A(\omega)\right|^2 d\omega = 2\pi \langle f \rangle = 2\pi \overline{f}\,. \qquad (5.9)
$$

If two functions, $\Psi(t)$ and $A(\omega)$, form a Fourier-integral pair, then they cannot both be of short duration. This is supported by the scaling theorem,

$$
\boldsymbol{\Psi(at) \leftrightarrow \frac{1}{|a|} A(\frac{\omega}{a})}, \qquad (5.10)
$$

*where **"a"** is a real constant. The above assertion, (5.10), also known as the **Uncertainty Principle**, can be given various interpretations, depending on the meaning of the term "duration".*

Using the time and frequency variances, (5.8), as the significant signal duration intervals, found for a finite wave function $\Psi(t)$, it is possible to prove validity of the following Uncertainty Principle (see **[7]** and **[8]**):

$$
\begin{aligned}
&\text{If } \ \sqrt{t}\,\Psi(t) \to 0 \text{ for } |t| \to \infty, \\
&\text{then } \ 2\pi \mathbf{T}F > \mathbf{T}F > \sigma_t \sigma_\omega = 2\pi \sigma_t \sigma_f = \sqrt{(\Delta^2 t)(\Delta^2 \omega)} = 2\pi \sqrt{(\Delta^2 t)(\Delta^2 f)} \geq \frac{1}{2}\,. \qquad (5.11)
\end{aligned}
$$

In the variance relations (5.11) we consider it obvious that absolute (or total) time and frequency durations, $\mathbf{T}$ and $\mathbf{F}$, can never be shorter than time and frequency variances, $\boldsymbol{\sigma_t}$ and $\boldsymbol{\sigma_f}$ (and usually they should be much larger than $\boldsymbol{\sigma_t}$ and $\boldsymbol{\sigma_f}$). It is also clear that statistical, (5.11), and Quantum Mechanic's aspect of Uncertainty should be fully integrated with absolute interval values Uncertainty Relations, (5.7)-(5.7.2), as for instance,

$$0 < \delta\, t \cdot \delta f = \delta\, x \cdot \delta f_x < \frac{1}{2} \leq \sigma_t \cdot \sigma_f = \sigma_x \cdot \sigma_{f\text{-}x} < F \cdot T = F_x \cdot L \leq \frac{1}{4 \cdot \delta\, t \cdot \delta f} = \frac{1}{4 \cdot \delta\, x \cdot \delta f_x},$$
$$0 < \delta\, t \cdot \delta \tilde{E} = \delta\, x \cdot \delta p < \frac{h}{2} \leq 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = \sigma_x \cdot \sigma_p < \tilde{E} \cdot T = P \cdot L \leq \frac{h}{4 \cdot \delta\, t \cdot \delta f} = \frac{h}{4 \cdot \delta\, x \cdot \delta f_x}, \quad (5.12)$$

or in cases when normal, Gauss distributions are applicable it should also be valid,

$$\frac{1}{2} \leq \sigma_t \cdot \sigma_f = \sigma_x \cdot \sigma_{f\text{-}x} < 36 \cdot \sigma_t \cdot \sigma_f = 36 \cdot \sigma_x \cdot \sigma_{f\text{-}x} < F \cdot T = F_x \cdot L,$$
$$\frac{h}{2} \leq 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = \sigma_x \cdot \sigma_p < 36 \cdot 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = 36 \cdot \sigma_x \cdot \sigma_p < \tilde{E} \cdot T = P \cdot L. \quad (5.12\text{-}1)$$

In other words, (5.12) extends (and explains) the meaning of Uncertainty relations (in connection with (5.7) and (5.11)), and presents a kind of atomization of a wave function.

This part is under reconstruction:

In Mathematics, modern Telecommunication Theory and Digital Signal Processing practice we can find a number of methods and formulas for discrete signal representations, meaning that time-continuous signals, or wave functions, could be fully represented (errorless, without residuals) if we just implement sufficiently short time increments sampling (of a certain continuous signal), and create discrete series of such signal samples. For instance, if a continuous wave function, $\mathbf{\Psi(t)}$ is frequency band limited (at the same time it should also be an energy finite function), by applying Kotelnikov-Shannon, or Sampling theorem, we can express a function $\mathbf{\Psi(t)}$ in terms of its sample values $\mathbf{\Psi(n \cdot \delta t)}$ as for instance (see **[8]**),

$$\Psi(t) = a(t)\cos\varphi(t) = \sum_{n=-\infty}^{+\infty} \Psi(n \cdot \delta t) \frac{\sin\ \Omega(t - n \cdot \delta t)}{\Omega(t - n \cdot \delta t)} =$$
$$= \sum_{n=-\infty}^{+\infty} a(n \cdot \delta t) \frac{\sin \Omega\ (t - n \cdot \delta t)}{\Omega(t - n \cdot \delta t)} \cos\varphi(n \cdot \delta t), \Psi(n \cdot \delta t) = a(n \cdot \delta t)\cos\varphi(n \cdot \delta t), \quad (5.12\text{-}2)$$
$$\delta t \leq \frac{\pi}{\Omega} = \frac{1}{2F},$$

where Ω is the highest frequency in the spectrum of $\mathbf{\Psi(t)}$, and we could also consider that Ω is the total frequency duration of the signal $\mathbf{\Psi(t)}$.

If in the relation (5.7) we take $\mathbf{\Delta t = \delta t}$, where $\mathbf{\delta t}$ is the sampling time interval taken from (5.12), then it will be $\mathbf{\Delta f = \delta f}$, where $\mathbf{\delta f}$ should be the frequency-sampling interval, when applying the Kotelnikov-Shannon sampling theorem on $\mathbf{A(f) = A(\omega / 2\pi)}$. It is also important to notice that $\mathbf{\delta t}$ and $\mathbf{\delta f}$, when found in (5.6) and (5.7), can take positive and negative values, but in the Sampling process they take only positive values. This way we can express the function $\mathbf{A(f)}$ in terms of its sample values $\mathbf{A(n \cdot \delta f)}$ as for instance,

$$\mathbf{A(f)} = \sum_{\mathbf{n}=-\infty}^{+\infty} \mathbf{A(n \cdot \delta f)} \frac{\mathbf{\sin 2\pi T(f - n \cdot \delta f)}}{\mathbf{2\pi T(f - n \cdot \delta f)}}, \ \mathbf{\delta f} \leq \frac{1}{2\mathbf{T}} \Leftrightarrow \mathbf{T \cdot \delta f} \leq \frac{1}{2},$$
$$\mathbf{(T + \delta t)(F + \delta f)} > \frac{1}{2}, \ \frac{1}{2} < \mathbf{TF} \leq \frac{1}{4\delta \mathbf{t} \cdot \delta \mathbf{f}}, \ \mathbf{\delta t \cdot \delta f} \leq \frac{1}{2}. \tag{5.13}$$

Now, from (5.12) and (5.13) we can get even more general Uncertainty and energy expressions than before, such as,

$$\left|\delta t\right| \cdot \left|\delta f\right| \leq \left(T \cdot \left|\delta f\right| \approx \left|\delta t\right| \cdot F\right) \leq \frac{1}{2} \leq \sigma_t \sigma_\omega = 2\pi\sigma_t\sigma_f < TF \leq \frac{1}{4\delta t \cdot \delta f} < 2\pi TF,$$
$$\frac{\left|\delta t\right| \cdot \left|\delta f\right|}{TF} \leq \left(\frac{\left|\delta f\right|}{F} \approx \frac{\left|\delta t\right|}{T}\right) \leq \frac{1}{2TF} \leq \frac{\sigma_t\sigma_\omega}{TF} = \frac{2\pi\sigma_t\sigma_f}{TF} \leq \frac{1}{4\delta t \cdot \delta f \cdot TF} < 2\pi,$$
$$T > \sigma_t > \left|\delta t\right|, \ F > \sigma_f > \left|\delta f\right|, \ \frac{\left|\delta t\right|}{T} \approx \frac{\left|\delta f\right|}{F}, \tag{5.14}$$
$$\tilde{E} = \int_{-\infty}^{+\infty} \Psi(t)^2 dt = \frac{1}{\pi}\int_{0}^{+\infty} \left|A(\omega)\right|^2 d\omega = \delta t \cdot \sum_{n=-\infty}^{+\infty} \left|\Psi(n \cdot \delta t)\right|^2 = \delta f \cdot \sum_{n=-\infty}^{+\infty} \left|A(n \cdot \delta f)\right|^2 =$$

$$\mathbf{2\delta t \cdot \delta f \leq 4\delta t \cdot \delta f \cdot TF \leq 1}$$

$$= \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty} \left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty} \Psi(t)\Psi^*(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)dt = \tilde{p}u = h\overline{f},$$

$$\overline{f} = \frac{\overline{\omega}}{2\pi} = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} f(t)\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} \frac{\partial\varphi}{\partial t}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2\pi^2\tilde{E}}\int_{0}^{+\infty} \omega\left|A(\omega)\right|^2 d\omega =$$
$$= \sqrt{\frac{1}{2\pi^2 h}\int_{0}^{+\infty} \omega\left|A(\omega)\right|^2 d\omega} = \sqrt{\frac{1}{h}\int_{-\infty}^{+\infty} f(t)\left|\overline{\Psi}(t)\right|^2 dt} = \sqrt{\frac{1}{h}\int_{-\infty}^{+\infty} \frac{\partial\varphi}{\partial t}\left|\overline{\Psi}(t)\right|^2 dt} .$$

The next possibility for analogical extending of uncertainty and energy relations (5.14), could be to introduce equivalent relations between linear and orbital mechanical moments and electromagnetic charges, in a similar way as (5.3) was created.

As we know from Signal Analysis, it should be clear that if certain signal (or wave) has a limited or very short duration in its time domain, then its duration in its frequency domain is unlimited, and vice versa (see (5.10)). Contrary to such general (mathematical) knowledge, here (in (5.14)), we are talking about energy finite and/or limited duration signals in both (time and frequency) domains. This is mathematically not quite correct, but under reasonable approximations (taking into account, for instance, 99% of the signal energy in both domains) could be practically imaginable and mathematically acceptable (and in cases of standing waves and stable resonant structures, it could be very much applicable). The position of the author of this paper is that Nature anyhow (intrinsically) implements or produces effective "signal filtering and time-frequency shrinking", during signal creation and propagation (also performing different signal modulations, modifications and attenuation), which eventually makes signal duration and energy content (carried by signals) limited both in time and frequency domain.

In order to illustrate what a finite and limited duration (elementary) wave function means in time and frequency domains, let us imagine that we can find (calculate) an equivalent, average wave function, which will replace real (arbitrary shaped) wave packet function, $\Psi(\mathbf{x},\mathbf{t})$, placing this new (effective) wave function into rectangular shape amplitude borders (both in time and frequency "rectangular frames"). <u>We shall also request that this elementary, finite wave function (or wave packet $\Psi(\mathbf{x},\mathbf{t})$) has energy equal to one energy quantum $\tilde{\mathbf{E}} = \mathbf{hf}$.</u> For instance, the real signal amplitude in a time domain, $\mathbf{a(t)}/\sqrt{2}$, will be replaced by its effective and constant amplitude, $\bar{\mathbf{a}}/\sqrt{2}$. The real signal duration, $\mathbf{T}$, in a time domain will be replaced by effective signal duration, $\overline{\mathbf{T}}$. The real signal amplitude in a frequency domain, $\mathbf{A}(\boldsymbol{\omega})/\sqrt{\boldsymbol{\pi}}$, will be replaced by its effective and constant amplitude, $\overline{\mathbf{A}}/\sqrt{\pi}$. The real signal duration, $\boldsymbol{F}$, in a frequency domain will be replaced by effective signal duration, $\overline{\mathbf{F}}$. Also, the signal mean (or central) frequency, $\mathbf{f}$, will be replaced by its effective central frequency $\bar{\mathbf{f}}$, (4.15), placed in the middle (center of gravity) point of the interval $\overline{\mathbf{F}}$. Now, based on general wave energy expressions found in (5.14), all relevant parameters of the one quantum wave packet, of an effective (or average), rectangular shape elementary wave packet signal, can be found as:

$$
\begin{aligned}
&\tilde{E} = \int_{[t]} [\Psi(t)]^2\, dt = \frac{\bar{a}^2}{2}\bar{T} = \frac{\bar{A}^2}{\pi}\bar{F} = \frac{\bar{A}\bar{a}}{2\sqrt{\pi}} = h\bar{f} = \overline{pu}, \\
&\bar{T} = \frac{\bar{A}}{\bar{a}\sqrt{\pi}} = \frac{1}{2\bar{F}} =, \ \bar{F} = \frac{\bar{a}\sqrt{\pi}}{2\bar{A}} = \frac{1}{2\bar{T}}\ , \ \bar{T}\bar{F} = \frac{1}{2}, \\
&\bar{f} = \frac{\bar{\omega}}{2\pi} = \frac{1}{2\pi^2\tilde{E}}\int_0^{+\infty} \omega|A(\omega)|^2\, d\omega = \sqrt{\frac{1}{2\pi^2 h}\int_0^{+\infty} \omega|A(\omega)|^2\, d\omega}\ , \\
&\bar{\lambda} = \frac{h}{\bar{p}},\ \bar{u} = \bar{\lambda}\,\bar{f} = \left\langle\frac{\omega}{k}\right\rangle = \left\langle\frac{\tilde{E}}{p}\right\rangle,\quad \bar{v} = \left\langle\frac{\Delta\omega}{\Delta k}\right\rangle = \left\langle\frac{d\tilde{E}}{dp}\right\rangle, \\
&h = \frac{\bar{A}\bar{a}}{2\bar{f}\sqrt{\pi}} = \frac{\bar{A}^2}{\pi}\cdot\frac{\bar{F}}{\bar{f}} = (\frac{\bar{a}}{2})^2\frac{1}{\bar{f}\cdot\bar{F}} = 6.62606876 \text{ x } 10^{-34}\,\text{Js}\ , \\
&\Psi(x,t) = a(x,t)\frac{\sin(\underline{\Delta\omega}\ t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\ t - \underline{\Delta k}\, x)}\cos(\omega t - kx)\ (\Leftrightarrow)\,\text{wave packet}.
\end{aligned}
\qquad (5.14\text{-}1)
$$

Obviously, several challenging questions and conclusions could be formulated related to results from (5.14-1), left to be analyzed some other time.

Yet another far-reaching aspect of uncertainty relations (and signals quantifying) could be developed if instead of relatively stable, mean-frequency found in (5.14) and (5.9), we use the time variable frequency definition $\mathbf{f(t)}$, from Analytical Signal model,

$$
\begin{aligned}
&\omega(t) = \partial\varphi/\partial t = 2\pi f(t), \\
&(\overline{\Psi}(x,t) = \Psi(x,t) + j\hat{\Psi}(x,t) = a(x,t)e^{j\varphi(x,t)}, \hat{\Psi} = H[\Psi]).
\end{aligned}
\qquad (5.14\text{-}2)
$$

that will also be left to be analyzed some other time.

Regarding here-presented concept/s of Uncertainty relations we should make a difference between Macro-Uncertainty relations valid between total interval lengths of mutually coupled or conjugate spectral domains (expressed in absolute or statistical conjugate variable intervals), and Micro-Uncertainty relations between minimal signal segments, when signal is atomized (sampled) by means of Digital Signal Processing. Consequently, we should formulate the complete set of united Micro and Macro Uncertainty relations, as well as merge them with the extended meaning of "de Broglie periodicity intervals" presented in T.5.4). Obviously, the contemporary concept of Uncertainty relations in Physics is still not in a full agreement with the above described objectives (and presents more a kind of confusing and uncertain, than clear view regarding that problematic).

5.2. Uncertainty Relations, Fields and Transformation Domains

In modern physics (starting from Heisenberg) we often find oversimplified (and sometimes mystified) presentations of Uncertainty relations without profound generalization or explanation between what kind of entities, values, phenomena, categories, signals, dimensions, domains, etc., such an Uncertainty exist (see for instance T.5.1 and T.5.2). The majority of contemporary physics books are presenting Uncertainty relations almost exclusively as something typical (and only valid) for micro-world of elementary particles and quantum entities, which is in reality not correct, because general time frequency domain signal analysis does not assume any limitations on the (time-space frequency) object-shape-size represented by certain wave function.

There are several important situations explaining the roots and background of Uncertainty relations. From the point of view of mathematics, there is not 1:1 or point-to-point imaging, or correspondence between certain function in its Original and its Spectral domain (valid in both directions, whatever we take as Original or Spectral domain). This property, first found and described in mathematics dealing with Signal Analysis, Spectrum Analysis, Fourier transformations, etc., is generally valid and applicable without limitations. From the point of view of Physics, the most important achievement was the formulation of generally valid conservation laws (and universal principles) as for instance: Energy conservation, Momentum conservation etc., found as consequences of space-time uniformity and isotropy (in isolated inertial systems). Euler-Lagrange-Hamilton Theory, as well as Quantum Mechanics differently and correctly contributed to the conclusion that time and energy domains are Original and Spectral domain to each other. Max Planck and Einstein also formulated the famous relation for photon energy, $\tilde{\mathbf{E}} = \mathbf{hf}$, this way connecting energy, frequency and time (and later, explanations of Photo-electric and Compton effects confirmed such a concept). Einstein showed that there is direct equivalence or proportionality between mass end energy, and that this conclusion extends its validity to any form of energy, to particles, quasiparticles, fields and waves. Luis de Broglie (indirectly) discovered another (Original Domain)-to-(Spectral Domain) couple, by formulating his matter wavelength $\lambda = h/p$, this way connecting position (distance, length, or space dimension) with momentum, also showing that $p = hf_s = \hbar\mathbf{k}$ (where $\mathbf{f_s}$ is the spatial frequency).

Physics found that “Nature obeys Fourier Spectrum Analysis”, or that predictions of Spectrum Analysis are confirmable by experimental evidence (this way precisely connecting time and frequency, and position momentum domains). It is (presently) obvious that Physics deals with two (mutually coupled and conjugate) worlds: the world of (our perceptible) Original domain/s, and the world of corresponding Spectral domain/s. Using this concept as a guiding idea (combined with already developed analogies from earlier chapters of this paper as a predictive platform for generating new physics related concepts) we can again formulate several essential Original-to-Spectral, conjugate domain couples, which should be the building blocs of our Universe, as for instance (see T.1.6, T.3.1, T.3.2, T.3.3 and T.5.1):

T. 5.5. Mutually conjugate variables

Original Domains $\leftrightarrow$	$\leftrightarrow$ Spectral Domains
Time = $\mathbf{t}$	**Energy =** $\tilde{\mathrm{E}}$ **, and/or frequency =** $\mathbf{f}$
Displacement = $\mathbf{x} = \mathrm{S}\dot{\tilde{\mathbf{p}}} = \mathrm{S}\tilde{\mathrm{F}}$, $(\tilde{\mathrm{F}}=\mathbf{force})$	**Momentum =** $\tilde{\mathbf{p}} = \tilde{\mathbf{m}}\dot{\mathbf{x}} = \tilde{\mathbf{m}}\mathbf{v}$
Angle = $\alpha = \mathbf{S_R}\dot{\mathbb{L}} = \mathbf{S_R}\tau$	**Angular momentum =** $\mathbb{L} = \mathrm{J}\dot{\alpha} = \mathrm{J}\omega$
Electric charge = $\mathbf{q}_{\mathbf{el.}} = \Phi_{\mathbf{el.}} = \mathbf{C}\dot{\mathbf{q}}_{\mathbf{mag.}} = \mathbf{C}\mathbf{i}_{\mathbf{mag.}}$	**Magn. charge =** $\mathbf{q}_{\mathbf{mag.}} = \Phi_{\mathbf{mag.}} = \mathbf{L}\dot{\mathbf{q}}_{\mathbf{el.}} = \mathbf{L}\mathbf{i}_{\mathbf{el.}}$

Since there is no chance to make 1:1, or point-to-point imaging or correspondence between Original and Spectral domain points, it is clear that there should be certain interval/s-relations between coupled domains, defining the amount of mutual interval matching or mismatching, named in physics the Uncertainty relations, (see (5.2), (5.3), (5.7) and (5.14)). In addition, most probably (in physics) we do not have too strong platform to say which domain is Original and which one is its Spectral domain (since both of them could coincidently exist, be equally important and experimentally verifiable). For instance, Quantum Mechanics (in connection with traditional Schrödinger's equation) formulates and exploits (at least) two of (bi-directional) Original to Spectral domain transformations (or associations), found also in T.1.1 and T.5.5, such as: (time)-(energy), given by $\mathbf{t} \leftrightarrow \mathbf{f}$, and (position)-(momentum), $\tilde{\mathbf{p}} \leftrightarrow -\mathbf{j}\hbar\frac{\partial}{\partial \mathbf{x}}$. By analogy (see **T.5.5**), we could also imagine (or propose) to introduce two more associations (concerning rotation and electromagnetic field), as for instance: (angle)-(angular momentum), $\mathbb{L} \leftrightarrow -\mathbf{j}\hbar\frac{\partial}{\partial \alpha}$, and (electric charge)-(magnetic charge), $\mathbf{q}_{\mathbf{mag.}} \leftrightarrow -\mathbf{j}\hbar\frac{\partial}{\partial \mathbf{q}_{\mathbf{el.}}}$.

Here we are in a strong position to explain the most interesting conceptual platform of this paper that predicts that the field of Gravitation should have its complement in certain (at present hypothetical) field caused by mass rotation, in the same way as Electric and Magnetic fields, which are mutually dependant and complementary fields (see also (4.30) and (4.31)). As we can conclude from the analogies given in T.5.5, as well as from $\mathbf{q}_{\mathbf{mag.}} \leftrightarrow -\mathbf{j}\hbar\frac{\partial}{\partial \mathbf{q}_{\mathbf{el.}}}$, we should be able to find the couple of Gravitation-Rotational, conjugate charges, $\mathbf{q}_{\mathbf{gravitat.}}$, $\mathbf{q}_{\mathbf{rotat.}}$ (analogue to electric and magnetic charges, $\mathbf{q}_{\mathbf{el.}}$, $\mathbf{q}_{\mathbf{mag.}}$), that will be mutually in the similar relation/s as electric and magnetic charges are. For instance such charges will be an Original and Spectral domain to each other, or satisfy the imaging and operator relation: $\mathbf{q}_{\mathbf{rotat.}} \leftrightarrow -\mathbf{j}\hbar\frac{\partial}{\partial \mathbf{q}_{\mathbf{gravitat.}}}$). Expressed in terms of direct and inverse Fourier transforms (of their generalized wave function), the above described relations between different charges (or between Original and Spectral domain-couples) will also satisfy uncertainty relation (5.2), and can be symbolically presentable as:

$$\mathcal{F}\left\{\Psi\left(\left\|S\right\|\right)\right\} = U\left(\left\|Q\right\|\right) = \int_{-\infty}^{+\infty} \Psi\left(\left\|S\right\|\right)\cdot e^{-j\|Q\|\cdot\|S\|} d\left\|S\right\|,$$

$$\mathcal{F}^{-1}\left\{U\left(\left\|Q\right\|\right)\right\} = \Psi\left(\left\|S\right\|\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U\left(\left\|Q\right\|\right)\cdot e^{-j\|Q\|\cdot\|S\|} d\left\|Q\right\|, \tag{5.15}$$

$$\left\|S\right\| = \begin{pmatrix} t \\ x \\ \alpha \\ q_{el.} \\ q_{gravitat.} \end{pmatrix} = \begin{pmatrix} \text{time} \\ \text{position} \\ \text{angle} \\ \text{el.chrg.} \\ \text{grv.chrg.} \end{pmatrix}, \left\|Q\right\| = \begin{pmatrix} f \\ \tilde{p} \\ L \\ q_{mag.} \\ q_{rotat.} \end{pmatrix} = \begin{pmatrix} \text{frequency} \\ \text{momentum} \\ \text{ang.momnt.} \\ \text{mag.chrg.} \\ \text{rotat.charg.} \end{pmatrix}; \ \Delta\left\|S\right\|\cdot\Delta\left\|Q\right\| \geq \frac{h}{2}.$$

It is interesting to notice that in the case of electric and magnetic charges (and fields) it is possible to have both, Original and Spectral domain equally and coincidentally present in the same real time (and in the same space), and that both of them create (electric and magnetic) fields around them. Is something like that possible (valid) for any other conjugate couple of Original-Spectral domains (5.15), it is the question to answer. What should be the Original-Spectral domain couple most relevant for gravitational phenomenology is also the question to answer. Anyhow, it would be difficult to show that **(t, x, α)** from (5.15) are sources of some presently known or unknown fields. Also we must find the answer about what should be the essential, minimal and complete sets of elements of Original and Spectral, conjugate domains ($\left\|\mathbf{S}\right\|, \left\|\mathbf{Q}\right\|$) relevant for description of our universe.

It is almost needless to say that Uncertainty relations similar to (5.2), (5.3) and (5.14) are applicable to all the values found in (5.15), T.5.1, T5.2 and T.5.5, belonging to vectors of Original and Spectral domains ($\left\|\mathbf{S}\right\|, \left\|\mathbf{Q}\right\|$). This time we can create another generalization of uncertainty relations (5.2) and (5.15), based on (4.32), as for instance:

$$\left\{\begin{array}{l} dP_{\Sigma} = \sum\limits_{(i)} \frac{\alpha_i}{\dot{q}_i} dE(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, \ldots, q_i^{(n)}, t) \Rightarrow \Delta P_{\Sigma} = \sum\limits_{(i)} \frac{\alpha_i}{\dot{q}_i}\Delta E_i \\ E_i = E(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, \ldots, q_i^{(n)}, t), \\ dX_{\Sigma} = V_{\Sigma}dt = \frac{\sum\limits_{(i)} dE_i}{dP_{\Sigma}} dt = \frac{\sum\limits_{(i)} \dot{E}_i}{F_{\Sigma}} dt = \frac{\sum\limits_{(i)} dE_i}{F_{\Sigma}} \Rightarrow \Delta X_{\Sigma} = \frac{\sum\limits_{(i)} \Delta E_i}{F_{\Sigma}} \\ \mathcal{F}\left\{\Psi\left(\left\|X_{\Sigma}\right\|\right)\right\} = U\left(\left\|P_{\Sigma}\right\|\right) = \int_{-\infty}^{+\infty} \Psi\left(\left\|X_{\Sigma}\right\|\right)\cdot e^{-j\|P_{\Sigma}\|\cdot\|X_{\Sigma}\|} d\left\|X_{\Sigma}\right\|, \\ \mathcal{F}^{-1}\left\{U\left(\left\|P_{\Sigma}\right\|\right)\right\} = \Psi\left(\left\|X_{\Sigma}\right\|\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} U\left(\left\|P_{\Sigma}\right\|\right)\cdot e^{-j\|P_{\Sigma}\|\cdot\|X_{\Sigma}\|} d\left\|P_{\Sigma}\right\| \end{array}\right\} \Rightarrow$$

$$\Delta\left\|X_{\Sigma}\right\|\cdot\Delta\left\|P_{\Sigma}\right\| = \left|\frac{\sum\limits_{(i)} \Delta E_i}{F_{\Sigma}} \cdot \sum\limits_{(i)} \frac{\alpha_i}{\dot{q}_i}\Delta E_i\right| \geq \frac{h}{2}. \tag{5.16}$$

Obviously, we could also make (almost) all components of vectors $\|\mathbf{S}\|, \|\mathbf{Q}\|$ and $\|\mathbf{X}_{\Sigma}\|, \|\mathbf{P}_{\Sigma}\|$ from (5.15) and (5.16) be Lorentz-covariant in Minkowski space (for instance to create 4-space vectors $\|\overline{\mathbf{S}}_{\mathbf{4}}\|, \|\overline{\mathbf{Q}}_{\mathbf{4}}\|$ and $\|\overline{\mathbf{X}}_{\Sigma-\mathbf{4}}\|, \|\overline{\mathbf{P}}_{\Sigma-\mathbf{4}}\|$, similar to (4.33)-(4.37)). Since the product between any couple of Lorentz-covariant vectors (in Minkowski space) always presents an invariant, <u>it is possible to show that uncertainty relations, or products made of such 4-vectors (in fact made between mutually related, conjugate intervals of such vectors),</u>

$$\Delta\|\overline{\mathbf{S}}_{\mathbf{4}}\| \cdot \Delta\|\overline{\mathbf{Q}}_{\mathbf{4}}\| = \Delta\|\overline{\mathbf{X}}_{\Sigma-\mathbf{4}}\| \cdot \Delta\|\overline{\mathbf{P}}_{\Sigma-\mathbf{4}}\| = \textbf{Constant } (= \textbf{invariant}) \,, \tag{5.17}$$

<u>are no more uncertain (and are equally applicable to the world of micro particles, or to planetary systems and galaxies)</u>. Also, when creating a New Topology and New Metrics of our universe, we should find a way to incorporate and merge results and predictions from T.5.2, (4.32), (4.37), (5.15), (5.16) and (5.17) in it.

*[♣ **COMMENTS & FREE-THINKING CORNER:** <u>If we continue to exploit the same idea regarding couples of Original and Spectral domains, we could reorganize and generalize all **analogy and symmetry** tables previously established in this paper, as well as to generalize Schrödinger-like equations, (4.25), to be equally applicable in all Physics domains (Quantum Mechanics, Gravity, Maxwell Electromagnetic Theory etc.).</u> For instance, since square of the wave function (in this paper) presents the power function, we know that power can be expressed (like in T.5.2) as the product of relevant velocity and force, or voltage and current, or angular velocity and torque, etc., or in some situations we will have sum of such power members (see also (4.30) and (4.31)). By applying generalized Schrödinger equation (4.25) on such wave functions we will be in a position to develop new wave equations (see also "**phase-group"** concept (4.19)), where currents, voltages, velocities, forces, torque, etc., are explicitly used (which effectively exists in Maxwell electromagnetic theory, and it was developed much before Schrödinger published anything). If the wave function is considered only as a (normalized) probability distribution (defined in the 4-dimensional space-time domain), there is no easy way to diversify Schrödinger-like equations, since for every new attribute, such as current, voltage, force, momentum, torque etc., we should formulate a proper operator. Unfortunately, the founders of Orthodox Quantum Mechanics devoted most of their professional careers modeling and fixing the meaning of the wave function only to a probability framework. Later, majority of their followers have just been repeating the verses from the bibles written a long time ago (and fortunately, this concept still works well, but it should not be considered the best, last, unique and only possible option, as often presented). **The modern Statistical Electrodynamics has already introduced a lot of similar, more implicitly than explicitly formulated doubts (like in this paper), into a unique and irreplaceable position of the Orthodox Quantum Mechanics.***

*Obviously, mass and different momentum (m, p and L) are very important, but not exactly the types of Gravity-Rotational charges we are now talking about, and this is the area where current Gravitation Theory could be modified and upgraded (while respecting predictions of here presented analogies). Consequently, we should pose the essential question about what the real sources of Gravity and Inertia are (if mass is not a sufficient attribute to describe all of such phenomena, according to (4.30)-(4.32), (5.15) and (5.16)). Most probably, the answer to such a question is in "energy interactions", similar to force expressions (2.1), (2.2) and (2.4), and maybe the same problem will be better explained in certain multidimensional space (see **[10]** and **[11]**). It is also clear that we cannot neglect the fact that mass always creates gravitation field or space deformation around itself (Newton force law, Einstein General Relativity), but here we only propose (based on analogies) searching for more general sources of gravitation that would be analog or symmetrical to electromagnetic field sources (and much wider than that). ♣]*

5.3. Central Differences, Uncertainty and Continuum

There is another platform closely related to quantifying, Uncertainty relations, and to the possibility of creating different analogies, which is a direct consequence of mathematics of finite differences, and whose significance has not been exposed and exploited enough in physics. In the table T.5.2, the replacement of infinitesimal differences with corresponding finite differences ($dt = \Delta t$, $dE = \Delta E$…) was obvious and relatively simply made, just following the need to create certain dimensional analogies. It will be good to pay attention to the background platform that makes such replacements possible, and to know when this is fully correct. ***If finite differences (of Δ - types) <u>belong to the class of central or symmetrical differences</u>, then, applying them to a large group of continual functions (often used in mathematical physics), we will obtain the same (or in some cases almost the same) results as in differential analysis with infinitesimal differences (of d - types).*** This situation gives us a chance, whenever something like that is applicable, to transform many differential equations of mathematical physics, almost directly (replacing d (=) Δ**,** $dy \to \Delta y$, $dx \to \Delta x$...), into much simpler algebraic, "quantified, discrete and finite" equations, respecting the rules of central and finite differences. In a number of cases the same method also replaces higher levels of infinitesimal derivatives $\mathbf{d^n}$ with their simple analog and discrete, differential $\Delta^{\mathbf{n}}$**-**operators.

The central difference of the function F(x) can be defined as $\Delta F(x) = F(x + \alpha\ \Delta x) - F(x - (1- \alpha)\Delta x)$, where $0 \leq \alpha \leq 1$. If $\alpha = 1/2$, than $\Delta F(x)$ presents central and symmetrical difference. There are cases of different functions and differential equations where the first derivation $dF(x)/dx$, is identical (or almost identical) to $\Delta F(x)/\Delta x$, where $\Delta F(x)$ is central and symmetrical difference, ***without applying $\Delta x \to 0$ (meaning that we can take Δx = Const.).*** Based on the previously mentioned specifics of central differences, we can try to explain the relations between the Physics of Continuum and Quantum Physics, and provide the part of the answer why, where, when and how nature made quantification of its elementary particles.

The mathematical models of reversible, continual, smooth and deterministic processes in physics (related to their differential equations) seem to belong to the family of functions where infinitesimal derivatives $\mathbf{d^n}$ *can be fully replaced by differential, central symmetrical* $\Delta^{\mathbf{n}}$***-****operators. In fact, this very much hypothetical statement should be taken as a starting platform for a new research task.*

The important message in this short Uncertainty relation review is also to show that there is only one fundamental Uncertainty Principle equally valid in Mathematics, as well as in Quantum Mechanics (or in overall Physics).

We can also conclude that Uncertainty relations, presently known in Physics, are not uniquely and generally treated, and that there is still a lot to be done in this field. The particle-wave dualism theory, as well as Schrödinger's equation should also be significantly upgraded (to cover all waving aspects regarding gravity, particle/s and field/s rotation and electromagnetic theory, as initiated in (5.15) and (5.16)).

Here we could stop, concluding that now we have many new elements for very profound conceptual understanding of the meaning and origin/s of Uncertainty relations, and try to formulate a more systematical and more general concept regarding Uncertainty than presently known.

[♣ __COMMENTS & FREE-THINKING CORNER:__ __Examples, … in process… only a reminder__

Analogies, if well and correctly established and applied, are the best possible platform for understanding and connecting different fields of physics. Knowing relevant analogies, we can (almost) start from every distinct physics theory and (try to) extend its most significant forms/laws to other fields.

*For instance, let us apply **central differences** on the simplest form of D. Bernoulli's fluid flow equation (valid for tube segment with negligible viscosity, laminar fluid flow):*

$$\frac{1}{2}\rho v^2 + \rho g z + p = \text{const.} = p_0 \tag{5.18}$$

where $\rho = \frac{\mathbf{m}}{\mathbf{V}}$ *-is fluid density,* $\boldsymbol{v}$ *-is fluid speed,* $\boldsymbol{g}$ *-is gravitational constant,* $\boldsymbol{p}$ *-is pressure in the fluid segment, and* $\boldsymbol{z}$ *-is a vertical coordinate/position of the analyzed fluid segment. Obviously, (5.18) can be transformed as follows (**applying central, Δ differences, introduced above**):*

$$\begin{aligned}
&\frac{\frac{1}{2}mv^2}{V} + \frac{mgz}{V} + p = \frac{E_k}{V} + \frac{E_p}{V} + p = \frac{\sum_{(i)}(E_{ki} + E_{pi})}{V} + p = \text{const.} = p_0 \Rightarrow \\
&\Rightarrow \sum_{(i)}(E_{ki} + E_{pi}) + pV = p_0 V \Leftrightarrow \sum_{(i)} E_i + pV = p_0 V \Leftrightarrow \sum_{(i)} E_i + (p - p_0)V = 0 \\
&\Rightarrow \sum_{(i)} \Delta E_i + (p - p_0)(\Delta V) + (\Delta p)V + \Delta p \Delta V = \sum_{(i)} \Delta E_i + \Delta(pV) = \\
&= \sum_{(i)} \Delta E_i + \Delta\left(\frac{F}{S} S \cdot \delta x\right) = \sum_{(i)} \Delta E_i + \Delta(F \cdot \delta x) = 0 \text{ , } E_i = E_{ki} + E_{pi} \,.
\end{aligned} \tag{5.19}$$

where $\boldsymbol{V}$ *is the volume of analyzed fluid/tube segment and* $\mathbf{E_k}$ *and* $\mathbf{E_p}$ *-corresponding kinetic and potential energy of the fluid segment. The last part of (5.19) can be taken as the starting point for creating different analogous forms, where the meaning of the fluid and energy could be extended to something else, as for instance to: fluid of electrons, fluid of photons, phonons… Certainly, we cannot say that (5.19) is fully and immediately applicable to other fields/phenomena in physics (before implementing necessary modifications to it), but we can be sure that the creative process (based on analogous thinking) has already started (even if the result regarding (5.19), in some later step, could be negative). ♣]*

6. DIFFERENT POSSIBILITIES FOR MATHEMATICAL FOUNDATIONS OF MULTIDIMENSIONAL UNIVERSE

Modern Physics has been establishing different theoretical platforms for a long time in order to explain that our universe could be multidimensional. Our natural, existential and perceptible reality shows that we live in a four-dimensional world (three spatial and one time dimension), and we are presently able to visualize and detect only such a world. We do not know precisely what could be higher or new dimensions of our universe (for instance, dimensions numbered as fifth, sixth, etc.). Estimating logically, if higher dimensions really exist, we can say with high certainty that such hidden reality produces some influence on certain physical state in our perceptual world. For instance, higher dimensions could influence measurable parameters of energy momentum states, have influence on spectral distributions of certain motional functions, make specific or unusual momentum energy channeling of particles created in different interactions (otherwise, new dimensions created only mathematically, without any measurable and experimentally verifiable momentum energy and spectral consequences, most probably are not realistic).

The simplest basis of an extended, multidimensional universe (**while staying in the theoretical frames of Riemannian metrics and 4-vector or n-vector rules of Minkowski-Space of the Theory of Relativity**) could be:

$\{(\mathbf{SPACE}) + \mathbf{TIME}\} \Leftrightarrow$

$$\left\{(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) + \mathbf{t}\right\} \Leftrightarrow \left\{(\mathbf{r}, \mathbf{t}), \mathbf{r} = \mathbf{r}(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)\right\} \Leftrightarrow (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n, \mathbf{t}) \Leftrightarrow (\mathbf{r}, \mathbf{t}). \quad (6.1)$$

There are many possibilities how to conceptualize and shape multidimensional or multi-coordinate basis (6.1), and here we will touch only a couple of them, sufficiently challenging. We can assume, as the first step, that every spatial dimension or coordinate from the multidimensional basis (6.1) could have its real and imaginary part, whatever that should mean,

$$\begin{aligned} &\overline{\mathbf{x}}_i = \mathbf{x}_{i(real)} + \mathbf{j}\,\mathbf{x}_{i(imaginary)},\ \mathbf{i} = 1,2,3,\ldots,\mathbf{n};\ \mathbf{j}^2 = -1, \\ &(\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_n, \mathbf{t}) \Leftrightarrow (\overline{\mathbf{r}}, \mathbf{t}) \Leftrightarrow \overline{\mathbf{r}}(\mathbf{t}), \\ &\overline{\mathbf{r}} = \overline{\mathbf{r}}(\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_n),\ \overline{\mathbf{x}}_i = \overline{\mathbf{x}}_i(\mathbf{t}), \end{aligned} \quad (6.2)$$

since a similar idea already got its legitimacy in the Minkowski space, where the time scale is placed on the imaginary axis.

1. Let us introduce the first pragmatic convention (compatible with (6.1) and (6.2)) saying that all measurable or detectable space dimensions of our world (three of them, $\overline{\mathbf{x}}_1 = \mathbf{x}_{1(real)} = \mathbf{x}_1, \overline{\mathbf{x}}_2 = \mathbf{x}_{2(real)} = \mathbf{x}_2, \overline{\mathbf{x}}_3 = \mathbf{x}_{3(real)} = \mathbf{x}_3$) have only real parts from (6.2), and all other non-measurable or non-detectable space dimensions have only imaginary parts from (6.2). **We will treat the remaining time dimension in the same way as it is treated in Minkowski space of the Theory of Relativity.** Thus, multidimensional basis (6.2) would be modified as,

$$\begin{aligned}
&(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4, \ldots, \bar{x}_n, t) \Leftrightarrow (\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}, \underleftrightarrow{x_{4(imag.)}, x_{5(imag.)}, \ldots, x_{n(imag.)}}, t) \Leftrightarrow \\
&\Leftrightarrow (\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}, \underleftrightarrow{jx_{4(imag.)}, jx_{5(imag.)}, \ldots, jx_{n(imag.)}}, jct) \Leftrightarrow \\
&\Leftrightarrow \left[(x_{1(real)}, x_{2(real)}, x_{3(real)}),\ j(x_{4(imag.)}, x_{5(imag.)}, \ldots, x_{n(imag.)}, ct)\right] \Leftrightarrow \\
&\Leftrightarrow \left[(x_{1(real)}, x_{2(real)}, x_{3(real)}),\ jct^*\right] \Leftrightarrow \left[(x_1, x_2, x_3),\ jct^*\right].
\end{aligned} \tag{6.3}$$

Now we can express the multidimensional (relativistic and coordinate invariant) space-time interval (using the analogy with space-time interval from the Relativity Theory) as,

$$(\Delta S)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2(\Delta t)^2 - (\Delta x_4)^2 - (\Delta x_5)^2 - \ldots - (\Delta x_n)^2, \tag{6.4}$$

or express the same interval in its infinitesimal form as,

$$(dS)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 - c^2(dt)^2 - (dx_4)^2 - (dx_5)^2 - \ldots - (dx_n)^2.$$

Obviously, since we know that our four-dimensional universe is the dominant reality we live in, there are at least two possibilities how to understand the space-time interval (6.4) in order to stay in agreement with the contemporary Relativity Theory.

1. a) The first possibility is that higher dimensions of our universe (if any) always present extremely short or small (negligible) space intervals,

$$(\Delta x_4)^2 + (\Delta x_5)^2 + \ldots + (\Delta x_n)^2 \cong \mathbf{0}, \tag{6.5}$$

and the second possibility is,

1. b) that higher dimensions somehow contribute to a time-scale modification, or to a certain time shift, $t^* \cong t + \tau$,

$$\begin{aligned}
&c^2(\Delta t^*)^2 = c^2(\Delta t)^2 + (\Delta x_4)^2 + (\Delta x_5)^2 + \ldots + (\Delta x_n)^2, \\
&(\Delta t^*)^2 = (\Delta t)^2 + \frac{(\Delta x_4)^2}{c^2} + \frac{(\Delta x_5)^2}{c^2} + \ldots + \frac{(\Delta x_n)^2}{c^2} = \left[\Delta(t + \tau)\right]^2.
\end{aligned} \tag{6.6}$$

Anyway, both options, (6.5) and (6.6) effectively produce the well-known space-time interval (from the Relativity Theory),

$$(\Delta S)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2(\Delta t^*)^2. \tag{6.7}$$

In (6.6), all space dimensions are presented by their real parts, and only the effective time dimension is associated to an imaginary coordinate axis. The effective time shift concept given by (6.6), where invisible (hidden or higher) dimensions create different time shifts,

$$(\Delta t^*)^2 = (\Delta t)^2 + \frac{(\Delta x_4)^2}{c^2} + \frac{(\Delta x_5)^2}{c^2} + \ldots + \frac{(\Delta x_n)^2}{c^2} = \left[\Delta(t + \tau)\right]^2,$$

could be a very interesting option, because it gives the chance for parallel existence of many different worlds having a part of common and a part of mutually different combinations of dimensions (and being mutually shifted in the time scale for different time intervals).

2. The options given above are very similar to the Minkowski space of the Relativity Theory, where we use ordinary complex numbers **(which have only one imaginary unit, $\mathbf{j}^2 = -1$)** as a convenient mathematical marker to make number of generalizations in physics. Let us now introduce new, Minkowski-equivalent space, operating with **hypercomplex numbers**, starting again from (6.2), which has (at least) three imaginary units (of course, hypercomplex numbers could also have an arbitrary number of imaginary units). For instance, just to give certain intuitive background, contemporary physivcs is demonstrating that elementary particles could be presentable as being composed of quarks, grouped (or classified) into sets of three elements (including additional sets of three antiparticles (of their kind), and additional sets of three of them in different “colors”... and each of them with two of some other attribute... This makes 36 quarks in total, up to present, and probably much more in the future), what could be an imaginative (and certainly not very strong) backing to try to apply the hyper-complex numbers, where the principal imaginary unit, $\mathbf{I}$, is composed of three, more elementary imaginary units, **i, j, k,** as follows:

$$\{(\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2, \ldots, \bar{\mathbf{x}}_n, \mathbf{t}) \Leftrightarrow (\bar{\mathbf{r}}, \mathbf{t}) \Leftrightarrow \bar{\mathbf{r}}(\mathbf{t}),\ \bar{\mathbf{x}}_i = \bar{\mathbf{x}}_i(\mathbf{t})\} \Rightarrow \{\bar{\mathbf{z}}, \mathbf{t}\} \Leftrightarrow \bar{\mathbf{z}}(\mathbf{t}) \Rightarrow$$

$$\left\{\begin{array}{l} (\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2, \bar{\mathbf{x}}_3, \mathbf{jct}) \Rightarrow (\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2, \bar{\mathbf{x}}_3, \mathbf{Ict}) \Rightarrow \{\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2, \bar{\mathbf{x}}_3, \mathbf{c}(\mathbf{it}_i + \mathbf{jt}_j + \mathbf{kt}_k)\} \\ \mathbf{Ict} = \mathbf{c}(\mathbf{it}_i + \mathbf{jt}_j + \mathbf{kt}_k),\ \mathbf{t}^2 = \mathbf{t}_i^{\ 2} + \mathbf{t}_j^{\ 2} + \mathbf{t}_k^{\ 2},\ \mathbf{t}_i\mathbf{t}_j\mathbf{k} - \mathbf{t}_i\mathbf{t}_k\mathbf{j} + \mathbf{t}_j\mathbf{t}_k\mathbf{i} = 0, \\ \mathbf{I}^2 = \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1, \end{array}\right\} \Rightarrow$$

$$\begin{aligned} &\bar{\mathbf{z}} = \mathbf{r} + \mathbf{a}_1 \cdot \mathbf{i} + \mathbf{a}_2 \cdot \mathbf{j} + \mathbf{a}_3 \cdot \mathbf{k} = \mathbf{r} + \mathbf{I} \cdot \mathbf{A} = |\bar{\mathbf{z}}|\mathbf{e}^{\mathbf{I}\varphi} = \bar{\mathbf{z}}_i + \bar{\mathbf{z}}_j + \bar{\mathbf{z}}_k = \\ &= |\bar{\mathbf{z}}_i|\mathbf{e}^{\mathbf{i}\varphi_i} + |\bar{\mathbf{z}}_j|\mathbf{e}^{\mathbf{j}\varphi_j} + |\bar{\mathbf{z}}_k|\mathbf{e}^{\mathbf{k}\varphi_k}, |\bar{\mathbf{z}}|^2 = \mathbf{r}^2 + \mathbf{A}^2, \\ &\mathbf{r} = |\bar{\mathbf{z}}|\cos\varphi,\ \mathbf{A} = |\bar{\mathbf{z}}|\sin\varphi, \end{aligned}$$

$$\begin{aligned} &\mathbf{I} \cdot \mathbf{A} = \mathbf{a}_1 \cdot \mathbf{i} + \mathbf{a}_2 \cdot \mathbf{j} + \mathbf{a}_3 \cdot \mathbf{k} = \mathbf{A}\mathbf{e}^{\mathbf{I}(\frac{\pi}{2} + 2\mathbf{m}\pi)} = \mathbf{I} \cdot |\bar{\mathbf{z}}|\sin\varphi = \\ &= \mathbf{a}_1\mathbf{e}^{\mathbf{i}(\frac{\pi}{2} + 2\mathbf{n}\pi)} + \mathbf{a}_2\mathbf{e}^{\mathbf{j}(\frac{\pi}{2} + 2\mathbf{p}\pi)} + \mathbf{a}_3\mathbf{e}^{\mathbf{k}(\frac{\pi}{2} + 2\mathbf{q}\pi)},\ \mathbf{m}, \mathbf{n}, \mathbf{p}, \mathbf{q} = 1, 2, 3, \ldots, \\ &\mathbf{I} = \frac{\mathbf{a}_1}{\mathbf{A}} \cdot \mathbf{i} + \frac{\mathbf{a}_2}{\mathbf{A}} \cdot \mathbf{j} + \frac{\mathbf{a}_3}{\mathbf{A}} \cdot \mathbf{k} = \mathbf{e}^{\mathbf{I}(\frac{\pi}{2} + 2\mathbf{m}\pi)}, \\ &\mathbf{I}^2 = \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1, \\ &\mathbf{i} \cdot \mathbf{j} = \mathbf{k},\ \ \mathbf{j} \cdot \mathbf{k} = \mathbf{i},\ \ \mathbf{k} \cdot \mathbf{i} = \mathbf{j},\ \ \mathbf{j} \cdot \mathbf{i} = -\mathbf{k},\ \ \mathbf{k} \cdot \mathbf{j} = -\mathbf{i},\ \ \mathbf{i} \cdot \mathbf{k} = -\mathbf{j}, \end{aligned} \tag{6.8}$$

What could be particularly interesting in the hypercomplex coordinate basis (6.8) is the possibility to unite linear and rotational (or torsion) aspects of certain motion in the same mathematical concept, since majority of elementary particles, quasiparticles and matter waves have (apart from their linear or translational motion parameters) also rotational attributes, like spin and orbital moments. Mentioned rotational or torsional parameters can be introduced, for instance, in the following way (just to start thinking about):

$$\overline{\mathbf{z}} = |\overline{\mathbf{z}}|\mathbf{e}^{\mathbf{I}\varphi} = |\overline{\mathbf{z}}|\mathbf{e}^{\mathbf{I}(\omega\mathbf{t}-\mathbf{kr})}, \omega = \frac{\partial\varphi}{\partial\mathbf{t}}$$
$$\overline{\mathbf{x}}_{\mathbf{n}} = |\overline{\mathbf{x}}_{\mathbf{n}}|\mathbf{e}^{\mathbf{i}\varphi_{\mathbf{n}}} = |\overline{\mathbf{x}}_{\mathbf{n}}|\mathbf{e}^{\mathbf{i}(\omega_{\mathbf{n}}\mathbf{t}-\mathbf{k}_{\mathbf{n}}\mathbf{x}_{\mathbf{n}})}, \omega_{\mathbf{n}} = \frac{\partial\varphi_{\mathbf{n}}}{\partial\mathbf{t}}, \mathbf{n} = 1,2,3. \tag{6.8-1}$$

Also, if motional and wave functions are presented in a hypercomplex space described by (6.8), we have a lot of freedom of motion and interaction modeling, such as: using of couples of mutually complex conjugated functions, algebraic sign variation in front of different imaginary units (**+** or **–** sign), etc., in order to represent different energy states, particles, antiparticles, etc.,

$$\overline{\mathbf{z}}^* = \mathbf{r} - \mathbf{a}_1\cdot\mathbf{i} - \mathbf{a}_2\cdot\mathbf{j} - \mathbf{a}_3\cdot\mathbf{k} = \mathbf{r} - \mathbf{I}\cdot\mathbf{A} = |\overline{\mathbf{z}}|\mathbf{e}^{-\mathbf{I}\varphi} = \overline{\mathbf{z}}_{\mathbf{i}}^* + \overline{\mathbf{z}}_{\mathbf{j}}^* + \overline{\mathbf{z}}_{\mathbf{k}}^* =$$
$$= |\overline{\mathbf{z}}_{\mathbf{i}}|\mathbf{e}^{-\mathbf{i}\varphi_{\mathbf{i}}} + |\overline{\mathbf{z}}_{\mathbf{j}}|\mathbf{e}^{-\mathbf{j}\varphi_{\mathbf{j}}} + |\overline{\mathbf{z}}_{\mathbf{k}}|\mathbf{e}^{-\mathbf{k}\varphi_{\mathbf{k}}}, |\overline{\mathbf{z}}^*|^2 = \mathbf{r}^2 + \mathbf{A}^2 = |\overline{\mathbf{z}}|^2, \tag{6.9}$$

whatever shows appropriate in certain case of interest (just to give an idea how to explore such modeling in physics).

3. In order to reach higher level of unity and applicability of the above-mentioned options **1.** and **2.** (for instance in relation to the quantum mechanical wave function), previously introduced complex functions (figuring explicitly or implicitly in (6.2)-(6.9)), can also be treated as multidimensional **Analytic Signal forms** (see: eq. (4.9), [**7**] and [**8**]), and presented in some of the following ways:

3. a) Using hypercomplex representation (6.8) and (6.9), we can transform the multidimensional basis (6.2) or (6.3), and express the hypercomplex (Analytic Signal) wave function as follows,

$$(\bar{r}, t) \Leftrightarrow (\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}, \underleftrightarrow{x_{4(imag)}, x_{5(imag)}, ..., x_{n(imag)}}, t) \Rightarrow$$
$$\Rightarrow (\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}, \underleftrightarrow{Ix_{4(imag)}, Ix_{5(imag)}, ..., Ix_{n(imag)}, Ict}) \Leftrightarrow$$
$$\Leftrightarrow \left\{r(\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}), Ict^*\right\} \Leftrightarrow (r, Ict^*),\ I^2 = -1$$
$$r = r(x_1, x_2, x_3),\ t^* = t^*(t, \frac{x_4}{c}, \frac{x_5}{c}, ..., \frac{x_n}{c}),$$
$$I = \frac{a_1}{A}i + \frac{a_2}{A}j + \frac{a_3}{A}k = e^{I(\frac{\pi}{2}+2m\pi m)},\ A^2 = a_1^2 + a_2^2 + a_3^2$$

$$\overline{\Psi}(\mathbf{r},\mathbf{t}) = \Psi(\mathbf{r},\mathbf{t}) + \mathbf{I}\cdot\mathbf{H}[\Psi(\mathbf{r},\mathbf{t})] = \Psi(\mathbf{r},\mathbf{t}) + \mathbf{I}\cdot\hat{\Psi}(\mathbf{r},\mathbf{t}) =$$
$$= \overline{\Psi}_{\mathbf{i}} + \overline{\Psi}_{\mathbf{j}} + \overline{\Psi}_{\mathbf{k}} = |\overline{\Psi}(\mathbf{r},\mathbf{t})|\cdot\mathbf{e}^{\mathbf{I}\cdot\varphi(\mathbf{r},\mathbf{t})},\ \overline{\Psi}_{\mathbf{i,j,k}} = \Psi_{\mathbf{i,j,k}} + \begin{bmatrix}\mathbf{i}\\ \mathbf{j}\\ \mathbf{k}\end{bmatrix}\cdot\hat{\Psi}_{\mathbf{i,j,k}} = |\overline{\Psi}_{\mathbf{i,j,k}}|\cdot\mathbf{e}^{\begin{bmatrix}\mathbf{i}\\ \mathbf{j}\\ \mathbf{k}\end{bmatrix}\cdot\varphi_{\mathbf{i,j,k}}},$$

$$\left|\overline{\Psi}(r,t)\right|^2 = \left[\Psi(r,t)\right]^2 + \left[\hat{\Psi}(r,t)\right]^2 = \Psi^2 + \hat{\Psi}^2 = \left|\overline{\Psi}\right|^2,\ \varphi(r,t) = \operatorname{arctg}\frac{\hat{\Psi}(r,t)}{\Psi(r,t)} = \varphi,$$

$$\left|\overline{\Psi}_{i,j,k}\right|^2 = \left[\Psi_{i,j,k}\right]^2 + \left[\hat{\Psi}_{i,j,k}\right]^2,\ \varphi_{i,j,k} = \operatorname{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi_{i,j,k}},\ \omega_{i,j,k} = \frac{\partial \varphi_{i,j,k}}{\partial t}$$

$$\left|\overline{\Psi}\right|^2 = \left|\overline{\Psi}_i\right|^2 + \left|\overline{\Psi}_j\right|^2 + \left|\overline{\Psi}_k\right|^2 = \Psi^2 + \hat{\Psi}^2,$$

$$\Psi = \left|\overline{\Psi}\right| \cdot \cos\varphi,\ \hat{\Psi} = \left|\overline{\Psi}\right| \cdot \sin\varphi = H\left[\Psi\right],$$

$$\Psi_{i,j,k} = \left|\overline{\Psi}_{i,j,k}\right| \cdot \cos\varphi_{i,j,k},\ \hat{\Psi}_{i,j,k} = H\left[\Psi_{i,j,k}\right] = \left|\overline{\Psi}_{i,j,k}\right| \cdot \sin\varphi_{i,j,k},$$

$$I \cdot \hat{\Psi}(r,t) = i \cdot \hat{\Psi}_i + j \cdot \hat{\Psi}_j + k \cdot \hat{\Psi}_\kappa = e^{I(\frac{\pi}{2}+2m\pi)} \cdot \hat{\Psi}(r,t),$$

$$I \cdot \varphi(r,t) = i \cdot \varphi_i + j \cdot \varphi_j + k \cdot \varphi_k = e^{I(\frac{\pi}{2}+2m\pi)} \cdot \varphi(r,t),$$

$$I = i \cdot \frac{\varphi_i}{\varphi} + j \cdot \frac{\varphi_j}{\varphi} + k \cdot \frac{\varphi_\kappa}{\varphi} = i \cdot \frac{\hat{\Psi}_i}{\hat{\Psi}} + j \cdot \frac{\hat{\Psi}_j}{\hat{\Psi}} + k \cdot \frac{\hat{\Psi}_\kappa}{\hat{\Psi}} = e^{I(\frac{\pi}{2}+2m\pi)},$$

$$\frac{\varphi_i}{\varphi} = \frac{\hat{\Psi}_i}{\hat{\Psi}},\ \frac{\varphi_j}{\varphi} = \frac{\hat{\Psi}_j}{\hat{\Psi}},\ \frac{\varphi_k}{\varphi} = \frac{\hat{\Psi}_k}{\hat{\Psi}},$$

$$\frac{\varphi_{i,j,k}}{\varphi} = \frac{\hat{\Psi}_{i,j,k}}{\hat{\Psi}} = \frac{H\left[\Psi_{i,j,k}\right]}{H\left[\Psi\right]} = \frac{\operatorname{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi}}{\operatorname{arctg}\frac{\hat{\Psi}}{\Psi}}, \tag{6.10}$$

$$\Psi = \Psi_i + \Psi_j + \Psi_k,\ \Psi^2 = \Psi_i^2 + \Psi_j^2 + \Psi_k^2,\ \Psi_i\Psi_j + \Psi_j\Psi_k + \Psi_i\Psi_k = 0,$$

$$\hat{\Psi} = \hat{\Psi}_i + \hat{\Psi}_j + \hat{\Psi}_k,\ \hat{\Psi}^2 = \hat{\Psi}_i^2 + \hat{\Psi}_j^2 + \hat{\Psi}_k^2,\ \hat{\Psi}_i\hat{\Psi}_j + \hat{\Psi}_j\hat{\Psi}_k + \hat{\Psi}_i\hat{\Psi}_k = 0,$$

$$\varphi = \varphi_i + \varphi_j + \varphi_k,\ \varphi^2 = \varphi_i^2 + \varphi_j^2 + \varphi_k^2,\ \varphi_i\varphi_j + \varphi_j\varphi_k + \varphi_i\varphi_k = 0,$$

What we see from (6.10) is that significant wave function elements (such as amplitude, phase, frequency, etc.) can not be found if we do not take into account both original wave function Ψ and its Hilbert couple $\hat{\Psi}$, meaning that in reality both such wave functions should coincidently exist (see Chapter 4.0 regarding more information about Analytic Signals).

The hypercomplex wave function, which has an arbitrary number of imaginary units (higher than 3), can also be expressed in a similar way, as:

$$\begin{aligned}\overline{\Psi}(t) &= \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)} = a_0(t)\left[\cos\varphi_0(t) + I\ \sin\varphi_0(t)\right] = \\ &= a_0(t)e^{\sum_{(k)} i_k\varphi_k(t)} = \sum_{(k)} a_k(t)e^{i_k\varphi_k(t)} = \sum_{(k)} \overline{\Psi}_k(t)\ ,\end{aligned} \tag{6.10-1}$$

$$\overline{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)} = \Psi_k(t) + i_k\hat{\Psi}_k(t) \ ,$$
$$\cos\varphi_k = \frac{1}{2}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}), \ \sin\varphi_k = \frac{1}{2i}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)}),$$
$$\varphi_k(t) = \operatorname{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)}, \ \varphi_0{}^2(t) = \sum_{(k)}\varphi_k{}^2(t), \quad \omega_k(t) = \frac{\partial\varphi_k(t)}{\partial t} = 2\pi f_k(t),$$
$$a_k{}^2(t) = a_{k-1}{}^2(t) + \hat{a}_{k-1}{}^2(t) = \Psi_{k+1}{}^2(t) = \Psi_k{}^2(t) + \hat{\Psi}_k{}^2(t) \ , \qquad (6.10\text{-}1)$$
$$a_0{}^2(t) = \left|\overline{\Psi}(t)\right|^2 = \Psi^2(t) + \hat{\Psi}^2(t) = \sum_{(k)} a_k{}^2(t) + 2\sum_{(i\neq j)}\Psi_i(t)\Psi_j(t), \ \forall i,j,k \in [1,n] \ .$$
$$\Psi(t) = \frac{a_n(t)}{2^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}), \hat{\Psi}(t) = \frac{a_n(t)}{(2i)^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)}) \ ,$$
$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = \frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}) + \frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\right\}.$$

$$H[\Psi_1\Psi_2] \ = \ (\Psi_1\Psi_2)\frac{\Psi_1\hat{\Psi}_2 + \hat{\Psi}_1\Psi_2}{\Psi_1\Psi_2 - \hat{\Psi}_1\hat{\Psi}_2}, \ H\left[\frac{\Psi_2}{\Psi_1}\right] \ = \ (\frac{\Psi_2}{\Psi_1})\frac{\Psi_1\hat{\Psi}_2 - \hat{\Psi}_1\Psi_2}{\Psi_1\Psi_2 + \hat{\Psi}_1\hat{\Psi}_2} \ .$$

After establishing a well-operating practice how to use easily analytic hypercomplex wave functions to present intrinsic structure of the elementary particles, quasiparticles, wave packets etc., we could conclude that what we have been trying to isolate, or define as a single elementary particle or entity, in reality presents ***a world*** that has its unbounded internal and external structure. Such structurized body of an analytic hypercomplex wave function, relative to energy level number (n = 1, 2, 3…) is recognizable from the expression:

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = \frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}) + \frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\right\},$$

where wave components going inwards and outwards are presented by different signs in exponent ($\mathbf{e^{I\varphi_k(t)}, e^{-I\varphi_k(t)}}$). This could be the main reason why our attempts to find and classify all elementary particles (of our universe) will never end. By going into higher energy levels (regarding particle impacts and relevant products in collider accelerators), we are just opening a countless number of new wave combinations of **mutually coupled**, inwards and outwards traveling waves of dynamically dependent wave structures (having open or self-closed structures). A much better strategy would be to find the right logical and conceptual structure regarding how to understand unbounded multidimensionality that should be in the background of the hypercomplex wave functions.

Here we could also intuitively address the elementary particle creation (when a particle with stable rest mass is created). Since certain wave function (matter wave) can be presented like amplitude modulating product of number of cosine functions, each of them having different frequency $\omega_{\mathbf{k}}(\mathbf{t}) = \frac{\partial\varphi_{\mathbf{k}}(\mathbf{t})}{\partial\mathbf{t}} = 2\pi\mathbf{f}_{\mathbf{k}}(\mathbf{t})$, some of the characteristic frequency members in a proper environment create a closed, stable structure with standing waves, which externally manifests as a stable particle.

3. b) Following some kind of a very simple analogy, based on the knowledge that our own universe has three spatial and one time dimension (x_1, x_2, x_3, t), this situation has been easily generalized (in modern Physics) creating multidimensional spaces analogically extending the basis of the three spatial dimensions into n spatial dimensions $(x_1, x_2, x_3, ..., x_n, t)$. In reality, this is the simplest choice and only one of numerous possibilities to imagine what could be a good starting platform to analyze hypothetical and multidimensional universes. Most probably we do not know what the actual choice of the Universe is, (we only think, or believe that basis $(x_1, x_2, x_3, ..., x_n, t)$ is the most realistic platform for a future research). We have seen that every spatial coordinate of our universe (x_1, x_2, x_3, t) could have its own real and imaginary component, $\left[(x_{1r}, x_{1i}), (x_{2r}, x_{2i}), (x_{3r}, x_{3i}), t\right]$ or that a time component, t, could also have its real and imaginary components, $\left[(x_{1r}, x_{1i}), (x_{2r}, x_{2i}), (x_{3r}, x_{3i}), (t_r, t_i)\right]$ **thus creating some kind of Minkowski space/s using hypercomplex 4-vectors,** or we could also use n-vectors (as already mentioned), etc. The most interesting situation (coming from hypercomplex functions) is that every single imaginary unit i, j, k from (6.8)-(6.10) can additionally be split into three new imaginary units, creating more and more levels of endlessly growing (dimensional, or sub-dimensional) triplets.

$$\begin{gathered} i \to (i_{11}, i_{12}, i_{13})..., j \to (j_{11}, j_{12}, j_{13})..., k \to (k_{11}, k_{12}, k_{13})... \\ i_{mn} \to (i_{mn1}, i_{mn2}, i_{mn3})..., j_{mn} \to (j_{mn1}, j_{mn2}, j_{mn3})..., k_{mn} \to (k_{mn1}, k_{mn2}, k_{mn3}).... \end{gathered} \tag{6.11}$$

Following the same patterns, we could also imagine the basis of multidimensional universe extending each of the spatial coordinate into a new set of three elements (associating different, hypercomplex imaginary units to every triplet of spatial coordinates), as for instance:

$$(x_1, x_2, x_3, t) \Rightarrow \left[(x_{11}, x_{21}, x_{23}), (x_{12}, x_{22}, x_{32}), (x_{13}, x_{23}, x_{33}), ...(x_{1n}, x_{2n}, x_{3n}), t)\right] . \tag{6.12}$$

Hypercomplex numbers and functions could be formulated in a more general way than in (6.8) and (6.9), using n elementary complex units (instead of three), and we could create new and more complex multi-dimensional coordinate systems...

*"**Playing**" with algebraic sign (**+** and/or **–**), with different combinations of hypercomplex imaginary units (**i, j, k**), and with couples of complex and complex-conjugated wave functions and their phase-shifted Hilbert couples mentioned in 3.a) and 3.b), we could also address various **symmetry structures** known in Physics.*

3. c) Until present the time dimension t or Ict was implicitly considered as a homogenous and isotropic, independent variable. Strictly mathematically, we can show that the time dimension and absolute speed c could also be treated as anisotropic, coordinate dependant, physical (and vectorial) values, **at least in one of three different ways**, as follows,

$$\mathbf{Ict} = \begin{cases} (\mathbf{i}_x \mathbf{ct} + \mathbf{i}_y \mathbf{ct} + \mathbf{i}_z \mathbf{ct}), & \mathbf{or} \\ (\mathbf{i}_x \mathbf{c}_x \mathbf{t} + \mathbf{i}_y \mathbf{c}_y \mathbf{t} + \mathbf{i}_z \mathbf{c}_z \mathbf{t}), & \mathbf{or} \\ (\mathbf{i}_x \mathbf{ct}_x + \mathbf{i}_y \mathbf{ct}_y + \mathbf{i}_z \mathbf{ct}_z), & \mathbf{or} \\ (\mathbf{i}_x \mathbf{c}_x \mathbf{t}_x + \mathbf{i}_y \mathbf{c}_y \mathbf{t}_y + \mathbf{i}_z \mathbf{c}_z \mathbf{t}_z) & \end{cases} \Rightarrow (=)\ \mathbf{I} \begin{cases} \mathbf{c}, \mathbf{t} & \mathbf{or} \\ (\mathbf{c}_x, \mathbf{c}_y, \mathbf{c}_z), \mathbf{t} & \mathbf{or} \\ \mathbf{c}, (\mathbf{t}_x, \mathbf{t}_y, \mathbf{t}_z) & \mathbf{or} \\ (\mathbf{c}_x, \mathbf{c}_y, \mathbf{c}_z), (\mathbf{t}_x, \mathbf{t}_y, \mathbf{t}_z) & \end{cases}$$

$$\mathbf{I}^2 = \mathbf{i}_x^{\ 2} = \mathbf{i}_y^{\ 2} = \mathbf{i}_z^{\ 2} = -\mathbf{1}\ , \tag{6.13}$$

$$(\mathbf{i}_x = \mathbf{i},\ \mathbf{i}_y = \mathbf{j},\ \mathbf{i}_z = \mathbf{k},\ \mathbf{x}_1 = \mathbf{x},\ \mathbf{x}_2 = \mathbf{y},\ \mathbf{x}_3 = \mathbf{z}),$$

while keeping the relativistic (and resulting) space-time interval $\Delta\mathbf{S}$, and absolute speed and time amplitude/s, $\mathbf{c}, \mathbf{t}$, unchanged,

$$(\Delta\mathbf{S})^2 = (\Delta\mathbf{x}_1)^2 + (\Delta\mathbf{x}_2)^2 + (\Delta\mathbf{x}_3)^2 - \mathbf{c}^2(\Delta\mathbf{t})^2 = \mathbf{const}.$$

In other words, there is a possibility that the constant and maximal absolute speed $\mathbf{c}$ (in vacuum) could have variable, and basically unlimited, mutually coordinate dependant, vectorial speed components $(\mathbf{c}_x, \mathbf{c}_y, \mathbf{c}_z)$, and that something similar could be valid for the time dimension components $(\mathbf{t}_x, \mathbf{t}_y, \mathbf{t}_z)$. Maybe some of the speed components, hypothetically higher than **c,** could be somehow related to the “Bell’s interconnections theorem” and to David Bohm’s concept of particle wave phenomenology of electron pilot waves.

In the Chapter 4.1 of this paper, we can find all relations between the group and the phase velocity applicable to any wave motion phenomenon that has sinusoidal wave components and harmonic nature of its spectrum (see eq. (4.2)).

$$\begin{cases} v = u - \lambda \dfrac{du}{d\lambda} = -\lambda^2 \dfrac{df}{d\lambda} = u + p\dfrac{du}{dp} = \dfrac{d\omega}{dk} = \dfrac{d\tilde{E}}{dp} = h\dfrac{df}{dp} = \dfrac{df}{df_s} = \dfrac{2u}{1 + \dfrac{uv}{c^2}}\ , \\ u = \lambda f = \dfrac{\omega}{k} = \dfrac{\tilde{E}}{p} = \dfrac{hf}{p} = \dfrac{f}{f_s} = \dfrac{v}{1 + \sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{E_k}{p} \Rightarrow \\ \Rightarrow \quad 0 \le 2u \le \sqrt{uv} \le v \le c\ , \\ d\tilde{E} = hdf = mc^2 d\gamma, \quad \dfrac{df}{f} = (\dfrac{dv}{v}) \cdot \dfrac{1 + \sqrt{1 - \dfrac{v^2}{c^2}}}{1 - \dfrac{v^2}{c^2}} \Rightarrow \dfrac{\Delta f}{\overline{f}} = (\dfrac{\Delta v}{\overline{v}}) \cdot \dfrac{1 + \sqrt{1 - \dfrac{\overline{v}^2}{c^2}}}{1 - \dfrac{\overline{v}^2}{c^2}} \end{cases} \tag{4.2}$$

It is clear that relations (4.2) are also applicable to photons, electromagnetic waves, and speed of light, where speed of light should correspond to the group velocity in (4.2). When an electromagnetic wave propagates in vacuum and free space, all three velocities become mutually (asymptotically) equal, $\mathbf{v} = \mathbf{u} = \mathbf{c} \cong \mathbf{300000\ km/s}$, and here should lie a big part of the explanation what the meaning of the universal constant $\mathbf{c}$ is. The other challenging question is if we would be able to send part of the total energy or meaningful information at speeds higher than $\mathbf{c}$.

3. d) Whatever we establish as the mathematically most convenient basis of multidimensional universe, it should be compatible with the energy aspects of dimensionality given earlier by (4.32), (5.15), (5.16), and (5.17). Complex or hypercomplex numbers (in connection with a chosen coordinate basis) will usually serve as a good mathematical tool, enabling elegant and concise formulations of physical laws.

The principal question here is based on what arguments do we almost uniquely take (or impose) the option: $(\mathbf{x_1, x_2, x_3, t}) \Rightarrow (\mathbf{x_1, x_2, x_3, \ldots, x_n, t})$ as the basis of an n-dimensional universe? Do we really know the choice of Nature (regarding n-dimensionality)? The leading idea here is not to criticize the existing mathematics (and physics) regarding n-dimensionality (which could be quite correct), but also to test other imaginable options, and to find out which one of them better fits into the actual structure of our n-dimensional Universe.

All of the above-introduced options regarding the frame of multidimensional universe can be generalized by modifying the multi-dimensional basis, given by (6.1), introducing only a hypercomplex space coordinates $(\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_n)$, where the time dimension is not explicitly present:

$$\{(\mathbf{SPACE}) + \mathbf{TIME}\} \Leftrightarrow \{(\mathbf{x_1, x_2, \ldots, x_n}) + \mathbf{t}\} \Leftrightarrow (\mathbf{r, t}) \Rightarrow (\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_n). \qquad (6.14)$$

In (6.14) the time dimension is created only from imaginary parts of generalized hypercomplex space coordinates $(\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_n)$, or n-vectors in the following way:

$$\begin{aligned}
&\overline{x}_i = x_{i(real)} + I\, x_{i(imaginary)} = x_i e^{I\theta_{xi}},\ \ x_i = \sqrt{x^2_{i(real)} + x^2_{i(imaginary)}}, \\
&\theta_{xi} = \arctan\frac{x_{i(imaginary)}}{x_{i(real)}},\ i = 1,2,3,\ldots,n;\ I^2 = -1, \\
&(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n) \Leftrightarrow \left\{(x_{1(real)}, x_{2(real)}, \ldots x_{n(real)}), (x_{1(imaginary)}, x_{2(imaginary)}, \ldots x_{n(imaginary)})\right\} \\
&\Leftrightarrow (r,t),\ r = r(x_{1(real)}, x_{2(real)}, \ldots x_{n(real)}),\ \ t = t(x_{1(imaginary)}, x_{2(imaginary)}, \ldots x_{n(imaginary)})
\end{aligned} \qquad (6.15)$$

There is a number of mathematical possibilities how to treat and analyze (6.15), operating with only one, three or more hypercomplex imaginary units. Practically, a kind of generalized multidimensional (hypercomplex) Minkowski n-space of the Relativity Theory is immediately recognizable in (6.15) and (6.16), when we introduce the following n-vector notation:

$$\begin{aligned}
&(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n) \Leftrightarrow \left\{(x_{1(real)}, x_{2(real)}, \ldots x_{n(real)}), (x_{1(imaginary)}, x_{2(imaginary)}, \ldots x_{n(imaginary)})\right\} \\
&\Leftrightarrow (r,ct),\ r = r(x_{1(real)}, x_{2(real)}, \ldots x_{n(real)}),\ \ t = \frac{1}{c}(x_{1(imaginary)}, x_{2(imaginary)}, \ldots x_{n(imaginary)})
\end{aligned} \qquad (6.16)$$

The 4-dimensional relativistic space-time interval based on (6.16), unified with Uncertainty relations from Chapter 5, could be expressed (of course hypothetically, just to give a freedom to unlimited imagination) as:

$$
\begin{aligned}
&(\Delta S)^2=(\Delta x_{1(real)})^2+(\Delta x_{2(real)})^2+(\Delta x_{3(real)})^2+(\delta s)^2-c^2(\Delta t)^2\,,\\
&(\delta s)^2=(\Delta x_{4(real)})^2+(\Delta x_{5(real)})^2+...+(\Delta x_{n(real)})^2\geq\left[\frac{h}{2\Delta p}\right]^2\approx 0\,,\\
&c^2(\Delta t)^2=(\Delta x_{1(imaginary)})^2+(\Delta x_{2(imaginary)})^2+...+(\Delta x_{n(imaginary)})^2\,,\\
&\delta s\cdot\Delta p=\Delta t\cdot\Delta\tilde{E}\geq\frac{h}{2}.
\end{aligned}
\qquad (6.17)
$$

Certainly, in some future analyses regarding coordinate systems of multidimensional space, it has to be shown which spatial coordinate frame ((6.1)-(6.17)... or something similar to them) would be the most realistic for systematic and consistent application in physics. Thus, we should be able to create a clear and logical structure, and correct introduction into any future analysis regarding Unified Field Theory, Superstrings Theory, and regarding all other theories operating with higher space dimensions (see [11]).

Here we were playing with a number of multidimensional space-time frame options similar or equivalent to Riemannian and Minkowski space and metrics of Relativity Theory, by exploring new possibilities in case when ordinary complex functions are replaced by analytic hypercomplex functions. In a similar fashion, the Quantum Mechanical wave function (of multidimensional space) could be formulated as a convenient Analytic Signal, a hypercomplex function leading to a closer integration between Relativistic and Quantum theory. The intellectually extrapolated future superiority and advantages of properly developed, multidimensional, hypercomplex analytic signal functions should be obvious (to open minded and visionary structured individuals) if we, at least, consider revolutionary and owerwealming superiority of present-days “ordinary Analytic Signal”, based on Hilbert Transform, compared to any other signal analysis framework established until introduction of the Analytic Signal (see chapters 4.0 and 4.3). Pitty that Quantum Theory in its early steps did not establish the Wave Function as an Analytic Signal (probably because the Analytic Signal concept was introduced just after Quantum Theory canonized its own wave functions processing, and later was looking too late to restart everything from zero, especially because Quantum Theory with all associated mathematics has been functioning sufficiently well). The opinion of the author of this paper about the same subject is that Quantum Theory will function equally well and much better in case if a Wave Function framework is based on multidimensional and hypercomplex Analytic Signals (just to start with).

[♣ COMMENTS & FREE-THINKING CORNER: **STILL IN PROCESS ⇒**

6.1. Hypercomplex, In-depth Analysis of the Wave Function

Starting from the signal representation in form of an analytical signal (**1.1**) and (**1.2**) from the chapter 4.0, (which reminds to a simple harmonic form of a signal), we can continue the same procedure of factorization unlimitedly, representing every new amplitude function a(t), by a new form of analytical signal. Thus it will be:

$$\Psi(t) = \frac{1}{\pi}\int_0^{\infty}[\mathbf{U}_c(\omega)\cos\omega t + \mathbf{U}_s(\omega)\sin\omega t]d\omega = \frac{1}{\pi}\int_0^{\infty}[\mathbf{A}(\omega)\cos(\omega t + \Phi(\omega))]d\omega =$$

$= a(t)\cos\varphi(t) = \Psi_0(t) = a_0(t)\cos\varphi_0(t) =$

$= a_1(t)\cos\varphi_1(t)\cos\varphi_0(t) = \Psi_1(t)\cos\varphi_0(t) =$

$= a_2(t)\cos\varphi_2(t)\cos\varphi_1(t)\cos\varphi_0(t) = \Psi_2(t)\cos\varphi_1(t)\cos\varphi_0(t) =$

....

$$= a_n(t)\prod_{i=0}^{n}\cos\varphi_i(t) = \Psi_n(t)\prod_{i=0}^{n-1}\cos\varphi_i(t) = a_n(t)\cdot\cos\varphi_0(t)\prod_{i=1}^{n}\cos\varphi_i(t),$$
$$\left[a(t) = a_0(t) = a_n(t)\prod_{i=1}^{n}\cos\varphi_i(t),\ \cos\varphi(t) = \cos\varphi_0(t)\right] . \tag{6.18}$$

Previously obtained, factorized signal form reminds us of a multiple amplitude modulation of the signal, where every following level represents an amplitude function of the previous level, so in that sense we can talk about in-depth structure of the signal, whose structural levels may be presented by following functions:

$$\Psi_1(t) = a_0(t) = a_1(t)\cos\varphi_1(t) = \left|\overline{\Psi}(\mathbf{t})\right| = \left|\overline{\Psi}_0(\mathbf{t})\right| = \frac{\Psi_0(\mathbf{t})}{\cos\varphi_0(\mathbf{t})},$$

$$\Psi_2(t) = a_1(t) = a_2(t)\cos\varphi_2(t) = \left|\overline{\mathbf{a}}_0(\mathbf{t})\right| = \left|\overline{\Psi}_1(\mathbf{t})\right| = \frac{\Psi_0(\mathbf{t})}{\cos\varphi_1(\mathbf{t})\cos\varphi_0(\mathbf{t})},$$

.... (6.19)

$$\Psi_n(t) = a_{n-1}(t) = a_n(t)\cos\varphi_n(t) = \left|\overline{\mathbf{a}}_{n-2}(\mathbf{t})\right| = \left|\overline{\Psi}_{n-1}(\mathbf{t})\right| = \frac{\Psi_0(\mathbf{t})}{\prod_{i=0}^{n-1}\cos\varphi_i(\mathbf{t})}.$$

From (3.15) and (3.16) one may notice that following relations remain:

$\Psi_{k+1}(t) = a_k(t) = a_{k+1}(t) \cos \varphi_{k+1}(t) = \mathbf{a_n(t)} \prod_{\mathbf{i=k+1}}^{\mathbf{n}} \mathbf{\cos\varphi_i(t)}$, k < n,

$$\prod_{i=1}^{n} \Psi_i(t) = \prod_{i=1}^{n} a_n(t) \cdot \prod_{i=1}^{n} \cos\varphi_i(t) \quad , \quad \frac{\Psi_k(t)}{\Psi_{k+1}(t)} = \cos\varphi_k \quad , \prod_{i=0}^{n} \cos\varphi_i(t) = \frac{\prod_{i=0}^{n} \Psi_i(t)}{\prod_{i=1}^{n+1} \Psi_i(t)} \quad , \quad (6.20)$$

$$\frac{\mathbf{a_1}}{\mathbf{a_0}} \mathbf{\cos\varphi_1} = \frac{\mathbf{a_2}}{\mathbf{a_1}} \mathbf{\cos\varphi_2} = \ldots\ldots = \frac{\mathbf{a_n}}{\mathbf{a_{n-1}}} \mathbf{\cos\varphi_n} = 1 .$$

If we briefly address signal velocities, we can conclude that the phase velocity of the basic signal $\mathbf{\Psi(t)}$ (as well as the common phase velocity of its components) is determined only by the $\mathbf{\cos\varphi_0}$. In addition, the group velocity of the signal $\mathbf{\Psi(t) = \Psi_0(t) = a_0(t)} \cos\varphi_0(t)$ propagation is determined only by the velocity of its amplitude function $\mathbf{a_0(t)}$. By considering the factorization of the signal, one may conclude that there are series of partial group velocities, which by appropriate addition (similar to the relativistic determination of the velocity of the inertia center) must form the resulting (central, group) velocity of the amplitude member $\mathbf{a_0(t)}$.

A problem that is very interesting to be solved mathematically is the case of factorized signal presentation (6.18), where we fix several $\mathbf{\cos\varphi_i(t)}$ *members in advance, giving them specific (or arbitrary) values for phase functions, regardless if such values could be found in the initially factorized signal form. Let us take the example where only three of such cosine-phase members (indexed with **A**, **B**, and **C**) are fixed by our intervention,*

$$\Psi(t) = a_n(t) \cdot \prod_{i=0}^{n} \cos\varphi_i(t) = a_n(t) \cdot \cos\varphi_A(t) \cdot \cos\varphi_B(t) \cdot \cos\varphi_C(t) \cdot \prod_{i=3}^{n} \cos\varphi_i^*(t)$$

The question here is if we could still find the right product of remaining cosine members $\prod_{\mathbf{i=3}}^{\mathbf{n}} \mathbf{\cos\varphi_i^*(t)}$ *that will fit exactly in order to represent the same signal (without any error). Why and where could be such factorization interesting? Let us imagine that we intend to present the process of disintegration of a certain micro particle, initially presentable by* $\Psi(t)$. *Based on numerous experimental data, we are able to know (in advance) the resulting amplitude function* $\mathbf{a(t) = a_0(t) = a_n(t)} \prod_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{\cos\varphi_i(t)}$ *of the same particle before its disintegration. In addition, knowing the experimental results of particle disintegration, we could find that three new particles (for the needs of this example) are clearly detectable (particles indexed with **A**, **B**, and **C**), and that possibly we have certain signal residuals, or energy dissipation, producing some other elements, not easy to detect. Hypothetically, we intend to present all such residual elements using the factorization* $\prod_{\mathbf{i=3}}^{\mathbf{n}} \mathbf{\cos\varphi_i^*(t)}$. *If here described scenario can be mathematically modeled to describe real situations from microphysics, this presents a way to start organizing elementary particles in a much better way than presently known.*

Every structural level of the signal from (6.19) can be associated with correspondent complex, analytical signal, by the same procedure as given in (4.02),

$$\{\Psi_1(t)\} \to \overline{\Psi}_1(t) = a_0(t) + i_1\hat{a}_0(t) = a_1(t)e^{i_1\varphi_1(t)},$$

$$\{\Psi_2(t)\} \to \overline{\Psi}_2(t) = a_1(t) + i_2\hat{a}_1(t) = a_2(t)e^{i_2\varphi_2(t)},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \tag{6.21}$$

$$\{\Psi_n(t)\} \to \overline{\Psi}_n(t) = a_n(t) + i_n\hat{a}_n(t) = a_n(t)e^{i_n\varphi_n(t)},$$

$$\{\Psi(t)\} \to \overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)},$$

where $\hat{\Psi}_{k+1}(t) = \hat{a}_k(t) = H[a_k(t)]$.

One may notice that in (6.19) an indexed complex unit is introduced, which for now should be understood and interpreted as an already known (common) complex unit:

$$I^2 = i^2 = j^2 = i_1^{\ 2} = i_2^{\ 2} = i_3^{\ 2} = ... = i_n^{\ 2} = -1, \tag{6.22}$$

but later (in extension) the indexed complex unit will serve for dividing of the complex space to a hyper-complex space with several complex units.

The specificity of penetrating into the structural depth of the signal (or into the wave function) becomes obvious if we determine energies of the structural level of the signal (in the same way as in (4.04)):

$$\tilde{E}_0 = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \int_{-\infty}^{\infty} a_0^{\ 2}(t)\cos^2\varphi_0(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_0^{\ 2}(t)\,dt,$$

$$\tilde{E}_1 = \int_{-\infty}^{+\infty} \Psi_1^{\ 2}(t)dt = \int_{-\infty}^{\infty} a_0^{\ 2}(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_1^{\ 2}(t)\,dt = 2\tilde{E}_0,$$

$$\tilde{E}_2 = \int_{-\infty}^{+\infty} \Psi_2^{\ 2}(t)dt = \int_{-\infty}^{\infty} a_1^{\ 2}(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_2^{\ 2}(t)\,dt = 4\tilde{E}_0, \tag{6.23}$$

....

$$\tilde{E}_n = \int_{-\infty}^{+\infty} \Psi_n^{\ 2}(t)dt = \int_{-\infty}^{\infty} a_{n-1}^{\ 2}(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_n^{\ 2}(t)\,dt = 2^n\tilde{E}_0 = 2\tilde{E}_{n-1}, \quad n = 0, 1, 2, ...$$

It is obvious that penetrating into deeper and deeper structural levels of the signal "**n**" demands consumption of more and more energy for factor 2^n, in order to isolate and analyze those levels (in a way similarly to the analysis of nuclear, elementary particles).

Let us go back now to the structural levels of the signal in the form of (6.19). We will call the basic level of the signal in its complex form a hypercomplex analytical function of the signal, and we will represent it as in (4.02), apart from that, the complex unit "j" will be replaced by the hypercomplex, imaginary unit "I":

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)} = a_0(t)[\cos\varphi_0(t) + I\sin\varphi_0(t)], \tag{6.24}$$

where the hypercomplex, imaginary unit "I" is defined as a linear combination over the base of elementary hypercomplex units "$i_1, i_2, i_3, ..., i_n$", as:

$$\mathbf{I}\varphi_0(\mathbf{t}) = \mathbf{i}_1\varphi_1(\mathbf{t}) + \mathbf{i}_2\varphi_2(\mathbf{t}) + ... + \mathbf{i}_n\varphi_n(\mathbf{t}) = \sum_{k=1}^{n}\mathbf{i}_k\varphi_k(\mathbf{t}),$$

$$\mathbf{I}^2 = \mathbf{i}_1{}^2 = \mathbf{i}_2{}^2 = \mathbf{i}_3{}^2 = ... = \mathbf{i}_n{}^2 = -1,$$

$$\mathbf{i}_j\mathbf{i}_k = 0, \forall \mathbf{j} \neq \mathbf{k} \Rightarrow \varphi_0{}^2(\mathbf{t}) = \sum_{k=1}^{n}\varphi_k{}^2(\mathbf{t}), \tag{6.25}$$

$$\mathbf{e}^{\mathbf{i}_k\varphi_k(\mathbf{t})} = \cos\varphi_k(\mathbf{t}) + \mathbf{i}_k \sin\varphi_k(\mathbf{t}), \forall \mathbf{k} \in [1, \mathbf{n}].$$

According to the introduction of the hypercomplex wave function $\overline{\Psi}(\mathbf{t})$ in the previous way, it is possible to show validity of the following relations:

$$\begin{aligned}&\overline{\Psi}(\mathbf{t}) = \Psi(\mathbf{t}) + \mathbf{I}\hat{\Psi}(\mathbf{t}) = \mathbf{a}_0(\mathbf{t})\mathbf{e}^{\mathbf{I}\varphi_0(\mathbf{t})} = \mathbf{a}_0(\mathbf{t})[\cos\varphi_0(\mathbf{t}) + \mathbf{I}\sin\varphi_0(\mathbf{t})] = \\ &= \mathbf{a}_0(\mathbf{t})\mathbf{e}^{\sum_{(k)}\mathbf{i}_k\varphi_k(\mathbf{t})} = \sum_{(k)}\mathbf{a}_k(\mathbf{t})\mathbf{e}^{\mathbf{i}_k\varphi_k(\mathbf{t})} = \sum_{(k)}\overline{\Psi}_k(\mathbf{t}),\end{aligned} \tag{6.26}$$

where:

$$\begin{aligned}&\overline{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)} = \Psi_k(t) + i_k\hat{\Psi}_k(t), \\ &\varphi_k(t) = \text{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)}, \ \varphi_0{}^2(t) = \sum_{(k)}\varphi_k{}^2(t), \\ &a_k{}^2(t) = a_{k-1}{}^2(t) + \hat{a}_{k-1}{}^2(t) = \Psi_{k+1}{}^2(t) = \Psi_k{}^2(t) + \hat{\Psi}_k{}^2(t), \\ &a_0{}^2(t) = \left|\overline{\Psi}(t)\right|^2 = \Psi^2(t) + \hat{\Psi}^2(t) = \sum_{(k)}a_k{}^2(t) + 2\sum_{(i\neq j)}\Psi_i(t)\Psi_j(t), \ \forall i, j, k \in [1, n].\end{aligned} \tag{6.27}$$

Based on the structural analysis of the signal, introducing its structural levels through (6.19), it is obvious that we are in a position that, if we know the function of the basic signal level $\Psi(t)$, we can determine its remaining (depth) levels of a higher order:

$$\Psi_0(t) = \Psi(t) \rightarrow \Psi_1(t) \rightarrow \Psi_2(t) \rightarrow \ ...... \ \rightarrow \Psi_n(t), \tag{6.28}$$

and also go back from any level $\Psi_k(t)$, $0 < k \leq n$ (using inverse transformations) to the basic level of the signal $\Psi_0(t)$,

$$\Psi_k(t) \rightarrow \Psi_{k-1}(t) \rightarrow \Psi_{k-2}(t) \rightarrow \ ...... \ \rightarrow \Psi_0(t) \ . \tag{6.29}$$

If we continue now the procedure of inverse transformations from (6.29) in the same direction, we will be in a position to determine substructural signal levels (or its history),

$\Psi_k(t) \to \Psi_{k-1}(t) \to \Psi_{k-2}(t) \to$......	$\to \Psi_0(t) \to$	$\Psi_{-1}(t) \to \Psi_{-2}(t) \to$ ...... $\to \Psi_{-n}(t)$
depth levels of the signal	basic level	substructural levels
(future of the signal, $\tilde{E}_k$)	(present state $\tilde{E}_0$)	(past, history of the signal, $\tilde{E}_{-k}$)

One can show that eigenenergies of the substructural signal levels (or its prehistory, i.e., energy spent to form that signal) will be, similarly to the formula for the energies of the structural (depth) levels (6.23), equal to

$$\tilde{\mathbf{E}}_{-n} = 2^{-n}\tilde{E}_0, \; n = 0, 1, 2, \ldots \tag{6.30}$$

One may immediately notice that sum of all energies of the substructural signal levels is equal to the sum of a convergent series,

$$\sum_{k=1}^{n}\tilde{\mathbf{E}}_{-k} = \sum_{k=1}^{n} 2^{-k}\tilde{\mathbf{E}}_0 \leq \tilde{\mathbf{E}}_0, \; (\mathbf{n} \to \infty)\,, \tag{6.31}$$

which obviously represents the form of the Law of energy conservation, while the series of the energies of the structural (depth) levels is divergent, i.e.,

$$\sum_{k=1}^{n}\tilde{\mathbf{E}}_{k} = \sum_{k=1}^{n} 2^{k}\tilde{\mathbf{E}}_0 \to \infty, (\mathbf{n} \to \infty)\,, \tag{6.32}$$

because, in order to penetrate deeper and deeper into the structure of the signal (or matter), we must spend more and more energy.

Now we can give a parallel between the previously exposed structural analysis of the signal, which reduces to the factorization of the signal and Fourier's analysis of a signal (which corresponds to the searching of appropriate sums of orthogonal functions). In a context of such classified signal analysis, we can picturesquely say that Fourier's analysis, or summational dissolution of a signal, is the **"surface"** analysis of a signal in the framework of one structural level. From the previously exposed procedure of dissolution of a signal there is a posssibility that a similar method (conveniently mathematically modified) can be used for the analysis of the matter structure (atom and subatomic particles), or for processing and transferring data in telecommunications, as well as for analyses of different real physical signals that carry information about some process.

ANALYTIC SIGNAL REPRESENTATION

$$\overline{H}\left[\Psi(t)\right]=\overline{\Psi}(t)=\Psi(t)+I\cdot\hat{\Psi}(t)=a(t)\cdot e^{I\cdot\varphi(t)}$$
$$\overline{H}=1+I\cdot H,\ I^2=-1$$

AS SUPERPOSITION	AS MULTIPLICATION
$\overline{\Psi}(t)=\sum_{(k)}\overline{\Psi}_k(t)=\sum_{(k)}\Psi_k(t)+I\sum_{(k)}\hat{\Psi}_k(t)$	$\overline{\Psi}(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t)+I\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t)$

RELATIONS BETWEEN ADDITIVE AND MULTIPLICATIVE ELEMENTS

$$\Psi(t)=\sum_{(k)}\Psi_k(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t)=a_0(t)\cos\varphi_0(t)=a(t)\cos\varphi(t)=-H\left[\hat{\Psi}(t)\right]$$

$$\hat{\Psi}(t)=\sum_{(k)}\hat{\Psi}_k(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t)=a_0(t)\sin\varphi_0(t)=a(t)\sin\varphi(t)=H\left[\Psi(t)\right]$$

$$\overline{\Psi}_k(t)=\Psi_k(t)+i_k\hat{\Psi}_k(t)=a_k(t)e^{i_k\varphi_k(t)}\ ,\Psi_k(t)=-H\left[\hat{\Psi}_k(t)\right],\ \hat{\Psi}_k(t)=H\left[\Psi_k(t)\right]$$

$$\overline{\Psi}(t)=\Psi(t)+I\hat{\Psi}(t)=\frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)}+e^{-I\varphi_k(t)})+\frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)}-e^{-I\varphi_k(t)})\right\}$$

Signal Amplitude

$$a(t)=a_0(t)=\left|\overline{\Psi}(t)\right|=\sqrt{\Psi^2(t)+\hat{\Psi}^2(t)}=a_n(t)\prod_{(i=1)}^{n}\cos\varphi_i(t)=\Psi_{n+1}(t)\prod_{(i=1)}^{n}\cos\varphi_i(t)$$

$$a_k(t)=\Psi_{k+1}(t)=\left|\overline{\Psi}_k\right|=\sqrt{\Psi_k^2(t)+\hat{\Psi}_k^2(t)}=a_{k+1}(t)\cos\varphi_{k+1}(t)=a_n(t)\prod_{(i=k+1)}^{n}\cos\varphi_i(t),\ k<n$$

$$a_{n-1}(t)=\Psi_n(t)=\left|\overline{\Psi}_{n-1}(t)\right|=\sqrt{\Psi_{n-1}^2(t)+\hat{\Psi}_{n-1}^2(t)}=a_n(t)\cos\varphi_n(t)=\frac{\Psi_0(t)}{\prod_{(i=0)}^{n-1}\cos\varphi_i(t)}$$

Signal Phase

$$\varphi_0(t)=\varphi(t)=\mathrm{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}=\sqrt{\sum_{(k)}\varphi_k^2(t)},\ \varphi_k(t)=\mathrm{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)}$$

$$I^2=i_1^2=i_2^2=...=i_n^2=-1\ ,\ i_j i_k=0,\forall\, j\neq k\ \ \text{(hypercomplex imaginary units)}$$

$$I\varphi_0(t)=I\varphi(t)=i_1\varphi_1(t)+i_2\varphi_2(t)+...+i_n\varphi_n(t)=\sum_{k=1}^{n}i_k\varphi_k(t)$$

$$e^{i_k\varphi_k(t)}=\cos\varphi_k(t)+i_k\sin\varphi_k(t),\ \ \varphi_0^2(t)=\sum_{k=1}^{n}\varphi_k^2(t)$$

Signal instant frequency

$$\omega_i(t)=2\pi f_i(t)=\frac{\partial\varphi_i(t)}{\partial t},\ \ i=0,1,2,...k,...n$$

Parallelism between Time and Frequency Domains	Analytic Signal	
	Time Domain	**Frequency Domain**
Complex Signal	$\overline{\Psi}(t)=a(t)e^{j\varphi(t)}$ $=\Psi(t)+j\hat{\Psi}(t)$ $=\int_{(0,+\infty)}\overline{U}(\omega)e^{-j\omega t}d\omega$ $=\int_{(0,+\infty)}A(\omega)e^{-j(\omega t+\Phi(\omega))}d\omega$	$\overline{U}(\omega)=A(\omega)e^{-j\Phi(\omega)}$ $=U_c(\omega)-jU_s(\omega)$ $=\int_{[t]}\overline{\Psi}(t)\,e^{j\omega t}dt$ $=\int_{[t]}a(t)e^{j(\omega t+\varphi(t))}dt$
Real and imaginary signal components	$\psi(t)=a(t)\cos\varphi(t)$ $=-H[\hat{\psi}(t)]$, $\hat{\psi}(t)=a(t)\sin\varphi(t)$ $=H[\psi(t)]$	$U_c(\omega)=A(\omega)\cos\Phi(\omega)$ $U_s(\omega)=A(\omega)\sin\Phi(\omega)$
Signal Amplitude	$a(t)=\sqrt{\psi^2(t)+\hat{\psi}^2(t)}$	$A(\omega)=\sqrt{U_c^{\,2}(\omega)+U_s^{\,2}(\omega)}$
Instant Phase	$\varphi(t)=\mathrm{arctg}\dfrac{\hat{\psi}(t)}{\psi(t)}$	$\Phi(\omega)=\arctan\dfrac{U_s(\omega)}{U_c(\omega)}$
Instant Frequency	$\omega(t)=\dfrac{\partial\varphi(t)}{\partial t}$	$\tau(\omega)=\dfrac{\partial\Phi(\omega)}{\partial\omega}$
Signal Energy	$E=\int_{[t]}\lvert\overline{\psi}(t)\rvert^2dt$ $=\int_{[t]}[a(t)]^2dt$	$E=\dfrac{1}{2\pi}\int_{-\infty}^{+\infty}\lvert\overline{U}(\omega)\rvert^2d\omega$ $=\dfrac{1}{\pi}\int_0^{\infty}[A(\omega)]^2d\omega$
Central Frequency	$\Omega=\dfrac{\int_{[t]}\omega(t)\cdot a^2(t)dt}{\int_{[t]}a^2(t)dt}$	$\Omega=\dfrac{\int_0^{\infty}\omega\cdot[A(\omega)]^2d\omega}{\int_0^{\infty}[A(\omega)]^2d\omega}$
“Central Time”	$T=\dfrac{\int_{[t]}t\cdot[a(t)]^2dt}{\int_{[t]}[a(t)]^2dt}$	$T=\dfrac{\int_{[\omega]}\tau(\omega)\cdot[A(\omega)]^2d\omega}{\int_{[\omega]}[A(\omega)]^2d\omega}$
Standard Deviation	$\sigma_\Omega^2=\dfrac{1}{\Delta t}\int_{[t]}\lvert\omega(t)-\Omega\rvert^2dt$	$\sigma_T^2=\dfrac{1}{\Delta\omega}\int_{[\omega]}\lvert\tau(\omega)-T\rvert^2d\omega$
Uncertainty Relations	$\omega(t)\cdot\tau(\omega)\cong\Delta\omega\cdot\Delta\tau(\omega)\cong\sigma_\Omega\cdot\sigma_T\cong\Omega\cdot T\cong\Delta\omega\cdot\Delta t\cong\pi\,(!?)$	

The hypercomplex wave function is a much richer concept regarding the usual complex functions, and on the following example, we can show some more advantages of such representations of the wave function. Let us start from the previously defined hypercomplex wave function (6.24) and let us represent it in one of the possible (modified) forms:

$$\overline{\Psi}(t) = \Psi(t) + I\,\hat{\Psi}(t) = a_0(t)e^{I\,\varphi_0(t)} = a_0(t)\left[\cos\varphi_0(t) + I\ \sin\varphi_0(t)\right] =$$
$$= a_{01}(t)e^{I_1\varphi_{01}(t)} + a_{02}(t)e^{I_2\varphi_{02}(t)} + a_{03}(t)e^{I_3\varphi_{03}(t)} =$$

$$= \Psi_1(t) + \Psi_2(t) + \Psi_3(t) + i_1\,\hat{\Psi}_1(t) + i_2\,\hat{\Psi}_2(t) + i_3\,\hat{\Psi}_3(t) =$$
$$= \left[\Psi_{11}(t) + \Psi_{12}(t) + \Psi_{13}(t)\right] + \left[\Psi_{21}(t) + \Psi_{22}(t) + \Psi_{23}(t)\right] + \left[\Psi_{31}(t) + \Psi_{32}(t) + \Psi_{33}(t)\right] +$$
$$+ \left[i_{11}\,\hat{\Psi}_{11}(t) + i_{12}\,\hat{\Psi}_{12}(t) + i_{13}\,\hat{\Psi}_{13}(t)\right] + \left[i_{21}\,\hat{\Psi}_{21}(t) + i_{22}\,\hat{\Psi}_{22}(t) + i_{23}\,\hat{\Psi}_{23}(t)\right] +$$
$$+ \left[i_{31}\,\hat{\Psi}_{31}(t) + i_{32}\,\hat{\Psi}_{32}(t) + i_{33}\,\hat{\Psi}_{33}(t)\right] =$$
$$= \left[\Psi_{11}(t) + i_{11}\,\hat{\Psi}_{11}(t)\right] + \left[\Psi_{12}(t) + i_{12}\,\hat{\Psi}_{12}(t)\right] + \left[\Psi_{13}(t) + i_{13}\,\hat{\Psi}_{13}(t)\right] +$$
$$+ \left[\Psi_{21}(t) + i_{21}\,\hat{\Psi}_{21}(t)\right] + \left[\Psi_{22}(t) + i_{22}\,\hat{\Psi}_{22}(t)\right] + \left[\Psi_{23}(t) + i_{23}\,\hat{\Psi}_{23}(t)\right] +$$
$$+ \left[\Psi_{31}(t) + i_{31}\,\hat{\Psi}_{31}(t)\right] + \left[\Psi_{32}(t) + i_{32}\,\hat{\Psi}_{32}(t)\right] + \left[\Psi_{33}(t) + i_{33}\,\hat{\Psi}_{33}(t)\right] =$$
$$= a_{01}(t)e^{I_1\varphi_{01}(t)} + a_{02}(t)e^{I_2\varphi_{02}(t)} + a_{03}(t)e^{I_3\varphi_{03}(t)} = \ \dots\ ,$$

so that following relations stand:

$$\Psi(t) = \Psi_1(t) + \Psi_2(t) + \Psi_3(t),$$
$$I\,\hat{\Psi}(t) = i_1\,\hat{\Psi}_1(t) + i_2\,\hat{\Psi}_2(t) + i_3\,\hat{\Psi}_3(t)\,,$$
$$\Psi_1(t) = \left[\Psi_{11}(t) + \Psi_{12}(t) + \Psi_{13}(t)\right],$$
$$\Psi_2(t) = \left[\Psi_{21}(t) + \Psi_{22}(t) + \Psi_{23}(t)\right],$$
$$\Psi_3(t) = \left[\Psi_{31}(t) + \Psi_{32}(t) + \Psi_{33}(t)\right],$$

$$i_1\,\hat{\Psi}_1(t) = \left[i_{11}\,\hat{\Psi}_{11}(t) + i_{12}\,\hat{\Psi}_{12}(t) + i_{13}\,\hat{\Psi}_{13}(t)\right],$$
$$i_2\,\hat{\Psi}_1(t) = \left[i_{21}\,\hat{\Psi}_{21}(t) + i_{22}\,\hat{\Psi}_{22}(t) + i_{23}\,\hat{\Psi}_{23}(t)\right],$$
$$i_3\,\hat{\Psi}_3(t) = \left[i_{31}\,\hat{\Psi}_{31}(t) + i_{32}\,\hat{\Psi}_{32}(t) + i_{33}\,\hat{\Psi}_{33}(t)\right].$$

The previous model of the wave function becomes a more interesting one, with following introduction of the rule of three-dimensional (vectorial) orthogonality of the imaginary axes:

$$i_1 \times i_2 = i_3\,,\quad i_2 \times i_3 = i_1\,,\quad i_3 \times i_1 = i_2\,,$$
$$i_2 \times i_1 = -i_3\,,\quad i_3 \times i_2 = -i_1\,,\quad i_1 \times i_3 = -i_2\,,$$
$$i_{1n} \times i_{2n} = i_{3n}\,,\ i_{2n} \times i_{3n} = i_{1n}\,,\ i_{3n} \times i_{1n} = i_{2n}\,,$$
$$i_{2n} \times i_{1n} = -i_{3n}\,,\ i_{3n} \times i_{2n} = -i_{1n}\,,\ i_{1n} \times i_{3n} = -i_{2n}\,,\quad n \in [1,2,3]\,.$$

....

It is obvious that (if we continue with the previous procedure) we can introduce different forms of analysis, synthesis, and divide the hypercomplex wave function into levels, which could probably be used in purpose of modeling of very specific structures that are characteristic for the world of elementary, subatomic particles, or for establishing the universal field theory, etc. ♣]

7. CONCLUSION

This paper has presented one specific system of overall electromechanical analogies (extended mobility type of analogies), which is established after:

1° Analogies taken from the similarity of relevant mathematical formula expressions and belonging differential equations;

2° Analogies coming from the same geometry or same topology of equivalent circuits or models;

3° Analogies created after comparing higher level of energy and force formulas, as well as comparing different Conservation Laws.

Based on the established system of analogies, several innovative ideas are formulated, regarding possible generalization and unification of Gravitation, Maxwell Electromagnetic Theory and Quantum Theory in the frames of new understanding of Particle Wave Duality. Of course, we have to be conscious that any conclusion based (only) on analogies is at the same time very powerful, if correct, and very risky and weak if it is not correctly developed and applied, and this is the reason why analogies in this paper are taken only as initial new idea generators.

The "fogy concept" of *Particle Wave Duality* known from our contemporary physics (or Quantum Theory) is very much elaborated in this paper, becoming the concept of "*Particle Wave Unity*" (here formulated also as **PWDC** (=) **P**article **W**ave **D**uality **C**ode). It is also shown that all important wave equations and Uncertainty relations of Orthodox Quantum Theory can be logically, naturally and step by step developed without even knowing that Quantum Theory exist, and without any of "*probability philosophy*" backing.

A wider and more general platform for extending and exploiting the analogies in physics is to be found in exploiting **Symmetries**. The following, supporting segment is taken from the book [**10**] written by Michio Kaku (see also [**11**], from the same author): *"… we are beginning to realize that nature, at the fundamental level, does not just prefer symmetry in a physics theory, nature demands it. Physicists now realize that* ***symmetry is the key to constructing physics laws without disastrous anomalies and divergences."***

WORKS CITED

[1] Hueter, F., and Richard H. Bolt. SONICS, Techniques for the Use of Sound and Ultrasound in Engineering and Science, New York: John Willey & Sons, Inc., 1955. Pages: 9-18. Library of Congress Catalog Card Number: 55-6388, USA

[2] Cyril M. Harris and Charles E. Crede. Shock and Vibration Handbook (Vol. 1 & 2): McGraw-Hill Book Company, 1961. (-Volume 1, Chapter 10, Pages: 10-1 to 10-59; -Volume 2, Chapter 29, Pages: 29-1 to 29-47). Library of Congress Catalog Card Number: 60-16636, USA

[3] Prokic Miodrag. "Energetic Parallelism" , Diploma Work (B.S. Thesis), November 1975. University of Nis, Faculty of Electronic Engineering, YU.

[4] L'ubomir Vlcek. New Trends in Physics: Slovak Academic Press, Slovakia, 1996. ISBN 80-85665-64-6.

[5] José-Philippe Pérez, Nicole Saint-Cricq-Chéry. Relativite et quantification: MASSON SA, Paris Milan Barcelone 1995. ISBN: 2-225-80799-X, ISSN: 0992-5538. Pages: 175-179

[6] R. Boskovic. Theoria Philosophiae naturalis; -theoria redacta ad unicam legem virium in natura existentium; Vienna, 1758 (and the same book in Venetian edition from 1763).

[7] Shie Qian and Dapang Chen. Joint Time-Frequency Analysis; PRENTICE HALL PTR, 1996, Upper Saddle River, NJ 07458. ISBN: 0-13-254384-2. Pages: 29-37

[8] Papoulis Athanasios. Signal Analysis; McGraw-Hill, USA, 1977. ISBN: 0-07-048460-0.

[9] Robert Eisberg and Robert Resnick. Quantum Physics, Second Edition; John Wiley & Sons, USA, 1985. ISBN: 0-471-87373-X.

[10] Michio Kaku & Jennifer Thompson. Beyond Einstein: Anchor Books Edition, USA, 1995. ISBN 0-385-47781-3. Pages: 100-110.

[11] Michio Kaku. Hyperspace: First Anchor Books Edition, USA, March 1995. ISBN 0-385-47705-8. Copyright by Oxford University Press. Printed in USA.

[12] Lawrence C. Lynnworth. Ultrasonic Measurements for Process Control: Panametrics, Inc., Waltham, Massachusetts. ACADEMIC PRESS, INC., 1989. ISBN 0-12-460585-0. Printed in USA. Pages: 325-335.

[13] Nick Herbert. Quantum Reality, Beyond the New Physics: Anchor Books, DOUBLEDAY, 1985. ISBN 0-385-23569-0. Printed in USA.

[14] **Heinz R. Pagels. The Cosmic Code: A Bantam New Age Book, Bantam DOUBLEDAY Dell Publishing, 1983-1990. ISBN 0-553-24625-9. Printed in USA and Canada.**

[15] **R. M. Kiehn. An Interpretation of the Wave Function as a Cohomological Measure of Quantum Vorticity, Physics Department, University of Houston. (Presented in Florida 1989) Houston TX 77004, USA. See also number of papers written by the same author regarding Torsion Fields and Topology.**

[16] Bergman, David L., and Lucas, Jr., Charles W., "Physical Models of Matter," Proceedings of Physics As A Science Workshop, Cologne, Germany (1997), to be published in Hadronic Press Supplement; preprint available at web site: http://www.cormedia.com/css.

[17] Bergman, David L., "Inertial Mass of Charged Elementary Particles", Proceedings of Physics As A Science Workshop, Cologne, Germany (1997), to be published in Hadronic Press Supplement; preprint available at web site: http://www.cormedia.com/css.

[18] Bergman, David. L., "Forces on Moving Objects," Proceedings of Physics As A Science Workshop, Cologne, Germany (1997), to be published in Hadronic Press Supplement; preprint available at web site: http://www.cormedia.com/css.

[19] Bergman, David. L, and Wesley, J. Paul, "Spinning Charg-ed Ring Model of Electron Yielding Anomalous Magnetic Moment", Galilean Electrodynamics 1:5 (Sept./Oct. 1990).

[20] Bergman, David L, "Spinning Charged Ring Model of Elementary Particles", Galilean Electrodynamics 2:2 (Mar./Apr. 1991).

[21] Barnes, Thomas G., Foundations of Electricity & Magnetism, Third Edition, Thomas G. Barnes, El Paso, TX, publisher (1975).

[22] Bergman, David L., "New Spinning Charged Ring Model of the Electron", Proceedings of the Twin-Cities Creation Conference, Minneapolis, MN (Jul./Aug. 1992).

[23] Wesley, J. Paul, "Weber Electrodynamics with Fields, Waves, and Absolute Space, "Progress in Space-Time Physics 1987, pp. 193-209, Benjamin Wesley, Publisher, Weiherdammstrasse 24, 7712 Blumberg, Germany, 1987; Assis, Andre K. T., "Modern Experiments Related to Weber's Electrodynamics," Proceedings of the Conference on Foundations of Mathematics and Physics, Benjamin Wesley, publisher, Weiherdammstrasse 24, 7712 Blumberg, Germany, 1989; Wesley, J. Paul, "Weber-Wesley Electrodynamics," pp. 212-272, Benjamin Wesley, Publisher, Weiherdammstrasse 24, 7712 Blumberg, Germany, 1991; Wesley, J. Paul, "Empirically Correct Electrodynamics," Classical Quantum Theory, pp. 284-322, Benjamin Wesley, Publisher, Weiherdammstrasse 24, 7712 Blumberg, Germany, 1996.

[24] Adey, A. I. A., "Non-Covariant Galilean Electrodynamics," Galilean Electrodynamics 6, 108-116 (November/December, 1995).

[25] Lucas, Jr., Charles Wm., and Lucas, Joseph W., "Electrodynamics of Real Particles vs. Maxwell's Equations, Relativity Theory and Quantum Mechanics," Proceedings of the 1992 Twin-Cities Creation Conference, The Twin-Cities Creation-Science Association, Minneapolis, Minnesota, USA (1992).

[26] Smulsky, Jozef J., "A New Approach to Electrodynamics and to the Theory of Gravity, (Published in Russian and English. Available from Jozef J. Smulsky, Institute of Earth's Cryosphere, Siberian Branch Russian Academy of Sciences, 625000, Tyumen, Box 1224, Russia.) (1992).

[27] J. M. T. Thompson, "Visions of the Future: Physics and Electronics, Published by the press syndicate of the University of Cambridge (The Royal Society, 2001). ISBN 0 521 80538 4 (pb). QC174.12.V47 2001. 530.12-dc21 00-064219. See chapters 1, 2 and 3.

[28] A K T ASSIS and HTORRE SSILVA, Comparison between Weber's electrodynamics and classical electrodynamics. Facultadd eIngenieria,Universidad de Tarapaca, Av. 18 de Septiembre 2222, Arica, Chile. PRAMANA, Journal of Physics, Vol. 55, No.3, pp. 393–404; September 2000. Indian Academy of Sciences.
Coupling of Electromagnetism and Gravitation in the Weak Field Approximation

[29] Charles W. Lucas, Jr. and Joseph C. Lucas - Class 2000. WEBER'S FORCE LAW FOR REALISTIC FINITE-SIZE ELASTIC PARTICLES. HARVARD UNIVERSITY, 281 Lowell Mail Center Cambridge, MA 02138-7527, jlucas@post.harvard.edu

[30] Master of Science Thesis by Lars Johansson: Longitudinal electrodynamic forces and their possible technological applications. Thesis Adviser, Professor Gerhard Kristensson, Department of Electromagnetic Theory, Lund Institute of Technology, P.O. Box 118 S-221 00 Lund, SWEDEN. CODEN:LUTEDX/(TEAT-5027)/1-55/(1996)

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APPENDIX:

8. BOHR's MODEL OF HYDROGEN ATOM AND PARTICLE-WAVE DUALISM (Still in preparation)

The basic idea of this appendix was formulated back in 1975, and many times after that supplemented with comments and new addenda. General synthesis and complete harmonization has not been performed yet, but the central message is still up-to-date and original enough...

The hydrogen atom represents almost perfect mini-laboratory for checking basic elements and principles of the particle-wave dualism. The importance of Bohr's hydrogen atom model (although it is obsolete and surpassed) is in the point that it gives certain results (linked to the light spectrum analysis), which are very accurate, and from which one may make many conclusions about the particle-wave dualism of the matter (in a very elementary and comprehensible way, which is close to the mechanistic and deterministic cognition of the world, with the possibility of visualization of the model topology). The purpose of the following analysis is to reinvestigate or summarize validity and logic of the particle-wave dualism on the simplest level, applying them onto Bohr's hydrogen atom model (in fact, to identify and prove the **P**article **W**ave **D**uality **C**ode = **PWDC**, already introduced in the earlier chapters of this paperwork). The following text indicates the roots of the modern particle-wave duality concepts of motion that are found even in Bohr's atom model, and sufficient care was not dedicated to that fact neither in the time of Bohr's atom model originating, nor today. Discovering the true sources of the ideas about the particle-wave dualism, we face the possibility of a critical attention to the later development of those ideas, so in that sense the following text sets and explains some dilemmas of the essential understanding of the particle-wave dualism that are still up-to-date, although they are bypassed by formalism and by the modern Quantum Physics methods. **In the context with the previous chapters of this paper, the following analysis of Bohr's hydrogen atom model will be limited only to the aspects of the model that check the foundations (presented till now) of the particle-wave dualism, i.e. they check and prove PWDC (introduced in the Chapter 4 of this paper).** Naturally, original Bohr's model will be partly modified and supplemented here.

It is almost needless to say that a number of times Physics explained the limits of Bohr's model in comparison with Quantum Mechanics, where the same problematic is treated using wave functions and Schrödinger's equation. Here we will not attempt to modify this well-known situation (regarding disadvantages and weak points of Bohr's model comparing it to the contemporary quantum wave mechanics). The ultimate objective here will be to prove that basics and step-stones of **P**article **W**ave **D**uality are in the following facts or statements (here classified as **PWDC**),

1 The matter waves or de Broglie waves are exclusive manifestations of motional energy (regardless of its origin). The rest energy (and rest particle mass) does not belong to de Broglie mater waves, or to a corresponding wave group, or a wave packet. Only the change of motional energy (of certain system) directly creates de Broglie, or matter waves.

2 De Broglie wavelength $\lambda = \frac{\mathbf{h}}{\mathbf{p}}$ and Planck-Einstein energy of the wave quantum $\tilde{E} = hf \, (= mc^2)$ are just the intrinsic and mutually compatible elements of relations between the group $\mathbf{v}$ and phase velocity $\mathbf{u}$ of the de Broglie matter waves, $\mathbf{v} = \mathbf{u} - \lambda \frac{\mathbf{du}}{\mathbf{d\lambda}} = \frac{d\tilde{E}}{dp} = \frac{\tilde{E}}{p} - \frac{h}{p} d(\frac{\tilde{E}}{p})/d(\frac{h}{p})$ (mathematically presentable as wave groups or wave packets). The group velocity is at the same time equal to the velocity of the particle that creates the wave group. The phase velocity (of the wave group) is never higher than its group velocity, and never higher than the speed of light.

3 The **P**article **W**ave **D**uality of matter (**PWD**) is the consequence of dynamic and intrinsic coupling between linear and/or torsion motions and their surrounding fields in any two or multi-body system. **PWD** is also in a very close relation to all mutually coupled action-reaction and inertial forces between relevant moving objects (regardless of the involved force-field nature, present in two-body or multiple-body systems), while respecting generalized energy and momentum conservation laws.

In the following text, we will try to identify and prove the above-formulated elements of the **PWDC**.

A short summary of Bohr's atom model (in the frameworks of the non-relativistic velocities of the electrons) is included in two postulates and one hypothesis about the electron motion in the atom field.

Let it be: $\mathbf{m}$ – mass of an electron, $\mathbf{e}$ – charge of an electron, $\mathbf{n}$ – main quantum number, $\mathbf{h}$ – Planck's constant, $\mathbf{v}$ - orbital velocity of an electron, $\mathbf{r}$ - radius of the electron orbit, $\mathbf{Z}$ – number of protons in the atom nucleus, ε - energy level (or energy) of the stationary electron orbit, $\mathbf{f_{12}}$ – frequency of the emitted or absorbed photon, and $\mathbf{f_s}$ - eigenvalue, stationary frequency of the electron wave (in the orbit denoted with quantum number $\mathbf{n}$).

Starting from the postulate of stationary electron atom orbits, which must satisfy the condition of quantization of the torque, Bohr introduces:

$$\mathbf{pr = mvr = n\frac{h}{2\pi}, \quad n = 1, 2, 3, \ldots} \tag{8.1}$$

The second postulate is related to the conditions when an electron may emit (or absorb) the quantum of electromagnetic emission, during the change of the energy level of the electron between its two stationary orbits:

$$\mathbf{hf_{12}} = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon, \quad \varepsilon_2 > \varepsilon_1 \ . \tag{8.2}$$

Finally, Bohr assumes that electron orbits around the nucleus of the (hydrogen) atom are circular and he sets the condition of the dynamic balance of the attractive and repulsive forces between the electron (that revolves) and the atom nucleus:

$$\frac{1}{4\pi\varepsilon_0}\frac{e(eZ)}{r^2} = \frac{mv^2}{r} \quad . \tag{8.3}$$

From the previous postulates and conditions (8.1), (8.2), and (8.3), one may get the elements that characterize the circular motion of the electrons around the atom nucleus, such as:

-radius of the electron orbit,

$$r = r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2Z} \quad , \tag{8.4}$$

-orbital velocity of the electron,

$$v = v_n = \frac{nh}{2\pi mr} = \frac{Ze^2}{2nh\varepsilon_0} \quad , \tag{8.5}$$

-kinetic energy of the electron,

$$E_k = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r_n} \tag{8.6}$$

and the potential (electrostatic) energy of the electron,

$$U = \int_{\infty}^{r} F dr = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} = -2E_k = -mv^2 \quad ,(F_\infty = 0) \quad . \tag{8.7}$$

Cumulative energy of the electron in the field of the hydrogen atom is equal to the sum of its kinetic and potential energy:

$$\varepsilon_B = E_k + U = -E_k = -\frac{Ze^2}{8\pi\varepsilon_0 r} = -\frac{mZ^2e^4}{8n^2h^2\varepsilon_0^{\ 2}} = -\frac{1}{2}mv^2 \quad . \tag{8.8}$$

Applying the second Bohr's postulate (8.2) onto the (8.8), one may find the energy of the electromagnetic emission of the atom, at transition of its electron between two stationary energy levels $\varepsilon_2 > \varepsilon_1, n_2 > n_1$,

$$hf_{12} = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon_B = \frac{mZ^2e^4}{8h^2\varepsilon_0^{\ 2}}\left(\frac{1}{n_1^{\ 2}} - \frac{1}{n_2^{\ 2}}\right) \quad . \tag{8.9}$$

The indexing with **"B"** in the previous (and following) expressions, where adequate, is introduced to emphatically indicate that certain values directly and exclusively emanate from the traditionally known Bohr's atom model (because later there will be other indexed symbols), due to the later comparison of the Bohr's results with results discussed in extension.

In reality, the electron and the nucleus revolve about their common center of gravity. Therefore, the mass of the nucleus $\mathbf{M}$ should enter the equations. Sommerfeld has shown that equation (8.9) still stands if $\mathbf{m}$ is replaced by the so-called reduced mass μ (contributing with very fine correction of (8.9), [9]),

$$\left.\begin{cases} \mathbf{m} \rightarrow \mathbf{m_r} = \mu = \dfrac{mM}{m+M} \ , \dfrac{M}{m} = 1836.13 \ , \\ (m \ = \ m_e = \text{electron mass}, \ M \ = \ m_p = \text{nucleus mass}) \end{cases}\right\} \Rightarrow \tag{8.9-1}$$

$$\mathbf{hf_{12}} = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon_B = \frac{\mu Z^2 e^4}{8h^2 {\varepsilon_0}^2} \left(\frac{\mathbf{1}}{\mathbf{n_1}^2} - \frac{\mathbf{1}}{\mathbf{n_2}^2}\right) .$$

Since the nucleus mass is much greater than the electron mass, if we analyze the same system (nucleus-electron) in the Laboratory and the center-of-mass system, **we can easily realize that the electron's stationary wave carries practically only the motional energy of the revolving electron**, meaning that we can easily and with high certainty find that this stationary wave energy (in the case of non-relativistic electron velocities: see (8.1)-(8.9-1)) will be expressed as,

$$\begin{aligned} &\varepsilon = \varepsilon_s = \mathbf{hf_s} = \frac{\mathbf{1}}{\mathbf{2}}\mu v^2 = \frac{\mathbf{1}}{\mathbf{2}} J_e \omega^2 = \frac{\mathbf{1}}{\mathbf{2}} \mathbf{L_e}\omega = \frac{\mu \mathbf{Z^2 e^4}}{\mathbf{8h^2 {\varepsilon_0}^2}} \cdot \frac{\mathbf{1}}{\mathbf{n^2}} \cong \frac{\mathbf{1}}{\mathbf{2}}\mathbf{mv^2} , \ J_e = \mu \mathbf{r^2} , \\ &\Leftrightarrow \mathbf{v} = \frac{\mathbf{Ze^2}}{\mathbf{2h\varepsilon_0}} \cdot \frac{\mathbf{1}}{\mathbf{n}} , \ \mathbf{L_e} = J_e \omega = \mu \mathbf{r^2}\omega = \frac{\mathbf{1}}{\omega} \cdot \frac{\mu \mathbf{Z^2 e^4}}{\mathbf{4h^2 {\varepsilon_0}^2}} \cdot \frac{\mathbf{1}}{\mathbf{n^2}} = \frac{\mathbf{r}}{\mathbf{v}} \cdot \frac{\mu \mathbf{Z^2 e^4}}{\mathbf{4h^2 {\varepsilon_0}^2}} \cdot \frac{\mathbf{1}}{\mathbf{n^2}} = \mathbf{n}\frac{\mathbf{h}}{\mathbf{2\pi}} . \end{aligned} \tag{8.9-2}$$

Later on it will be shown and proved that (8.9-2) is fully correct and that it contains very essential information regarding the **PWDC** (see also (8.25)-(8.30)).

Originally, Bohr introduced the idea about discrete, particle-like revolving of an electron mass around the atom center, and until certain level, such concept started producing verifiable results. The experimentally found spectral lines of the hydrogen atom spectrum are in the perfect agreement with theoretical predictions described by (8.9) and (8.9-1), confirming the level of validity of Bohr's atom model.

Starting from the fact that Bohr's model reveals little about the nature of the atom and about the essence of the accompanying PWD phenomenology, because it is based on two postulates and one hypothesis, but also from the fact that the result (8.9) enabled theoretical predictions of the experimentally obtained spectral distributions, one may conclude that by some more comprehensive analysis of Bohr's model, we may probably recognize some physical essence not exposed till now. This would additionally explain Bohr's postulates and shed some new light on the nature of the matter and the essence of the particle-wave dualism (in this paper, we call that unexposed physical essence PWDC = Particle Wave Duality Code).

Primarily, retrospectively looking, Bohr's atom model does not take into account possibilities of the wave interpretation of the nature of electron, although the postulate (8.2) implies emission or absorption of the electromagnetic wave quantum exploiting transition of the electrons between the stationary orbits, which implicitly qualifies that some accompanying wave properties are connected with the orbital motion of the electrons, even before the energy change occurs (8.9).

Since the momentum of inertia of a rotating electron (initially considered a discrete particle) in the framework of Bohr's atom model is $J = mr^2$***, and the same is valid if we consider that the same electron mass rotates while being distributed around a thin-walled equivalent torus (which covers the same space of electron orbit or path), we will be able to make the next step in conceptualizing such electron as a matter wave on a certain toroidal form. Here we approach somewhat similar, very significant and original ideas introduced by David L. Bergman and his colleagues [16] - [22]: -"Common Sense Science". In fact, D. L. Bergman's electron (and atom) concept is much more advanced and better comparing to original Bohr's model and to models and concepts of contemporary Quantum Theory (apart from their ideological elaborations and messages patched along explaining such concept). Since historically Bohr's model was the first one of relatively successful atom models, we will try to draw the maximum of PWDC-relevant conclusions from it. Later, an attempt will be made to find the maximum of unifying, updating and migrating areas in order to merge the most successful and best aspects of Bohr's and Bergman's concepts.***

Starting from an intuitive, simplified geometric concept of the interpretation of **stationary** or stable electron orbits, which we directly connect with the assumed existence of a stable orbital electron wave, limiting ourselves for now only on circular orbits, we will say that the stationary electron orbit (in the internal atom space) is the orbit to which the stationary (standing) electron wave belongs, such that the middle perimeter of that orbit is equal to the integer of the wavelengths of the stationary electron wave, i.e.,

$$2\pi \mathbf{r} = \mathbf{n}\lambda_s \quad . \qquad (8.10)$$

Comparing the condition of "stationarity" (8.10) and Bohr's postulate (8.1), one may notice a direct and indisputable analogy from which it takes only one step to the posing of the de Broglie's wave hypothesis,

$$2\pi \mathbf{r} = \mathbf{n}\frac{\mathbf{h}}{\mathbf{mv}} = \mathbf{n}\frac{\mathbf{h}}{\mathbf{p}} = \mathbf{n}\lambda_s \quad , \qquad (8.11)$$

i.e., wavelength of an orbital electron wave equals:

$$\lambda_s = \frac{\mathbf{h}}{\mathbf{p}} \quad . \qquad (8.12)$$

Since it seems clear that electron on its stationary orbit (whatever that means) is a kind of mixed or dual entity (having mutually equivalent and commuting properties of a particle and a wave), for the purpose of literal identification of such dual object (or de Broglie matter wave), in this paper we will use the following wave expressions and symbolism for the wavelength and wave energy of de Broglie matter waves: $\tilde{\mathbf{E}} = \mathbf{hf}$, $\lambda = \frac{\mathbf{h}}{\tilde{\mathbf{p}}}$, $(\mathbf{E_k} \to \tilde{\mathbf{E}}, \mathbf{p} \to \tilde{\mathbf{p}})$. In order to support the use of wave symbols, $\tilde{\mathbf{E}}, \tilde{\mathbf{p}}$, we can remember that energy $\mathbf{E_f} = \tilde{\mathbf{E}}_\mathbf{f}$, mass $\mathbf{m_f} = \tilde{\mathbf{m}}_\mathbf{f}$, and wavelength λ, of an electromagnetic quantum, or photon, can also be given by relations,

$$\tilde{E}_f = hf = (m_f - m_{f0})c^2 = m_f c^2 = \tilde{m}_f c^2 = E_{kf}\ ,\ \ \tilde{p}_f = \frac{hf}{c} = p_f\ ,\ \ \tilde{m}_f = m_f = \frac{hf}{c^2},\ \ \lambda_f = \frac{h}{\tilde{p}_f}\ ,$$

and all of them are effectively proved valid in analyses of Photoelectric, Compton and other familiar effects (without explicit using of here proposed wave symbolic).

By previous results, (8.11)-(8.12), and (8.9-2), it is showed that Bohr's atom model and de Broglie's wave hypothesis have become mutually complementary with elements that simultaneously support and prove correctness of both concepts.

For now, for the assumed electron wave we may say that we know only its wavelength. Of course, we know that the difference between the eigenenergies of two stationary electron waves, (8.9) very precisely predicts emissive or absorptive atom spectral lines, but we do not know the absolute amounts of the energies of stationary electron waves (hf_s), which take part in that difference. Yet, formally, we may write Bohr's postulate (8.2) in a form that takes into consideration (for now unknown) orbital stationary frequencies of that wave, so it will be

$$hf_{12} = h(f_{s2} - f_{s1}) = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = \Delta\varepsilon_B\ . \tag{8.13}$$

In addition, we will try to determine (in absolute values) all elements of the stationary electron wave, such as its phase and group velocity, frequency, wavelength and energy. Of course, the term stationary electron wave is quite justified with the fact that atom can absorb or emit an electromagnetic wave (i.e. quantum or photon), i.e. it is likely that appropriate interferences and superposition of the two familiar wave phenomena will happen. Thereupon a possibility intuitively obtruded is that electron in orbital revolving around the atom nucleus possesses, beside particle attributes and some wave attributes, i.e. that it is no longer a particle localized in some point, but some distributed mass and distributed charge of an electron across the perimeter of the corresponding stationary orbit (which we will here call de Broglie's mater wave).

Until here we didn't care about relativistic mass-energy velocity dependencies, since the first objective was to recognize and prove the most important relations belonging to the **P**article **W**ave **D**uality **C**ode (= **PWDC**), $\tilde{E} = hf$, $\lambda = \frac{h}{\tilde{p}}$. There are some other elements of **PWDC** that will be mentioned later.

In order to identify the orbital eigenfrequency of the electron wave (before any energy jumps, emissions, or photon absorptions) it is necessary to assume the nature of that wave, i.e. what energy the electron stationary wave exists on.

For example, we may express total energy of the electron in orbital motion as,

$$\varepsilon_t = mc^2\ ,\ \ \left(m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0\ ,\ m_0 = \text{const.}\right), \tag{8.14}$$

while its total motion energy is,

$$\varepsilon_k = (m - m_0)c^2 = (\Delta m)c^2 = \frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}}, \; p = mv = \gamma m_0 v, \qquad (8.15)$$

We may notice at once that energy ε_B from (8.8), which we consider the motional energy of the electron in electric field of the atom, must be included in some amount in (8.14) or (8.15). In addition, we will notice that we may now represent Bohr's postulate (8.2) by the difference of two energy levels (which are in direct and mutually translated dependence, i.e. which mutually differ in some constant) in several ways, using energy expressions (8.8), (8.13), (8.14), or (8.15):

$$hf_{12} = h(f_{s2} - f_{s1}) = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = \Delta\varepsilon_B = \Delta\varepsilon_t = \Delta\varepsilon_k \; . \qquad (8.16)$$

In expressions (8.14)-(8.16) and later on, the symbol of electron's non-relativistic kinetic or total energy E_k, E, is simply transformed into its relativistic motional or total energy $\varepsilon_k, \varepsilon$, implicitly indicating that relativistic motional and/or total energy could (or should) have more complex nature, containing particle and wave manifestations of electron's energy, for instance.

If we are not sure which one of the previous energies from (8.16) is in absolute amount equal to the energy of the stationary electron wave, we could not then determine what the frequency of the stationary electron wave is. The thing we know at start (or may determine) is the set of all possible assumptions (where some of them figure only fictively and dimensionally as frequencies) among which at least one will represent the required stationary, eigenfrequency. Later, by elimination, we will find one real eigenfrequency (in its absolute amount) that satisfies the selection criterion (particularly by satisfying the criterion of group and phase velocity connection, and other elements of **PWDC**).

Now we may quote the following frequencies (connected to the state of the electrons on the stationary orbit):

-the frequency of the mechanical rotation of the electron, as a particle, around the atom nucleus, which is found in (8.4) and (8.5),

$$f_m = f_B' = \frac{v}{2\pi r} = \frac{mZ^2e^4}{4n^3h^3\varepsilon_0^2} = \frac{2\varepsilon_B}{nh} = \frac{2}{n} f_B \; , \qquad (8.17)$$

- the frequency that is dimensionally found in (8.8),

$$f_B = f_B'' = \frac{\varepsilon_B}{h} = \frac{mZ^2e^4}{8n^2h^3\varepsilon_0^2} = \frac{n}{2} f_B' = \frac{n}{2} f_m \; , \qquad (8.18)$$

- the frequency that is obtained based on the total energy of the electron (8.14),

$$f_t = \frac{\varepsilon_t}{h} = \frac{mc^2}{h} \; , \qquad (8.19)$$

- and the frequency that is obtained from the motion energy (8.15),

$$f_k = \frac{\varepsilon_k}{h} = \frac{(\Delta m)c^2}{h} \quad . \tag{8.20}$$

Considering all previous frequencies from (8.17) to (8.20), the second Bohr's postulate (8.16) formally becomes:

$$\begin{aligned} & hf_{12} = h(f_{s2} - f_{s1}) = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = \Delta\varepsilon_B = \Delta\varepsilon_t = \Delta\varepsilon_k = \\ & = \frac{1}{2} h(n_2 f_{B2}{}' - n_1 f_{B1}{}') = h(f_{B2} - f_{B1}) = h\Delta f_B = \\ & = h(f_{t2} - f_{t1}) = h\Delta f_t = \\ & = h(f_{k2} - f_{k1}) = h\Delta f_k \; . \end{aligned} \tag{8.21}$$

As one may see, in (8.21) several different frequencies (that are mutually linearly translated) figure here, which leads to the equality of their differences.

Now one may determine the phase velocity of a stationary electron wave, as a product of its corresponding wavelength, (8.12), and some of the frequencies from (8.17) to (8.20). Thus is possible to determine several (following) phase velocities, from which only one (real and exact) will be required:

$$\begin{aligned} & u_m = \lambda_s f_m = \lambda_s f_B{}' = u_B{}' = \frac{Ze^2}{2n^2 h\varepsilon_0} \quad , \\ & u_B = \lambda_s f_B = \lambda_s f_B{}'' = u_B{}'' = \frac{n}{2} u_B{}' = \frac{n}{2} u_m = \frac{Ze^2}{4nh\varepsilon_0} \quad , \\ & u_t = \lambda_s f_t = \frac{c^2}{v} \quad , \\ & u_k = \lambda_s f_k = c\sqrt{\frac{m - m_0}{m + m_0}} \; . \end{aligned} \tag{8.22}$$

Phase velocities from (8.22) are mutually very much different, but the physical essence of the electron orbital wave may describe only one of them. In order to form the selection criterion, let's start from fact that the orbital velocity of an electron (8.5) is equal to the group velocity of the stationary electron wave associated to it. Then, we will search for the phase velocity of a stationary electron wave as the phase velocity that satisfies the general equation of the group and phase velocity relation of (any) wave group,

$$v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} \quad . \tag{8.23}$$

The equation (8.23) is obtained from definitional expressions for group and phase velocity of the wave group, which are generally valid for any form of harmonic wave motions,

$$\mathbf{v} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{d\tilde{p}}\ ,\ \mathbf{u} = \frac{\omega}{k} = \frac{\tilde{E}}{\tilde{p}} = \lambda f\ ,\ \omega = 2\pi f\ ,\ k = \frac{2\pi}{\omega}\ , \tag{8.24}$$

So it is obvious that relation (8.23) may become the key arbiter in the explanation of the energy essence of the electron wave (i.e. in finding its phase velocity and frequency).

In fact, the relations (8.23) and (8.24) also belong to **PWDC** (**P**article-**W**ave-**D**uality-**C**ode), including $\tilde{E} = hf$, $\lambda = \frac{h}{\tilde{p}}$, too.

One may check and prove that equations and relations (8.23)-(8.24) describe a stationary electron wave that has:

-phase velocity (see (8.22)),

$$u_s = u_k = \lambda_s f_k = \lambda_s f_s = c\sqrt{\frac{m - m_0}{m + m_0}} = c\sqrt{\frac{\gamma - 1}{\gamma + 1}} = \frac{v}{1 + \sqrt{1 - (\frac{v}{c})^2}},\ m = \gamma m_0\ ,$$

-frequency,

$$f_s = f_k = \frac{\varepsilon_k}{h} = \frac{(\Delta m)c^2}{h} = n\frac{f_m}{2}(1 + \frac{u^2}{c^2}) = n\frac{mZ^2e^4}{8n^3h^3\varepsilon_0^{\ 2}}(1 + \frac{u^2}{c^2})\ ,$$

-energy, (8.25)

$$\varepsilon_s = hf_s = \varepsilon_k = (\Delta m)c^2 = muv\ ,$$

and wavelength,

$$\lambda_s = \frac{h}{p} = \frac{h}{c\sqrt{m^2 - m_0^{\ 2}}} = \frac{h}{m_0 c\sqrt{\gamma^2 - 1}}\ .$$

Due to obtaining greater evidence in checking the previous solutions, taking into account the cases when the orbital velocity of an electron is relatively small, $v << c$, previous expressions from (8.25), after adequate approximations, are reduced to the results known from Bohr's atom model, but one also obtains some new values (that Bohr's model does not generate) and which are typical for the wave concept of electron motion. Therefore, it is:

- phase velocity of the electron wave,

$$u_s = u_k = \lambda_s f_k = \lambda_s f_s = \frac{\omega}{k} = c\sqrt{\frac{m - m_0}{m + m_0}} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} \approx \tag{8.26}$$
$$\approx \frac{1}{2}v = u_B = \frac{n}{2}u_m = \frac{Ze^2}{4nh\varepsilon_0} ,$$

-group velocity of the electron wave (see (4.0.17), (4.0.26), and (4.0.28)),

$$v = \frac{d\omega}{dk} = \frac{2u_s}{1+\frac{{u_s}^2}{c^2}} \approx 2u_s = \frac{Ze^2}{2nh\varepsilon_0} , \tag{8.27}$$

- frequency of the orbital electron wave (see (4.0.31) and (4.0.39)),

$$f_s = f_k = \frac{\varepsilon_k}{h} = \frac{(\Delta m)c^2}{h} = n\frac{f_m}{2}(1 + \frac{u^2}{c^2}) \approx \tag{8.28}$$
$$\approx \frac{mv^2}{2h} = f_B = f_B'' = \frac{\varepsilon_B}{h} = \frac{mZ^2e^4}{8n^2h^3{\varepsilon_0}^2} = \frac{n}{2}f_m ,$$

-wavelength of the orbital electron wave (see (4.7), (4.8), and (8.12)),

$$\lambda_s = \frac{h}{p} = \frac{h}{c\sqrt{m^2 - {m_0}^2}} = \frac{2\pi r}{n} \approx \frac{2nh^2\varepsilon_0}{mZe^2} , \tag{8.29}$$

-and energy of the electron wave (see T4.1 and (4.0.8)),

$$\varepsilon_s = hf_s = \varepsilon_k = (\Delta m)c^2 = muv = \frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = nh\frac{f_m}{2}(1 + \frac{u^2}{c^2}) \approx . \tag{8.30}$$
$$\approx \frac{1}{2}mv^2 = \varepsilon_B = \frac{mZ^2e^4}{8n^2h^2{\varepsilon_0}^2} .$$

Conclusion that obtrudes from the previous analysis is that sources of the wave nature of the matter (precisely of the electron) are included only in the motional energy, and that rest mass of the electron does not enter the content of the accompanying wave energy. This is against the official presentation found in modern quantum theory, and which takes into account total energy and rest mass of the particle as elements of the wave equivalent of the particle. Almost all

possible misapprehensions and errors connected with questions about particle-wave dualism (in the atom world) originate from the differences of the corresponding relative energy levels (8.21) equal to the differences of their absolute energy levels. One could not claim in advance, which levels are absolute, and which relative (because all of them are mutually linearly translated for some constant energy level). As we have already seen, in a very close relation with the mentioned situation there was also the relation of the group and phase velocity.

In fact, the results (8.25)-(8.30) are completely in agreement with (8.9-2), mutually confirming and supporting each other, and fully describing the PWDC (Particle **W**ave **D**uality **C**ode**). The same expressions also explain the essential relations and intrinsic connections between mechanical rotation and de Broglie matter waves (when electron mass is treated as reduced mass, as in (8.9-1)).**

Bohr's second postulate (8.2) now becomes entirely determined (as a difference between absolute stationary energy levels),

$$\tilde{\mathbf{E}}_{\mathbf{f}} = \mathbf{hf}_{12} = \mathbf{h}(\mathbf{f}_{s2} - \mathbf{f}_{s1}) = \varepsilon_{s2} - \varepsilon_{s1} = \Delta\varepsilon_{s} = \Delta\varepsilon_{k} \quad , \tag{8.31}$$

because it is obvious that it relates only to the (relativistic) motion energy of the electron and to the orbital eigenfrequencies of the electron (8.28). Of course, applying Bohr's postulate (8.2), i.e. (8.31) to the frequency (8.28), with approximation $\mathbf{v} \ll \mathbf{c}$, one gets the known result for the emissive spectral distribution of the hydrogen atom (8.9).

Accuracy and applicability of the spectral formula (8.9) is far too great, yet one could consider Bohr's model not current and completely surpassed. On the contrary, analyzing that model we return to the origins of the atom science and face with some essential truths that apparently remained hidden (until now). By this occasion we also face with the possibility of the revision of some states valid hitherto, which have arisen during the construction of the concept of the particle-wave dualism.

------------------------ (This part, from here, should be additionally modified) ----------------
Let us try now to completely associate Bohr's atom model (in the context of the previous interpretation) with the general concept of the particle-wave dualism, which is presented in Fig.4.1 and relations (4.9) to (4.22). In the expression for the second Bohr's postulate, (8.31) figures a stationary orbital energy of the electron, which is in fact equal to the total (relativistic) motional energy of the electron (4.12). Of course, we may now consider that the electron has its orbital and stationary state (for which the main quantum number $\mathbf{n}$ is associated) with the total motional energy,

$$\varepsilon_{kn} = \mathbf{E}_{kn} + \tilde{\mathbf{E}}_{n} = \varepsilon_{sn} \quad . \tag{8.32}$$

If we now say that the electron crossed from its stationary state n = n_2 = 2, to its second stationary state n = n_1 = 1, and there emitted a photon $\tilde{\mathbf{E}}_{\mathbf{f}} = \mathbf{hf}_{12}$, the second Bohr's postulate (8.31) may be developed as,

$$\begin{aligned} \tilde{\mathbf{E}}_{\mathbf{f}} &= \mathbf{hf}_{12} = \mathbf{h}(\mathbf{f}_{s2} - \mathbf{f}_{s1}) = \Delta\varepsilon_{k} = (\mathbf{E}_{k2} + \tilde{\mathbf{E}}_{2}) - (\mathbf{E}_{k1} + \tilde{\mathbf{E}}_{1}) = \\ &= (\mathbf{E}_{k2} - \mathbf{E}_{k1}) + (\tilde{\mathbf{E}}_{2} - \tilde{\mathbf{E}}_{1}) = \Delta\mathbf{E}_{k} + \Delta\tilde{\mathbf{E}} \quad . \end{aligned} \tag{8.33}$$

As the emitted photon, $\tilde{\mathbf{E}}_{\mathbf{f}} = \mathbf{hf}_{12}$, is a form of wave energy that leaves the atom on the whole (and not only one its stationary orbit), we have a case of action and reaction, or better to say, a case of shooting a missile from a gun called an atom. It is obvious that the atom will in such case experience something like a reactive recoil, i.e. then we must apply the general balance of energy and impulse of the particle-wave dualism, which are given by expressions (4.14) and (4.20), but it is also clear that this general balance of energy will differ from the balance represented by (8.33). Of course, the difference between the total balance of the atom energy on the whole, and the case described by (8.33) is most likely, quantitatively, negligibly small, considering the relations of total energies among the electron, nucleus, and the emitted photon, but it should not be forgotten (or neglected) due to the qualitatively understanding of the situation in the full complexity of the particle-wave dualism.

Let us determine now the most general case of the balance of the atom energy that the photon will emit. Of course, if an atom, i.e. some of its electrons, before emitting a photon was excited (because according to the assumption it will emit photon in the next moment and return to its basic state), then such atom may be characterized by its total motional energy that is equal to the sum of the motional energy of its excited electron and the motional energy of the atom on the whole (let's say the motional energy of its nucleus). So, state of an atom before de-excitation may be presented as:

$$\varepsilon_{\mathbf{ka2}} = \mathbf{E}_{\mathbf{k2}} + \tilde{\mathbf{E}}_{2} + \mathbf{E}_{\mathbf{ka2}} + \tilde{\mathbf{E}}_{\mathbf{a2}} \quad , \tag{8.34}$$

where $\mathbf{E}_{\mathbf{k2}} + \tilde{\mathbf{E}}_{2}$ is total stationary motion energy of the excited electron, as given in (8.33), and $\mathbf{E}_{\mathbf{ka2}} + \tilde{\mathbf{E}}_{\mathbf{a2}}$, is total motion energy of an atom as a whole (or let's say the motion energy of the atom nucleus), which was neglected till now (or one started from an unsaid assumption that it was equal to zero). In a way analogous to the previous, the state of the whole situation after de-excitation (emitting a photon) may be expressed by the summary motional energy:

$$\varepsilon_{\mathbf{ka1}} = \mathbf{E}_{\mathbf{k1}} + \tilde{\mathbf{E}}_{1} + \mathbf{E}_{\mathbf{ka1}} + \tilde{\mathbf{E}}_{\mathbf{a1}} + \mathbf{hf}_{12} \quad , \tag{8.35}$$

where $\mathbf{E}_{\mathbf{k1}} + \tilde{\mathbf{E}}_{1}$ is total stationary motional energy of an electron in a new stationary orbit, and $\mathbf{E}_{\mathbf{ka1}} + \tilde{\mathbf{E}}_{\mathbf{a1}}$ is total motional energy of an atom as a whole (or let's say the motional energy of the atom nucleus) after de-excitation.

Let's apply now the law of conservation of total energy (4.14) to the cases given by (8.34) and (8.35), assuming that the rest energy (or rest mass) of an atom (before and after de-excitation) did not change, wherefrom one will get:

$$\begin{aligned} \varepsilon_{\mathbf{ka2}} - \varepsilon_{\mathbf{ka1}} &= \mathbf{E}_{\mathbf{k2}} + \tilde{\mathbf{E}}_{2} + \mathbf{E}_{\mathbf{ka2}} + \tilde{\mathbf{E}}_{\mathbf{a2}} - (\mathbf{E}_{\mathbf{k1}} + \tilde{\mathbf{E}}_{1} + \mathbf{E}_{\mathbf{ka1}} + \tilde{\mathbf{E}}_{\mathbf{a1}} + \mathbf{hf}_{12}) = \\ &= (\mathbf{E}_{\mathbf{k2}} - \mathbf{E}_{\mathbf{k1}}) + (\tilde{\mathbf{E}}_{2} - \tilde{\mathbf{E}}_{1}) + (\mathbf{E}_{\mathbf{ka2}} - \mathbf{E}_{\mathbf{ka1}}) + (\tilde{\mathbf{E}}_{\mathbf{a2}} - \tilde{\mathbf{E}}_{\mathbf{a1}}) - \mathbf{hf}_{12} = \mathbf{0} \,. \end{aligned} \tag{8.36}$$

It is now possible from (8.36) to determine the energy of the emitted photon as,

$$\begin{aligned} \tilde{\mathbf{E}}_{\mathbf{f}} = \mathbf{hf}_{12} &= (\mathbf{E}_{\mathbf{k2}} - \mathbf{E}_{\mathbf{k1}}) + (\tilde{\mathbf{E}}_{2} - \tilde{\mathbf{E}}_{1}) + (\mathbf{E}_{\mathbf{ka2}} - \mathbf{E}_{\mathbf{ka1}}) + (\tilde{\mathbf{E}}_{\mathbf{a2}} - \tilde{\mathbf{E}}_{\mathbf{a1}}) \approx \\ &\approx (\mathbf{E}_{\mathbf{k2}} - \mathbf{E}_{\mathbf{k1}}) + (\tilde{\mathbf{E}}_{2} - \tilde{\mathbf{E}}_{1}) \,. \end{aligned} \tag{8.37}$$

One may notice that the second Bohr's postulate (8.33) is identical to the situation described by (8.37), under condition that all forms of the motional energy of an atom as a whole, $(\mathbf{E_{ka2}} - \mathbf{E_{ka1}}) + (\tilde{\mathbf{E}}_{\mathbf{a2}} - \tilde{\mathbf{E}}_{\mathbf{a1}})$, may be neglected or mutually annulled, which is in essence correct (because the mass of an electron is negligible considering the mass of the atom nucleus, and rest masses of the electron and the nucleus did not change). If we now apply the law of conservation of the impulse (4.20), it would appear that the atom (under the same assumptions as hither) would receive approximately the impulse that corresponds to the impulse of the emitted photon with an opposite sign.

The nature of the wavelength of the total stationary electron wave (that is expressed through the wave impulse of an electron) remains to be explained. During the previous analysis classical de Broglie's definition of the wavelength was used for the wavelength of the electron wave ((8.12), (8.25), (8.29)), compared with the fact that in this paper we also exercise with the new definition of that same wavelength, through the wave impulse. **It is obvious that in the case of Bohr's atom model the equality of the absolute values of the particle and wave impulse of an electron that is in motion in its stationary orbit, i.e., $|\mathbf{p}| = |\tilde{\mathbf{p}}|$ is achieved. We can explain this situation in the following way. If the atom as a whole is in the state of rest, or, let the velocity of its center of mass equal zero, then orbital and stationary revolution of an electron around the atom nucleus is very balanced in the sense that some uniform, centrally-symmetrical, spatial distribution of the electron mass (across the whole area of the stationary orbit) exists. The total impulse of the electron, $\mathbf{P_e}$, which is equal to the vector sum of its impulse as a particle $\mathbf{p_e}$ and its wave impulse $\tilde{\mathbf{p}}_{\mathbf{e}}$, must be equal to zero (in opposite, if it is not equal to zero, the center of the atom mass would not be in the state of rest),**

$$\mathbf{P_e} = \mathbf{p_e} + \tilde{\mathbf{p}}_{\mathbf{e}} = \mathbf{p} + \tilde{\mathbf{p}} = \mathbf{0} \ . \tag{8.38}$$

From (8.38) we have:

$$\Delta\mathbf{p} + \Delta\tilde{\mathbf{p}} = \mathbf{0}\,,\ \mathbf{p} = -\tilde{\mathbf{p}} \ \Rightarrow\ |\mathbf{p}| = |\tilde{\mathbf{p}}| \ \Rightarrow\ \lambda_s = \frac{\mathbf{h}}{|\tilde{\mathbf{p}}|} = \frac{\mathbf{h}}{|\mathbf{p}|}\,. \tag{8.39}$$

In reality, when we search for the de Broglie's wavelength (of some energy state) we should know that every stable, uniform and/or stationary state (or state of relative rest) regarding some «Laboratory system» may be characterized either by its particle $\mathbf{p}$*, or by its wave momentum,* $\tilde{\mathbf{p}}$ *(*$\lambda = \frac{\mathbf{h}}{\tilde{\mathbf{p}}}$ **and / or** $\lambda = \frac{\mathbf{h}}{\mathbf{p}}$*), under the condition that we are not interested in its vector nature. Substantial differences (looking "vectorially and scalarly") between the wave and particle momentum originate in transient regimes when certain state experiences a modulation of its motion*. According to (8.39), quantizing the orbital momentum of an electron, or applying the first Bohr's postulate, (8.1), in context with the wave attributes of an electron is more logical to present by quantizing its wave orbital momentum,

$$\tilde{\mathbf{m}}\mathbf{vr} = \tilde{\mathbf{p}}\mathbf{r} = \mathbf{n}\frac{\mathbf{h}}{2\pi} \ . \tag{8.40}$$

In reality, instead of (8.38), for the stable atom in the state of rest we should take into account the total momentum of the atom $\mathbf{P_a}$ (including the nucleus momentum $\mathbf{P_p} = \mathbf{p_p} + \tilde{\mathbf{p}}_\mathbf{p}$). The result or the conclusion above will approximately stay the same, since the nucleus of the stable atom has a much bigger mass than the electron, and it will be almost in the state of rest (comparing to the revolving electron: see also (8.9-1) and (8.9-2)),

$$\begin{aligned} &\mathbf{P_a} = \mathbf{P_e} + \mathbf{P_p} = (\mathbf{p_e} + \tilde{\mathbf{p}}_\mathbf{e}) + (\mathbf{p_p} + \tilde{\mathbf{p}}_\mathbf{p}) = \mathbf{0} \Rightarrow \\ &\Rightarrow \mathbf{P_e} = \mathbf{p_e} + \tilde{\mathbf{p}}_\mathbf{e} \cong \mathbf{0} \text{ and } \mathbf{P_p} = \mathbf{p_p} + \tilde{\mathbf{p}}_\mathbf{p} \cong \mathbf{0} \Rightarrow \\ &\Rightarrow \Delta\mathbf{p_e} + \Delta\tilde{\mathbf{p}}_\mathbf{e} \cong \mathbf{0} \text{ and } \Delta\mathbf{p_p} + \Delta\tilde{\mathbf{p}}_\mathbf{p} \cong \mathbf{0}\,. \end{aligned} \tag{8.38-1}$$

As we know, the electron and the nucleus both revolve around their common center of gravity. Consequently, the total orbital momentum of a stable and neutral atom $\mathbf{L_a} = \mathbf{L_p} + \mathbf{L_e}$ (in the state of rest, analogously to (8.38-1)) should also be equal to zero,

$$\begin{aligned} &\mathbf{L_a} = \mathbf{L_e} + \mathbf{L_p} = (\ell_\mathbf{e} + \tilde{\ell}_\mathbf{e}) + (\ell_\mathbf{p} + \tilde{\ell}_\mathbf{p}) = \mathbf{0} \Rightarrow \\ &\Rightarrow (\Delta\ell_\mathbf{e} + \Delta\tilde{\ell}_\mathbf{e}) + (\Delta\ell_\mathbf{p} + \Delta\tilde{\ell}_\mathbf{p}) = \mathbf{0}\,, \end{aligned} \tag{8.38-2}$$

implicitly accepting that between the atom nucleus and the revolving electron there is always certain energy momentum coupling (and wave exchange).

By previous results and conclusions, Bohr's atom model fits into the general concept of the particle-wave dualism of this paper, which was the purpose of former analysis.

8.1. Some New Aspects of Atom Configuration from the view of the Bohr's Model

By further exploitation of the previously quoted database, one may draw some other interesting consequences that result from the richness of content of Bohr's atom model. For example, the frequency of the quantum of inter-orbital, electron exchange for stationary orbits with big quantum numbers is approximately equal to:

$$\mathbf{f_{12} = \frac{mZ^2e^4}{8h^3\varepsilon_0^{\ 2}}(\frac{1}{n_1^{\ 2}} - \frac{1}{n_2^{\ 2}}) \approx \frac{mZ^2e^4}{4n^3h^3\varepsilon_0^{\ 2}} = f_m \approx \frac{2f_s}{n}} \quad , \tag{8.41}$$

implying that it is possible to use the following approximations:

$$\mathbf{n_1 = n_2 + 1 \ >> \ 1 , \quad n_1 \approx n_2 \approx n \approx \sqrt{n_1 n_2} \approx (n_1 + n_2)/2 , \quad v << c} .$$

Looking from the other side, the frequency of the stationary electron wave (in the adequate orbit) is:

$$\begin{aligned} & f_s = nf_m \frac{1}{2}(1 + \frac{u^2}{c^2}) \approx \frac{mZ^2e^4}{8n^2h^3\varepsilon_0^{\ 2}} = \frac{n}{2} f_m \\ & \frac{n}{2} \leq \frac{f_s}{f_m} \leq n \ , \ \frac{1}{2} \leq \frac{f_s}{nf_m} \leq 1 \ , \ 0 \leq u \leq c. \end{aligned} \tag{8.42}$$

One may notice that for border cases, (8.41), when $\mathbf{n}$ is a great number, the frequency of the quantum of the electron emission is approximately equal to the corresponding orbital, mechanical (or rotational) frequency of an electron treated like a particle, f_m , (see (8.17)). By this, the known quantum mechanical principle of correspondence is really evoked by which Bohr intended to show where the borders between the macroscopic and quantum treatment of the matter are in the world of physics. However, one should pay attention that wave frequency or electromagnetic quantum frequency does not have very much in common with the revolution number of the particle around some center, (8.42), and this again represents something else for a stationary electron wave (the only common term is the term frequency and that they may be dimensioned by same units). **In fact, Bohr's principle of correspondence is now corrected and it differs from what Bohr at one moment claimed.**

Based on the expressions found for the group and phase velocity of an electron, (8.25), (8.26), and (8.27), as well as based on (4.0.27), (4.0.28), and (4.0.29), one checks that the following relations remain:

$$\mathbf{0 \leq u_s \leq v \leq c, \ \ 0 \leq u_s^{\ 2} \leq u_s v \leq v^2 \leq c^2} \ . \tag{8.43}$$

The most interesting is that by solving differential equations that connect group and phase velocity of an electron wave (which may be found in the same form as in expressions (4.0.27), (4.0.28), and (4.0.29)) one comes to adequate spectral distributions of radiating energies that remind us of the Planck's law of heated black-body emission.

Let us start from the expression for the group velocity of an electron wave (8.27) and let us connect it with (8.43), where one will get:

$$\mathbf{v} = \frac{2\mathbf{u}_s}{1+\frac{\mathbf{u}_s^2}{\mathbf{c}^2}} \approx 2\mathbf{u}_s = \frac{\mathbf{Ze}^2}{2\mathbf{nh}\varepsilon_0} < \frac{\mathbf{Z}_{max}\mathbf{e}^2}{2\mathbf{n}_{min}\mathbf{h}\varepsilon_0} < \mathbf{c}\ ,\ \mathbf{n}_{min} = 1,$$
$$\Rightarrow \mathbf{Z}_{max} < \frac{2\mathbf{c}\varepsilon_0\mathbf{h}}{\mathbf{e}^2} = \frac{1}{\alpha} = \frac{2\mathbf{h}}{\mu_0\mathbf{ce}^2} = 137.03604\ ,(\varepsilon_0\mu_0 = \frac{1}{\mathbf{c}^2}) \tag{8.44}$$
$$\Rightarrow \alpha\mathbf{Z}_{max} < 1.$$

In the previous expression, α is one of the known universal physical constants and it is called the constant of a thin (fine) structure. Also from (8.44) one may conclude that the maximal possible protons' number of some element is $\mathbf{Z}_{max} \leq \mathbf{137}$. Since it is not a much bigger number than hitherto known maximal protons number of the last known element from the periodic table, one may expect discovering around twenty new elements of the periodic table. Of course, concluding based on the previous (simplified) analysis is approximate by its character to be on the proof level, but it is logically non-contradictory and interesting.

Since in the Bohr's planetary atom model some circular motion of electrons (or their stationary waves) is immanently present, it is important to determine mechanical and magnetic orbital moment of the electron (in the stationary orbit).

Mechanical orbital moment or torque of the electron is,

$$L = L_e = J_e\omega_m = \gamma mvr = \gamma m\omega_m r^2 = 2\gamma m\pi r^2 f_m\, , J_e = \gamma mr^2\, , \tag{8.45}$$

and magnetic moment of the electron is,

$$\mathbf{M} = \mathbf{I}_e\mathbf{S} = \mathbf{I}_e\pi\mathbf{r}^2\ , \tag{8.46}$$

where $\mathbf{I}_e$ is an elementary current of a rotating electron across its orbit, and $\mathbf{S}$ is the surface encompassed by that orbit.

If we now determine the elementary current of a rotating electric charge as the product of the electron charge $\mathbf{e}$ and corresponding frequency by which that charge really orbits $\mathbf{f}_e$, there will be,

$$\mathbf{I}_e = \mathbf{ef}_e \ \Rightarrow\ \mathbf{M} = \pi\mathbf{r}^2\mathbf{ef}_e\, . \tag{8.47}$$

Quotient of the magnetic and mechanical momentum of an electron is called gyromagnetic ratio and it has great significance in describing the magnetic properties of materials. Such is

$$\frac{\mathbf{M}}{\mathbf{L}} = \frac{\pi\mathbf{r}^2\mathbf{e}}{2\pi\mathbf{r}^2\gamma\mathbf{m}}\frac{\mathbf{f}_e}{\mathbf{f}_m} = (\frac{\mathbf{e}}{2\gamma\mathbf{m}})\frac{\mathbf{f}_e}{\mathbf{f}_m}\ . \tag{8.48}$$

There remains an open question how to treat the frequency $\mathbf{f_e}$ by which the charge orbits around the atom nucleus. In general case we have two possibilities, and they are: **a)** that the electron charge and its mass are always concentrated in the same space spot, i.e. that there is some compact material object that orbits around the atom nucleus by its mechanical frequency, so there will be $\mathbf{f_e = f_m}$, or **b)** that electron in the stationary orbit is a somewhat different form of spatially distributed mass and electric charge (across the whole orbit area), when the notion of mechanical rotational frequency of the electron mass differs from the frequency by which the distributed electric charge of the electron orbits $\mathbf{f_e \neq f_m}$, and when we may consider that the orbiting frequency of the electron is equal to the frequency of its stationary electron wave $\mathbf{f_e = f_s}$, (see (8.42)), and c) that all previously mentioned frequencies are mutually dependent, but principally different $\mathbf{f_e \neq f_m \neq f_s}$.

In the first case, if the electron charge would orbit by its mechanical rotation frequency around the atom nucleus, $\mathbf{f_e = f_m}$, the gyromagnetic ratio would be:

$$\begin{aligned}
&\frac{\mathbf{M}}{\mathbf{L}} = \left(\frac{\mathbf{e}}{2\gamma\mathbf{m}}\right)\frac{\mathbf{f_e}}{\mathbf{f_m}} = \frac{\mathbf{e}}{2\gamma\mathbf{m}} = \frac{\mathbf{e}}{2\mathbf{m}}\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}\,, \\
&0 < \frac{\mathbf{M}}{\mathbf{L}} \leq \frac{\mathbf{e}}{2\mathbf{m}}\,,\ 0 \leq \mathbf{v} \leq \mathbf{c}\,,
\end{aligned} \qquad (8.49)$$

which is the known case found in the existing textbook literature.

In the second case, when the electron charge forms a stationary orbital electron wave and orbits by eigenfrequency of that wave around the atom nucleus, $\mathbf{f_e \approx f_s}$, the gyro-magnetic ratio will be:

$$\begin{aligned}
&\frac{\mathbf{M}}{\mathbf{L}} = \left(\frac{\mathbf{e}}{2\gamma\mathbf{m}}\right)\frac{\mathbf{f_e}}{\mathbf{f_m}} = \frac{\mathbf{n}}{2}\left(\frac{\mathbf{e}}{2\gamma\mathbf{m}}\right)\left(1+\frac{\mathbf{u_s}^2}{\mathbf{c}^2}\right)\frac{\mathbf{f_e}}{\mathbf{f_s}} = \frac{\mathbf{n}}{2}\left(\frac{\mathbf{e}}{2\mathbf{m}}\right)\left(1-\frac{\mathbf{u_s}^2}{\mathbf{c}^2}\right)\frac{\mathbf{f_e}}{\mathbf{f_s}}\,, \quad (0 \leq \mathbf{u_s} \leq \mathbf{c}), \\
&\mathbf{f_s} = \mathbf{n}\mathbf{f_m}\frac{1}{2}\left(1+\frac{\mathbf{u}^2}{\mathbf{c}^2}\right), \mathbf{f_m} = 2\mathbf{f_s}/\mathbf{n}\left(1+\frac{\mathbf{u}^2}{\mathbf{c}^2}\right) \\
&\Rightarrow\ 0 < \frac{\mathbf{M}}{\mathbf{L}} \leq \frac{\mathbf{n}}{2}\left(\frac{\mathbf{e}}{2\mathbf{m}}\right)\frac{\mathbf{f_e}}{\mathbf{f_s}} \cong \frac{\mathbf{n}}{2}\left(\frac{\mathbf{e}}{2\mathbf{m}}\right)\,,\ \mathbf{L} = \mathbf{n}\hbar\,,\ \left(\hbar = \frac{\mathbf{h}}{2\pi}\right), \\
&\Rightarrow\ \mathbf{M} = (\mathbf{n}\hbar)\frac{\mathbf{n}}{2}\left(\frac{\mathbf{e}}{2\gamma\mathbf{m}}\right)\left(1+\frac{\mathbf{u_s}^2}{\mathbf{c}^2}\right)\frac{\mathbf{f_e}}{\mathbf{f_s}} = (\mathbf{n}\hbar)\frac{\mathbf{n}}{2}\left(\frac{\mathbf{e}}{2\mathbf{m}}\right)\left(1-\frac{\mathbf{u_s}^2}{\mathbf{c}^2}\right)\frac{\mathbf{f_e}}{\mathbf{f_s}} = (\mathbf{n}\hbar)\frac{\mathbf{n}}{2}\left(\frac{\mathbf{e}}{2\mathbf{m}}\right)\left(1-\frac{\mathbf{u_s}^2}{\mathbf{c}^2}\right)\,, \\
&0 \leq \mathbf{M} \leq (\mathbf{n}\hbar)\frac{\mathbf{n}}{2}\left(\frac{\mathbf{e}}{2\mathbf{m}}\right)\frac{\mathbf{f_e}}{\mathbf{f_s}} \cong (\mathbf{n}\hbar)\frac{\mathbf{n}}{2}\left(\frac{\mathbf{e}}{2\mathbf{m}}\right).
\end{aligned} \qquad (8.50)$$

There is an obvious difference between possible treatments of the gyromagnetic ratio, depending on how we treat the frequency of an electron charge rotation around the atom nucleus (8.49) and (8.50), and in what relation the orbital eigenfrequency of the electron (de Broglie's) wave is with the (mechanical) rotation frequency of its mass and charge. It is probable that in this area it is possible to look for the weak points of Bohr's atom model and quantum theory, which was later added to the fact that the frequency of the mechanical rotation of an electron was wrongly equalized with the frequency of its orbital (de Broglie's) wave.

The point is that the so-called mechanical rotational frequency represents the revolution number of the material point around some center of rotation in the unit of time, and that the frequency of the wave motion (or some harmonic form) is characterized through relations (8.23) and (8.24). It is obvious that we have here two different notions and phenomena for which in certain situations an identical term has been used: frequency, which later lead to all other consequences found now in literature about quantum physics. Sometimes is very hard to make nuance between the senses of certain (dimensionally equal) close notions.

Of course, after the previous plot, we can make the situation with the gyromagnetic ratio more complex, introducing the spin (local) rotation of the electron (around itself), which will be superimposed to the total magnetic momentum of the electron (that orbits around the nucleus), etc. Analyzing the problem of the gyromagnetic ratio in the previous way easily leads to the hypothesis that an electron addressed integrally as part of the atom shell is not a particle that orbits around the nucleus any more, but some diffused and distributed particle-wave state, which binds to itself characteristics of rotation, but not any more in a sense of motion of strictly spatially localized particles, which is a logical way the quantum mechanics took. We may only add that electron-mass distribution and electron-charge distribution involved in such orbital motion, most likely comply with mutually dependent but different distribution laws. The confirmation of the previous opinion is also the fact that submitting the atoms to high pressures (or low temperatures) leads to their phase transformations of the material properties and to body and energy restructuring of their electron shells. In addition, it is clear that we must find expressions different from those done hitherto for the electron radius, or for the area that the electron occupies.

The most important implication of the situation mentioned above is related to PWDC, or to the situation that linear motion and rotation are intrinsically coupled (similar to the coupling of electric and magnetic field).

[♣ <u>The aim of such seemingly exceeded analyses is to point out that in essence there is no problem of different view of the structure of matter from the point of view of classical mechanics or quantum wave mechanics, but that we know insufficiently some expressions, i.e. their logical links and couplings, or that some theoretical concepts are partially set incorrectly from the beginning, to formally construct those, in certain historical moment, unknown links.</u> **<u>Of course, erroneous theoretical postulates must be added onto by new erroneous concepts in order to be corrected according to the natural truth given by experiment, unless we are ready to reject primary and basic theoretical hypothesis (that was erroneous).</u>** The most ungrateful moment during writing such papers is that they may be superiorly criticized from <u>the point of view</u> of the existing, formed and generally adopted theoretical concepts and models that have already entered the history of certain scientific opinion, encyclopedias, and textbooks and seemingly fit into the experimental database (and that from <u>the modern point of view</u> such papers are neither relevant, nor interesting too much any more). Namely, it is known that for certain, relevant set of facts a model good enough may be formed, which connects all facts, but that in essence that model is very bad if it is later considered from some substantially broader set of relevant facts, which will be discovered by experimental practice. Intellectual rigidity, inertia of thinking and dogmatic compliance to the adopted standpoints that haven't changed for years, represent great obstacle in introducing new ideas that undermine the structure of the formed models. Because of that, especially in physics, it happens that some models are simply exhausted at certain moment, abandoned, and replaced by new ones, because they serve no more to any new, objective of scientific predictions, and they cannot explain all phenomena from some field. Of course, this has the good and the bad side, because all abrupt jumps are risky and problematic, and scientific opinion demands continuity of the cognitive process from simpler models to the more complex, which approximately imply their predecessors. ♣]

8.2. Bohr's Atom Model and Stationary States of the Nucleus

Until now a somewhat static atom model has been discussed, assuming that electron orbits around the stable and immobile nucleus (which is almost correct, regarding the ratio of the mass of the nucleus and the mass of the electron), and it is more precise to say that the electron and the proton orbit around the common center of inertia.
As an illustration of the general validity of here introduced platform for treating de Broglie waves, let us analyze a hydrogen atom in the center-of-mass coordinate system, similar to the one shown in Fig.4.1. Let us apply (4.1), (4.2), and (4.3) to describe the movement of the electron and the proton around their common center of gravity, and to describe their associated de Broglie waves, assuming that the electron and atom nuclei are treated as rotating particles (slightly modified Bohr's model), having the following characteristics: m_e-electron mass, $\mathbf{m_p}$-nucleus or proton mass, $\mathbf{v_e = \omega_{me} r_e}$-electron velocity around the common center of gravity, $\mathbf{v_p = \omega_{mp} r_p}$-proton velocity around the common center of gravity, r_e-radius of revolving electron, $\mathbf{r_p}$-radius of revolving proton, $\mathbf{\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m}$ -mechanical, revolving frequency of electron and proton, $\mathbf{\lambda_e = h / \gamma_e m_e v_e = h / p_e}$ -de Broglie wavelength of electron wave, $\mathbf{\lambda_p = h / \gamma_p m_p v_p = h / p_p}$ -de Broglie wavelength of proton wave, $\mathbf{f_e}$-de Broglie frequency of electron wave, $\mathbf{f_p}$ -de Broglie frequency of proton wave, $\mathbf{u_e = \lambda_e f_e}$-phase velocity of the de Broglie electron wave and $\mathbf{u_p = \lambda_p f_p}$-phase velocity of the de Broglie proton wave. In addition, the following basic relations also belong to the above-described Bohr's hydrogen atom model: $\mathbf{\gamma_e m_e r_e^2 = \gamma_p m_p r_p^2}$, $\mathbf{\gamma_e m_e r_e^2 \omega_m = \gamma_p m_p r_p^2 \omega_m}$, $\mathbf{\gamma_e m_e v_e r_e = \gamma_p m_p v_p r_p}$, $\mathbf{n\lambda_e = 2\pi r_e}$, $\mathbf{n\lambda_p = 2\pi r_p}$ (see **[4]** regarding the same subject).

In a few steps, the following correct relations (between all the parameters described above, see (4.2) and (4.3)), valid for Bohr's atom model, can easily be found (see also **[4]**):

$$\mathbf{\frac{m_p}{m_e} = \frac{\gamma_e}{\gamma_p}(\frac{r_e}{r_p})^2 = \frac{\gamma_e}{\gamma_p}(\frac{v_e}{v_p})^2 = \frac{\gamma_e}{\gamma_p}(\frac{\lambda_e}{\lambda_p})^2 = \frac{\gamma_e}{\gamma_p}(\frac{u_e}{u_p})^2 \cdot (\frac{f_p}{f_e})^2 = \frac{\gamma_e}{\gamma_p}(\frac{u_e}{u_p})^2 \cdot (\frac{\tilde{E}_p}{\tilde{E}_e})^2 =}$$

$$\mathbf{= \frac{\gamma_e v_e \lambda_e}{\gamma_p v_p \lambda_p} = \frac{\gamma_e v_e r_e}{\gamma_p v_p r_p} = 1836.13 = \frac{m_p c^2}{m_e c^2}, \ f_m = f_{me} = f_{mp} = f_{m(e,p)} = \omega_m / 2\pi,}$$

or for $(v_e, v_p, u_e, u_p << c) \Rightarrow$

$$\frac{m_p}{m_e} = 1836.13 \cong \left[(\frac{r_e}{r_p})^2 = (\frac{v_e}{v_p})^2 = (\frac{\lambda_e}{\lambda_p})^2 = (\frac{u_e}{u_p})^2 \cdot (\frac{f_p}{f_e})^2 = \frac{v_e \lambda_e}{v_p \lambda_p} = \frac{v_e r_e}{v_p r_p} \right],$$

$$\mathbf{f_e = n\frac{f_m}{2}(1 + \frac{u_e^2}{c^2}) = n\frac{m_e Z^2 e^4}{8n^3 h^3 \varepsilon_0^2}(1 + \frac{u_e^2}{c^2}) \cong (f_p, \text{ for } u_e << c, u_e = c),}$$

$$\mathbf{f_p = n\frac{f_m}{2}(1 + \frac{u_p^2}{c^2}) = n\frac{m_e Z^2 e^4}{8n^3 h^3 \varepsilon_0^2}(1 + \frac{u_p^2}{c^2}) \cong (f_e, \text{ for } u_p << c, u_p = c),}$$

$$1 \le \frac{v_e}{u_e} = n \cdot \frac{f_m}{f_e} = \frac{2}{(1+\frac{u^2{}_e}{c^2})} = 1 + \sqrt{1 - \frac{v^2{}_e}{c^2}} \le 2,$$
$$1 \le \frac{v_p}{u_p} = n \cdot \frac{f_m}{f_p} = \frac{2}{(1+\frac{u^2{}_p}{c^2})} = 1 + \sqrt{1 - \frac{v^2{}_p}{c^2}} \le 2,\ n = 1, 2, 3, \ldots \tag{4.4}$$
$$\frac{f_p}{f_e} = \frac{1+\frac{u^2{}_p}{c^2}}{1+\frac{u^2{}_e}{c^2}} = \frac{1+\sqrt{1-\frac{v^2{}_e}{c^2}}}{1+\sqrt{1-\frac{v^2{}_p}{c^2}}} = \frac{1+\frac{1}{\gamma_e}}{1+\frac{1}{\gamma_p}} = \frac{v_e}{v_p}\frac{u_p}{u_e}.$$

It is important to underline that the revolving mechanical frequency of an electron and proton around their common center of gravity, $\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m$, is something that should not be mixed or directly (quantitatively) associated with de Broglie wave frequency of the (stationary or orbital) electron and/or proton wave/s, i.e., $\omega_m = 2\pi f_m \ne (\omega_e = 2\pi f_e \ne \omega_p = 2\pi f_p)$, $f_{e,p} \le n \cdot f_m = f_{e,p} \cdot (1 + \sqrt{1 - v^2{}_{e,p} / c^2}) \le 2 f_{e,p}$.

From (4.4) we can also conclude that (wave) energy of the stationary electron wave $(hf_e = (\gamma_e - 1) m_e c^2 = \gamma_e m_e v_e u_e = p_e u_e = E_k)$ *is fully equal to the electron's motional energy (the rest electron mass or its rest energy have no participation in this energy).* Obviously, a relation similar to (4.4) should be valid for planets rotating around their suns, except that the mass ratio is a different number (and planets in their solar systems should also have their associated de Broglie waves). For instance, our planet Earth rotates around its sun and at the same time rotates around its own planetary axis, performing similar motion as presented in Fig.4.1 (but numerical values of the de Broglie wavelength and frequency are meaningless for such big objects).

It is obvious that the atom nucleus also has some form of complex motional energy (inherent internal motional energy, including the adequate rest energy, too). Let's observe now the possible influence nucleus-electron in the laboratory system, and in the center-of-mass system. Since we are interested only in aspects of photon emission and absorption, we know that on this occasion there is no grasp of the rest masses (similarly as in (8.36) and (8.37)), so the law of conservation of the total energy is identical to the law of conservation of the motional energy, and we can apply it in the following way (as in (4.0.44) and (4.0.47)):

$$\varepsilon_{kj} + \varepsilon_{ke} = \varepsilon_{kc} + \varepsilon_{je} \Leftrightarrow E_{kj} + \tilde{E}_j + E_{ke} + \tilde{E}_e = E_{kc} + \tilde{E}_c + E_{je} + \tilde{E}_{je}\ , \tag{8.51}$$

This part, starting here, should be reformulated and generalized.

In the following expressions we will use the following symbols and values:

m_e, M_p -masses of the electron and the proton,

v_e, v_j, v_c -velocities of the electron, the nucleus, and the mass-center of the electron-nucleus system,

$$v_c = \frac{m_e v_e + M_p v_j}{m_e + M_p} \approx (\frac{m_e}{M_p})v_e + v_j$$ -center of mass velocity.

Now the corresponding energies (given by their non-relativistic representatives),

$\boldsymbol{\varepsilon_{kj} = E_{kj} + \tilde{E}_j}$ - total energy of the motion of the nucleus in the laboratory system,

$\boldsymbol{\varepsilon_{ke} = E_{ke} + \tilde{E}_e \approx \frac{1}{2} m_e v_e^2}$ - energy of the motion of the electron in the laboratory system,

$$\boldsymbol{\varepsilon_{kc} = E_{kc} + \tilde{E}_c \approx \frac{1}{2}(M_p + m_e)v_c^2 \approx \frac{1}{2}M_p v_c^2 \approx}$$
$$\boldsymbol{\approx (\frac{m_e}{M_p})\frac{1}{2}m_e v_e^2 + \frac{1}{2}M_p v_j^2 + m_e v_e v_j \approx (\frac{m_e}{M_p})\varepsilon_{ke} + \frac{1}{2}M_p v_j^2 + m_e v_e v_j}$$ - energy of the

motion of the mass-center of the electron-nucleus system,

$\boldsymbol{\varepsilon_{je} = E_{je} + \tilde{E}_{je} = \varepsilon_r \approx \frac{m_e M_p}{2(m_e + M_p)}\left|v_e - v_j\right|^2 \approx \frac{1}{2}m_e v_e^2 \approx \varepsilon_{ke}}$ -relative energy of the motion of the electron towards the nucleus, or electron-nucleus reaction energy (see (4.0.62)).

In non-relativistic case (when all figuring velocities of the nucleus and the electron are negligible regarding the light speed), the balance of energy (8.51) becomes,

$$\boldsymbol{\varepsilon_{kj} + \frac{1}{2}m_e v_e^2 \approx \frac{1}{2}(M_p + m_e)v_c^2 + \frac{m_e M_p}{2(m_e + M_p)}\left|v_e - v_j\right|^2} . \tag{8.52}$$

In case of the hydrogen atom, it is logical to assume that $\boldsymbol{v_j << v_e \text{ i } m_e << M_p}$, so (8.51) or (8.52) becomes:

$$\boldsymbol{\begin{aligned} \varepsilon_{kj} = E_{kj} + \tilde{E}_j &\approx \frac{1}{2}M_p v_j^2 + (\frac{m_e}{M_p})\frac{1}{2}m_e v_e^2 + m_e v_e v_j \approx \\ &\approx \frac{1}{2}M_p v_j^2 + (\frac{m_e}{M_p})\varepsilon_{ke} + m_e v_e v_j . \end{aligned}} \tag{8.53}$$

In the previous expressions one should distinguish the total motional energy ε_k from the energy of particle motion E, in the same way it differentiated in (4.12). If we start from the fact that photo-excitation of the atom has no interference with the rest mass of the nucleus, i.e. that it is relevant only for the electron stationary states in the shell, which is correct in essence, then one may transform (8.53) in the following way:

$$\varepsilon_{kj} = E_{kj} + \tilde{E}_j \approx \frac{1}{2}M_p v_j^{\ 2} + (\frac{m_e}{M_p})\varepsilon_{ke} + m_e v_e v_j \approx$$
$$\approx E_{kj} + (\frac{m_e}{M_p})\varepsilon_{ke} + m_e v_e v_j , \qquad (8.54)$$
$$\Rightarrow \tilde{E}_j = (\frac{m_e}{M_p})\varepsilon_{ke} + m_e v_e v_j \approx (\frac{m_e}{M_p})\varepsilon_{ke} .$$

From (8.54) we can determine the total motional energy of an electron,

$$\varepsilon_{ke} \approx (\frac{M_p}{m_e})(\varepsilon_{kj} - E_{kj} - m_e v_e v_j) . \qquad (8.55)$$

In the first approximation, starting from the fact that the atom nucleus observed as a particle, is in essence at rest, i.e. that eventual velocity of the nucleus motion is almost equal to zero, and undoubtedly far smaller than the velocity of the electron (thereby the particle energy of the motion of the nucleus, E_{kj}, will be equal to zero) the previous relation, (8.55), will become,

$$\varepsilon_{ke} \approx (\frac{M_p}{m_e})\varepsilon_{kj} \quad , \ (E_{kj} + m_e v_e v_j \approx 0) . \qquad (8.56)$$

We may now ask ourselves what happens if an atom is hit (excited) by a photon, or emits a photon, i.e. we can apply the second Bohr‘s postulate (8.31) on (8.56), so one gets,

$$\tilde{E}_f = hf_{e12} = h(f_{s1} - f_{s2}) = \varepsilon_{s2} - \varepsilon_{s1} = \Delta\varepsilon_s = \Delta\varepsilon_k =$$
$$= \Delta\varepsilon_{ke} \approx (\frac{M_p}{m_e})\Delta\varepsilon_{kj} = 1836.13\Delta\varepsilon_{kj} \approx 1836.13 \cdot \frac{\gamma_p}{\gamma_e} \cdot \frac{(u_p\tilde{E}_e)^2}{(u_e\tilde{E}_p)^2}\Delta\varepsilon_{ke} \approx 1836.13 \cdot \frac{(u_p\tilde{E}_e)^2}{(u_e\tilde{E}_p)^2} \cdot \Delta\varepsilon_{ke}$$
$$(\frac{M_p}{m_e} \approx \frac{\Delta\varepsilon_{ke}}{\Delta\varepsilon_{kj}}) = 1836.13 = \frac{\gamma_e \cdot (u_e\tilde{E}_p)^2}{\gamma_p \cdot (u_p\tilde{E}_e)^2} \approx \frac{(u_e\tilde{E}_p)^2}{(u_p\tilde{E}_e)^2} ,$$
$$\Rightarrow m_e\Delta\varepsilon_{ke} \approx M_p\Delta\varepsilon_{kj} , \ \frac{\tilde{E}_e^{\ 2}\Delta\varepsilon_{ke}}{\gamma_e u_e^{\ 2}} = \frac{\tilde{E}_p^{\ 2}\Delta\varepsilon_{kj}}{\gamma_p u_p^{\ 2}} , \ 1836.13 \cdot \gamma_p \cdot (u_p\tilde{E}_e)^2 = \gamma_e (u_e\tilde{E}_p)^2 .$$
$$(8.57)$$

Starting from the fact that every change of motional energy is accompanied by creation of corresponding element of wave energy (neglecting the change of internal energy or rest mass of particles, which is at the time quite justified, we may formally transform (8.57) into

$$m_e\Delta\varepsilon_{ke} \approx M_p\Delta\varepsilon_{kj} \Leftrightarrow m_e hf_{e12} \approx M_p hf_{j12}$$
$$\Leftrightarrow m_e f_{e12} \approx M_p f_{j12} . \qquad (8.58)$$

Based on (8.57) and (8.58) a conclusion obtruded is that every inter-orbital change of stationary energy of an electron is accompanied by some similar response in a form of a coincident change of (some) stationary energy levels of the nucleus (regardless what it meant), and that all those changes of the stationary levels are accompanied by emission or absorption of corresponding photons of interchange (as in the electron shell, so in the area of the atom nucleus). In fact, (8.58) represents the reaction or echo of the atom nucleus to the change that takes place in the atom shell, which is logically (as well as from an angle of the law of conservation of impulses) quite reasonable.

Of course, "vectorially" looking, if there is some wave reaction of the atom nucleus to the change that hits its electron shell, they will be vectors (impulses) with same direction, and with opposite senses.

Since the masses of the electron and the proton are known, it is possible to give (8.58) in a form of numerical relations,

$$\begin{aligned} & \mathbf{f_{j12} \approx 5.446 \cdot 10^{-4} f_{e12}} \ , \\ & \mathbf{f_{e12} \approx 1836.1 \cdot f_{j12}} \ . \end{aligned} \qquad (8.59)$$

By (8.58) and (8.59) we see that it is likely that there are radiating and absorptive spectra of the atom nucleus that are coincident with absorptive and radiating spectra of the electron shell (and the relations of proportionality between corresponding frequencies are given). There is another question- how to detect previously predicted (hypothetical) nucleus spectrum, and whether some interferences between the nucleus spectrum and electron spectrum occur or not. **It is immediately noticeable that hypothetical nucleus spectrum (or spectral ECHO of the nucleus) will be in the infrared and micro-wave part of the spectrum (partly overtaking the area of millimeter wavelengths and cosmic, background radiation), and that further means that Planck's law of blackbody radiation will be supplemented by a similar law of radiation, which is translated into a domain of lower frequencies for a factor $\mathbf{f_{j12} \approx 5.446 \cdot 10^{-4} f_{e12}}$, with a much smaller intensity and greater frequency density of corresponding wave components.** Perhaps on this occasion we could also pose a question about the existence of communication between stationary levels of the atom nucleus and shell, by the existence of some quantized spectra of interchange of corresponding photons, which all falls down into the hypothetical aspects of the previous process of analysis and concluding.

We can now present (8.59) in a complete form that defines combinational spectrum of the electromagnetic emission or absorption of the atom, using (8.9) or (8.41),

$$\begin{aligned} & \mathbf{f_{e12} = \frac{m_e Z^2 e^4}{8h^3 \varepsilon_0^{\ 2}} \left(\frac{1}{n_1^{\ 2}} - \frac{1}{n_2^{\ 2}}\right)} , \\ & \mathbf{f_{j12} \approx \left(\frac{m_e}{M_p}\right) \frac{m_e Z^2 e^4}{8h^3 \varepsilon_0^{\ 2}} \left(\frac{1}{n_1^{\ 2}} - \frac{1}{n_2^{\ 2}}\right)} . \end{aligned} \qquad (8.60)$$

Knowing the relations between the masses of the electron and the proton, as well as between frequency and the wavelength of the electromagnetic quantum (based on calculations from (8.60)), it is possible to obtain the following domains of defining the hydrogen spectrum,

$$\frac{f_{e12}}{f_{j12}} = \frac{\lambda_{j12}}{\lambda_{e12}} \approx (\frac{M_p}{m_e}) \approx 1836.1 \,,$$
$$f_{e12} \leq 3.2898417 \cdot 10^{15}\,\text{Hz} \,,\, \lambda_{e12} \geq 0.091126714\mu\text{m} \,, \tag{8.61}$$
$$f_{j12} \leq 1.7917551 \cdot 10^{12}\,\text{Hz} \,,\, \lambda_{j12} \geq 167.31776\mu\text{m} \,.$$

Since the hydrogen atom is one of the most prevalent elements in the cosmic universe, the previously given quantitative relations (8.61) should be almost universally valid, but if we would like to encompass all possible atoms (heavier than the hydrogen atom), we could broaden the meaning of the relation (8.58), considering that the nucleus mass of any atom is equal to the sum of masses of the protons and the neutrons that constitute the nucleus, so we shall have:

$$Z \cdot m_e f_{e12} \approx \left[ZM_p + (A - Z)M_n\right] f_{j12} \approx AM_p f_{j12} \,. \tag{8.62}$$

If we assume that all possible atoms and their isotopes (known and unknown) will be encompassed with atom number $A \leq 6Z$, (8.58) and (8.62) will generate the following relations,

$$\frac{f_{e12}}{f_{j12}} = \frac{\lambda_{j12}}{\lambda_{e12}} \leq 6(\frac{M_p}{m_e}) \approx 6 \cdot 1836.1 = 11016.6 \,,$$
$$f_{e12} \leq Z \cdot 3.2898417 \cdot 10^{15}\,\text{Hz} \,,\, \lambda_{e12} \geq \frac{1}{Z} \cdot 0.091126714\mu\text{m} \,, \tag{8.63}$$
$$f_{j12} \leq \frac{Z}{6} \cdot 1.7917551 \cdot 10^{12}\,\text{Hz} \,,\, \lambda_{j12} \geq \frac{6}{Z} \cdot 167.31776\mu\text{m} \,.$$

At the end one could pose a question: is the relict, background radiation of the universe an exclusive reflection of the primary cosmic explosion (**BIG-BANG**), or it is maybe strictly connected (or superimposed) with real-time «**ECHO**» radiation of the atom/s nuclei (as previously described)? If we try to apply general relations of elementary certainty (4.0.64) from the chapter 4.0, and (5.1) from the chapter 5., i.e. Relations of Uncertainty (Heisenberg), to intervals of the spectral (and time) duration of the background radiation of the universe (known from the measurements of the background radiation and other theoretical estimates), it will be shown that those relations are not satisfied, i.e. that the current pattern of the primary cosmic explosion, even though the concept is completely qualitatively justified, has significant (at best for that concept, quantitative) faults.

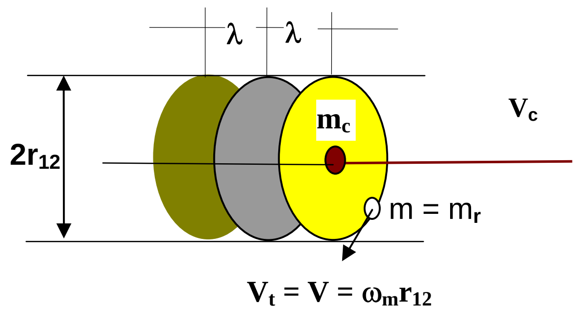

Fig. 4.1 Two particle problem presented in equivalent center of mass system

$$E_k = pu = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{J\omega_m^{\ 2}}{1+\sqrt{1-\frac{v^2}{c^2}}} = (\frac{J4\pi^2 f_m^{\ 2}}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{J4\pi^2}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{v^2}{(2\pi r_{12})^2} =$$

$$= \frac{Jv^2}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{1}{(r_{12})^2} = \frac{\gamma m(r_{12})^2 v^2}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{1}{(r_{12})^2} = \frac{\gamma m v^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}}) \Leftrightarrow$$

$$\Leftrightarrow \tilde{E} = \tilde{p}u = h\frac{\omega}{2\pi} = hf = \hbar\omega,$$

$$\Delta E_k = -\Delta\tilde{E},\ p = \gamma m v \Leftrightarrow \tilde{p} = h\frac{k}{2\pi} = hf_s = \hbar k,\ J \approx \gamma m r_{12}^{\ 2},$$
$$\text{where: } \omega_m = 2\pi f_m \neq \omega = 2\pi f,\ f_m = \text{frequency of mechanical rotation},$$
$$f = \text{freq. of associated de Broglie wave},\ \hbar = \frac{h}{2\pi},\ 2\pi r_{12} = n\lambda = n\frac{h}{\tilde{p}} = \frac{v}{f_m}, \qquad (4.3)$$
$$u = \lambda f,\ 1 \leq \left[\frac{v}{u} = \frac{\tilde{p}v}{\tilde{E}} = \frac{n\cdot f_m}{f} = 1+\sqrt{1-\frac{v^2}{c^2}}\right] \leq 2,\ n = 1, 2, 3, \ldots,$$
$$\frac{1}{2} \leq \left[\frac{u}{v} = 1+\frac{\lambda}{v}\frac{du}{d\lambda} = 1+(\frac{h}{\tilde{p}v})\cdot(\frac{du}{d\lambda}) = \frac{hf}{\tilde{p}v} = \frac{\hbar\omega}{\tilde{p}v}\right] \leq 1\ .$$

Bohr's hydrogen atom-model is a very simple one, often (experimentally) tested and proved applicable in all frames of its definition. By combining Bohr's planetary model with here introduced concept of de Broglie waves (see Fig. 4.1 and equations (4.1), (4.2) and (4.3)), we are indirectly testing and proving the hypothesis (of this paper) claiming that every linear motion should be accompanied with rotation (and such rotation naturally creates de Broglie waves, producing a correct result (4.4)).

8.3. Structure of the Field of Subatomic Forces

Let's try now to add on to the concept of Bohr's atom model, from an angle of the field theory (mathematically).

Let's assume that all occurrences connected with stationary energy states of the electron and the atom nucleus take place in the space where specific and complex structure of material force field exists. Let's denote, in spherical coordinates, the function of force of that complex (atomic) field as $\mathbf{F(r,\theta,\phi,t)}$, where: $\mathbf{r}$ is radius, θ,ϕ are angular coordinates and $\mathbf{t}$ is time coordinate.

Let's assume that the function of the force of an atomic field is continuous and limited (i.e. finite):

$$\mathrm{Lim}\,F(X)_{x\to A} = F(A), \quad \mathrm{Lim}\,F(r)_{r\to R\le\alpha} = 0 \tag{8.64}$$

Let $\mathbf{F(r,\theta,\phi,t)}$ be the radius vector of possible orbits of inter-atomic states in the force field of the atomic field. The element of the orbit described by the radius vector $\mathbf{F(r,\theta,\phi,t)}$ may be presented as,

$$\mathbf{dR^2 = dr^2 + r^2\cdot d\theta + r^2\cdot \sin^2\theta\cdot d\phi,\quad 0\le\theta\le 2\pi,\quad -\frac{\pi}{2}\le\phi\le\frac{\pi}{2}.} \tag{8.65}$$

Based on the previously introduced elements we will give a simplified definition of the stationary electron orbit $\mathbf{C_n}$, as an orbit whose perimeter $\mathbf{a_n}$ is equal to the mean value of the wavelength $\overline{\lambda}_n$, of a stationary electron wave that exists in orbit $\mathbf{C_n}$, multiplied by the main quantum number $\mathbf{n}$, (which results from the first Bohr's postulate, as well as from the definition of the notion of standing waves, and in agreement with de Broglie's wavelength, see (8.10)-(8.11)),

$$\begin{aligned} &a_n = \oint_{C_n} dR = \oint_{C_n} dr = n\overline{\lambda}_n = n\frac{h}{\tilde{p}_n}, \\ &\overline{\lambda}_n = \frac{1}{n}\oint_{C_n} dR = \frac{1}{n}\oint_{C_n} dr = \frac{a_n}{n} = \frac{1}{n}\sum_{i=1}^{n}\lambda_{ni} = \frac{h}{\tilde{p}_n}. \end{aligned} \tag{8.66}$$

A more general replacement for (8.66) was introduced (or postulated and proved valid for any periodical motion on a closed and stationary orbit) by Wilson-Sommerfeld action integral (as the new quantifying rule, see **[9]**), applied over one period of the motion. Here, the same Wilson-Sommerfeld action integral will be formulated in a bit modified form (to be consistent with the concept of Particle-Wave Duality presented in this paper) as,

$$\oint_{C_n} p_q dq = \left[\oint_{C_n} d(p_q q) - \oint_{C_n} q dp_q\right] = n_q h = \frac{\tilde{E}'_{nq}}{f'_q} = \frac{hf'_{nq}}{f'_q}, \tilde{E}'_{nq} = hf'_{nq} = hn_q f'_q = \left[\oint_{C_n} p_q dq\right]\cdot f'_q,$$
$$\left[\oint_{C_n} d(p_q q) = 0, \oint_{C_n} q dp_q = -n_q h,\ f'_{nq} = n_q f'_q\right],$$

$$\oint_{C_n} L_q dq = \left[\oint_{C_n} d(L_q q) - \oint_{C_n} q\, dL_q\right] = n_q h = \frac{\tilde{E}''_{nq}}{f''_q} = \frac{hf''_{nq}}{f''_q}, \tilde{E}''_{nq} = hf''_{nq} = hn_q f''_q = \left[\oint_{C_n} L_q dq\right] \cdot f''_q$$

$$\left[\oint_{C_n} d(L_q q) = 0, \oint_{C_n} q\, dL_q = -n_q h, f_{nq} = n_q f''_q\right],$$

$$q = (r, \theta, \varphi, ...), \; r = r(x, y, z...), \; n_q = \text{integer } (=1, 2, 3, ...).$$

Since in the Wilson-Sommerfeld action integral we have generalized the momentum (concerning linear motion and rotation), using electromechanical analogies (established in the first three chapters of this paper), we can (at this time hypothetically) extend the Wilson-Sommerfeld's action integral to electric and magnetic charges, as follows,

$$\left.\begin{array}{l} \oint_{C_n} \Phi_{electr.} d\Phi_{magn.} = \left[\oint_{C_n} d(\Phi_{electr.}\Phi_{magn.}) - \oint_{C_n} \Phi_{magn.} d\Phi_{electr.}\right] = n_{electr.} h = \frac{\tilde{E}_{n-electr.}}{f_{electr.}} = \frac{hf_{n-electr.}}{f_{electr.}}, \\ \oint_{C_n} \Phi_{magn.} d\Phi_{electr.} = \left[\oint_{C_n} d(\Phi_{electr.}\Phi_{magn.}) - \oint_{C_n} \Phi_{electr.} d\Phi_{magn.}\right] = n_{magn.} h = \frac{\tilde{E}_{n-magn.}}{f_{magn.}} = \frac{hf_{n-magn.}}{f_{magn.}}, \end{array}\right\rangle \Rightarrow$$

$$\oint_{C_n} d(\Phi_{electr.}\Phi_{magn.}) = 0, \oint_{C_n} \Phi_{magn.} d\Phi_{electr.} = -n_{electr.} h, \oint_{C_n} \Phi_{electr.} d\Phi_{magn.} = -n_{magn.} h$$

$$n_{electr.} = -n_{magn.}, f_{n-electr.} = n_{electr.} f_{electr.}, \; f_{n-magn.} = n_{magn.} f_{magn.},$$

$$\tilde{E}_{n-electr.} = hf_{n-electr.}, \tilde{E}_{n-magn.} = hf_{n-magn.}, (n_{electr.}, n_{magn.}) = \text{integer } (= 1, 2, 3, ...) .$$

By mathematical manipulation of the Wilson-Sommerfeld action integral, an attempt is made here to introduce the idea that Plank's quantization law $\mathbf{\tilde{E} = nhf}$ *is wrongly interpreted (even by Planck itself), and that more correct interpretation should be based on* $E = hf(n), f(n) = nf$ *(or using some other frequency function* **f(n)***). More precisely, in this paper has already underlined that mechanical revolving frequency of a particle should not be mixed with the matter wave frequency (or with the frequency of associated de Broglie matter wave, see equations (8.28) and (8.42)), as the consequence of the correct treatment of the phase and group velocity. The same comment should be automatically addressed to Planck's postulate regarding the energy of simple harmonic oscillator (in relation with blackbody radiation). Of course, the intention of the author of this paper is to say that modern Quantum Mechanics treatment of phase velocity (of de Broglie waves) is not fully correct. Luckily, Planck (and Einstein, too) produced a smart, tricky and, only as the final result, correct fitting function for the energy density of a blackbody radiator, and this result was so good that basic quantization law* $\mathbf{\tilde{E} = nhf}$ *was easily and wrongly interpreted as correct (as explained before). Here we also see a very simple picture of the quantum nature of matter, as the consequence of proper gearing and fitting of discrete matter constituents (or fitting between their particle and field characteristics).*

We may now define the energy of the stationary electron wave (i.e. the second Bohr's postulate) through the work of atomic field force (4.1.64), as well as through the wave function of the stationary electron wave (see (4.0.1), to (4.0.5) from the chapter 4.0 and (4.9) to (4.10-5) from the chapter 4.3), in the following way,

$$\tilde{\varepsilon}_{sn} = \oint_{C_n} \mathbf{F}\mathbf{dR} = \int_{C_n:[\Delta t]} \Psi_n^{\ 2}(t)dt = \int_{C_n:[\Delta p]} v_n dp = \iint_{\sigma_n} (\nabla\times\mathbf{F})d\sigma = hf_{sn} \neq 0 \ , \tag{8.67}$$
$$\Rightarrow \nabla\times\mathbf{F} \neq 0 \ .$$

We shall determine the energy of an electromagnetic quantum of inter-orbital interchange (absorption or emission of photons), as (see (4.0.1), to (4.0.5) from the chapter 4.0 and (4.9) to (4.10-5) from the chapter 4.3),

$$\tilde{\varepsilon}_{m,n} = \oint_{[C_m - C_n]} \mathbf{F}\mathbf{dR} = \int_{[\Delta t]} \Psi_{m\text{-}n}^{\ 2}(t)dt = \int_{[\Delta p]} v_{m\text{-}n} dp = h(f_n - f_m) = hf_{mn} \neq 0 \ , \tag{8.68}$$
$$\Rightarrow \nabla\mathbf{F} \neq 0 \ .$$

Let's go back now to the atomic force field $\mathbf{F(r,\theta,\phi,t)}$. From (8.67) and (8.68) we see that $\nabla\times\mathbf{F}\neq\mathbf{0}$ and $\nabla\mathbf{F}\neq\mathbf{0}$, so according to the general classification of fields in differential geometry, we must consider force field $\mathbf{F(r,\theta,\phi,t)}$ as a complex vector field, which may be always decomposed into two more elementary fields, and they are,

$$\mathbf{F(r,\theta,\phi,t)} = \mathbf{F_1(r,\theta,\phi,t)} + \mathbf{F_2(r,\theta,\phi,t)} \ , \tag{8.69}$$

from which the first is **potential field**:

$$\nabla\times\mathbf{F_1} = \mathbf{0}, \nabla\mathbf{F_1} \neq \mathbf{0} \ , \tag{8.70}$$

and the second is **solenoidal field**:

$$\nabla\times\mathbf{F_2} \neq \mathbf{0}, \nabla\mathbf{F_2} = \mathbf{0} \ , \tag{8.71}$$

so that now, instead of (8.67) and (8.68), we may give new definitional expressions which replace them,

$$\tilde{\varepsilon}_{sn} = \oint_{C_n} \mathbf{F_2}\mathbf{dR} = \int_{C_n:[\Delta t]} \Psi_{2n}^{\ 2}(t)dt = \int_{C_n:[\Delta p]} v_{2n} dp =$$
$$= \iint_{\sigma_n} (\nabla\times\mathbf{F_2})d\sigma = hf_{sn} \neq 0 \ , \tag{8.72}$$
$$\Rightarrow \nabla\times\mathbf{F_2} \neq 0 \ ,$$

and

$$\tilde{\varepsilon}_{m,n} = \oint_{[C_m - C_n]} \mathbf{F_1}\mathbf{dR} = \int_{[\Delta t]} S_{m\text{-}n}(t)dt = \int_{[\Delta t]} \Psi_{m\text{-}n}^{\ 2}(t)dt = \int_{[\Delta p]} v_{m\text{-}n} dp = \tag{8.73}$$
$$= h(f_n - f_m) = hf_{mn} \neq 0 \ , \ \Rightarrow \ \nabla\mathbf{F_1} \neq 0 \ .$$

In order to treat an atom as a stabile structure, one should add to the previous definitional expressions the condition of balance of the potential energy of all attractive and all repulsive forces that set the structure of an atomic field as,

$$\oiiint\limits_{\left\{r\in[0,\infty],\theta\in[0,2\pi],\phi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right\}} \mathbf{FdR} = \mathbf{0}. \tag{8.74}$$

The condition of the balance of the potential energy of all attractive and all repulsive forces (8.74) may be added on by a hypothesis about the existence of permanent communication by interchange of electromagnetic quanta between stationary states of the nucleus and shell of an atom (synchronously and coincidently, in both directions), so that this interchange is always captured in the internal field of an atom (i.e. couldn't be noticed in the external space out of an atom, if an atom is neutral and non-excited). **Embryonic roots of the previous interpretation of an atomic field are found even in the papers of Rudjer Boskovic (Principles of the Natural Philosophy), as well as in some papers published in «Herald of Serbian Royal Academy of Science» between 1924 and 1940 (J. Goldberg 1924; V. Žardecki 1940).** One may just imagine how much more picturesque, interesting and richer would be the concept of nuclear physics, if the previously introduced ideas (or hypotheses) have not been completely missed, bypassed or ignored in modern quantum physics.

Of course, in order to complete, make precise and revive the previously described atom model, it would be the most properly to analyze the structure of an atomic field using the modified, general Schrödinger's equation (4.22) to (4.28), which is formulated in the chapter 4.3 of this paper:

$$\left\{\begin{array}{l} \frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi} + L\overline{\Psi} = 0 \; ; \; \Delta\overline{\Psi} = (\frac{\tilde{E}}{L})\frac{1}{u^2}\frac{\partial^2 \overline{\Psi}}{\partial t^2} = jk(\frac{\tilde{E}}{L})\nabla\overline{\Psi} \; ; \; (\frac{L}{\hbar})^2\overline{\Psi} + \frac{\partial^2 \overline{\Psi}}{\partial t^2} = 0, \; \tilde{E} \Leftrightarrow E_k \\ \frac{L}{\hbar}\overline{\Psi} = j\frac{\partial\overline{\Psi}}{\partial t} = -\frac{\hbar}{L}\frac{\partial^2 \overline{\Psi}}{\partial t^2} ; \; \frac{\partial\overline{\Psi}}{\partial t} + u\nabla\overline{\Psi} = 0 ; \overline{\Psi} = \overline{\Psi}(t,r), \; j^2 = -1 \end{array}\right\} \Rightarrow$$

$$\begin{array}{l} \frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta = \frac{(\hbar u)^2}{\tilde{E}}\Delta = \frac{\tilde{E}}{k^2}\Delta = j(\frac{\tilde{E}^2}{kL})\nabla = \frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2} \; , \Delta = jk(\frac{\tilde{E}}{L})\nabla, \; \tilde{E} - U_p \le L < \infty , \\ S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i ..., t)dt = \text{extremum} \; , \\ (\frac{u}{v} = 1 + \frac{\lambda}{v}\frac{du}{d\lambda} = 1 + \frac{h}{\tilde{p}v}\frac{du}{d\lambda} = \frac{hf}{\tilde{p}v} = \frac{\hbar\omega}{\tilde{p}v} = \frac{\tilde{E}}{\tilde{p}v} \; , \; \tilde{E} = \tilde{m}uv = \tilde{p}u = hf = \hbar\omega) \; . \end{array} \tag{4.25}$$

On the occasion of the use of the wave equation (4.25) one should not forget that it is about wave function that has a physical dimension (it does not represent only the probability of finding electrons in some part of the atomic field space) and which is an integral part of the equations and expressions that describe stability of an atom structure, starting from (8.67) until (8.74).

If one takes a look from a different angle how the structural modeling of an atom field is done (starting from (8.64) to (8.75)), he may conclude that it may be a general approach to modeling, applicable to any elementary particle (with respect to relevant physical, mathematical and quantum particularities). For example, one may always pose questions if the field of some elementary particle is "solenoidal", potential or complex, if there is a particle nucleus and particle shell, and what the communication between them is like, and/or what the communication of that elementary particle with outer space is like, etc. After that, there come analyses from an angle of connection between group and phase velocity, then determination of amplitude and phase wave functions (in time and frequency domain), as well as application of all relevant relations of uncertainty (2.10). In order to successfully and coherently connect all previous steps into a whole, and thus define the global model of some elementary particle, the whole previous procedure should be fit into the frames of the *concept of particle-wave dualism (4.9) - (4.22). Regardless of the fact that modern quantum physics treats the analysis of the atom and elementary particles in a very complex and quantitatively predictable way, one may yet conclude that it is not (nor nearly) the analysis that is previously described or proposed (in this paper). Against previous statements, we may justifiably ask ourselves what happens when we name/make the wave function a probability function, and we mathematically (by normalization, etc.) completely conform it to (additionally define it with) functions of probability and statistic distributions (that stand for sets with large number of elements). Thus, we made some kind of generalized (in-average) localization of the domain in which certain force or field exists (with respect to all spatial, time, and other parameters), so it is no wonder that such probabilistic treatment of the wave function gives correct results.*

9. WAVE FUNCTION OF THE BLACK BODY RADIATION AND A PHOTON

Since we know the spectral nature of the Black Body radiation only in its frequency domain, let us try to find its time domain wave function.

The integral forms of Stefan-Boltzman and Wien's radiation laws of a black body heated to the temperature T presents a surface power density of emitted electromagnetic waves from that black body,

$$
\begin{aligned}
& R_T = \int_0^{\infty} dR(f) = d\tilde{P}/dS = \sigma \cdot T^4 (=) \left[\frac{W}{m^2}\right], \\
& P \ = \ \sigma \cdot S \cdot T^4 = \ \text{Power radiated in } [W] \ = \left[\frac{J}{s}\right], \\
& \lambda_{peak} \ = \ \frac{2.898 \cdot 10^{-3}}{T} \ (=) \ [m] \ (=) \ \text{Peak Wavelength}, \ \ T(=)[K], \qquad (9.1) \\
& \sigma \ = \ \text{Stefan's Constant} \ = \ 2\pi^5 k^4/15c^2h^3 = \ 5.67 \text{ x } 10\text{-}8 \left[\frac{W}{m^2K^4}\right], \\
& S \ = \ \text{Surface area of body } [m^2], \\
& T \ = \text{Surface Temperature of body } [K].
\end{aligned}
$$

Max Planck successfully created (or better to say fitted) the differential expression of the radiation spectral function **dR(f)** as,

$$
dR(f) = \frac{2\pi f^2}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} df \ (=) \left[\frac{W}{m^2}\right]; \frac{dR(f)}{df} = \frac{2\pi f^2}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} (=) \left[\frac{Ws}{m^2}\right] \qquad (9.2)
$$

where the total power of a heat radiation from the black body surface **S** can be found as,

$$
\tilde{P}(t) = \Psi^2(t) = d\tilde{E}/dt = \int_{[S]} R_T dS = R_T S = S\int_0^{\infty} dR(f) = S\sigma T^4 (=)[W] \qquad (9.3)
$$

Radiated heat energy from a black body, after the time period Δt, would be,

$$
\begin{aligned}
& \tilde{E} = \int_{-\infty}^{+\infty} \tilde{P}(t)dt = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |U(\omega)|^2 d\omega = \frac{1}{\pi}\int_0^{+\infty} |A(\omega)|^2 d\omega = S\Delta t\int_0^{+\infty} dR(f) = \\
& = S\Delta t \int_0^{+\infty} \frac{dR(f)}{df} df \ = S \ \Delta t \cdot \sigma T^4 (=)[Ws = J], \qquad (9.4) \\
& d\tilde{E} = \left[S\int_0^{+\infty} dR(f)\right] dt = \Psi^2(t)dt = vdp \ .
\end{aligned}
$$

From (9.4) is possible to determine the amplitude spectral function as,

$$A^2(\omega) = \pi\ S\ \Delta t\left[\frac{dR(f)}{d\omega}\right] = \frac{1}{2} S\ \Delta t\left[\frac{dR(f)}{df}\right] = S\ \Delta t\frac{\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1} =$$
$$= S\ \Delta t\frac{\pi}{\lambda^2}\cdot\frac{\tilde{E}}{e^{\tilde{E}/kT}-1} = S\ \Delta t\frac{h}{8\pi^2c^2}\cdot\frac{\omega^3}{e^{h\omega/2\pi kT}-1} = \frac{1}{2}\cdot\frac{d\tilde{E}}{df} = \pi\frac{d\tilde{E}}{d\omega} =$$
$$= U_c^2(\omega) + U_s^2(\omega\ )\ (=)\ \left[Ws = Js\right],\ (c = \lambda f\ ,\ \tilde{E} = hf)\ ,$$

$$A(\omega\ )e^{-j\Phi\ (\omega\ )} = \iint\limits_{[-\infty,+\infty]} \overline{\Psi}(t)e^{j\omega\ t}dt = U(\omega\ ) = U_c(\omega\ ) - jU_s(\omega\ )$$
$$\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = a(t)e^{j\varphi(t)} = \int\limits_{(0,+\infty)} A(\omega\ )e^{-j(\omega\ t+\Phi(\omega))}d\omega\ .$$
$$\hat{\Psi}(\mathbf{t}) = \mathbf{H}\left[\Psi(\mathbf{t})\right],\ \mathbf{a}(\mathbf{t}) = \sqrt{\Psi^2 + \hat{\Psi}^2}\ ,\ \Psi(\mathbf{t}) = \mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})$$

$$\omega_{mean} = \overline{\omega} = \langle\omega\rangle = (\int_0^\infty \frac{\omega^4}{e^{h\omega/2\pi kT}-1}d\omega)/(\int_0^\infty\frac{\omega^3}{e^{h\omega/2\pi kT}-1}d\omega). \qquad (9.5)$$

Based on the amplitude spectral function $A^2(\omega)$ we should be able to find a family of possible time-domain wave functions $\mathbf{\Psi(t) = a(t)\cos\varphi(t)}$ that comply with Planck's radiation law.

Since we know relations between group and phase velocity, and we know the wavelength expression of mater waves, we should also be able (again by applying analogies) to find velocity-dependent <u>*spectral distribution of moving particles*</u> *(that are inside a black body cavity), as for instance,*

$$\left\{\begin{array}{l} \lambda = h/p,\ \tilde{E} = hf = pu,\ u = \lambda f = \dfrac{v}{1+\sqrt{1-(\frac{v}{c})^2}}\ ,\ v = \dfrac{2u}{1-\frac{uv}{c^2}} = u - \lambda\dfrac{du}{d\lambda} = -\lambda^2\dfrac{df}{d\lambda},\ \omega = 2\pi\ f \\ 0 \le 2u \le \sqrt{uv} \le v \le c \end{array}\right\} \Rightarrow$$

$$A^2(\omega) = A^2(\frac{2\pi}{h}\tilde{E}) = A^2(\frac{2\pi}{h}pu) = S\ \Delta t\frac{\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1} =$$
$$= S\ \Delta t\frac{\pi}{\lambda^2}\cdot\frac{\tilde{E}}{e^{\tilde{E}/kT}-1} = S\ \Delta t\frac{\pi}{h^2}\cdot\frac{p^3u}{e^{pu/kT}-1},$$
$$u = \lambda f = \frac{f}{T}\cdot 2.898\cdot 10^{-3}\left[mK\right] = \frac{E_k}{p} = \frac{\tilde{E}}{p} \le c\ (=)\left[\frac{m}{s}\right],$$
$$\lambda_{peak} = \frac{h}{p} = \frac{2.898\cdot 10^{-3}\left[mK\right]}{T}(=)\ \left[m\right], \qquad (9.6)$$
$$E_k = \tilde{E} = pu = hf = \frac{huT}{2.898\cdot 10^{-3}\left[mK\right]}\ (=)\ \left[J\right],$$
$$T = \frac{2.898\cdot 10^{-3}\left[mK\right]}{h}p \ge \frac{2.898\cdot 10^{-3}\left[mK\right]}{hc}E_k (=)\ \left[K\right]$$

For non-relativistic motions (lower temperature ranges), we know that phase velocity is two times lower than group or particle velocity, and we could approximate the amplitude spectral distribution (9.6) as,

$$A^2(\omega) = A^2(\frac{\pi}{h}pv) = S\ \Delta t\frac{\pi}{2h^2}\cdot\frac{p^3 v}{e^{pv/2kT}-1}. \tag{9.7}$$

9.1. Wave Function of a Single Photon

Since Planck radiation law says that the heat radiation is composed of numbers of elementary wave packets or energy quanta, called photons, we could try to determine the most probable waveforms describing photons (in a time and frequency domains) and see up to which level such concept is mathematically defendable.

It is generally accepted (in Quantum Theory) that a single photon has wave energy equal to the product of Planck's constant h and photon's frequency $\mathbf{f}$. Let us analyze additional mathematical background of Planck's radiation law, regarding single photon energy, $\mathbf{E} = \mathbf{hf} = \frac{\mathbf{h}}{\mathbf{2\pi}}\omega$, since we know that initially such radiation law was found almost empirically and by best curve fitting (without having some more general mathematical and theoretical background). Photon is an electromagnetic wave state or wave packet, known as the carrier of one quantum of energy ($\mathbf{E} = \mathbf{hf}$), and it should be presentable using certain time-domain wave function $\psi(\mathbf{t}) = \mathbf{a(t)}\cos\varphi(\mathbf{t})$ that can be expressed in the form of an Analytic signal. Here, we will make an effort to define the framework, which will give the possibility to find the family of wave functions, or wave packets that intrinsically or naturally carry one quantum of energy (in order to support Planck's radiation law).

Since the Analytic Signal presentation gives the chance to extract immediate (or instant) signal amplitude $\mathbf{a(t)}$, phase $\varphi(\mathbf{t})$, and frequency $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial\mathbf{t}} = \mathbf{2\pi f(t)}$, let us also extend and test the meaning of Planck's energy quantum ($\mathbf{E} = \mathbf{hf}$) where, instead of constant photon frequency $\mathbf{f} = \omega/2\pi$ (valid for a single photon), we will take the mean wave frequency, $\Omega = \langle\omega(\mathbf{t})\rangle = \mathbf{2\pi}\langle\mathbf{f(t)}\rangle$, of the time-domain, arbitrary wave function $\psi(\mathbf{t})$, meanwhile not reducing the applicability of Plank's energy formula. In other words, we assume that photons or elementary energy quanta are limited time and frequency duration (or short) wave packets, and as the representative photon frequency we are taking its energy dominant, central or mean frequency. Practically, this way we are formulating the realistic concept that a wave energy quantum could be represented by a frequency band limited wave function, without installing any limits regarding the width of its frequency band. It is also obvious that such time-domain elementary wave functions (carrying one quantum of energy, $\mathbf{E} = \mathbf{hf}$) could be used as basis functions for very natural Spectral Signal Analysis and Synthesis (of course after we succeed to formulate them precisely). The platform for searching for analytic expressions of elementary wave function/s will be established by connecting Analytic Signal wave function, $\psi(\mathbf{t}) = \mathbf{a(t)}\cos\varphi(\mathbf{t})$ with Planck energy formula, as follows.

$$\mathbf{E = hf = \frac{h}{2\pi}\omega \Rightarrow E = \frac{h}{2\pi}\langle\omega\rangle = h\langle f\rangle},$$

$$\mathbf{E = \int_{[t]} \psi^2(t)dt = \int_{[t]} [a(t)\cos\varphi(t)]^2 dt = \int_{[t]} a^2(t)dt = hf\ (=)\frac{h}{2\pi}\langle\omega\rangle = h\langle f\rangle}$$

$$\mathbf{\overline{\Psi}(t) = a(t)e^{j\varphi(t)} = \Psi(t) + j\hat{\Psi}(t)}$$

$$\mathbf{\psi(t) = a(t)\cos\varphi(t) = -H[\hat{\psi}(t)],\quad \hat{\psi}(t) = a(t)\sin\varphi(t) = H[\psi(t)]}$$

$$\mathbf{a(t) = \sqrt{\psi^2(t) + \hat{\psi}^2(t)}\ ,\ \varphi(t) = arctg\frac{\hat{\psi}(t)}{\psi(t)}}$$

$$\mathbf{\omega(t) = \dot{\varphi}(t) = \frac{\partial\varphi(t)}{\partial t} = 2\pi f(t) = \frac{\psi(t)\cdot\dot{\hat{\psi}}(t) - \dot{\psi}(t)\cdot\hat{\psi}(t)}{a^2(t)} = Im\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right]}$$

$$\mathbf{\dot{a}(t) = \frac{\partial a(t)}{\partial t} = \frac{\psi(t)\cdot\dot{\psi}(t) + \hat{\psi}(t)\dot{\hat{\psi}}(t)}{a^2(t)} = a(t)\cdot Re\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right]}$$

It is obvious that we still do not know the shape of the elementary wave function (neither in its time or frequency domain), and the only thing we know is that each of such wave functions should have the energy content of one energy quantum,

$$\mathbf{E = hf = \frac{h}{2\pi}\omega, or\ E = \frac{h}{2\pi}\langle\omega\rangle = h\langle f\rangle.} \Leftrightarrow \frac{\tilde{E}}{\omega} = \frac{\tilde{E}}{\langle\omega\rangle} = \frac{d\tilde{E}}{d\omega} = \frac{h}{2\pi}\ .$$

One of the ways for testing general applicability of Planck's photon energy expression will be to find the signal energy for different analytic forms of wave packets, and calculate mean frequency and Planck energy based on that mean frequency. We also know that the mean wave function frequency (in a time domain) can be found by applying different definitions of mean values, and two of the very common are:

$$\mathbf{a)\ \Omega = \langle\omega(t)\rangle = \frac{\int_{[t]} a^2(t)\omega(t)dt}{\int_{[t]} a^2(t)dt} = 2\pi\langle f(t)\rangle\ ,\ \ b)\ \Omega = \langle\omega(t)\rangle = \frac{1}{\Delta t}\int_{[t]}\omega(t)dt = 2\pi\langle f(t)\rangle}$$

Before we are able to distinguish (or test) precisely which definition of the mean frequency works better, we will keep both options, **a)** and **b)**, as possible solutions. It will also be interesting to find how big a difference exist between two definitions of the same mean frequency, since in different literature (Quantum Theory, Spectral Analysis, wave motion analyzes) for the mean frequency is more often considered definition under **a)**.

$$\mathbf{E}=\mathbf{hf}=\frac{\mathbf{h}}{2\pi}\omega\Rightarrow\mathbf{E}=\frac{\mathbf{h}}{2\pi}\Omega=\mathbf{h}\left\langle\mathbf{f}\right\rangle\Rightarrow$$

$$\Rightarrow\left\{\mathbf{E}=\frac{\mathbf{h}}{2\pi}\frac{\int_{[t]}\mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}}{\int_{[t]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}}=\int_{[t]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}\text{, or }\mathbf{E}=\frac{\mathbf{h}}{2\pi}\frac{\mathbf{1}}{\Delta\mathbf{t}}\int_{[t]}\omega(\mathbf{t})\mathbf{dt}=\int_{[t]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}\right\}\Rightarrow$$

$$\Rightarrow\left\{\frac{\left[\int_{[t]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}\right]^2}{\int_{[t]}\mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}}=\frac{\mathbf{h}}{2\pi}=\mathbf{Const.},\quad\text{or}\quad\frac{\int_{[t]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}}{\frac{\mathbf{1}}{\Delta\mathbf{t}}\int_{[t]}\omega(\mathbf{t})\mathbf{dt}}=\frac{\mathbf{h}}{2\pi}=\mathbf{Const.}\right\}.$$

Depending how we define the mean frequency, we should be able to find the family of specific wave functions, $\psi(\mathbf{t})=\mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})$, that describe any photon, or some other elementary quantum of wave energy in a time domain, and to prove validity of at least one of the above given relations. All such elementary wave functions should have at least one common characteristic, which is that each of them has the energy content equal to a Planck photon energy quantum, $\mathbf{E}=\mathbf{h}\left\langle\mathbf{f}\right\rangle=\int_{[t]}\psi^2(\mathbf{t})\mathbf{dt}=\int_{[t]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}$. Continuing this way, we should be able to see how universal Planck's energy law ($\mathbf{E}=\mathbf{hf}$) could be regarding quantifying energy of arbitrary wave functions, where the same energy law, $\mathbf{E}=\mathbf{h}\left\langle\mathbf{f}\right\rangle$ is applicable. It is also clear that by creating a set of specific additional conditions and objectives (in advance), we are able to search and eventually find certain family of specific wave functions that satisfies such conditions.

The principal objective here is to formulate (or find) the family of elementary wave functions that could present the quantum reality of all matter waves in our universe, or to come to the conclusion that something like that is generally impossible. In fact Quantum Theory already uses the concept of quantized energy wave packets, which mathematically works well in explaining number of experimental facts in Physics (some of them, well-known, are Compton and Photoelectric effect), regardless of the fact that we still do not know their analytic forms (and the fact is that in many cases we do not need to know them).

The question here is: do we (in Physics) really deal with the universal law of naturally quantized wave packets, or with discrete energy exchanges between discrete quantum nature of resonant and standing waves building blocks which are inside the matter constituents, and which in different situations absorb or emit certain wave packets, as a packaging wise non-acceptable energy surplus and/or deficit amounts, giving only a false impression that such wave-elements are intrinsically quantized and generally applicable. In other words, if the stable matter presents only certain spatial and complex standing waves inside resonant-like energy packaging structures, it would be logical that the communication between such structures looks to an external observer like an exchange of quantized energy amounts (which should not be generalized to all other kinds of energy exchanges). The position of the author of this paper is that present Quantum Theory concepts regarding quantizing are overstated, and too widely generalized, taking also places where something like that is not universally valid.

The far reaching consequences of mastering the above-presented ideas are to prove that we are able to control selectively the number of matter wave energy modulations and mater wave energy flow, as well as to influence structural integrity and stability of matter states by modulating certain characteristic frequency, or modulating a set of characteristic signal frequencies of the same matter wave signal, this way amplifying and modifying selected mechanical and structural effects in matter structures.

It is also known that by applying Parseval's identity (see below) we can connect the time and frequency domain of the signal energy expressions, this way formulating the form of Energy Conservation Law. The most challenging aspect of Parseval's identity is related to our knowledge (from Physics or Quantum Theory) that photons, or wave packets (that are without rest mass) under certain conditions could be transformed into particles having rest masses. Effectively, by searching for the more general meaning of wave energy frequency dependences, we touch the profound ontological essence of the intrinsic matter structure.

$$\mathbf{E} = \int_{[t]} \psi^2(\mathbf{t})\mathbf{dt} = \int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt} = \frac{\mathbf{1}}{2\pi}\int_{-\infty}^{+\infty} \left|\bar{\mathbf{U}}(\omega)\right|^2 \mathbf{d}\omega = \frac{\mathbf{1}}{\pi}\int_{\mathbf{0}}^{\infty} \left[\mathbf{A}(\omega)\right]^2 \mathbf{d}\omega = \frac{\mathbf{h}}{2\pi}\left\langle \omega \right\rangle = \mathbf{h}\left\langle \mathbf{f} \right\rangle$$

$$\bar{\Psi}(\mathbf{t}) = \mathbf{a}(\mathbf{t})\mathbf{e}^{\mathbf{j}\varphi(\mathbf{t})} = \Psi(\mathbf{t}) + \mathbf{j}\hat{\Psi}(\mathbf{t}) = \int_{(0,+\infty)} \bar{\mathbf{U}}(\omega)\mathbf{e}^{-\mathbf{j}\omega\mathbf{t}}\mathbf{d}\omega = \int_{(0,+\infty)} \mathbf{A}(\omega)\mathbf{e}^{-\mathbf{j}(\omega\mathbf{t}+\Phi(\omega))}\mathbf{d}\omega$$

$$\bar{\mathbf{U}}(\omega) = \mathbf{A}(\omega)\mathbf{e}^{-\mathbf{j}\Phi(\omega)} = \mathbf{U}_c(\omega) - \mathbf{j}\,\mathbf{U}_s(\omega) = \int_{[t]} \bar{\Psi}(\mathbf{t})\,\mathbf{e}^{\mathbf{j}\omega\mathbf{t}}\mathbf{dt} = \int_{[t]} \mathbf{a}(\mathbf{t})\mathbf{e}^{\mathbf{j}(\omega\mathbf{t}+\varphi(\mathbf{t}))}\mathbf{dt}$$

$$\mathbf{A}^2(\omega) = \mathbf{U}_c{}^2(\omega) + \mathbf{U}_s{}^2(\omega),\ \Phi(\omega) = \mathbf{arctan}\frac{\mathbf{U}_s(\omega)}{\mathbf{U}_c(\omega)},\ \pi = \left\{\int_{\mathbf{0}}^{\infty}\left[\mathbf{A}(\omega)\right]^2\mathbf{d}\omega\right\} / \left\{\int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}\right\}$$

In order to close the loop regarding testing the applicability of Planck energy formula ($\mathbf{E} = \mathbf{h}\left\langle \mathbf{f} \right\rangle$), the mean frequencies previously found using a time signal domain, could be extended or linked to the signal frequency domain (as the result of applying Parseval's identity), what will give the chance to reinforce the most appropriate definition of the mean frequency, by proving validity of the following expressions:

$$\Omega = \left\langle \omega(\mathbf{t}) \right\rangle = 2\pi\left\langle \mathbf{f}(\mathbf{t}) \right\rangle = \frac{\int_{\mathbf{0}}^{\infty} \omega \cdot \left[\mathbf{A}(\omega)\right]^2 \mathbf{d}\omega}{\int_{\mathbf{0}}^{\infty} \left[\mathbf{A}(\omega)\right]^2 \mathbf{d}\omega} = \frac{\int_{[t]} \omega(\mathbf{t}) \cdot \mathbf{a}^2(\mathbf{t})\mathbf{dt}}{\int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}} =$$

$$= \frac{\mathbf{2}}{\mathbf{h}}\int_{\mathbf{0}}^{\infty}\left[\mathbf{A}(\omega)\right]^2\mathbf{d}\omega = \frac{2\pi}{\mathbf{h}}\int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt},\quad \sigma_\Omega^2 = \frac{\mathbf{1}}{\Delta\mathbf{t}}\int_{[t]} \left|\omega(\mathbf{t}) - \Omega\right|^2 \mathbf{dt} \Rightarrow$$

$$\Rightarrow \frac{\left[\int_{\mathbf{0}}^{\infty}\left[\mathbf{A}(\omega)\right]^2\mathbf{d}\omega\right]^2}{\pi\int_{\mathbf{0}}^{\infty} \omega \cdot \left[\mathbf{A}(\omega)\right]^2\mathbf{d}\omega} = \frac{\left[\int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}\right]^2}{\int_{[t]} \omega(\mathbf{t}) \cdot \mathbf{a}^2(\mathbf{t})\mathbf{dt}} = \frac{\mathbf{h}}{\mathbf{2}}\frac{\int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}}{\int_{\mathbf{0}}^{\infty}\left[\mathbf{A}(\omega)\right]^2\mathbf{d}\omega} = \frac{\mathbf{h}}{2\pi} = \mathbf{Const.}$$

Now, we are in a position to test applicability of different (single energy quantum) elementary functions. Any of such elementary quantum functions should also be presentable in the form of an Analytic Signal function, as for instance:

$$\mathbf{\Psi(t) = a(t)\cos\varphi(t) = \omega^{0.5}A(t)\cos\varphi(t),\ [a(t)]^2 = \omega[A(t)]^2,}$$

$$\int_{[t]}[\Psi(t)]^2\,dt = \int_{[t]}[a(t)]^2\,dt = \int_{[t]}\omega[A(t)]^2\,dt = \omega\int_{[t]}[A(t)]^2\,dt =$$

$$= \frac{h\omega}{2\pi} = h\,f\,,\ \ f = \frac{\omega}{2\pi}\ ,\ \omega = \langle\omega(t)\rangle,\ h = const.,\ \int_{[t]}[A(t)]^2\,dt = \frac{h}{2\pi} \Rightarrow$$

$$\frac{\left[\int_{[t]} a^2(t)dt\right]^2}{\int_{[t]}\omega(t)\cdot a^2(t)dt} = \frac{\left[\int_{[t]} a^2(t)dt\right]^2}{\omega\int_{[t]} a^2(t)dt} = \frac{1}{\omega}\int_{[t]} a^2(t)dt = \int_{[t]}[A(t)]^2\,dt = \frac{hf}{2\pi f} = \frac{h}{2\pi} = Const.$$

The condition $[a(t)]^2 = \omega[A(t)]^2$ is chosen simply because this looks (for the time being) like the only available mathematical option to get logical results regarding single quantum energy, $\int_{[t]}[\Psi(t)]^2\,dt = hf$, and presently we do not have any better explanation.

Let us just jump back to the Plank's radiation law (which should deal with number of elementary quanta), and in order to find possible amplitude spectral functions of black body radiation let us apply the relation: $\frac{\tilde{E}}{\omega} = \frac{\tilde{E}}{\langle\omega\rangle} = \frac{d\tilde{E}}{d\omega} = \frac{h}{2\pi} \Rightarrow$

$$A^2(\omega) = \pi\ S\ \Delta t\left[\frac{dR(f)}{d\omega}\right] = \frac{1}{2}S\ \Delta t\left[\frac{dR(f)}{df}\right] = S\ \Delta t\frac{\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1} =$$

$$= S\ \Delta t\frac{\pi}{\lambda^2}\cdot\frac{\tilde{E}}{e^{\tilde{E}/kT}-1} = S\ \Delta t\frac{h}{8\pi^2c^2}\cdot\frac{\omega^3}{e^{h\omega/2\pi kT}-1} = \frac{1}{2}\cdot\frac{d\tilde{E}}{df} = \pi\frac{d\tilde{E}}{d\omega} = \frac{h}{2} =$$

$$= U_c^2(\omega) + U_s^2(\omega)\ (=)\ [Ws = Js]\ ,\ (c = \lambda f\ ,\ \tilde{E} = hf) \Rightarrow$$

$$\frac{2A^2(\omega)}{h} = 2S\ \Delta t\frac{\pi f^2}{hc^2}\cdot\frac{hf}{e^{hf/kT}-1} = S\ \Delta t\frac{1}{4\pi^2c^2}\cdot\frac{\omega^3}{e^{h\omega/2\pi kT}-1} = 1$$

$$A^2(\omega) = A^2(\frac{\pi}{h}pv) = S\ \ \Delta t\frac{\pi}{2h^2}\cdot\frac{p^3v}{e^{pv/2kT}-1} = \frac{h}{2} \Rightarrow$$

$$\frac{p^3v}{e^{pv/2kT}-1} = \frac{h^3}{\pi S\ \Delta t}$$

In fact, we can only be sure that (above-described) energy quantization primarily exists in inter-atomic stationary and standing wave states and structures (and in all energy transformations in relation with such inter-atomic states), generally described by Wilson-Sommerfeld rules (see chapter 5, (5.4.1)).

Since we know that in reality the square of (non-normalized) wave function $\Psi(\mathbf{t}) = \mathbf{a(t)cos\varphi(t)} = \omega^{0.5}\mathbf{A(t)cos\,\varphi(t)}$ presents the signal power function, and we also know that every power function presents the product of some other mutually conjugated functions (presently unknown), we eventually come to the conclusion that frequency has the chance to stand in front of a power function, $\left[\Psi(\mathbf{t})\right]^2 = \left[\mathbf{a(t)cos\varphi(t)}\right]^2 = \omega\left[\mathbf{A(t)}\right]^2 \mathbf{cos}^2\,\varphi(\mathbf{t})$. Maybe somehow the nature of so-called "**1/f –** noise" is in certain relation with this situation.

By creating an analogy with immediate (or instant) frequency definition $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial\,\mathbf{t}}$**,** $\varphi(\mathbf{t}) = \mathbf{arctg}\frac{\hat{\psi}(\mathbf{t})}{\psi(\mathbf{t})}$, we could also analyze the meaning of the analogous time expression in the frequency domain, $\tau(\omega) = \frac{\partial\Phi(\omega)}{\partial\,\omega}$**,** $\Phi(\omega) = \mathbf{arctan}\frac{\mathbf{U}_s(\omega)}{\mathbf{U}_c(\omega)}$, and try to find how they are mutually related, $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial\,\mathbf{t}}$ $(\sim \mathbf{?} \sim)$ $\tau(\omega) = \frac{\partial\Phi(\omega)}{\partial\,\omega}$ (most probably as a kind of Uncertainty relation, well known in Quantum Theory and Spectrum Analysis, for instance, $\omega(\mathbf{t})\cdot\tau(\omega) \geq \pi$, or $\Delta\omega(\mathbf{t})\cdot\Delta\tau(\omega) \geq \pi$, or $\sigma_\Omega \cdot \sigma_\mathbf{T} \geq \pi$ … which one is correct, remaining to be proved later).

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